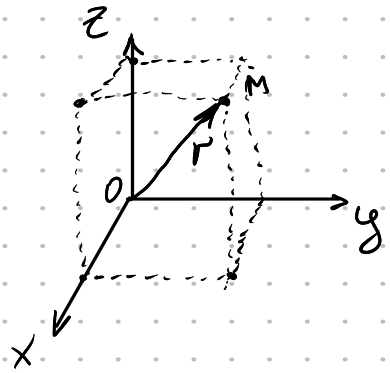
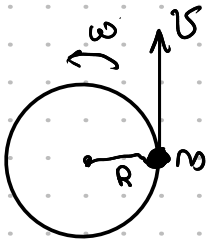
Момент инерции

Вращение тела



Вращательное движение:



$\bar{L} = [\bar{r}, \bar{p}]$

$L = mr^2\omega$ - момент импульса

$I = \sum_i m_i r_i^2$ - момент инерции

$L = I\omega$

$E_{вращ} = \frac{1}{2} I\omega^2 = \frac{L^2}{2I}$

- Теорема Гюйгенса - Штейнера: $I_A = I_C + ma^2$

- Вычисление моментов инерции тел:

I_A - инерция оси A || оси у.м.

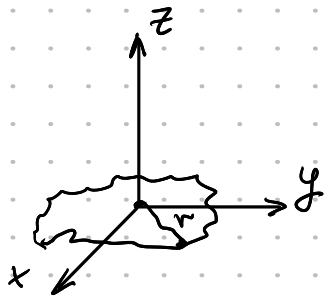
I_C - инерция оси у.м.

a - расстояние от A до C

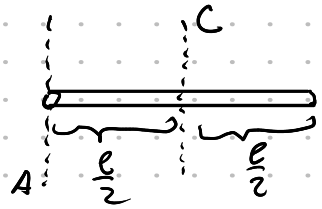
$I_x = M(y^2+z^2)$, $I_y = M(x^2+z^2)$, $I_z = M(x^2+y^2)$

$I_x + I_y + I_z = 2Mr^2 = 2\Theta$

Для плоскости: $I_x + I_y = I_z$ (см. рис)



(1) Стержень тонкий:

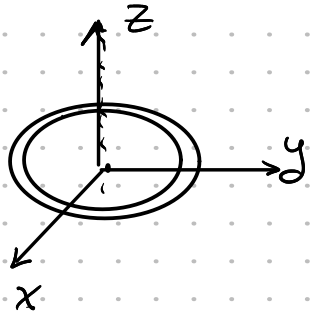


$I_A = kml^2$

По т. Гюйг: $I_A = I_C + m\left(\frac{l}{2}\right)^2$

$I_C = \frac{km}{2}\left(\frac{l}{2}\right)^2 \Rightarrow I_A = \frac{1}{3}ml^2$, $I_C = \frac{1}{12}ml^2$

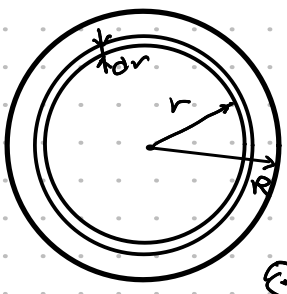
(2) Кольцо тонкое:



$[I_z = mR^2]$

$\left[I_x = I_y = \frac{I_z}{2} = \frac{mR^2}{2}\right]$

(3) Диск тонкий:



Делим на кольца: $dS = 2\pi r \cdot dr$

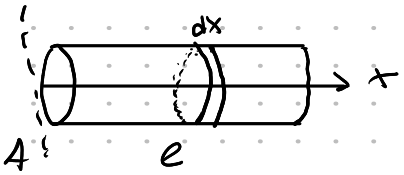
$dI_z = dmr^2$

$\frac{dm}{m} = \frac{dS}{S}$

$\left[I_z = \int dmr^2 = \int \frac{m}{S} dS\, r^2 = \frac{2\pi m}{S}\int_0^R r^3 dr = \frac{2m}{R^2}\frac{R^4}{4} = \frac{mR^2}{2}\right]$

$$\left[I_z = \frac{mR^2}{2}\right] \quad \left[I_x = I_y = \frac{I_z}{2} = \frac{mR^2}{4}\right]$$

④ **Цилиндр:**



Делим на тонкие диски: $dI_A = dmx^2 + \frac{1}{4}dmR^2$ — т. Гюйг. Штейнера

$$\frac{dm}{m} = \frac{dV}{V} = \frac{\pi R^2 dx}{e \cdot \pi R^2} = \frac{dx}{e} \rightarrow I_A = \int x^2 dm + \int \frac{1}{4} dm R^2$$

$$I_A = \int \frac{m}{e} x^2 \cdot dx + \frac{R^2}{4} \int \frac{m}{e} dx$$

$$\left[I_A = \frac{1}{3} m e^2 + \frac{1}{4} m R^2\right]$$

⑤ **Шар полый:**

$\Theta = mR^2$, $I_x = I_y = I_z = I$ $\rightarrow$ $3I = 2\Theta$

$$\left[I = \frac{2}{3}\Theta = \frac{2}{3} mR^2\right]$$

⑥ **Шар сплошной:** Разбиваем на слои dm:

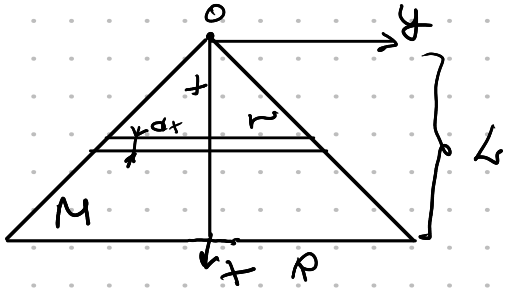
$$\frac{dm}{m} = \frac{dV}{V} = \frac{4\pi r^2 \cdot dr}{\frac{4}{3}\pi R^3} = \frac{3r^2 dr}{R^3} \rightarrow dI = \frac{2}{3} dm r^2 =$$

$$= 2mr^2 \frac{dr}{R^3} r^2$$

$$dI = \frac{2mr^4 dr}{R^3} \rightarrow I = \int dI = \int \frac{2mr^4 dr}{R^3}$$

$$\left[I = \frac{2m}{R^3} \cdot \frac{R^5}{5} = \frac{2}{5} mR^2\right]$$

⑦ **Конус:** $\frac{x}{L} = \frac{r}{R} \Rightarrow r = \frac{R}{L} x$



Разбиваем на диски:

$$\frac{dm}{M} = \frac{dV}{V} = \frac{\pi r^2 dx}{\frac{1}{3}\pi R^2 L} = \frac{3r^2 dx}{R^2 L} = \frac{3 \cdot x^2 dx}{L^3}$$

По т. Гюйг. Штейн.: $dI_y = dmx^2 + \frac{dmr^2}{4}$

$$I_y = \int dm x^2 + \int \frac{dm r^2}{4} = \int \frac{3}{L^3} \cdot M \cdot x^2 dx \cdot x^2 + \int \frac{3}{4} \frac{M}{L^3} x^2 dx \cdot \frac{R^2}{L^2} \cdot x^2$$

$$I_y = \frac{3}{L^3} M \cdot \frac{L^5}{5} + \frac{3}{4} \frac{M}{L^3} \cdot \frac{R^2}{L^2} \cdot \frac{L^5}{5}$$

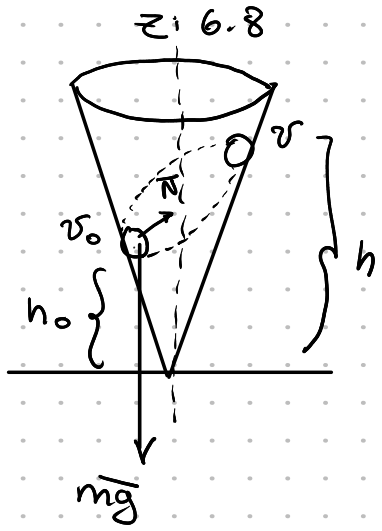
$$\left[I_y = \frac{3}{5} ML^2 + \frac{3}{20} MR^2\right] \quad \left[x_c = \frac{\int_0^L x dm}{m} = \frac{3}{4} L\right]$$

№ 6.8.

v_0 – горизонтально

Известно h – макс. высота

h_0 – мин.



1) Движется по воронке

$\vec{N}$ – направлена к оси z

$m\vec{g} \parallel z$

Значит $\Sigma M = 0$

2) Тогда верен ЗСМИ на z:

Получаем $mvr = const$

↑ расстояние от шара до оси

3) Свойства конуса

$$\frac{r_0}{h_0} = \frac{r_к}{h} \rightarrow v_0 r_0 = v \cdot r_к$$

$$v_0 \cdot \frac{h_0}{h} r_к = v r_к \rightarrow \boxed{v_0 h_0 = v h}$$

4) ЗСЭ:

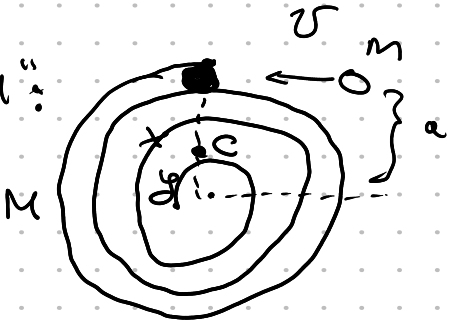
$$\boxed{\frac{v_0^2}{2} + g h_0 = \frac{v^2}{2} + g h}$$

$$\hookrightarrow v_0 = h\sqrt{\frac{2g}{h_0 + h}}, \quad v = h_0\sqrt{\frac{2g}{h_0 + h}}$$

№ 9.95

M, a, v, m, I

$\omega - ?$



1) Найдём центр масс "m+M":

$$x = \frac{M}{M+M} a$$

$$y = \frac{m}{M+m} a$$

$$v_C = \frac{mv + 0}{M+m} = \frac{mv}{M+m}$$

2) В СЦМ:

$$v_{1c} = v - v_C = \frac{Mv}{M+m}$$

$$v_{2c} = 0 - v_C = -\frac{mv}{M+m}$$

3) Система замкнута ⇒ ЗСМИ отн. ц.м.

$$m v_{1c} x + M v_{2c} y = I\omega$$

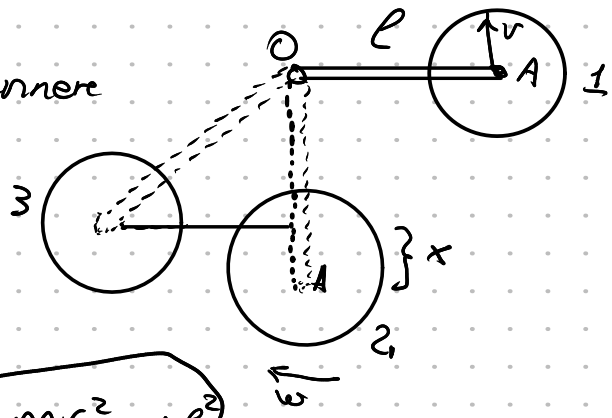
$$\frac{mMv}{M+m} \cdot \frac{M}{M+M} a + M \frac{mv}{M+m} \cdot \frac{ma}{M+m} = I\omega$$

$$\left[\omega = \frac{Mm \cdot a \cdot v}{I(M+m)}\right]$$

№ 9.121.

l, r

Сначала закреплен 1-2
Затем свободен 2-3
Стержень лёгкий



1) 1-2: диск закреплен ось вращения O.

по т. Гюйг. Шт.

$I_O = I_A + ml^2$

$I_A = \frac{mr^2}{2} \Rightarrow$ $\boxed{I = \frac{mr^2}{2} + ml^2}$

Тогда ЗСЭ от 1 к 2:

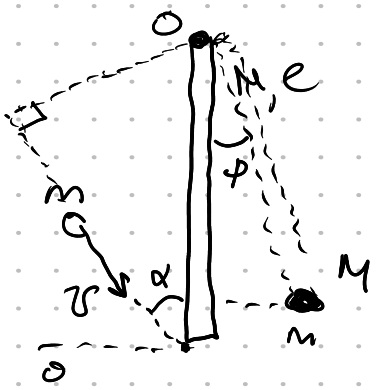
$$mgl = \frac{I\omega^2}{2} = \frac{m(r^2+2l^2)}{2\cdot 2}\omega^2 \Rightarrow \omega^2 = \dots$$

2) ЗСЭ от 2 к 3: $mgx = \frac{mv_A^2}{2} = \frac{m\omega^2 l^2}{2}$

$$\left[x = \frac{\omega^2 l^2}{2g} = \frac{4gl\ l^2}{(r^2+2l^2)2g} = \frac{2l^3}{(r^2+2l^2)}\right]$$

№ 9.105

M, l



1) ЗСМИ отн. O:

$$0 + mv\cdot\sin\alpha\, l = I\omega = L$$

$$I = ml^2 + \frac{1}{3}Ml^2$$

2) ЗСЭ: $\frac{I\omega^2}{2} = mg(l - l\cos\varphi) + Mg\left(\frac{l}{2} - \frac{l}{2}\cos\varphi\right)$

$$\frac{L^2}{2I} = \frac{mv\sin\alpha\, l)^2}{2(ml^2 + \frac{1}{3}Ml^2)} = \underbrace{mgl(1-\cos\varphi)}_{x} + \underbrace{\frac{Mgl}{2}(1-\cos\varphi)}_{x}$$

$$\frac{m^2v^2\sin^2\alpha}{2(m+\frac{1}{3}M)} = x\,gl\left(m+\frac{M}{2}\right)$$

$$1-\cos\varphi = \frac{m^2v^2\sin^2\alpha}{2(m+\frac{1}{3}M)gl(m+\frac{M}{2})}$$

$$\left[\cos\varphi = 1 - \frac{3m^2v^2\sin^2\alpha}{(3m+M)gl(2m+M)}\right]$$

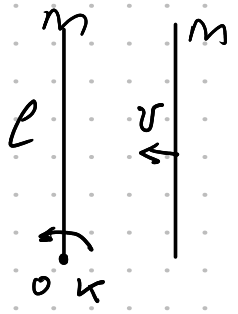
№ 9.126

$l = 0{,}3\ м$

$\kappa = 0{,}1$

$v = 3\ \frac{м}{с}$

$n - ?$

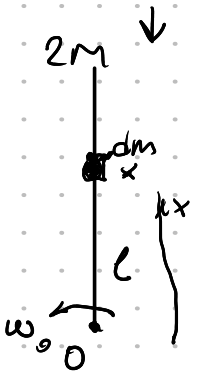


1) ЗСМИ:

$$mv \cdot \frac{l}{2} = I\omega_0, \quad I = \frac{1}{3}2ml^2$$

$$\left[\omega_0 = \frac{mvl}{\frac{1}{3}2ml^2} = \frac{3v}{2l}\right]$$

2) $E_0 = \frac{L^2}{2I} = \frac{m^2v^2l^2}{4 \cdot 4 \cdot \frac{2}{3}ml^2} = \frac{3mv^2}{32}$



ЗЭК: $dA_{тр} = n \cdot 2\pi x \cdot dm g \mu$

$$\frac{dm}{m} = \frac{dx}{l} \rightarrow dm = \frac{m}{l} dx$$

$$A_{тр} = -\int n \cdot 2\pi x \cdot \frac{m}{l} dx\, g\mu = -\frac{2\pi n mg}{l}\mu \int x\,dx = -\frac{\pi n\, mg \cdot \mu l^2}{l}$$

$$\Delta E = 0 - E_0 = -\pi n mg\mu l \rightarrow \frac{3v^2}{32} = \pi n g \mu l$$

$$n = \frac{3v^2}{32\pi g \mu l}$$

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(97) $I\frac{d\omega}{dt} = M$

№ 9.71

M, R

$a, N - ?$

один полый, один полый

1) Инерция полого – MR^2

Инерция полного – $\frac{MR^2}{2}$

$I_{полого} \cdot \varepsilon_{полого} = M$

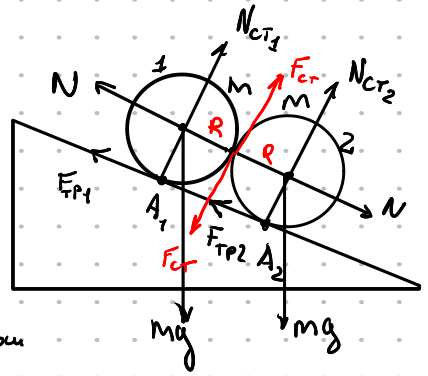
$I_{сплош} \cdot \varepsilon_{сплош} = M$

$\Rightarrow I_{пол} > I_{сплош}$

$\downarrow$

$\varepsilon_{пол} < \varepsilon_{сплош} \Rightarrow$ полый спереди чтобы не отстал

$I_1 = \frac{mR^2}{2}, \quad I_2 = mR^2$



2) Уравнение Вращ. движ. отн. A_1, A_2:

$$I_{A_1} = I_1 + MR^2 = \frac{3}{2}mR^2, \quad I_{A_2} = I_2 + mR^2 = 2mR^2$$

- $I_{A_1} \cdot \varepsilon_1 = M_{A_1} = mgR\sin\alpha - NR - F_{тр}R$
- $I_{A_2} \cdot \varepsilon_2 = M_{A_2} = mgR\sin\alpha + NR - F_{тр}R$

$\Big\}$: $* \varepsilon_1 = \varepsilon_2$ – вместе

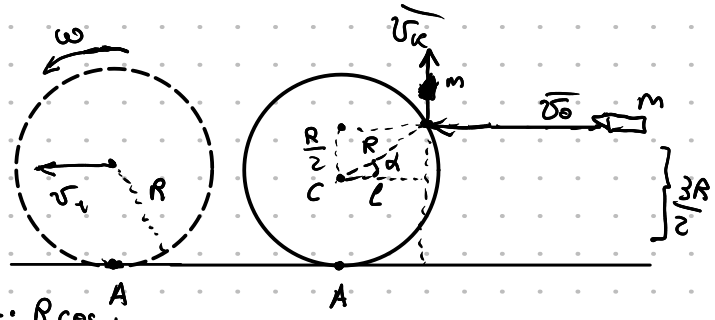
$$\frac{I_{A_1}}{I_{A_2}} = \frac{M_{A_1}}{M_{A_2}} = \ldots \rightarrow \varepsilon = \ldots, \ N = \ldots$$

3) $\varepsilon = \frac{d\omega}{dt} = \frac{d(\omega \cdot R)}{dt \cdot R} = \frac{dv}{dt \cdot R} = \frac{a}{R} \rightarrow a = \varepsilon R = \dots$

№ 9.89

$h = \frac{3R}{2}$
m, v_0
v_1
$v_к - ?$

1) $\sin\alpha = \frac{\frac{R}{2}}{R} = \frac{1}{2}$, $\omega = \frac{v_1}{R}$
$\cos\alpha = \frac{\sqrt{3}}{2}$



2) ЗСМИ отн. A:

$mv_0 \cdot \frac{3R}{2} = I_A\omega + mv_к \cdot R\cos\alpha$

$\left[I_A = MR^2 + \frac{2}{5}MR^2 = \frac{7}{5}MR^2\right]$

$mv_0 \cdot \frac{3R}{2} = \frac{7}{5}MR^2 \frac{v_1}{R} + mv_к R\frac{\sqrt{3}}{2}$

$\frac{3}{2}mv_0 - \frac{7}{5}Mv_1 = mv_к\frac{\sqrt{3}}{2}$

$$\left[v_к = \frac{3mv_0 - \frac{14}{5}Mv_1}{m\sqrt{3}}\right]$$

№ 9.187

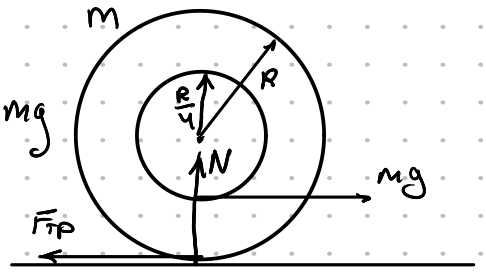
$I_0 = \frac{MR^2}{8}$
$S = 1\text{ м}$
$S_x - ?$
$\kappa = \frac{3}{10}$

1) ЗЗН для ц.м.:

$ma = mg - \kappa_0 N$, $N = mg$

$ma = mg(1-\kappa_0)$

$a = g(1-\kappa_0)$



2) $I\varepsilon = M$, $\varepsilon = \frac{a}{R}$

$I\frac{a}{R} = M = -mg\frac{R}{4} + \kappa_0 mgR$

$\frac{Ig(1-\kappa_0)}{R} = -\frac{mgR}{4} + \kappa_0 mgR$

$\frac{1}{8}(1-\kappa_0) = \kappa_0 - \frac{1}{4}$

$\frac{1}{8} + \frac{1}{4} = \frac{9}{8}\kappa_0 \rightarrow \kappa_0 = \frac{8}{9}\cdot\frac{3}{8} = \frac{1}{3}$

$\kappa_0 = \frac{1}{3} > \frac{3}{10}$ ⇒ есть проскальзывание

$a = \frac{d\omega}{dt} = \frac{a}{R}$

3) Для κ: $mg - \kappa mg = ma$, $a = g(1-\kappa) = \frac{7}{10}g$

$I\varepsilon = M$ $\quad I\varepsilon = -mg\frac{R}{4} + \kappa mgR$

$\frac{mR^2}{8}\cdot\varepsilon = -mg\frac{R}{4} + \kappa mgR$

$\varepsilon = \frac{8g(\kappa - \frac{1}{4})}{R} = \frac{8g}{R}\left(\frac{3}{10} - \frac{1}{4}\right)$

$\left[\varepsilon = \frac{8\cdot 9}{20}\frac{g}{R} = \frac{2}{5}\frac{g}{R}\right] = \text{const}$ $\quad \omega = \varepsilon t$

4) $S = \frac{at^2}{2}$, $\left[t = \sqrt{\frac{2S}{a}}\right]$

5) Угол поворота итоговый: $d\varphi = \omega dt = \varepsilon t dt$

$$\left[\varphi = \varepsilon \frac{t^2}{2} = \frac{2}{5}\frac{g}{R} \cdot \frac{2S}{2a} = \frac{2}{5}\frac{gS}{Ra} = \frac{2}{5}\frac{gS}{R\frac{7}{10}g} = \frac{4S}{7R}\right]$$

$$l = \frac{1}{4}\varphi R = \frac{1}{7}S \rightarrow \boxed{S_x = S - l = \frac{6}{7}S}$$