Assignment 2

Course: MAD101

Deadline: *31 October, 2023*

3.1 Describe an algorithm that takes as input a list of n integers and finds the location of the last even integer in the list or returns 0 if there are no even integers in the list.

3.2 Use the definition of $f(x)$ is $O(g(x))$ to show that $2^x + 17$ is $O(3^x)$.

3.3 Find the least integer n such that $f(x)$ is $O(x^n)$ for the function

$$f(x) = \frac{x^4 + x^2 + 1}{x^2 + 1}.$$

3.4 Give a big-O estimate for the number of operations (where an operation is an addition or a multiplication) used in this segment of an algorithm.

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t := 0
for i := 1 to 3
     for j := 1 to 4
       t := t + ij
```

4.1 Find the integer a such that

$$a \equiv 13 \mod 31 \text{ and } -150 \leq a \leq 150.$$

4.2 Find the value of the following expression:

$$(99^2 \mod 32)^2 \mod 15.$$

4.3 Convert $(1100001100011)_2$ from its binary expansion to its hexadecimal expansion.

4.4 Find the sum and product of each of the folowing pair of num- bers. Express your answers as a base 3 expansion.

$$(112)_3 \text{ and } (210)_3$$

4.5 Decrypt the message

$$LO\ WI\ PBSOXN$$

encrypted using the shift cipher $f(p) = (p + 10) \mod 26$.

5.1 Prove that for every positive integer n,

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{n}} > 2(\sqrt{n+1} - 1).$$

5.2 Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. Show that

$$A^n = \begin{bmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{bmatrix}$$

when n is a positive integer and f_n is the $n-$th term of the Fibonacci numbers.

6.1 There are 18 mathematics majors and 32 computer science majors at a falculty.

a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?

b) In how many ways can one representative be picked who is either a mathematics major or a computer science major?

6.2 How many positive integers between 1000 and 9999 inclusive are not divisible by 3 or 5 or 7?

Remark. *Students must complete the assignment on paper and do as much as possible.*