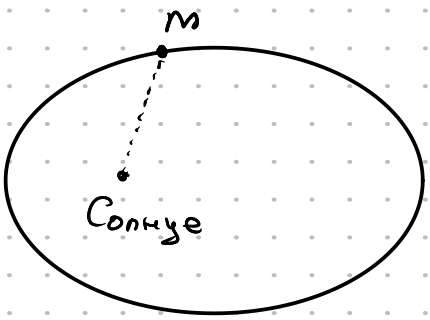
Всё о Гравитации

$\bar{M} = [\bar{r}, \bar{F}]$, $\bar{L} = [\bar{r}, \bar{p}]$, $\dot{\bar{L}} = \bar{M}$



• В поле центральных сил $\bar{M} = \bar{0}$ =>

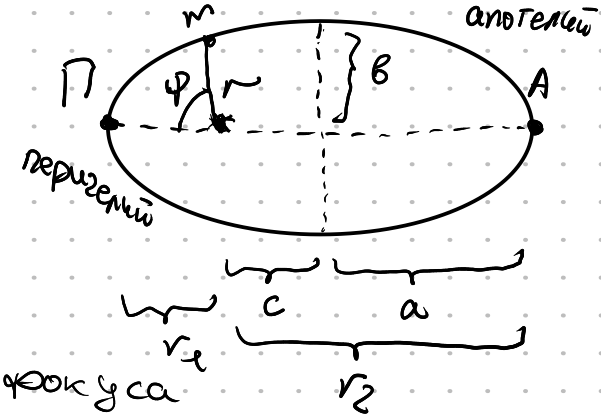
$\bar{L} = const$

1) Верен ЗСМЧ: $L_1 = L_2 = const = mVr$

2) $\bar{F} = -\frac{GmM}{r^2} \cdot \frac{\bar{r}}{r}$, $\bar{g} = -\frac{GM}{r^3}\bar{r}$

3) $\Pi = -\frac{GmM}{r}$

4) Т. Гаусса для замкн. поверхности: $\Phi = -4\pi GM$



(1) Закон Кеплера:

– движется тело по эллипсу

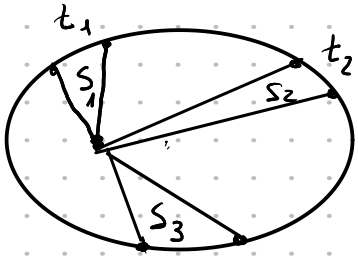
$e = \frac{c}{a}$

$\left[r = \frac{p}{1 + e\cos(\varphi)}\right]$ расстояние от фокуса до точки на траектории

$p = \frac{L^2}{GMm^2}$, $e = \frac{c}{a}$, φ – угол считаем от перигелия

эксцентриситет

(2) Закон Кеплера:



– Секториальная скорость постоянна.

$u = \dot{\varphi} = \frac{dS}{dt}$, – заметается площадь за время

$u = const = \frac{L}{2m}$

(3) Закон Кеплера: $\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$ – периодов обращения двух планет и длины больших полуосей

5) Энергии:

$E_к = \frac{mv^2}{2} = \frac{L^2}{2mr^2}$; $E_{полн} = \frac{mv^2}{2} - \frac{GMm}{r} = const$

$E_{полн} = -\frac{GmM}{2a}$; $e = \frac{L}{\sqrt{2mE}}$

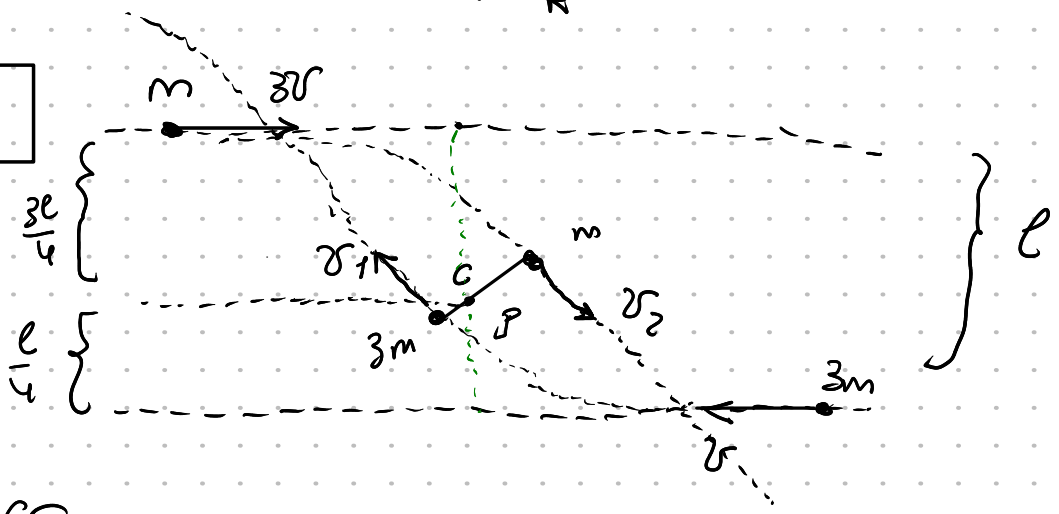
Если с бесконечности летит, то $E = \frac{mv_\infty^2}{2} = K_1 + П_1 = K_2 + П_2$

ЗСЭ + ЗСМИ

6) I космическая: $v_I = \sqrt{\frac{GM}{R}}$

II космическая: $v_{II} = \sqrt{\frac{2GM}{R}}$

Т2



1) Сблизились на ρ

Пишем ЗСМИ отн. С-у.ш.:

$\frac{3\ell}{4} \cdot 3v \cdot m + \frac{\ell}{4} \cdot v \cdot 3m = \frac{\rho}{4} \cdot 3m v_1 + \frac{3\rho}{4} \cdot m v_2$

$3mv\ell = \frac{3}{4} m\rho (v_1 + v_2)$

$4\ell v = \rho(v_1 + v_2)$ (1)

2) ЗСЭ: $\frac{1}{2} m (3v)^2 + \frac{1}{2} 3m v^2 = -\frac{G \cdot 3m \cdot m}{\rho} + \frac{1}{2} m v_2^2 + \frac{1}{2} 3m v_1^2$

$6v^2 = -\frac{3Gm}{\rho} + \frac{1}{2} v_2^2 + \frac{1}{2} \cdot 3 v_1^2$

$12v^2 = -\frac{6Gm}{\rho} + v_2^2 + 3v_1^2$ (2)

ЗСИ

3) $3mv_1 = mv_2 \Rightarrow v_2 = 3v_1$ (3)

4) ... Math

Т3

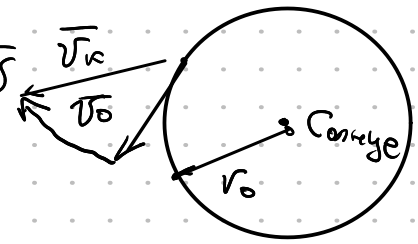
$\Delta v = 15 \frac{км}{с^2}$

$T = ?$

$T_0 = 1$ год

$r_0 = 1$ а.е. $= 1{,}5 \cdot 10^8$ км

1) $v_0 = \frac{2\pi r_0}{T_0} = \sqrt{\frac{GM}{r_0}}$



2) $\Delta E = \frac{m\Delta v^2}{2}$

$E_0 = -\frac{GNm}{2r_0} = \frac{mv_0^2}{2} - \frac{GMm}{r_0}$

$E_0 = -\frac{mv_0^2}{2}$

3) $\frac{T^2}{T_0^2} = \frac{a^3}{r_0^3}$, $E_к = E_0 + \Delta E = -\frac{GMm}{2a}$

$a = -\frac{GMm}{2(E_0+\Delta E)}$

$r_0 = -\frac{GMm}{2E_0}$

$T^2 = T_0^2 \cdot \frac{E_0^3}{(E_0+\Delta E)^3}$

$T^2 = T_0^2 \cdot \left(\frac{1}{1+\frac{\Delta E}{E_0}}\right)^3 = T_0^2 \cdot \left(\frac{1}{1-\frac{\Delta v^2}{v_0^2}}\right)^3$

$\left[T = T_0 \left(\frac{1}{1-\frac{\Delta v^2}{v_0^2}}\right)^{\frac{3}{2}} \right]$

Т4

$v_ф = 30 \frac{км}{с} = v_0$

$v_r = 40 \frac{км}{с}$

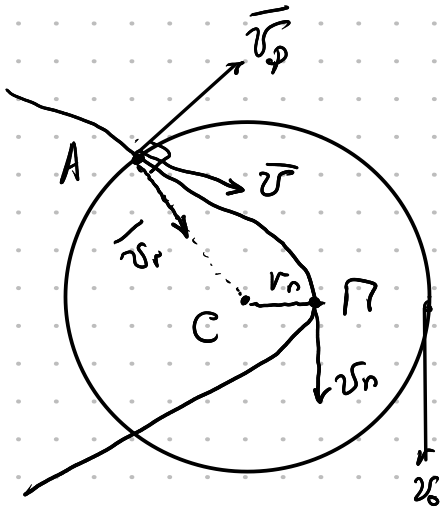
$v = 50 \frac{км}{с}$

$r = 1$ а.е. $= R_З$

$v_п - ?$

$v_r - ?$

1) $E = \frac{mv^2}{2} - \frac{GMm}{r} = \frac{mv^2}{2} - mv_0^2$



2) ЗСМИ

$mv_п r_п = mv_ф \cdot r$

$v_п r_п = v_ф r \Rightarrow \frac{r}{r_п} = \frac{v_п}{v_ф}$

$v_0 = \sqrt{\frac{GM}{r}}$

3) ЗСЭ:

$\frac{mv^2}{2} - mv_0^2 = -\frac{GMm}{r_п} + \frac{mv_п^2}{2}$

$v^2 - 2v_0^2 = -\frac{GM}{r_п} \cdot 2 + v_п^2 = -v_0^2 \frac{r}{r_п} \cdot 2 + v_п^2$

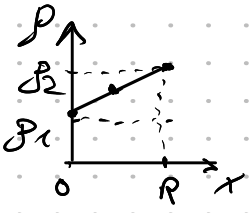
$v^2 - 2v_0^2 = -2v_0^2 \cdot \frac{v_п}{v_ф} + v_п^2$

$v_п^2 - 2v_п \cdot \left(v_0^2 - \frac{v_0^2}{v_ф}\right) - v^2 = 0$

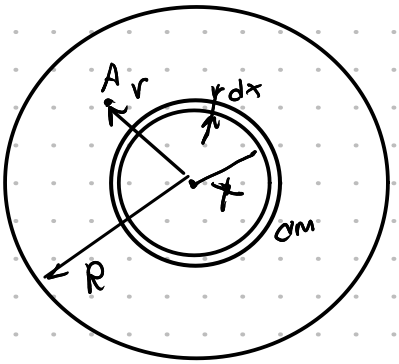
...

№7.139

1) Теорема Гаусса



$$\rho(x) = \rho_1 - \frac{\rho_1 - \rho_2}{R} x$$



$$g(r) = \frac{G}{r^2} \cdot \int_0^r dm \quad \text{- уравнение}$$

$$dm = dx \cdot 4\pi x^2 \cdot \rho(x)$$

$$g(r) = \frac{G}{r^2} \cdot \int_0^r 4\pi x^2 \left(\rho_1 - \frac{\rho_1 - \rho_2}{R} x\right) dx$$

$$g(r) = \frac{4\pi G \rho_1}{r^2} \cdot \int_0^r x^2 dx - \frac{4\pi G}{Rr^2} \cdot \frac{\rho_1 - \rho_2}{} \cdot \int_0^r x^3 dx$$

$$g(r) = \frac{4\pi G \rho_1}{r^2} \cdot \frac{r^3}{3} - \frac{4\pi G}{Rr^2} \cdot (\rho_1 - \rho_2) \frac{r^4}{4}$$

$g'(r) = 0$ - макс.

$$\left(\frac{\rho_1 r}{3} - \frac{(\rho_1 - \rho_2)}{R} \cdot \frac{r^2}{4}\right)' = 0$$

$$\frac{\rho_1}{3} - \frac{(\rho_1 - \rho_2)}{R} \frac{r}{2} = 0$$

$$r = \frac{2}{3} \frac{\rho_1 R}{\rho_1 - \rho_2}$$