Actuary Problems Solution Set
Probability for Risk Management

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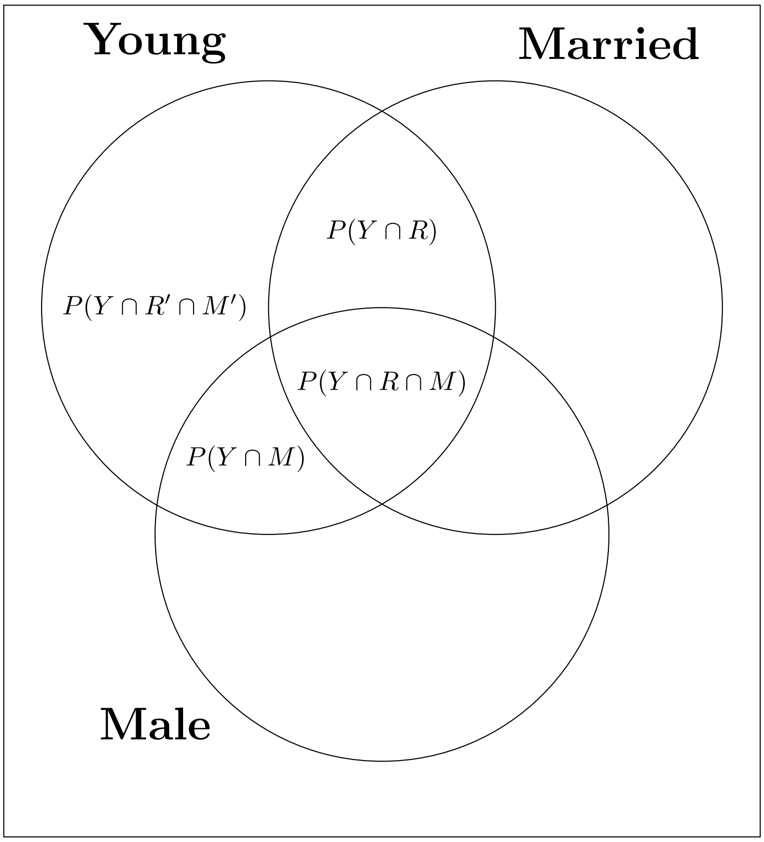
Contents

Chapter 2
Counting for Probability

2-47

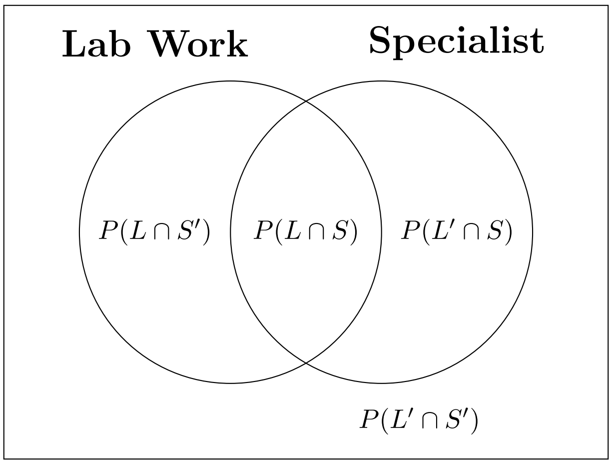


According to the constructed venn diagram, we want $P(Y \cap R' \cap M')$.

$$\begin{aligned} P(Y \cap R \cap M) &= 600 \\ P(Y \cap R) &= 1400 - 600 = 800 \\ P(Y \cap M) &= 1320 - 600 = 720 \\ P(Y \cap R' \cap M') &= 3000 - (600 + 800 + 720) = \boxed{880} \end{aligned}$$

Chapter 3
Elements of Probability

3-46



According to the constructed venn diagram, we want $P(L \cap S)$.

$$P(L' \cap S') = 0.35 = P(L \cup S)'$$
$$P(L) = 0.40 \quad P(S) = 0.30$$
$$P(L') = 0.60 \quad P(S') = 0.70$$

$$\begin{aligned} P(L \cap S) &= P(L) + P(S) - P(L \cup S) \\ &= [1 - P(L')] + [1 - P(S')] - [1 - P(L \cup S)'] \\ &= [1 - 0.60] + [1 - 0.70] - [1 - 0.35] \\ &= \boxed{0.05} \end{aligned}$$

3-47

$$\begin{aligned} P(A \cup B) &= 0.7 = P(A) + P(B) - P(A \cap B) \\ P(A \cup B') &= 0.9 = P(A) + P(B') - P(A \cap B') \end{aligned}$$

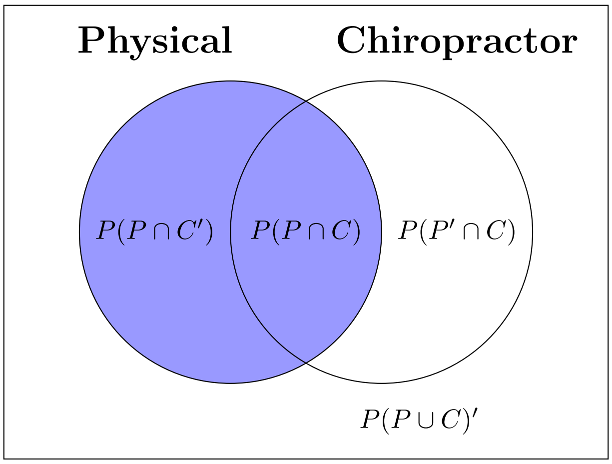
$$\begin{aligned} 1.6 &= 2P(A) + P(B) + P(B') - P(A \cap B) - P(A \cap B') \\ 1.6 &= 2P(A) + P(B) + [1 - P(B)] - P(A \cap B) - [P(A) - P(A \cap B)] \\ 1.6 &= P(A) + 1 \\ P(A) &= \boxed{0.6} \end{aligned}$$

3-48

$$P(>1) = 0.64 \quad P(S) = 0.2$$
$$P(1) = 0.36 \quad P(S') = 0.8$$

$$\begin{aligned} P(S') &= P(>1 \cap S') + P(1 \cap S') \\ P(1 \cap S') &= P(S') - P(>1 \cap S') \\ &= 0.8 - (0.64 \times 0.85) \\ &= \boxed{0.256} \end{aligned}$$

3-49

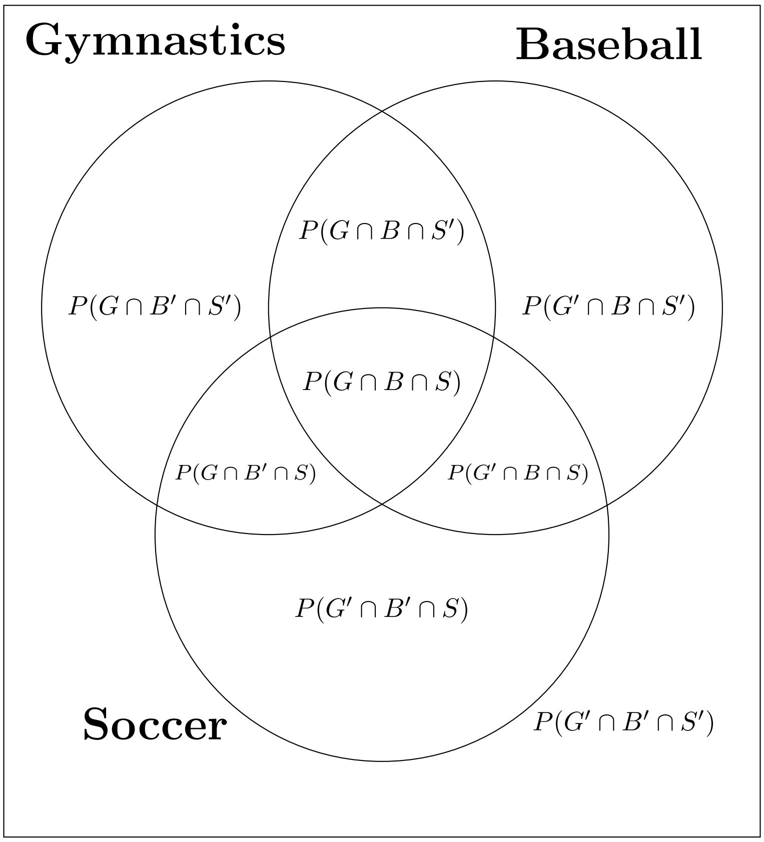


According to the constructed venn diagram, we want the blue shaded area $P(P)$.

$$P(P \cap C) = 0.22 \quad P(P \cup C)' = 0.12$$
$$P(C) = P(P) + 0.14$$

$$\begin{aligned} P(P \cap C) &= P(P) + P(C) - P(P \cup C) \\ 0.22 &= P(P) + [P(P) + 0.14] - [1 - 0.12] \\ P(P) &= \boxed{0.48} \end{aligned}$$

3-50



According to the constructed venn diagram, we want $P(G' \cap B' \cap S')$.

$$P(G \cap B \cap S) = 0.08$$

$$P(G \cap B' \cap S) = 0.10 - 0.08 = 0.02$$
$$P(G' \cap B \cap S) = 0.12 - 0.08 = 0.04$$
$$P(G \cap B \cap S') = 0.14 - 0.08 = 0.06$$

$$P(G \cap B' \cap S') = 0.28 - 0.06 - 0.02 - 0.08 = 0.12$$
$$P(G' \cap B \cap S') = 0.29 - 0.06 - 0.04 - 0.08 = 0.11$$
$$P(G' \cap B' \cap S) = 0.19 - 0.02 - 0.04 - 0.08 = 0.05$$

$$P(G' \cap B' \cap S') = 1 - (0.05 + 0.11 + 0.12 + 0.06 + 0.04 + 0.02) = \boxed{0.52}$$

3-51

The events are independent and that $P(C) = 2P(D)$.

$$\begin{aligned} P(C \cap D) &= P(C) \cdot P(D) = 0.15 \\ 2P(D) \cdot P(D) &= 0.15 \\ P(D) &= \sqrt{\frac{0.15}{2}} = 0.274 \\ P(C) &= 2 \cdot 0.274 = 0.548 \end{aligned}$$

$$\begin{aligned} P(C \cup D) &= P(C) + P(D) - P(C \cap D) \\ 1 - P(C \cup D)' &= 0.548 + 0.274 - 0.15 \\ P(C \cup D)' &= \boxed{0.328} \end{aligned}$$

3-52

The events are independent.

$$P(E \cup O) = 0.85$$

$$P(E') = 0.25 \quad P(E) = 0.75$$

$$\begin{aligned} P(E \cup O) &= P(E) + P(O) - P(E \cap O) \\ 0.85 &= 0.75 + P(O) - 0.75 \cdot P(O) \\ P(O) &= \boxed{0.4} \end{aligned}$$

3-53

N	0	1	2	3	4
p(n)	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{20}$	$\frac{1}{30}$

$$P(N \geq 1 | N \leq 4) = \frac{P(N \geq 1 \cap N \leq 4)}{P(N \leq 4)} = \frac{\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30}}{\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30}} = \boxed{\frac{2}{5}}$$

3-54

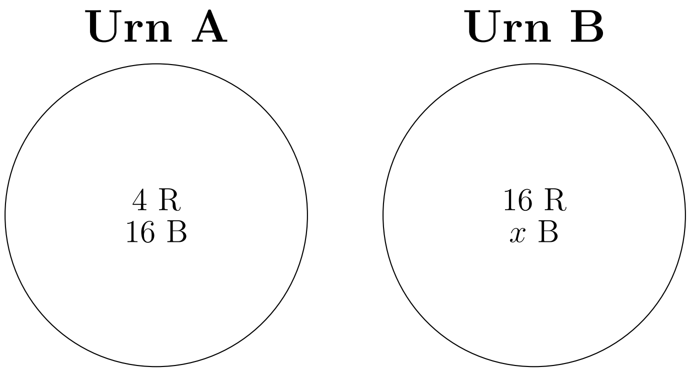
We want $P(H|P') = \dfrac{P(H \cap P')}{P(P')}$

$$P(H) = \frac{210}{937} = P(H \cap P) + P(H \cap P')$$

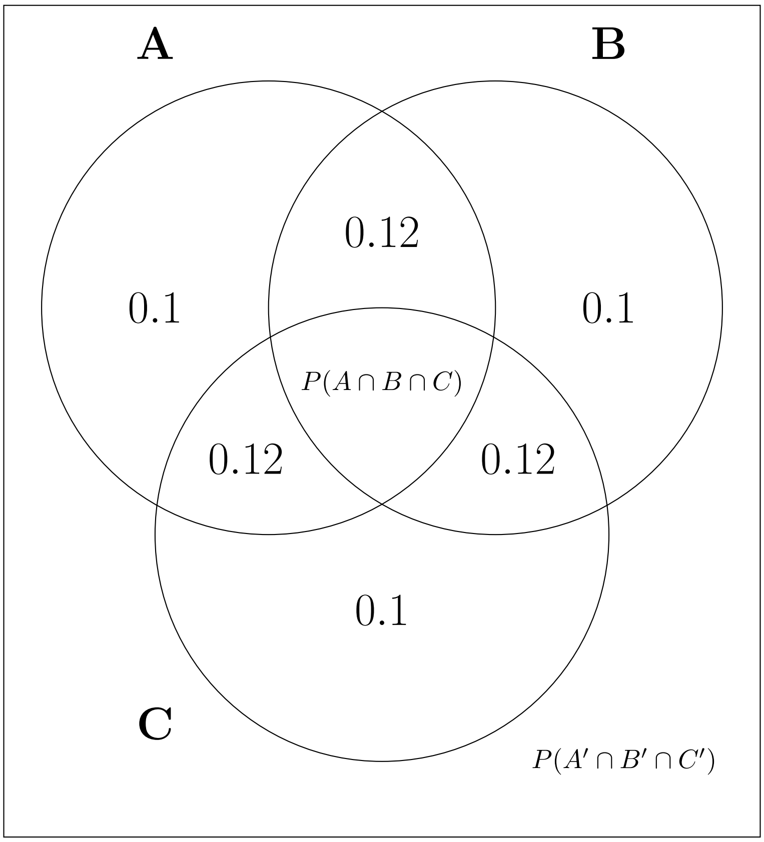
$$P(P) = \frac{312}{937} \quad P(H \cap P) = \frac{102}{937}$$

$$P(H \cap P') = \frac{\frac{210}{937} - \frac{102}{937}}{\frac{625}{937}} = \frac{108}{625} = \boxed{0.1728}$$

3-55



3-56



We want $P((A' \cap B' \cap C')|A') = \dfrac{P((A' \cap B' \cap C') \cap A')}{P(A')} = \dfrac{P(A' \cap B' \cap C')}{P(A')}$

$$0.10 = P(A \cap B' \cap C') = P(A' \cap B \cap C') = P(A' \cap B' \cap C)$$

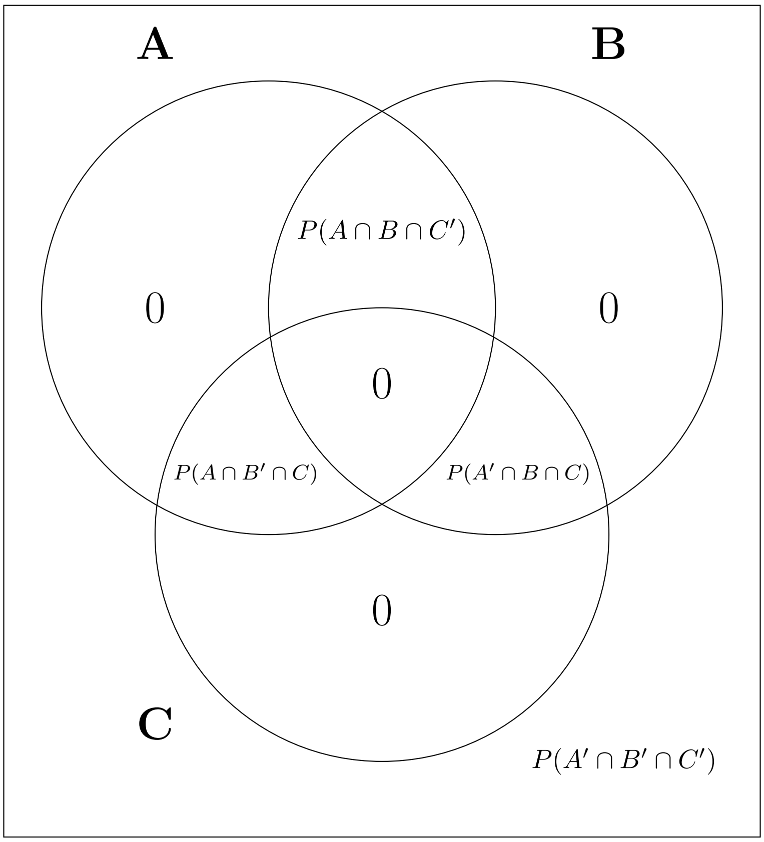
$$0.12 = P(A \cap B \cap C') = P(A \cap B' \cap C) = P(A' \cap B \cap C)$$

$$\frac{1}{3} = P(A \cap B \cap C|A \cap B) = \frac{P(A \cap B \cap C \cap A \cap B)}{P(A \cap B)} = \frac{P(A \cap B \cap C)}{P(A \cap B)} = \frac{P(A \cap B \cap C)}{P(A \cap B \cap C) + 0.12}$$

$$\begin{aligned} P(A \cap B \cap C|A \cap B) = \frac{P(A \cap B \cap C \cap A \cap B)}{P(A \cap B)} = \frac{P(A \cap B \cap C)}{P(A \cap B)} &= \frac{1}{3} \\ \frac{P(A \cap B \cap C)}{P(A \cap B \cap C) + 0.12} &= \frac{1}{3} \\ P(A \cap B \cap C) &= 0.06 \end{aligned}$$

$$\frac{P(A' \cap B' \cap C')}{P(A')} = \frac{1 - (3(0.1) + 3(0.12) + 0.06)}{1 - (0.1 + 2(0.12) + 0.06)} = \frac{0.28}{0.6} = \boxed{0.467}$$

3-57



$$P(A) = \frac{1}{4} = P(A \cap B \cap C') + P(A \cap B' \cap C)$$
$$P(B) = \frac{1}{3} = P(A' \cap B \cap C) + P(A \cap B \cap C')$$
$$P(C) = \frac{5}{12} = P(A' \cap B \cap C) + P(A \cap B' \cap C)$$
$$P(A' \cap B' \cap C') = 1 - [P(A \cap B \cap C') + P(A \cap B' \cap C) + P(A' \cap B \cap C)]$$

Chapter 4
Discrete Random Variables

4-18

Chapter 5
Commonly Used Discrete Distributions

5-36

Chapter 6
Applications for Discrete Random Variables

6-17

x	0	1	2	3	⋯
p(x)					⋯

$\Rightarrow$

y	0	1000	2000	2000	⋯
p(y)					⋯

2000x	0	2000	2000	2000	⋯
p(x)					⋯

Chapter 7
Continuous Random Variables

7-12

$$f(x) \propto (10+x)^{-2}$$
$$f(x) = K\,(10+x)^{-2} \quad ; \quad 0 \le x \le 40$$

$$\int_0^{40} K\,(10+x)^{-2} = 1$$
$$1 = K\left[\frac{(10+x)^{-1}}{-1}\right]_0^{40}$$
$$\frac{1}{K} = \left[\left(-\frac{1}{50}\right) - \left(-\frac{1}{10}\right)\right] = \frac{2}{25}$$
$$K = \frac{25}{2}$$

$$f(x) = \frac{25}{2}\,(10+x)^{-2} \quad ; \quad 0 \le x \le 40$$

$$P(X<6) = \int_0^6 \frac{25}{2}\,(10+x)^{-2}$$
$$= \frac{25}{2}\left[-(10+x)^{-1}\right]_0^6$$
$$= \frac{25}{2}\left[\left(-\frac{1}{16}\right) - \left(-\frac{1}{10}\right)\right]$$
$$= \frac{15}{32} = \boxed{0.468\,75}$$

7-13

$$k^{th} \text{ Percentile : } \int_{200}^{P_k} \frac{2.5(200)^{2.5}}{x^{3.5}}\,dx = \frac{k}{100}$$
$$\frac{k}{100} = (2.5(200)^{2.5})\int_{200}^{P_k} x^{-3.5}$$
$$\frac{k}{100(2.5(200)^{2.5})} = \left[\frac{x^{-2.5}}{-2.5}\right]_{200}^{P_k}$$
$$\frac{(k)(-2.5)}{100(2.5(200)^{2.5})} = \left[P_k^{-2.5} - 200^{-2.5}\right]$$
$$P_k^{-2.5} = 200^{-2.5} - \frac{k}{100(200)^{2.5}} = \frac{100(200)^{2.5}(200)^{-2.5} - k}{100(200)^{2.5}}$$
$$P_k^{2.5} = \frac{100(200)^{2.5}}{100-k}$$
$$P_k = \left[\frac{100(200)^{2.5}}{100-k}\right]^{1/2.5}$$

$$P_{70} - P_{30} = 323.73 - 230.670 = \boxed{93.06}$$

7-14

$$f(x) \propto (1+x)^{-4}$$
$$f(x) = K\,(1+x)^{-4} \quad ; \quad 0 \le x \le \infty$$

$$\int_0^\infty K\,(1+x)^{-4} = 1$$
$$1 = K\left[\frac{(1+x)^{-3}}{-3}\right]_0^\infty$$
$$\frac{1}{K} = \left[0 - \left(-\frac{1}{3}\right)\right] = \frac{1}{3}$$
$$K = 3$$

$$f(x) = 3\,(1+x)^{-4} \quad ; \quad 0 \le x \le \infty$$
$$E\,[X] = \int_0^\infty 3x\,(1+x)^{-4}$$
$$= 3\left[\frac{(1+x)^{-3}(1-3x)}{6}\right]_0^\infty = \left[0 - \left(\frac{1}{2}\right)\right] = \boxed{\frac{1}{2}}$$

7-15

$$f(x) = \frac{|x|}{10} \quad \text{for } -2 \le x \le 4 \begin{cases} \dfrac{-x}{10} & \text{for } -2 \le x \le 0 \\ \dfrac{x}{10} & \text{for } 0 \le x \le 4 \end{cases}$$

$$E\,[X] = \int_{-2}^{0} \frac{-x^2}{10}\;dx + \int_0^4 \frac{x^2}{10}\;dx$$
$$= \left[\frac{-x^3}{30}\right]_{-2}^{0} + \left[\frac{x^3}{30}\right]_0^4 = \left[\left(\frac{0}{30}\right) - \left(\frac{8}{30}\right)\right] + \left[\left(\frac{64}{30}\right) - \left(\frac{0}{30}\right)\right] = \boxed{\frac{28}{15}}$$

7-16

$$P(X > 16 \mid X > 8) = \frac{P(X > 16)}{P(X > 8)} = \frac{1 - P(X < 16)}{1 - P(X < 8)} = \frac{1 - \int_0^{16} 0.005(20-x)\;dx}{1 - \int_0^{8} 0.005(20-x)\;dx}$$

$$P(X < x) = \int_0^x 0.005(20-x)\;dx = 0.005\left[\int_0^x 20\;dx - \int_0^x x\;dx\right]$$
$$= 0.005\left[[20x]_0^x - \left[\frac{x^2}{2}\right]_0^x\right] = 0.1x - 0.0025x^2$$

$$P(X > 16 \mid X > 8) = \frac{1 - 0.96}{1 - 0.64} = \boxed{\frac{1}{9}}$$

7-17

$$P(X < 2 \mid X \geq 1.5) = \frac{P(1.5 \leq X \leq 2)}{P(X \geq 1.5)} = \frac{P(1.5 < X < 2)}{1 - P(X < 1.5)} = \frac{\int_{1.5}^{2} 3x^{-4}\ dx}{1 - \int_{1}^{1.5} 3x^{-4}\ dx}$$

$$P(a \leq X \leq b) = \int_a^b 3x^{-4}\ dx = \left[-x^{-3}\right]_b^a = (b)^{-3} - (a)^{-3}$$

$$P(X < 2 \mid X \geq 1.5) = \frac{\left(\frac{37}{216}\right)}{1 - \left(\frac{19}{27}\right)} = \frac{37}{64} = \boxed{0.578125}$$

Chapter 8
Commonly Used Continuous Distributions

8-56

$$F(4) = 1 - e^{-4\lambda} = 0.5$$
$$\lambda = -\frac{\ln(0.5)}{4}$$

$$\begin{aligned} P(X \geq 5) &= 1 - P(X \leq 5) \\ &= 1 - \left(1 - e^{-5\left(-\frac{\ln(0.5)}{4}\right)}\right) \\ &= \boxed{0.420\,45} \end{aligned}$$

8-57

$$E[G] = 6 = \frac{1}{\lambda_{\mathrm{G}}} \Rightarrow \lambda_{\mathrm{G}} = \frac{1}{6}$$
$$E[B] = 3 = \frac{1}{\lambda_{\mathrm{B}}} \Rightarrow \lambda_{\mathrm{B}} = \frac{1}{3}$$

$$\begin{aligned} P(G \leq 3 \cap V \leq 2) &= P(G \leq 3) \cdot P(V \leq 2) \\ &= F_{\mathrm{G}}(3) \cdot F_{\mathrm{B}}(2) \\ &= \left(1 - e^{-\frac{3}{6}}\right) \cdot \left(1 - e^{-\frac{2}{3}}\right) \\ &= \boxed{0.191\,46} \end{aligned}$$

8-58

$$E[G] = 2 = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{2}$$
$$F(x) = 1 - e^{-\frac{1}{2}x}$$

Probability model for each printer

x	1	2
p(x)	$F(1)$	$F(2) - F(1)$

$\Rightarrow$

y	200	100
p(y)	$F(1)$	$F(2) - F(1)$

$$\begin{aligned} E[Y] &= \sum Y \cdot P(Y) \\ &= 200\left(1 - e^{-\frac{1}{2}}\right) + 100\left[\left(1 - e^{-1}\right) - \left(1 - e^{-\frac{1}{2}}\right)\right] \\ &= 102.56 \\ 100 \cdot E[Y] &= \boxed{10\,256} \end{aligned}$$

8-59

$$
\begin{aligned}
P(X \le 50) &= 0.3 \\
F(50) = 1 - e^{\frac{-50}{\lambda}} &= 0.3 \\
\lambda &= -\frac{50}{\ln(0.7)} \\
P(X \le 80) = F(80) &= 1 - e^{\frac{-80}{\lambda}} \\
&= 1 - e^{\frac{-80}{\left(-\frac{50}{\ln(0.7)}\right)}} \\
&= \boxed{0.434\,86}
\end{aligned}
$$

8-60

$$
\begin{aligned}
f(x) &= ce^{-0.004x} \text{ for } x \ge 0 \\
f(x) &= \lambda e^{-\lambda x} \text{ for } x \ge 0 \\
c &= 0.004 \\
f(x) &= 0.004e^{-0.004x} \\
F(x) &= 1 - e^{-0.004x} \\
F(M) = 1 - e^{-0.004M} &= 0.5 \\
M &= \frac{\ln(0.5)}{-0.004} \\
&= \boxed{173.29}
\end{aligned}
$$

8-61

N	0	1	> 1
p(n)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

Three different probability densities for each N. $P(4 < S < 8)$ means at least 1 claim was insured for a given year, therefore $P(N = 0)$ is not included.

$$
\begin{aligned}
F_{\exp(5)}(x) &= 1 - e^{-\frac{x}{5}} \\
F_{\exp(8)}(x) &= 1 - e^{-\frac{x}{8}}
\end{aligned}
$$

$$
\begin{aligned}
P(4 < S < 8) &= \frac{1}{3}\left[F_{\exp(5)}(8) - F_{\exp(5)}(4)\right] + \frac{1}{6}\left[F_{\exp(8)}(8) - F_{\exp(8)}(4)\right] \\
&= \frac{1}{3}\left[\left(1 - e^{-\frac{8}{5}}\right) - \left(1 - e^{-\frac{4}{5}}\right)\right] + \frac{1}{6}\left[\left(1 - e^{-\frac{8}{8}}\right) - \left(1 - e^{-\frac{4}{8}}\right)\right] \\
&= \boxed{0.122\,25}
\end{aligned}
$$

8-62

$$X \sim Po(2) \quad \lambda = 2 = \sigma^2$$
$$n\lambda = 1250 \cdot 2 = 2500 = \sigma^2 \quad \Rightarrow \quad \sigma = 50$$
$$P(2450 \leq X \leq 2600)$$

Since independence is assumed, the Central Limit Theorem can be used.

$$P(z_1 \leq z \leq z_2)$$
$$z_1 = \frac{2450 - 2500}{50} = -1 \quad , \quad z_2 = \frac{2600 - 2500}{50} = 2$$
$$P(-1 \leq z \leq 2) = 0.9772 - (1 - 0.8413) = \boxed{0.8185}$$

8-63

$$f(x) = \frac{1}{1000}e^{-\frac{x}{1000}} \quad \Rightarrow \quad \lambda = 1000$$

$$P(x) = E[X] + 100 = 1100$$
$$100P(x) = 100 \cdot 1100 = 110000$$
$$100E[X] = 100 \cdot 1000 = 10^5$$
$$100Var(X) = 100 \cdot 1000^2 = 10^8$$
$$\sigma = \sqrt{10^8} = 10^4$$

$$X \sim N(10^5, 10^4) \quad P(X > 110000)$$
$$z = \frac{110000 - 100000}{10000} = 1$$
$$P(z > 1) = 1 - P(z < 1) = 1 - 0.8413 = \boxed{0.1587}$$

8-64

x	0	1	2
p(x)	0.6	0.1	0.3

$$E[X] = 0.7 = \mu$$
$$E[X^2] = 1.3$$
$$Var(X) = 0.81$$
$$\sigma = 0.9$$

$$100 \cdot \mu = 70$$
$$100 \cdot Var(X) = 81$$
$$\sigma = 9$$

$$X \sim N(70, 9) \quad P(X \leq 90)$$

Since we approximate a discrete random variable with a continuous one, an adjustment is needed by continuity correction, which includes the entire block of probability for that value.

$$P(X \leq 90) \Rightarrow P(X \leq 90.5)$$
$$z = \frac{90.5 - 70}{9} = 2.28$$
$$P(z \leq 2.28) = \boxed{0.9887}$$

8-65

$$X \sim U(-2.5, 2.5) \quad , \quad \mu = 0 \quad , \quad \sigma^2 = \frac{25}{12}$$

Because the sample size $n = 48$ is large, the sample mean $\bar{x}$ is approximately normally distributed, with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$.

$$\begin{aligned} P(-0.25 \leq \bar{x} \leq 0.25) &= P\left(\frac{-0.25 - \mu}{\frac{\sigma}{\sqrt{n}}} \leq z \leq \frac{0.25 - \mu}{\frac{\sigma}{\sqrt{n}}}\right) \\ &= P(-1.2 \leq z \leq 1.2) \\ &= 0.8849 - (1 - 0.8849) \\ &= \boxed{0.7698} \end{aligned}$$

8-66

For individual contributions:

$$\mu = 3125 \quad , \quad \sigma = 250 \quad , \quad Var(x) = \sigma^2 = 62500$$

For $n = 2025$:

$$n\mu = 6328125 \quad , \quad nVar(x) = 126562500 \quad , \quad \sigma = 11250$$

$$X \sim N(6328125, 11250) \quad , \quad P(X \leq z) = 0.9$$

0.9 is found between $z = 1.28$ and $z = 1.29$, with z-scores 0.8997 and 0.9015 respectively.

$$(0.01)\left(\frac{0.0003}{0.9015 - 0.8997}\right) = \frac{1}{600}$$

$$\begin{aligned} z = \frac{x - 6328125}{11250} &= 1.28 + \frac{1}{600} \\ x &= \boxed{6\,342\,543.75} \end{aligned}$$

Chapter 9
Applications for Continuous Random Variables

9-26

$$P(Y < 0.5) = 0.64 \quad ; \quad Y = X - C$$
$$P(X - C < 0.5) = 0.64$$
$$P(X < 0.5 + C) = 0.64$$
$$0.64 = \int_0^{0.5+C} 2x\ dx = \left[x^2\right]_0^{0.5+C}$$
$$0.64 = (0.5 + C)^2$$
$$C = \pm 0.8 - 0.5 \quad ; \quad 0 < C < 1$$
$$C = \boxed{0.3}$$

9-27

Set Y = losses paid by the manufacturer with deductible

$$Y = \begin{cases} X & ; \quad 0.6 < x \le 2 \\ 2 & ; \quad x > 2 \end{cases}$$

$$E[Y] = \int_{0.6}^{2} x \left(\frac{2.5(0.6)^{2.5}}{x^{3.5}}\right) + \int_2^{\infty} 2\left(\frac{2.5(0.6)^{2.5}}{x^{3.5}}\right)$$
$$= 2.5(0.6)^{2.5}\left[\left(\frac{1}{-1.5(2)^{1.5}} - \frac{1}{-1.5(0.6)^{1.5}}\right) + \left(0 - \frac{2}{-2.5(2)^{2.5}}\right)\right]$$
$$E[Y] = \boxed{0.93427}$$

9-28

$$X \sim U[0, 1000] \quad , \quad f(x) = \frac{1}{1000 - 0} = \frac{1}{1000} \quad , \quad E[X] = \frac{1000 + 0}{2} = 500$$

$$E[Y] = 0.25 \cdot E[X] = 125$$
$$E[Y] = \int_0^d (0)\left(\frac{1}{1000}\right)\ dx + \int_d^{1000} (x - d)\left(\frac{1}{1000}\right)\ dx = 125$$
$$125 = \left[\frac{(x-d)^2}{2000}\right]_d^{1000}$$
$$125 = \left[\left(\frac{(1000 - d)^2}{2000}\right) - \left(\frac{(d-d)^2}{2000}\right)\right]$$
$$(1000 - d)^2 = 250000$$
$$1000 - d = \pm 500$$
$$d = 1000 \pm 500 \quad ; \quad 0 < d < 1000$$
$$d = \boxed{500}$$

9-29

$$X \sim \exp(10) \quad ; \quad f(x) = \frac{1}{10}\exp\Big(-\frac{x}{10}\Big)$$

Y	1	2	3	4	$\cdots$
p(y)	x	$\frac{x}{2}$	$\frac{x}{2}$	0	$\cdots$

$$\begin{aligned}
E[Y] &= \int_0^1 x \cdot \frac{1}{10}\exp\Big(-\frac{t}{10}\Big)\ dt + \int_1^3 \frac{x}{2} \cdot \frac{1}{10}\exp\Big(-\frac{t}{10}\Big) + \int_3^\infty 0 \cdot \frac{1}{10}\exp\Big(-\frac{t}{10}\Big)\ dt \\
1000 &= x\left[-\exp\left(-\frac{t}{10}\right)\right]_0^1 + x\left[-\frac{1}{2}\exp\left(-\frac{t}{10}\right)\right]_1^3 \\
1000 &= x\left[\left(-\exp\left(-\frac{1}{10}\right)+1\right) + \left(-\frac{1}{2}\exp\left(-\frac{3}{10}\right) + \frac{1}{2}\exp\left(-\frac{1}{10}\right)\right)\right] \approx 0.17717x \\
x &= \boxed{5664.3}
\end{aligned}$$

9-30

$$X \sim \exp(3) \quad ; \quad f(x) = \frac{1}{3}\exp\left(-\frac{x}{3}\right) \quad ; \quad F(x) = 1 - \exp\left(-\frac{x}{3}\right)$$

$$\begin{aligned}
E[X] &= \int_0^2 2 \cdot \frac{1}{3}\exp\left(-\frac{x}{3}\right)\ dx + \int_2^\infty x \cdot \frac{1}{3}\exp\left(-\frac{x}{3}\right)\ dx \\
&= 2\left[1 - \exp\left(-\frac{2}{3}\right)\right] + \frac{1}{3}\left[-3x\exp\left(-\frac{x}{3}\right) - 9\exp\left(-\frac{x}{3}\right)\right]_2^\infty \\
&= \left[2 - 2\exp\left(-\frac{2}{3}\right)\right] + \left[2\exp\left(-\frac{2}{3}\right) + 3\exp\left(-\frac{2}{3}\right)\right] \\
E[X] &= 2 + 3\exp\left(-\frac{2}{3}\right) \approx \boxed{3.5403}
\end{aligned}$$

9-31

$$\begin{aligned}
E[Y] &= \int_1^{10} y \cdot 2y^{-3}\ dy + \int_{10}^\infty 10 \cdot 2y^{-3}\ dy \\
&= \left[-\frac{2}{y}\right]_1^{10} + \left[-\frac{10}{y^2}\right]_{10}^\infty \\
&= \left(-\frac{2}{10} + \frac{2}{1}\right) + \left(0 + \frac{10}{100}\right) \\
E[Y] &= \boxed{1.9}
\end{aligned}$$

9-32

$$
\begin{aligned}
E[Y] &= \int_0^4 x \cdot \frac{1}{5}\ dx + \int_4^5 4 \cdot \frac{1}{5}\ dx \\
&= \left[\frac{x^2}{10}\right]_0^4 + \left[\frac{4x}{5}\right]_4^5 \\
E[Y] &= \frac{12}{5} \\
E[Y^2] &= \int_0^4 x^2 \cdot \frac{1}{5}\ dx + \int_4^5 4^2 \cdot \frac{1}{5}\ dx \\
&= \left[\frac{x^3}{15}\right]_0^4 + \left[\frac{16x}{5}\right]_4^5 \\
E[Y^2] &= \frac{112}{15} \\
Var(Y) &= E[Y^2] - (E[Y])^2 \\
&= \frac{112}{15} - \left(\frac{12}{5}\right)^2 \\
Var(Y) &= \frac{128}{75} \approx \boxed{1.7067}
\end{aligned}
$$

9-33

$$
X \sim U(0, 1500) \quad , \quad f(x) = \frac{1}{1500 - 0} = \frac{1}{1500}
$$

$$
\begin{aligned}
E[Y] &= \int_0^{250} 0 \cdot \frac{1}{1500}\ dx + \int_{250}^{1500} (x - 250) \cdot \frac{1}{1500}\ dx \\
&= \left[\frac{(x-250)^2}{3000}\right]_{250}^{1500} \\
E[Y] &= \frac{3125}{6} \\
E[Y^2] &= \int_0^{250} 0^2 \cdot \frac{1}{1500}\ dx + \int_{250}^{1500} (x - 250)^2 \cdot \frac{1}{1500}\ dx \\
&= \left[\frac{(x-250)^3}{4500}\right]_{250}^{1500} \\
E[Y^2] &= \frac{3906250}{9} \\
Var(Y) &= E[Y^2] - (E[Y])^2 \\
&= \frac{3906250}{9} - \left(\frac{3125}{6}\right)^2 \\
Var(Y) &= 162\,760.4167 = \sigma^2 \\
\sigma &= \sqrt{162\,760.4167} = \boxed{403.436}
\end{aligned}
$$

9-34

$$X \sim \exp(300) \quad , \quad f(x) = \frac{1}{300}\exp\left(-\frac{x}{300}\right) \quad , \quad F(x) = 1 - \exp\left(-\frac{x}{300}\right)$$

$$\begin{aligned}
P(X > x|X > 100) = 0.95 &= \frac{P(X > x \cap X > 100)}{P(X > 100)} \\
0.95 &= \frac{\int_{100}^{x} \frac{1}{300}\exp\left(-\frac{x}{300}\right)}{\int_{100}^{\infty} \frac{1}{300}\exp\left(-\frac{x}{300}\right)} = \frac{\left[-\exp\left(-\frac{x}{300}\right)\right]_{100}^{x}}{\left[-\exp\left(-\frac{x}{300}\right)\right]_{100}^{\infty}} = \frac{-\exp\left(-\frac{x}{300}\right) + \exp\left(-\frac{1}{3}\right)}{0 + \exp\left(-\frac{1}{3}\right)} \\
0.95 &= \frac{-\exp\left(-\frac{x}{300}\right)}{\exp\left(-\frac{1}{3}\right)} + 1 \\
\exp\left(-\frac{x}{300}\right) &= 0.05\exp\left(-\frac{1}{3}\right) \\
x &= -300\ln\left[0.05\exp\left(-\frac{1}{3}\right)\right] \approx \boxed{998.72}
\end{aligned}$$

9-35

$$\begin{aligned}
G(y) &= P(Y < y) = P(T^2 < y) = P(T < \pm\sqrt{y}) \quad , \quad y > 4 \\
G(y) &= P(T < \sqrt{y}) = F(\sqrt{y}) = 1 - \left(\frac{2}{\sqrt{y}}\right)^2 \\
g(y) = G'(y) = F'(\sqrt{y}) &= \frac{d}{dy}\left[1 - \frac{4}{y}\right] = \boxed{\frac{4}{y^2}}
\end{aligned}$$

9-36

$$\begin{aligned}
R \sim (0.04, 0.08) \quad &, \quad f(x) = \frac{1}{0.08 - 0.04} = 25 \\
F(v) = P(V < v) &= P(10000\exp(R) < v) = P(R < \ln\left(\frac{v}{10000}\right)) \\
F(v) &= \int_{0.04}^{\ln\left(\frac{v}{10000}\right)} 25\ dx = \boxed{25\left[\ln\left(\frac{v}{10000}\right) - 0.04\right]}
\end{aligned}$$

9-37

$$X \sim \exp(1) \quad , \quad f(x) = \exp(-x) \quad , \quad F(x) = 1 - \exp(-x)$$

$$\begin{aligned}
G(y) &= P(Y < y) = P(10X^{0.8} < y) = P\left(X < \left(\frac{y}{10}\right)^{1.25}\right) = F\left(\left(\frac{y}{10}\right)^{1.25}\right) \\
G(y) &= F\left(\left(\frac{y}{10}\right)^{1.25}\right) = 1 - \exp\left(-\left(\frac{y}{10}\right)^{1.25}\right) \\
g(y) = G'(y) &= \frac{d}{dy}\left[1 - \exp\left(-\left(\frac{y}{10}\right)^{1.25}\right)\right] = -\left[-(1.25)\left(\frac{y}{10}\right)^{0.25}\left(\frac{1}{10}\right)\right]\exp\left(-\left(\frac{y}{10}\right)^{1.25}\right) \\
g(y) &= \boxed{0.125\exp\left((-0.1y)^{1.25}\right)(0.1y)^{0.25}}
\end{aligned}$$

9-38

Chapter 10
Multivariate Distributions

10-27