Discrete Mathematics
THE FOUNDATIONS: LOGIC & PROOFS

FPT University

Department of Mathematics

Quynhon, 2023

Outline of Lesson

1. **Propositional Logic**
2. **Propositional Equivalences**
3. **Predicates and Quantifiers**
4. **Nested Quantifiers**
5. **Rules of Inference**

Textbook: Discrete Mathematics and Its Applications, Seventh edition, K.Rosen.

Upcoming ...

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Motivations

Barber Paradox. The barber is the "one who shaves all those, and those only, who do not shave themselves". The question is, does the barber shave himself?

1. Logic is the basis of all mathematical reasoning.
2. Logic is foundation of automated reasoning.
3. Applications in: design of computer hardware (logic circuits), artificial intelligence, programming languages, machine translation, ... and many other computer science fields.
4. Propositional logic helps us to understand mathematical arguments and proofs.

Proposition

A **proposition** is a statement that is either true or false (but not both).

Example. Which of the following sentences are propositions?

1. The water boils at 100 degrees Celsius.
2. There are 15 students in Discrete Mathematics class.
3. $x^2 + 4 \geq y$.
4. It is now 10pm.
5. $1 + 7 = 9$.
6. Do you have a cat?
7. She is so sexy!

Note.

- Only affirmative sentences are propositions.
  $\rightarrow$ Exclamation, imperative or interrogative sentences are NOT propositions.
- A proposition can be viewed as a number with value 1 (true) or 0 (false).
  $\rightarrow$ Propositional logic is a mathematical system that works with 0 and 1 numbers.

Upcoming ...

1. Propositional Logic
2. **Propositional Equivalences**
3. Predicates and Quantifiers
4. Nested Quantifiers
5. Rules of Inference

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Propositional Operators: Negation

- **Negation** $\neg$
  $\neg p =$ not $p =$ proposition that is true if p is false, and is false if p is true.

- **Examples.**
  1. He is rich $\rightarrow$ He is NOT rich.
  2. The girl is pretty $\rightarrow$ The girl is NOT pretty.
  3. Discrete maths is not difficult $\rightarrow$ Discrete maths is difficult.

- **Truth table.**

p	$\neg p$
0	1
1	0

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Propositional Operators: Conjunction

- **Conjunction** $\wedge$
  $p \wedge q = p$ and $q =$ proposition that is true when both p and q are true, and is false otherwise.

- **Examples.**
  1. $p =$ "He is handsome.", $q =$ "He is super rich."
     $\rightarrow p \wedge q =$ "He is handsome AND super rich."
  2. $p =$ "The school is closing.", $q =$ "The kids are not happy."
     $\rightarrow p \wedge q =$ "The school is closing AND the kids are not happy."

- **Truth table.**

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

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Propositional Operators: Disjunction

- **Disjunction** $\vee$
  $p \vee q = p$ or $q =$ proposition that is false when both p and q are false, and is true otherwise.

- **Examples.**
  1. $p =$ "She is smart.", $q =$ "She is hard-working."
     $\rightarrow p \vee q =$ "She is smart OR hard-working."
  2. At least one of the suspects is the murder.

- **Truth table.**

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

Propositional Operators: Exclusive-or

- **Exclusive-or** $\oplus$
  $p \oplus q =$ only p or only $q =$ proposition that is true when exactly one of p and q is true and is false otherwise.

- **Examples.**
  1. $p =$ "She will marry Peter.", $q =$ "She will marry Tom."
     $\rightarrow p \oplus q =$ "She will marry EITHER Peter OR Tom."
  2. You never lose. You EITHER win OR learn.

- **Truth table.**

p	q	$p \oplus q$
0	0	0
0	1	1
1	0	1
1	1	0

Propositional Operators: Conditional statement

- **Conditional statement (Implication) $\rightarrow$**
  $p \rightarrow q =$ proposition that is false when p is true and q is false, and is true otherwise.

- **Examples.**
  1. $p =$ "You dont water your plants.", $q =$ "The plants die."
     $\rightarrow p \rightarrow q =$ "If you dont water your plants, they will die."
  2. If I am a billionaire, I will buy each of my students a Mercedes G63.

- **Truth table.**

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Language Bank: Express the conditional statement

There several ways to express the conditional statement $p \to q$:

- If p, then q
- If p, q
- p is sufficient for q
- q if p
- q when p
- a necessary condition for p is q
- q unless $\neg p$
- p implies q
- p only if q
- a sufficient condition for q is p
- q whenever p
- q is necessary for p
- q follows from p
- q provided that p.

Propositional Operators: Biconditional statement

- **Biconditional statement** $\leftrightarrow$
  $p \leftrightarrow q =$ proposition that is true when p and q have the same truth values[1], and is false otherwise.

- **Examples.**
  1. $x \geq y \leftrightarrow x - y \geq 0$.
  2. Two triangles are congruent if and only if all three pairs of corresponding sides are congruent.

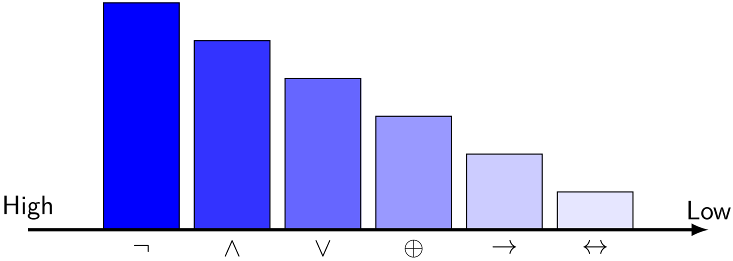
- **Truth table.**

p	q	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

[1] The meaning of truth value is the truth or falsity of a proposition or statement.

Precedence of Logical Operators

From high to low priority.



Truth table.

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \rightarrow q$	$p \leftrightarrow q$
0	0	1	0	0	0	1	1
0	1	1	0	1	1	1	0
1	0	0	0	1	1	0	0
1	1	0	1	1	0	1	1

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Translating Sentences Into Logical Expressions

Quiz

"I watch football only if Arsenal play or I have no homework."

$p =$ "I watch football".
$q =$ "Arsenal play."
$r =$ "I have homework."

Quiz

1. "You can not pass this course if you miss more than 20% of lectures."
2. "You can not pass this course if you miss more than 20% of lectures unless you provide reasonable excuses."

$p =$ "You pass this course."
$q =$ "You miss more than 20% of lectures."
$r =$ "You provide reasonable excuses ".

Logic and Bit Operations

Computers represent information using bits. A **bit** is a symbol of two possible values, 0 and 1. A bit can represent a truth value, that is, **1 represents T (true)** and **0 represents F (false)**. Information is often represented using bit strings, and operations on bit strings can be used to manipulate this information.

A **bit string** is a sequence of zero or more bits. The length of this string is the number of bits in the string.

Table for the Bit Operators OR, AND, and XOR.

x	y	$x \wedge y$	$x \vee y$	$x \oplus y$
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

The bitwise OR, bitwise AND, and bitwise XOR of two strings of the same length to be the strings that have as their bits the OR, AND, and XOR of the corresponding bits in the two strings, respectively.

Example. $1001100 \wedge 0011001 = 0001000.$

Evaluating Compound Proposition

Evaluate

$$A = \neg p \to (q \oplus r \to \neg p \wedge q) \leftrightarrow s,$$

where $p = 0, q = 1, r = 0, s = 1.$

Solution. Substitute p, q, r and s to the expression:

$$\begin{aligned} A &= \neg 0 \to (1 \oplus 0 \to \neg 0 \wedge 1) \leftrightarrow 1 \\ &= 1 \to (1 \oplus 0 \to 1 \wedge 1) \leftrightarrow 1 \\ &= 1 \to (1 \oplus 0 \to 1) \leftrightarrow 1 \\ &= 1 \to (1 \to 1) \leftrightarrow 1 \\ &= 1 \to 1 \leftrightarrow 1 \\ &= 1 \leftrightarrow 1 \\ &= 1 \end{aligned}$$

Formalize a proposition

Let p, q and r be the propositions where

- p : Grizzly bears have been seen in the area.
- q : Hiking is safe on the trail.
- r : Berries are ripe along the trail.

Write the following propositions using p, q and r and logical connectives (including negations).

1. *Berries are ripe along the trail, but grizzly bears have not been seen in the area.* $r \wedge \neg p$
2. *Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.* $\neg p \wedge q \wedge r$
3. *If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.* $r \to (q \leftrightarrow \neg p)$

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Formalize a proposition

4. *It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.* $\neg q \wedge \neg p \wedge \neg r$
5. *For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.* $q \rightarrow (\neg r \wedge \neg p)$
6. *Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.* $p \wedge r \rightarrow \neg q$

Propositional Equivalences

- A compound proposition is called a **tautology** if it is always true regardless of the truth values of the propositions that occur in it.
- A compound proposition is called a **contradiction** if it is always false.
- A compound proposition that is neither tautology nor contradiction is called a **contingency**.

Two propositions p and q are **logically equivalent** if the biconditional statement $p \leftrightarrow q$ is a tautology. In this case, we use notation $p \equiv q$.

Note. Two methods for proving logical equivalences:

1. Use truth table.
2. Use other logical equivalences.

Proving logical equivalences

Prove that $p \to q$ is logically equivalent to $\neg p \vee q$.

Solution. We can use truth table as follows

p	q	$\neg p$	$p \to q$	$\neg p \vee q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	0	1	1

Hence,

$$p \to q \equiv \neg p \vee q.$$

Some logical equivalences

- Double negation law $\neg(\neg p) \equiv p$
- Identity laws $p \wedge T \equiv p, \quad p \vee F \equiv p$
- Domination laws $p \vee T \equiv T, \quad p \wedge F \equiv F$
- Negation laws $p \vee \neg p \equiv T, \quad p \wedge \neg p \equiv F$
- Idempotent laws $p \vee p \equiv p, \quad p \wedge p \equiv p$
- Commutative laws $p \vee q \equiv q \vee p, \quad p \wedge q \equiv q \wedge p$
- Associative laws $(p \vee q) \vee r \equiv p \vee (q \vee r),$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Distributive laws $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r),$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- De Morgan's laws $\neg(p \wedge q) \equiv \neg p \vee \neg q, \quad \neg(p \vee q) \equiv \neg p \wedge \neg q$

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Logical equivalences involving (bi)conditional statements

Involving conditional statements

- $p \to q \equiv \neg p \vee q$
- $p \to q \equiv \neg q \to \neg p$
- $p \vee q \equiv \neg p \to q$
- $p \wedge q \equiv \neg(p \to \neg q)$
- $\neg(p \to q) \equiv p \wedge \neg q$
- $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
- $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
- $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
- $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

Involving biconditional statements

- $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
- $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
- $p \oplus q \equiv \neg(p \leftrightarrow q)$
- $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
- $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Questions.

1. Show that $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$.
2. Show that $(p \wedge q) \to (p \vee q)$ is a tautology.

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Predicates

Let $P(x) = "x^2 \geq 1"$.

- $P(x)$ is not a proposition since its truth value is unknown.
- But if $x = x_0$ is given, then $P(x_0)$ is a proposition. $\rightarrow$ It means that $P(x)$ will become a proposition when a value is assigned to x. And $P(x) = "x^2 \geq 1"$ is called a **propositional function**.
- It is easy to see that

$$P(0) = F, \quad P(3) = T.$$

$\rightarrow$ x is called a variable and $" \geq 1"$ is the **predicate**, P denotes the predicate.

A **propositional function** is a mapping $P : \mathbb{X} \rightarrow \{0, 1\}$ where $\mathbb{X}$ is a given set of parameters.

Example. $R(x, y, z) = "x + y \geq 2z"$ is a propositional function with variables x, y, z and R is the predicate.

Note. A propositional function can be multi-variable.

Predicates: Example

Some propositional functions:

- $P(x)$: "x is the best student in our class."
- $Q(x, y)$: "x loves y."
- $R(x, y, z)$: "$x^2 + y^2 + z^2 \leq 1$."

Question. Let $P(x, y)$: "$x^2 + y^2 \geq 3xy$". What is the truth values of the propositions $P(1, 2)$ and $P(3, 0)$?

Quantifiers $\forall, \exists$

Let $P(x)$ be a propositional function where x gets values in a particular domain.

1. **Universal quantification** $\forall x\ P(x) =$ $P(x)$ for all values of x in the domain.
2. **Existential quantificatio** $\exists x\ P(x) =$ There exists an element x in the domain such that $P(x)$.
3. **Uniquely existential quantification** $\exists! x\ P(x) =$ There exists uniquely an element x in the domain such that $P(x)$.

Example.

1. $\forall x\ x$ passes the MAD101 exam.
2. $\exists x\ x$ wins "Best Student Award".
3. $\exists! x\ x^3 = 1$.

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Truth value of propositions involving $\forall, \exists$

Statement	***When true?***	***When false?***
$\forall x\ P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x\ P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

Question. Let x represent a real number. Determine the truth value of the following propositions:

1. $\forall x\ ((x > 0) \to (x^2 \geq x))$
2. $\forall x\ ((x > 0) \land (x^2 \geq x))$
3. $\forall x\ ((x > 0) \lor (x^2 \geq x))$
4. $\exists x\ ((x > 0) \to (x^2 \geq x))$
5. $\exists x\ ((x > 0) \land (x^2 \geq x))$
6. $\exists x\ ((x > 0) \lor (x^2 \geq x))$

Negating Quantified Expressions

$$\neg\forall x\ P(x) \equiv \exists x\ \neg P(x) \qquad \neg\exists x\ P(x) \equiv \forall x\ \neg P(x)$$

Example. The negation of the statement $\forall x\ (x^2 > x)$ is

$$\neg\forall x\ (x^2 > x) \equiv \exists x\ \neg(x^2 > x) \equiv \exists x\ (x^2 \leq x)$$

Question. What is the negation of the statement $\exists x\ (x^2 = 1)$?

Question. Rewrite the expression $\neg\forall x\ (P(x) \rightarrow Q(x))$ so that the negation precedes the predicates.

Translating Sentences into Logical Expressions

Example. Express the statement "Every student in this class has studied English" using predicates and quantifiers.

Solution. First, we rewrite the statement so that we can clearly identify the appropriate quantifiers to use. Doing so, we obtain: "For every student in this class, that student has studied English."

Next, we introduce a variable x so that our statement becomes "For every student x in this class, x has studied English."

Continuing, we introduce $P(x)$, which is the statement "x has studied English." Consequently, if the domain for x consists of the students in the class, we can translate our statement as $\forall x\ P(x)$.

Question. Translating the following sentences into Logical Expression.

- "Each student of FPTU has visited Canada or Mexico".
- "Some students of FPTU have visited Canada or Mexico".

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Nested Quantifiers

- $\forall x \, \forall y \, P(x, y) =$ For all x and for all $y, P(x, y)$.
- $\forall x \, \exists y \, P(x, y) =$ For all x there is y such that $P(x, y)$.
- $\exists x \, \forall y \, P(x, y) =$ There exists x such that for all $y, P(x, y)$.
- $\exists x \, \exists y \, P(x, y) =$ There exist x and y such that $P(x, y)$.

Note. The order of the quantifiers is important!

Determine the truth values of the following propositions on set of real numbers.

1. $\forall x \, \forall y \, (x + y = 1)$
2. $\forall x \, \exists y \, (x + y = 1)$
3. $\exists x \, \forall y \, (x + y = 1)$
4. $\exists x \, \exists y \, (x + y = 1)$

Quantifications of Two Variables

Statement	**When true?**	**When false?**
$\forall x\ \forall y\ P(x,y)$ $\forall y\ \forall x\ P(x,y)$	$P(x,y)$ is true for every pair x,y.	There is a pair x,y for which $P(x,y)$ is false.
$\forall x\ \exists y\ P(x,y)$	For every x there is a y for which $P(x,y)$ is true.	There is an x such that $P(x,y)$ is false for every y.
$\exists x\ \forall y\ P(x,y)$	There is an x for which $P(x,y)$ is true for every y.	For every x there is a y for which $P(x,y)$ is false.
$\exists x\ \exists y\ P(x,y)$ $\exists y\ \exists x\ P(x,y)$	There is a pair x,y for which $P(x,y)$ is true.	$P(x,y)$ is false for every pair x,y.

Translate Logical Expressions into Sentences

Example.

$$\forall x \in \mathbb{R}\ \forall y \in \mathbb{R}\ [(x > 0) \wedge (y > 0) \rightarrow (x + y > 0)]$$

means "The sum of two positive real numbers is always positive".

Question. Translate the following Logical Expressions into Sentences:

- $\forall x \in \mathbb{R}\ \forall y \in \mathbb{R}\ [(x > 0) \wedge (y > 0) \rightarrow (xy > 0)]$.
- Let x, y represent students in a university, and

  $$C(x) = \text{"}x \text{ has a laptop"} \quad F(x, y) = \text{"}x \text{ and } y \text{ are friends"}.$$

  Translate the logical expressions:

  - $\forall x\ [C(x) \vee \exists y\ (C(y) \wedge F(x, y))]$.
  - $\exists x\ \forall y\ \forall z\ [(F(x, y) \wedge F(x, z) \wedge (y \neq z)) \rightarrow \neg F(y, z)]$.

Negating Nested Quantifiers

- $\neg(\forall x \forall y\ P(x,y)) \equiv \exists x \exists y\ \neg P(x,y)$
- $\neg(\forall x \exists y\ P(x,y)) \equiv \exists x \forall y\ \neg P(x,y)$
- $\neg(\exists x \forall y\ P(x,y)) \equiv \forall x \exists y\ \neg P(x,y)$
- $\neg(\exists x \exists y\ P(x,y)) \equiv \forall x \forall y\ \neg P(x,y)$

Example. Translate the following statements into logical expressions, then find the negation statements.

1. "For all real numbers x there is a real number y such that $x = y^3$".
2. "For all $\varepsilon > 0$, for all real numbers x there exists a rational number r such that $|r - x| < \varepsilon$".

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Valid argument

1. An argument is a sequence of statements that end with a **conclusion**. An argument is **valid** if the conclusion follows from the truth of the preceding statements (**premises** or **hypotheses**).
2. In propositional logic, an argument is valid if it is based on a tautology.
3. Arguments that are not based on tautology are called **fallacies**.

Rules of Inference

Rule	***Tautology***	***Name***
p $\underline{p \to q}$ $\therefore\ q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $\underline{p \to q}$ $\therefore\ \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $\underline{q \to r}$ $\therefore\ p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\underline{\neg p}$ $\therefore\ q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism

Rules of Inference (2)

Rule	Tautology	Name
$\begin{array}{rl} & p \\ \therefore & \overline{p \vee q} \end{array}$	$p \rightarrow (p \vee q)$	Addition
$\begin{array}{rl} & \underline{p \wedge q} \\ \therefore & p \end{array}$	$(p \wedge q) \rightarrow p$	Simplification
$\begin{array}{rl} & p \\ & q \\ \therefore & \overline{p \wedge q} \end{array}$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$\begin{array}{rl} & p \vee q \\ & \underline{\neg p \vee r} \\ \therefore & q \vee r \end{array}$	$((p \vee q) \wedge \neg p) \rightarrow q$	Resolution

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Example. "It is below freezing now. Therefore, it is either below freezing or raining now."

Solution. Let p be the proposition It is below freezing now and q the proposition It is raining now. Then this argument is of the form

$$\begin{array}{l} \quad p \\ \therefore \overline{p \vee q} \end{array}$$

This is an argument that uses the addition rule.

Example. "It is below freezing and raining now. Therefore, it is below freezing now."

Solution. Let p be the proposition It is below freezing now, and q be the proposition It is raining now. This argument is of the form

$$\begin{array}{l} \quad \underline{p \wedge q} \\ \therefore p \end{array}$$

This argument uses the simplification rule.

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Questions

1. State which rule of inference is used in the argument: "If it rains today, then we will not have a barbecue today. If we do not have a barbecue today, then we will have a barbecue tomorrow. Therefore, if it rains today, then we will have a barbecue tomorrow."

2. Given the hypotheses:
   - "It is not sunny and is cold."
   - "We go swimming only if it is sunny."
   - "If we do not go swimming then we will play soccer."
   - "If we play soccer then we will go home by sunset."

   Show that these hypotheses lead to the conclusion: "We will go home by sunset".

3. Given the hypotheses:
   - "If you send me an email, I will finish writing the program."
   - "If you do not send email then I will go to bed early."
   - "If I go to bed early then I will go jogging tomorrow morning."

   Show that these hypotheses lead to the conclusion: "If I do not finish writing the program then I will go jogging tomorrow morning".

Some Fallacies

Caution!

1. Fallacy of affirming the conclusion: $[(p \to q) \land q \to p]$
2. Fallacy of denying the hypothesis: $[(p \to q) \land \neg p \to \neg q]$

Example. Is the following argument valid?
"If you do every problem in this book, then you will learn Discrete Mathematics.You learned Discrete Mathematics. Therefore, you did every problem in this book."

Solution. Let p be the proposition You did every problem in this book and q be the proposition You learned Discrete Mathematics. Then this argument is of the form:

$$[(p \to q) \land q \to p].$$

This is an example of an incorrect argument using the fallacy of affirming the conclusion. Indeed, it is possible for you to learn Discrete Mathematics in some way other than by doing every problem in this book.

Fallacies: Extra example

Is the following argument valid?

"You did not learn discrete mathematics if you did not do every problem in the book, assuming that if you do every problem in this book, then you will learn Discrete Mathematics".

Rules of Inference for Quantified Statements

Name	***Rule of Inference***
Universal instantiation	$\begin{array}{rl} & \forall x\ P(x) \\ \hline \therefore & P(c), c \text{ is arbitrary} \end{array}$
Universal generalization	$\begin{array}{rl} & P(c), c \text{ is arbitrary} \\ \hline \therefore & \forall x\ P(x) \end{array}$
Existential instantiation	$\begin{array}{rl} & \exists x\ P(x) \\ \hline \therefore & P(c), \text{ for some } c \end{array}$
Existential generalization	$\begin{array}{rl} & P(c), \text{ for some } c \\ \hline \therefore & \exists x\ P(x) \end{array}$

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Questions

Example. Show that the premises "Everyone in this discrete mathematics class has taken a course in computer science" and "Marla is a student in this class" imply the conclusion "Marla has taken a course in computer science."

Solution. Let $D(x)$ denote x is in this discrete mathematics class, and $C(x)$ denote x has taken a course in computer science. Then the premises are $\forall x\ [D(x) \to C(x)]$ and $D(\text{Marla})$. The conclusion is $C(\text{Marla})$.

1. Given the hypotheses:
   - Each student of SE0000 must take Discrete Maths,
   - Jenifer is a student of SE0000.

   Show that these hypotheses lead to the conclusion Jenifer must take Discrete Math.
2. Given the hypotheses:
   - Some student of SE has not read this book,
   - Every student of SE passed the exam.

   Show that these hypotheses lead to the conclusion Some student of SE who passed the exam has not read this book.

Thank you!

Call it a day

"Pure logical thinking cannot yield us any knowledge of the empirical world. All knowledge of reality starts from experience and ends in it."

ALBERT EINSTEIN

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