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HW 5 probability

1. $\frac{1}{15} \cdot \frac{1}{14} \cdot \frac{1}{13} \cdot \frac{1}{12} \cdot \frac{1}{11} \cdot \frac{1}{10} \cdot \frac{1}{9} \cdot \frac{1}{8}$

Pool of possible students to fit criteria decreases by 1 each question

2. Probability of single number meeting criteria:

$$\left(\frac{1}{5} \cdot \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{5}\right) + \left(\frac{1}{5} \cdot \frac{1}{4} \cdot \frac{1}{5} \cdot \frac{1}{4}\right) = x$$

odd digits; # odds, pool decreased; 3rd digit is odd; 3rd digit is even

Exactly 5/8:

one combo: $5x \cdot 3(1-x)$

all combos: $5x \cdot 3(1-x) \cdot \frac{8!}{5!\,3!}$

3. $P(B|A) \overset{?}{=} P(B)$

$$A = \left(\frac{3}{6} \cdot \frac{3}{6} \cdot \frac{3}{6}\right) \cdot \frac{3!}{2!} + \frac{3}{6} \cdot \frac{3}{6} \cdot \frac{3}{6} = \frac{1}{2}$$

$$B = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot 6 = \frac{1}{36} \qquad P(A \cap B) = \frac{1}{6} \cdot \frac{1}{6} \frac{1}{6} \cdot 3$$

$$P(B|A) = \frac{\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot 3}{\frac{1}{2}} = \frac{1}{36}$$

They are independent

4/1. $P(\text{flush}) = \left(\frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \cdot \frac{9}{48}\right) 4 = 0.00198$

$\frac{1}{P(\text{flush})} = 504.8$

505 games must be played to get a flush on average

5/2. $P(4/5 \text{ w/ star}) = (0.7^4 \cdot 0.3^1)\frac{5!}{4!} = 0.36015$

$P(4/5 \text{ w/out star}) = (0.5^4 \cdot 0.5^1) \cdot \frac{5!}{4!} = 0.15625$

$$0.75 \cdot \frac{0.36015}{0.15625 + 0.36015} + 0.25 \cdot \frac{0.15625}{0.36015 + 0.15625}$$