

Misha Khmelkov

## HW 5 Counting

1. Unique strings:

$r$ -permutation without duplicates

$$\downarrow$$
$$\frac{7!}{(7-5)!}$$

$\downarrow$   
division rule,  $\cdot \frac{1}{3!}$

$$\frac{7!}{(7-5)! \cdot 3!} = 420 \text{ unique strings}$$

Unique subsets:

No duplicates allowed in a formal set

So, parent set =  $\{u, n, s, a, l\}$

$2^n$  subsets =  $2^5 = 32$  unique subsets

2. Single pair:

2/4 cards that have same value =  $r$ -permutation

$$\frac{4!}{(4-2)!} \text{ ways to form a single pair}$$

Hand w/ 2 different pairs:

$$\frac{13 \cdot 4!}{(4-2)!} \cdot \frac{12 \cdot 4!}{(4-2)!} \cdot (52-8)$$

pair 1

pair 2

values not yet used

3. 1 song + 15 indistinguishable into 6 distinguishable fighting couple

$$1 + \frac{(6+15-1)!}{15!(6-1)!}$$

4. 2 ways to form 2-node tree

3 ways to form a 3-node tree from each 2-node tree



4 ways to form a 4-node tree from each 3-node tree

12-node tree, root 3 right 9 means we start with a <sup>single</sup> 2-node tree and add until 12 nodes

$$12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \quad \text{or} \quad \frac{12!}{2!}$$

5. Minimum number served is 3 if that one nurse is on break and the other 3 go at minimum speed

Max number is 10

$10 - 3 + 1 = 8$  possible variations of number of patients served