plot 1:

Fitting h_i^{model} parameter: for countercurrent:

$$\frac{1}{U_{Exp}} = \frac{a}{G_i^b} + c + \frac{d}{G_s^e}$$

where: $U_{Exp} = \frac{Q}{A_i \Delta T_{LM}}$, $\Delta T_{LM} = \frac{(T_{h,in} - T_{c,out}) - (T_{h,out} - T_{c,in})}{\ln\left(\frac{T_{h,in} - T_{c,out}}{T_{h,out} - T_{c,in}}\right)}$

$Q = \frac{|Q_c| + |Q_H|}{2}$, $Q_c = -Q_H$.

$Q_c = w_c \cdot C_p \cdot (T_{c,out} - T_{c,in})$

$Q_H = w_H \, C_p \, (T_{H,out} - T_{H,in})$

$C_p = \cancel{4.183}\ 4.182 \ \frac{kJ}{kg\,C}$

w_c and w_H is measured with bucket and stop watch: for 1 run.

$M_{c_1} = 1.5\ kg$, $t_1 = 40.1 s$. $\Rightarrow$

$M_{H_1} = 8.53 kg$, $t_1 = 20.05 s$ $\Rightarrow$

$\boxed{w_{c_1} = 0.038\ \frac{kg}{s} \quad w_{h_1} = 0.42\ \frac{kg}{s}}$

$T_{c,out} = 38.66 °C$, $T_{c,in} = 16.27 °C$

$T_{h,out} = 53.69 °C$, $T_{h,in} = 55.82 °C$ ~~$T_{c,ou}$~~

$Q_{c_1} = 0.037 \left[\frac{kg}{s}\right] \cdot 4.182 \frac{kJ}{kg\,°C} \cdot (38.7 - 16.3) °C = \boxed{3.5\ \frac{kJ}{s}}$ 3.

$\boxed{Q_{H_1} = -3.76 \frac{kJ}{s}}$ (same calculation)

T difference convert to Kelvin

$$\boxed{Q_1 = \frac{|Q_{c_1}| + |Q_{H_1}|}{2} = 3.65 \frac{kJ}{s}}$$

$N_T = 31$ (no. tubes)

$A_i = \pi D L\ N_T = \pi \cdot \left(0.21 in \cdot \frac{in}{39.37 m}\right) \cdot \left(7.875 in \cdot \frac{in}{39.37 m}\right) \cdot (31)$

$= 0.104\ m^2$.

$$\Delta T_{LM} = \frac{(\cancel{16.27}\ 55.82 - 38.66) - (53.69 - \cancel{55.82}\ 16.27)}{\ln\left(\frac{\cancel{16.27}\ 55.82 - 38.66}{53.69 - \cancel{55.82}\ 16.27}\right)} = \cancel{25.0}\ 26.0\ °K$$

$$\frac{Q}{A_i \Delta T_{LM}} = U_1 = 3.65 \left[\frac{kJ}{s}\right] \cdot \frac{1}{0.10\ m^2} \cdot \frac{1}{26\ K} = \boxed{1.35 \left[\frac{kJ}{m^2 K s}\right]}$$

accounted error:

$$M_{c,2} = 1.52\,kg \quad , t = 40 \Rightarrow w_{c,2} = 0.038\left[\frac{kg}{s}\right]$$

$$\text{Avg } w_{c} = \frac{w_{c,1} + w_{c,2}}{2} = \frac{0.037 + 0.038}{2} = 0.0377\left[\frac{kg}{s}\right]$$

$$\text{std } w_c = \sqrt{\frac{(w_{c,Avg} - w_{c,1})^2 + (w_{c,Avg} - w_{c,2})^2}{2}} = \sqrt{\frac{(0.0377 - 0.37)^2 + (0.037 - 0.38)^2}{2}}$$

$$= 3.00 \cdot 10^{-4}\,\frac{kg}{s}$$

$$\boxed{w_c = 0.037 \pm 0.0003\,\frac{kg}{s}}$$

$$\text{Avg } Q_{c} = \frac{Q_{c,1} + Q_{c,2}}{2} = 3.53\left[\frac{kJ}{s}\right]$$

$$\text{std } Q_c = 0.0027\left[\frac{kJ}{s}\right]$$

$$\Rightarrow Q_c = 3.53 \pm 0.0027\,\frac{kJ}{s}$$

$$\text{std } Q_c = \frac{|0.027| + |0|}{2} = 0.0135\,\frac{kJ}{s}$$

$$\boxed{Q = 3.65 \pm 0.0135\,\frac{kJ}{s}}$$

$$\text{std } U = 0.0135\,\frac{kJ}{s} \cdot \frac{1}{0.10\,m^2} \cdot \frac{1}{26K}$$

$$= 0.0049\,\frac{kJ}{m^2 K s}$$

$$\boxed{U = 1.35 \pm 0.0049\left[\frac{kJ}{m^2 K s}\right]}$$

$\Rightarrow$ shown error on graph

All future calculation will include errors

$$G_i = \frac{w_c}{N_T \pi \cdot \frac{D_i^2}{4}} = \frac{0.037 \frac{kg}{s}}{31 \cdot \pi \cdot \frac{\left(\frac{0.21}{39.37}\right)^2}{4} m^2} = \boxed{53.4 \pm 0.43 \frac{kg}{m^2 s}}$$

$D_H = 2.025$

$$G_H = \frac{w_s}{\frac{\pi D_H^2}{4} - \frac{N_T \pi D_i^2}{4}} = \frac{0.42 \frac{kg}{s}}{\frac{\pi \left(2.025 in \cdot \frac{m}{39.37 in}\right)^2}{4} - 31 \cdot \frac{\left(\pi \cdot \left(\frac{0.21}{39.37}\right)^2 m^2\right)}{4}}$$

$$\boxed{G_H = 388. \frac{kg}{m^2 s}}$$

$$\frac{1}{U} = \frac{1}{1.35 \frac{kJ}{m^2 K s}} = \boxed{0.74 \pm 0.0004 \frac{m^2 K s}{kJ}} \Rightarrow$$ ~~show~~

fit into: $\frac{1}{U} = \frac{a}{G_i^b} + c + \frac{d}{G_s^e}$

$$\Rightarrow 0.74 = \frac{a}{53.4^b} + c + \frac{d}{388^e}$$

(need at least 5 data point to get all parameter
fit into 36 run data. use curve-fit on python to determine).

$a = 40.09$, $b = 1.076$, $c = 0.228$, $d = 0.442$, $e = 5.94$.

calibrate unit for variables:

$$\frac{1}{U} = \left[\frac{m^2 K s}{kJ}\right] = \frac{a}{\left[\frac{kg}{m^2 s}\right]^{1.076}} \Rightarrow a = \left[\frac{m^2 K s}{kJ}\right]\left[\frac{m^2 s}{kg}\right]^{1.076}$$

$$= \left[\frac{m^{2(1+1.076)} K s^{(1+1.076)}}{kJ\ kg^{1.076}}\right]$$

$$\boxed{a = \left[\frac{s^{2.08}\ m^{4.16}\ K}{kJ\ kg^{1.08}}\right]}$$

so $\Rightarrow$ $a = 40.09\left[\frac{s^{2.08}\, m^{4.16} K}{kJ\ kg^{1.08}}\right]$, $b = 1.076$, $c = 0.0228\left[\frac{m^2 K s}{kJ}\right]$,

$d = 0.442\left[\frac{s^{6.93}\, m^{13.86} K}{kJ\ kg^{5.93}}\right]$, $e = 5.93$.

Same is done for co-current.

figure 3 fitting:

~~refit~~ use parameters to find $h_i^{Exp} = \frac{1}{a} G_i^{b}$

$$\Rightarrow\ h_i^{Exp} = \frac{1}{40.09} \cdot 53.4^{1.076} \left[\frac{1}{\left(\frac{s^{2.08}\, m^{4.16} K}{kJ\ kg^{1.08}}\right)} \cdot \frac{kg^{1.076\ (1.08)}}{m^{2\cdot 1.076}\ s^{1.076}} \approx (1.08)\right]$$

$$\boxed{h_i^{Exp} = 0.458 \frac{kJ}{m^2 \cdot s \cdot K}}$$

h_i^{S-T} using Sieder-Tate correlation: $\mu = 1cp = 0.001 \frac{kg\, s}{m}$

$k = 0.000598 \frac{kJ}{s \cdot m \cdot K}$, $D_i = 0.21\ in \cdot \frac{m}{39.37\ in} = 0.0053\ m$.

$$h_i^{S-T} = 0.023 \cdot \frac{5.98 \cdot 10^{-4}\left[\frac{kJ}{s \cdot m \cdot K}\right]}{0.0053[m]} \cdot \underbrace{\left[\frac{0.0053[m] \cdot 53.4\left[\frac{kg}{m^2 s}\right]}{0.001\ \frac{kg\ s}{m}}\right]^{0.8}}_{\text{D-less}} \cdot \underbrace{\left[\frac{4.182\left[\frac{kJ}{kg\,K}\right] \cdot 0.001\left[\frac{kg\ s}{m}\right]}{5.98 \cdot 10^{-4}\left[\frac{kJ}{s \cdot m \cdot K}\right]}\right]^{0.33}}_{\text{D-less}}$$

$$\boxed{h_i^{S-T} = 0.46 \left[\frac{kJ}{s\, m^2 K}\right]}$$

both water.

$$\left(\frac{h_i^{S-T} D_i}{k} = 0.023 \cdot \left(\frac{D_i G_i}{\mu_i}\right)^{0.8} \left(\frac{\hat{C}_p \mu}{k}\right)^{\frac{1}{3}} \cancelto{1}{\left(\frac{\mu_b}{\mu_o}\right)^{0.14}}\right)$$

Figure 5.
Error Propagation

$Q_c = \dot{m}_c C_p dT_c \quad , \quad Q_H = m_H C_p dT_H$

Normalized ~~denom~~ numerator $Q = \frac{|Q_c| + |Q_H|}{2}$. $\quad \frac{dQ}{dQ_c} = \frac{1}{2} \quad , \quad \frac{dQ}{dQ_H} = \frac{1}{2}$.

$$dQ = \cancel{\frac{|dQ_c| + |dQ_H|}{2}} \frac{\sqrt{\left|\frac{dQ}{dQ_c} \Delta Q_c\right|^2 + \left|\frac{dQ}{dQ_H} \Delta Q_H\right|^2}}{2}$$

$$\cancel{\Delta Q_c} = \Delta Q = \frac{\sqrt{\left|\frac{1}{2} \Delta Q_c\right|^2 + \left|\frac{1}{2} \Delta Q_H\right|^2}}{}$$

$$= \frac{1}{2} \sqrt{|\Delta Q_c|^2 + \Delta Q_H|^2}$$

$$\Delta Q_c = \sqrt{\left|\frac{dQ_c}{d\dot{m}_c} \Delta \dot{m}_c\right|^2 + \left|\frac{dQ_c}{dT_c} \Delta T_c\right|^2} \qquad \frac{dQ_c}{d\dot{m}_c} = C_p dT \quad , \quad \frac{dQ_c}{dT_c} = \dot{m}_c C_p$$

$$\Delta Q_c = \sqrt{\left[C_{p_c}(T_{c,out} - T_{c,in}) \cdot \Delta \dot{m}_c\right]^2 + \left[\dot{m}_c C_p \Delta T\right]^2}$$

$C_{p_c} = 4.182 \frac{kJ}{kg\,K}$, $\Delta \dot{m}_c = \text{std } m_c = 0.000297 \frac{kg}{s}$

$T_{c,out} - T_{c,in} = 22.39\,K \qquad \Delta T = 0.5$ (systematic fluctuation)

$\dot{m}_c = 0.0377 \frac{kg}{s}$

$$\Delta Q_c = \sqrt{\left(4.182 \left[\frac{kJ}{kg\,K}\right] \cdot 22.39[K] \cdot \overset{0.0003}{\cancel{0.0377}} \left[\frac{kg}{s}\right]\right)^2 + \left[0.0377 \cancel{4.182} \frac{kg}{s} \cdot 4.182 \frac{kJ}{kg\,K} \cdot 0.5K\right]^2}$$

$= 0.084 \frac{kJ}{s}$

ΔQ_H derived similar. $\Delta Q_H = 0.39 \frac{kJ}{s}$.

$$\Delta Q = \sqrt{(0.084)^2 + (0.39)^2} = \cancel{0.4} \; 0.398$$

Normalized error bar:

$$\frac{\Delta Q}{0.5(|Q_H| + |Q_c|)} = \frac{0.398}{3.65} = \cancel{0.10} \; 0.11$$

Normalized data:

$$\frac{Q_c + Q_H}{0.5(|Q_H| + |Q_c|)} = \frac{\cancel{3.50}\;3.53 - 3.76}{3.65} = \boxed{-0.0635}$$

Done for all data point.