Unit root tests for Panel Data and Application to World Temperature Data

Kihyun Han

Abstract

We analyze various unit root tests for panel data, which can be considered as time series for multiple objects and is common in a wide variety of fields, including econometrics. We introduce the Dickey-Fuller test (DF) and the Augmented Dickey-Fuller test (ADF), which are well-known unit root tests in time series analysis, by stating the asymptotic distributions by each of their conditions. Moreover, we investigate the methodologies of unit root tests for panel datasets, including the IPS test of Im et al. (2003). In temperature data analysis, we consider the model of linearity and seasonality and apply the panel unit root tests after removing the seasonal term. In this process, we fill up missing values and verify the autocorrelation in the error term with the Bonferroni test. We conclude that the unit root does not exist in the data, which implies that the non-seasonal term is stationary.

1. Introduction

This research illustrates the theoretical background of unit root tests that verify the existence of unit roots in time series analysis. We focus on the panel data, which is a time series of same variable with multiple objects, and investigate the model specification and methods of unit root tests. Moreover, we thoroughly analyze one of the widely-used unit root tests for panel data, Im *et al.* (2003). Further, we consider the monthly average of daily maximum temperature from 1981 to 2020 observed at weather stations all over the world. We apply the unit root tests to this data in practice and analyze the existence of the unit root. Accordingly, we conclude that the unit root does not exist in the non-seasonal term of monthly temperature, and the temperature change is stationary.

2. Theoretical Background

2.1. Dickey-Fuller test and Augmented Dickey-Fuller test

The existence of the unit root in the time series y_t implies that the data is not stationary and that the values of variance $Var(y_t)$ and covariance $Cov(y_t, y_{t-l})$ of time lag l change with respect to time t. The origin of unit root tests for time series traces back to the Dickey-Fuller Test (DF test), which first appeared at Dickey and Fuller (1979), and the Augmented Dickey-Fuller test (ADF test).

First, the DF test assumes the following model with a time series y_t with an autocorrelation and a white noise error term u_t.

$$y_t = \alpha + \beta t + \rho y_{t-1} + u_t$$

This model is equivalent to $\Delta y_t = \alpha + \beta t + (\rho - 1)y_{t-1} + u_t$ where $\Delta y_t = y_t - y_{t-1}$. There are three forms in the DF test. Note that (1) and (2) consider the null hypothesis $H_0: \rho = 1$ and the alternative hypothesis $H_1: |\rho| < 1$, whereas (3) considers $H_0: \rho = 1 \; and \; \beta = 0$ and $H_1 : not \; H_0$.

(1) $\alpha = \beta = 0$: $y_t = \rho y_{t-1} + u_t$ ($AR(1)$ model)
(2) $\beta = 0$: $y_t = \alpha + \rho y_{t-1} + u_t$ ($AR(1)$ model with drift)
(3) In general: $y_t = \alpha + \beta t + \rho y_{t-1} + u_t$ ($AR(1)$ model with drift and the linear deterministic trend)

In the similar way to the DF test, which was specified for the $AR(1)$ model, the ADF test is a test for $AR(p+1)$ model and is expressed as the following formula.

$$y_t = \alpha + \beta t + \rho y_{t-1} + \theta_1 \Delta y_{t-1} + \theta_2 \Delta y_{t-2} + \cdots + \theta_p \Delta y_{t-p} + u_t$$

$$\Delta y_t = \alpha + \beta t + (\rho - 1)y_{t-1} + \theta_1 \Delta y_{t-1} + \theta_2 \Delta y_{t-2} + \cdots + \theta_p \Delta y_{t-p} + u_t$$

The ADF test also has three different forms. In the similar manner as the DF test, (1) and (2) consider $H_0: \rho = 1$ and $H_1: |\rho| < 1$ while (3) is the test for $H_0: \rho = 1 \; and \; \beta = 0$ and $H_1: not \, H_0$. We denote the following tests by $ADF(p)$. Then, the DF test can be represented as $ADF(0)$.

(1) $\alpha = \beta = 0$: $y_t = \rho y_{t-1} + \theta_1 \Delta y_{t-1} + \theta_2 \Delta y_{t-2} + \cdots + \theta_p \Delta y_{t-p} + u_t$ ($AR(p+1)$ model)

(2) $\beta = 0$: $y_t = \alpha + \rho y_{t-1} + \theta_1 \Delta y_{t-1} + \theta_2 \Delta y_{t-2} + \cdots + \theta_p \Delta y_{t-p} + u_t$ ($AR(p+1)$ model with drift)

(3) In general: $y_t = \alpha + \beta t + \rho y_{t-1} + \theta_1 \Delta y_{t-1} + \theta_2 \Delta y_{t-2} + \cdots + \theta_p \Delta y_{t-p} + u_t$ ($AR(p+1)$ model with drift and the linear deterministic trend)

The DF and the ADF tests adopt the *t* test statistic derived from the maximum likelihood ratio test. The formula is

$$t = \frac{\rho - 1}{\sqrt{\widehat{\sigma^2}}},$$

where $\widehat{\sigma^2}$ is the ordinary least squares estimate of the variance.

Since this statistic is computed from the least squares method, it has a similar formula as the well-known t-distribution. Nevertheless, it does not exactly follow the *t*-distribution due to the existence of autocorrelation of the time series. Furthermore, the distribution of the statistic converges to a distribution expressed with the standard Brownian motion $W(t)$ where T, the total length of y_t, gets sufficiently large. We state the asymptotic distributions ζ and η for (1) and (2), respectively.

$$\zeta = \frac{\frac{1}{2}[W(1)^2 - 1]}{\int_0^1 W(r)^2 dr}, \qquad \eta = \frac{\frac{1}{2}[W(1)^2 - 1] - W(1)\int_0^1 W(r)dr}{\int_0^1 W(r)^2 dr - \left[\int_0^1 W(r)dr\right]^2}$$

For each test, the asymptotic distribution of *t* test statistic is given by Brownian motions, so some features, such as the quartiles, cannot be precisely computed. Therefore, only the table of critical values for the significance level 0.01 and 0.05 are displayed by Monte Carlo simulations for each of the three forms.

2.2. Unit Root Test for Panel Data

Panel data is the data of a particular variable $y_{i,t}$ for N units with i ($i \in \{1,2,\cdots,n\}$) with respect to time t ($t \in \{0,1,\cdots,T_i\}$). The unit can correspond to each nation, enterprise, or individual, and the variable encompasses a wide array of factors, which can be found in social sciences, economics, and cultural indicators. The panel data analysis exhibits the common characteristics of a variable throughout various units.

Panel data is called ‘balanced’ if the time interval where the data exists is identical for each unit and ‘unbalanced’ otherwise. Moreover, we classify the panel data as “independent” and “cross-sectional” by whether the variable is independent between each unit or there is a correlation between the units.

In general, the model for panel data can be represented as the following formula, which is similar to the ADF test. In this formula, $d_{i,t}$ indicates the deterministic trend, such as linearity or existence of a drift.

$$\Delta y_{i,t} = d_{i,t} + (\rho_i - 1)y_{i,t-1} + \sum_{j=1}^{p_i} \theta_{i,j}\Delta y_{i,t-j} + u_{i,t}$$

For example, Levin and Lin (1993) propose a unit root test (LL test) for balanced and independent panel data. The LL test assumes that ρ_i's are equal to ρ ($\rho_1 = \rho_2 = \cdots = \rho_N = \rho$) and that $u_{i,t}$'s are independent and identically distributed as a normal distribution. The test displays the result for the null hypothesis $H_0: \rho = 1$ and $H_1: |\rho| < 1$. As in the ADF test, they deal with several models modifying $d_{i,t}$, specifically the existence of the drift and the deterministic trend ($d_{i,t} \in \{\emptyset, \alpha_i, \alpha_i + \beta_i t\}$). The research proves that $\hat{\rho}$ for each model and t_ρ, the test statistic, converge to normal distributions as $N \to \infty$, $T \to \infty$, and $N / T \to \infty$ under certain assumptions, applying the central limit theorem. Then, they introduce a table of critical values with respect to finite N and T with different significance levels obtained by the Monte Carlo simulations.

2.3. Understanding of the IPS test

K. Im, M. Peseran. Y. Shin develop a unit root test (IPS test) for the model of independent data where the error term $u_{i,t}$ has a variance σ_i^2, which is heterogenous with respect to i. Unlike the LLC test, they did not assume that ρ_i are identical, and set the null hypothesis $H_0: \rho_1 = 1, \rho_2 = 1, \cdots, \rho_N = 1$ and the alternative hyptothesis H_1: "at least one value among $\rho_1, \rho_2, \cdots, \rho_N$ is not 1".

First of all, the research assumes that $y_{i,t}$ for each i satisfies (2) of the DF test. If we write $\rho_i - 1 = \gamma_i$, the formula is represented as follows.

$$\Delta y_{i,t} = \alpha_i + \gamma_i y_{i,t-1} + u_{i,t}, \quad i = 1, \cdots, N, \quad t = 1, \cdots, T$$

If we re-write the above utilizing T-dimensional vectors $\Delta y_i = \left(\Delta y_{i,1}, \cdots, \Delta y_{i.T}\right)'$, $\tau_T = (1,1,\cdots,1)'$, $y_{i,(-1)} = \left(y_{i,0}, y_{i,1}, \cdots, y_{i,T-1}\right)'$, and $u_i = \left(u_{i,1}, \cdots, u_{i,T}\right)'$, then the following is attained.

$$\Delta y_i = \alpha_i \tau_T + \gamma_i y_{i,(-1)} + u_{i,t}, \quad i = 1, \cdots, N, \quad t = 0, \cdots, T$$

Since we assume that the values are independent between different i's, we obtain the estimate $\widehat{\gamma_{iT}}$ of γ_i and the t-statistic $\widehat{\gamma_{iT}}$ as a consequence of the least squares method for $y_{i,t}$. For the $T \times T$ matrix $M_\tau = I_T - \frac{1}{T}\tau_T\tau_T'$, $\widehat{\gamma_{iT}}$ and t_{iT} are described as follows.

$$\widehat{\gamma_{iT}} = \frac{\Delta y_i' M_\tau y_{i,(-1)}}{y_{i,(-1)}' M_\tau y_{i,(-1)}}$$

$$t_{iT} = \frac{\widehat{\gamma_{iT}}\left(y'_{i,(-1)} M_\tau y_{i,(-1)}\right)^{1/2}}{\sqrt{\widehat{\sigma^2_{iT}}}}$$

Here, $\widehat{\sigma^2_{iT}} = \frac{\Delta y_i' M_{X_i} \Delta y_i}{T-2}$ is the estimated MSE (Mean Squared Error) from the least squares method where $X_i = (\tau_T, y_{i,-1})$ and $M_{X_i} = I_T - X_i(X_i'X_i)^{-1}X_i'$. Since the data is balanced, T is fixed for each i, and each t_{iT} follows an identical distribution under H_0. In order to take advantage of the identicalness, they denote the $tbar$ statistic of the IPS test by the following.

$$tbar_{NT} = \frac{1}{N}\sum_{i=1}^{N} t_{iT}$$

It is well known that t_{iT} converges to η_i, the asymptotic distribution of the *t*-statistic for the model (2) in DF test, as T increases to infinity. Moreover, it is proved that the second moment of t_{iT} exists for $T > 5$, by applying the Cauchy-Schwarz inequality. As a result, if we obtain Z_{tbar} by standardizing the $tbar$ statistic, then Z_{tbar} converges to the standard normal distribution as N increases by the central limit theorem.

$$Z_{tbar} = \sqrt{N}\frac{tbar_{NT} - E(t_{iT})}{\sqrt{var(t_{iT})}} \Rightarrow N(0,1)$$

Meanwhile, applying the actual computation, they mention that the difference between quantiles of Z_{tbar} and the standard normal distribution is minute for small N. That is, Z_{tbar} guarantees the validity of the IPS test for small N.

Furthermore, for a model that $y_{i,t}$ is fitted for $AR(p_i + 1)$, similar to the $ADF(p_i)$ test, $t_{iT}(p_i, \theta_i)$ converges in distribution to η_i as T increases to infinity. Then, it is proved that Z_{tbar}, which is developed similarly, converges to the standard normal distribution as $N, T \to \infty$ and $N/T \to k$ for some nonnegative k.

In the similar manner, Im *et al.* (2003) proposes that W_{tbar}, which is obtained by the standardization of $tbar_{NT}$ under $\gamma_i = 0$, converges to the standard normal distribution as T and N gets large.

$$W_{tbar} = \sqrt{N}\frac{tbar_{NT} - E(t_{iT}(p_i, \theta_i)|\gamma_i = 0)}{\sqrt{var(t_{iT}(p_i, \theta_i)|\gamma_i = 0)}} \Rightarrow N(0,1)$$

In practice, the optimal value of p_i is achieved by AIC (Akaike Information Criterion) or SIC (Schwarz Bayesian Information Criterion). When applying to a typical panel data with ARMA model, the LL test mentioned above has a drawback that the power for $ADF(p_i)$ condition is considerably low for $p_i \geq 1$. This implies that there is a high chance that we accept the null hypothesis even though the data does not contain a unit root. On the contrary, W_{tbar} statistic of the IPS test has a high power, which resolves this problem.

2.4. Unit root test for Cross-sectional panel data

M. Pesaran proposes a unit root test for time series models of cross-sectional panel data. Here, an assumption that the error term $u_{i,t}$ of the model can be represented as a multiplication of a time factor f_t and a unit factor g_t for each time t and unit i. In this case, $\epsilon_{i,t}$ is independent for different i's and t's.

$$u_{i,t} = g_i f_t + \epsilon_{i,t}$$

Under another assumption that f_t is a linear combination of $\overline{y_t} = \frac{1}{N}\sum_{i=1}^{N} y_{i,t}$, the model is transformed into the following form.

$$\Delta y_{i,t} = d_{i,t} + (\rho_i - 1)y_{i,t-1} + \sum_{j=1}^{p_i} \theta_{i,j}\Delta y_{i,t-j} + c_i\overline{y_{t-1}} + \sum_{j=0}^{p_i} \Psi_{i,j}\overline{\Delta y_{t-j}} + \epsilon_{i,t}$$

While $t_{\rho,i}$ is obtained similarly in this model, the mean $C = \frac{1}{N}\sum_{i=1}^{N} t_{\rho,i}$ for all i does not follow the normal distribution as in the IPS test. Hence, the critical values of the statistic are attained via simulation.

3. Results

3.1. Data Descriptiom

In this report, we analyze the monthly average of daily maximum temperatures for each region around the world from January 1981 to December 2020. The data was attained by National Centers for Environmental Information, which stores the temperature and atmospheric data observed in weather stations worldwide and provides it online. The temperature data on the website is recorded in Fahrenheit instead of Celsius (https://www.ncdc.noaa.gov/data-access)

The data can be interpreted as panel data, where each weather station corresponds to a unit, and it has time series characteristics. Most of the weather stations having missing values higher than 5% of the monthly temperature data for forty years from January 1981 to December 2020. The stations with missing values lower than 5% of 480 months were scarce, and we discovered seven weather stations among these to apply for the research. The name, location, and the number and proportion of missing values of each weather station is stated in Table 1.

Table 1 List of weather stations

Index (i)	Weather Station	Location	Latitude (°)	Longitude (°)	Missing values	Proportion (%)
1	BAKERSFIELD AIRPORT	California, United States	35.43	-119.05	0	0
2	BARROW AIRPORT	Alaska, United States	71.28	-156.78	0	0
3	CHUUK WEATHER SERVICE OFFICE AIRPORT	Micronesia	7.46	151.85	9	1.88
4	CONDOBOLIN AG RESEARCH STATION	Australia	-33.06	147.22	7	1.46
5	KOKPEKTY	Kazakhstan	48.75	82.36	5	1.04
6	SPRINGFIELD WEATHER SERVICE OFFICE AIRPORT	Missouri, United States	37.23	-93.39	0	0
7	TURAIF	Saudi Arabia	31.69	38.73	20	4.17

For each weather station, a month of missing value is filled up with other data of the weather station. In order to find the missing values, we assumed that the monthly averages of the daily maximum temperature contain a linear trend and a seasonal trend without an autocorrelation in the error term. In other words, the monthly average $y_{i,t}$ of daily maximum temperatures for the i-th weather station ($i = 1,2,\cdots,7$) in the t-th month ($t = 1,2,\cdots,480$) satisfies the following model. Here, the error terms $\epsilon_{i,t}$ are independent between different i's and t's, and follows $N(0, \sigma_i^2)$.

$$y_{i,t} = \beta_{i,0} + \beta_{i,1}\mathrm{t} + \sum_{j=0}^{11} \gamma_{i,j}\, cos\left(\frac{2\pi}{12}(t-j)\right) + \epsilon_{i,t}$$

For a weather station i involving missing values, we attain $\widehat{\beta_{i,0}}, \widehat{\beta_{i,1}}, \widehat{\gamma_{i,j}}$ by applying the least squares method based on observed values $y_{i,t}$. Since we assume that the data in each weather station i is independent, the estimates $\widehat{\beta_{i,0}}, \widehat{\beta_{i,1}}, \widehat{\gamma_{i,j}}$ are derived only from the data $y_{i,t}$ of the i-th weather station, not elsewhere. If the t^*-th month is not observed, we set $\widehat{y_{i,t^*}}$ as the evaluation of $y_{i,t}$ with the estimates above. Through this process, we may transform the data into balanced panel data.

$$\widehat{y_{i,t*}} = \widehat{\beta_{i,0}} + \beta_{i,1}t^* + \sum_{j=0}^{11} \gamma_{i,j}\, cos\left(\widehat{\frac{2\pi}{12}(t^*-j)}\right)$$

For instance, the 7th weather station (Turaif) has a record for only 460 among 480 months that we are interested in, and the rest 20 months are missing. Figure 1 displays a graph that the missing value is filled with the estimates (red curve).

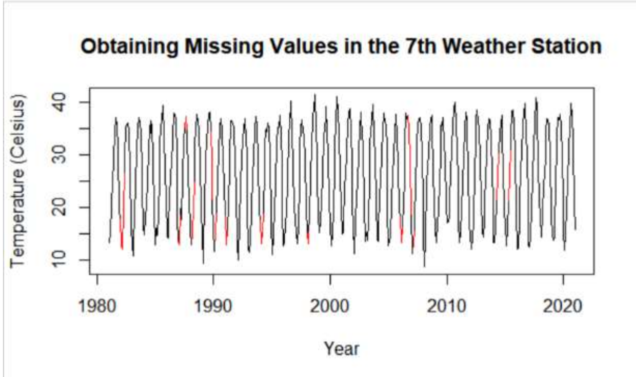


Figure 1 Estimates of the monthly temperature in the Station 7 (Turaif)

3.2. Test for Autocorrelation

In order to check whether the monthly average of the daily maximum temperature data contains autocorrelation, we consider the following model. Note that $T_{i,t}$ is the linear deterministic trend, and $S_{i,t}$ is the seasonal deterministic trend. The model has a similar form to the one we used to fill up the missing values above, but this model assumes the autocorrelation of the error term $u_{i,t}$.

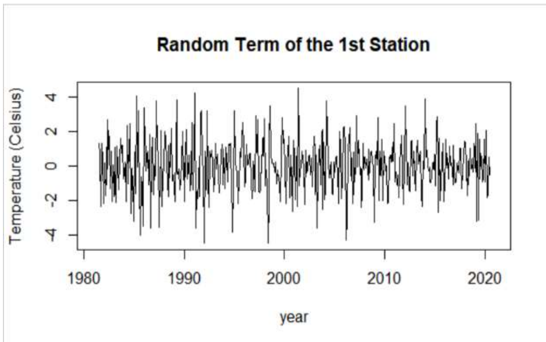
$$y_{i,t} = T_{i,t} + S_{i,t} + u_{i,t}$$

After obtaining $T_{i,t}$ and $S_{i,t}$ by applying the least squares method, we implemented the Ljung-Box test independently on the error terms $u_{i,t}$. The p-values of each weather station i are given in Table 2. For the significance level $\alpha = 0.01$, the Ljung-Box test implies that we may reject the null hypothesis that there is no autocorrelation of $u_{i,t}$ for only $i = 2,3,4$. Meanwhile, $y_{i,t}$ is a panel data; we may assume that there is a universal feature across all i's, so $u_{i,t}$ must contain autocorrelation for all i or no i. In other words, we can apply the Bonferroni test to control the family-wise error rate and set a null hypothesis as H_0: "no autocorrelation for all $u_{i,t}$." By applying the test for significance level $\alpha = 0.01$, the p-value for the 3rd weather station, which is the lowest value, is less than $\alpha/N = \frac{1}{700}$. Hence, we may reject the null hypothesis. That is, it means that we may conclude that $u_{i,t}$ has autocorrelation for all i.

Table 2 Result of the Ljung-Box test

Station (i)	1	2	3	4	5	6	7
p-value	0.986	5.64e-4	1.67e-5	2.18e-3	0.158	0.691	0.210

The error term $u_{i,t}$ after fitting can be exhibited as the following for the 1st and the 4th weather stations (Figure 2, 3). The p-values for the Ljung-Box test are 0.986 on the left and 0.002 on the right, but we cannot tell these by simply watching the plot.



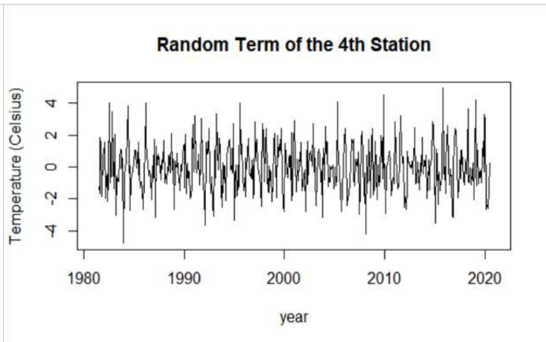


Figure 2 Error term $u_{i,t}$ of the 1st station **Figure 3 Error term $u_{i,t}$ of the 4th station**

3.3. Unit root tests for Panel data

The unit root tests which appeared in the papers that we referred to can be applied for the model with at most linear trend − after removing the seasonal deterministic trend. Hence, we apply the panel data analysis by introducing new variable $v_{i,t} = y_{i,t} - S_{i,t}$ in order to apply the unit root test to the model $y_{i,t} = T_{i,t} + S_{i,t} + u_{i,t}$. We may assume the existence of the linear deterministic trend and the drift for $v_{i,t}$. We suppose that $v_{i,t}$ is independent panel data between different i's.

When we regard $v_{i,t}$ as a panel data and apply the unit root tests, the existence of the unit root would directly imply that there is a stochastic trend as a random walk as well as the seasonal trend in $y_{i,t}$. Specifically, this indicates that the variance of the temperature data increases over time, and in meteorology, the trend of global warming would be unexpected and unable to predict.

$$v_{i,t} = \alpha_i + \beta_i t + \rho_i v_{i,t-1} + \theta_{i,1}\Delta v_{i,t-1} + \theta_{i,2}\Delta v_{i,t-2} + \cdots + \theta_{i,p_i}\Delta v_{i,t-p_i} + \epsilon_{i,t}$$

We applied the LL test, which assumes that all ρ_i's are equal for all weather stations $i = 1,2,\cdots,7$, and the IPS test, which admits that ρ_i can be different. By utilizing the 'plm' package of R, we applied the tests for a model with "drift" ((2) of the ADF test) and a model with "drift and the deterministic trend" ((3) of the ADF test). Also, p_i's are determined by comparing the AIC. The detailed information about the statistics and the p-values are stated below in Table 3.

Table 3 Results for the LL test and the IPS test

	LL test (statistic: t_ρ)		IPS test (statistic: W_{tbar})	
	drift	drift & trend	drift	drift & trend
statistic	-39.116	-65.988	-37.992	-47.772
p-value	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16

From the result, all tests show significantly low p-values $-$ smaller than 2.2×10^{-16}. Thus, we may reject the null hypothesis that the unit root exists for significance level $\alpha = 0.001$. Hence, for the confidence level 99.9%, we may deduce that the unit root for $v_{i,t}$ as panel data does not exist. That is, $v_{i,t}$ is stationary.

4. Conclusions and Recommendations

In this report, we analyzed the monthly average of the daily maximum temperatures for forty years $-$ from January 1981 to December 2020 $-$ by considering it as panel data. This data has shown the autocorrelation when removing the seasonal and the linear trend. Also, by applying the unit root tests, including the LL test and the IPS test, to the data after eliminating the seasonal trend, we strongly verify that the unit root does not exist and the data is stationary. That is, the monthly temperature data is consisted of a seasonal trend and a stationary term which does not have a temporal change of variance. In the sense of earth sciences, climate change, involving global warming, does not have a significant change in the fluctuation of the temperature.

In the case of the LL test, it is well known that the model from the $ADF(p_i)$ test has lower power for $p_i \geq 1$, which means that the probability of accepting the null hypothesis even though the alternative is true $-$ the type 2 error $-$ is high. Recall that $ADF(0)$ test model was used in the ordinary Dickey-Fuller test. In our research, some of the seven units LL test has fitted the model from $ADF(1)$ test as a result of comparing AIC. Since the p-values were considerably low in Table 3, we may adopt the alternative hypothesis with higher confidence level. Thus, we discover that the non-existence of the unit root in the temperature is way more significant.

Moreover, the test statistics of the LL test and the IPS test converge to the limit distribution as N and T goes to infinity. While the length of the time interval T in this research was large (480), the number of units N is 7. If we reduce or modify the time interval (from 1981 to 2020) and discover other weather stations that preserve the data with minimal missing values, then N will increase, and the confidence for the results of both unit root tests will rise. We add a conjecture that the possible reason for low p-values of two tests is that N is relatively small than T.

The unit root tests in this research admit at most linearity for the deterministic trend. Hence, we applied the test after removing the seasonal trend. The unit root tests for panel data considering the seasonal trend appear in Otero (2005) and Otero (2007). We hope that we can apply this technique to the temperature data to find whether the unit root exists.

References

[1] Im, K.S., Pesaran, M.H., Shin, Y., 2003. Testing for unit roots in heterogeneous panels,

Journal of Econometrics, Volume 115, Issue 1, 53-74,

[2] Dickey, D.A., Fuller, W.A., 1979. Distribution of the estimators for autoregressive time series with a unit root. Journal of the American Statistical Association 74, 427–431.

[3] Hamilton, J.D., 1994. Time Series Analysis. Princeton University Press, Princeton.

[4] Levin, A., Lin, C.F., 1993. Unit root tests in panel data: asymptotic and finite-sample properties. Unpublished manuscript, University of California, San Diego.

[5] National Centers for Environmental Information, 과거 세계 기상 자료 제공, U.S. Government - National Oceanic and Atmospheric Administration, https://www.ncdc.noaa.gov/data-access

[6] Croissant, Y., 2021, Linear Models for Panel Data (R Package 'plm'), https://cran.r-project.org/web/packages/plm/plm.pdf

[7] 이상열, 2013, 시계열분석 이론 및 SAS 실습, 자유아카데미

[8] 조신섭, 손영숙, 2019, SAS/ETS를 이용한 시계열분석, 율곡출판사

[9] Kirchgässner, G., Wolters, J., Hassler, U., 2007, Introduction to Modern Time Series Analysis. Springer