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a.

Iteration k	1	2	3	4	k	
i after iteration	4	16	256	65536	2^{2^k}	stop at n

$2^{2^k} = n$ $\quad \log(2^{2^k}) = \log n$

$2^k = \log n$ $\quad \log(2^k) = \log(\log n)$

$k = \log(\log n)$

Inside of while is $\theta(1)$

$$T(n) = \sum_{k=1}^{\log(\log n)} (\theta(1)) = \theta(\log \log n)$$

b.

$$T(n) = \sum_{i=1}^{n} \left(\theta(1) + O\left(\sum_{k=0}^{i^3-1} \theta(1)\right)\right)$$

$$T(n) = \sum_{i=1}^{n} \theta(1) + \sum_{i} \sum_{k=0}^{i^3-1} \theta(1)$$

j	1	2	3	
i	1(int)$\sqrt{n}$	2(int)$\sqrt{n}$	3(int)$\sqrt{n}$	Stop when i=n

$i = j\sqrt{n}$ $\quad j\sqrt{n} = n$ $\quad j = \sqrt{n}$

$$T(n) = \theta(n) + \sum_{j=1}^{\sqrt{n}} \sum_{k=0}^{(j\sqrt{n})^3-1} \theta(1)$$

$$T(n) = \theta(n) + \sum_{j=1}^{\sqrt{n}} \theta((j\sqrt{n})^3) = \theta(n) + n^{3/2} \cdot \sum_{j=1}^{\sqrt{n}} \theta(j^3)$$

$$T(n) = \theta(n) + \theta\left(n^{3/2} \cdot \sqrt{n}^{3+1}\right) = \theta(n) + \theta(n^{7/2}) = \theta(n^{7/2})$$

c.

Iteration x	1	2	3	4	
m after iteration	2	4	8	16	Stop at n

$m = 2^x \quad n = 2^x \quad \log n = x$

$$T(n) = \sum_{i=1}^{n} \sum_{k=1}^{n} \left(\theta(1) + O\left(\sum_{x=1}^{\log n} \theta(1) \right) \right)$$

Worst case, the if statement is triggered n times (if each A[k] matches some i in the range once)

$$T(n) = \theta(n^2) + \sum_{y=1}^{n} \sum_{x=1}^{\log n} \theta(1)$$

$$T(n) = \theta(n^2) + \sum_{y=1}^{n} \theta(\log n) = \theta(n^2) + \theta(n \log n) = \theta(n^2)$$

d. $$T(n) = \theta(1) + \sum_{i=0}^{n-1} \left(\theta(1) + O\left(\theta(1) + \sum_{j=0}^{size-1} \theta(1) \right) \right)$$

Sizes: 10, 15, 22.5 → 22, 33, 49.5 → 49, ...

$$10 \cdot \left(\frac{3}{2}\right)^{x-1} = n \qquad \left(\frac{3}{2}\right)^{x-1} = \frac{n}{10} \qquad (x-1) \log \frac{3}{2} = \log \frac{n}{10}$$

$$x = \frac{\log(n/10)}{\log(3/2)} + 1 \qquad size = 10\left(\frac{3}{2}\right)^{x-1}$$

$$T(n) = \theta(1) + \theta(n) + \sum_{x=0}^{\frac{\log n/10}{\log 3/2} + 1} \left(\theta(1) + \theta\left(10\left(\frac{3}{2}\right)^{x-1}\right) \right)$$

$$T(n) = \theta(n) + \theta\left(\frac{\log n/10}{\log 3/2} + 1 + 10 \cdot \left(\frac{3}{2}\right)^{\frac{\log n/10}{\log 3/2} + 1} \right)$$

$$T(n) = \theta(n) + \theta(\log n) + \theta\left(\left(\frac{3}{2}\right)^{\log n} \right)$$

$$T(n) = \theta(n) + \theta(\log n) + \theta\left(\frac{3^{\log n}}{2^{\log n}} \right)$$

$$T(n) = \theta(n) + \theta(\log n) + \theta\left(\frac{3^{\log n}}{n} \right) = \theta(n)$$