Assignment 1

Course: MAD101
Deadline: ***16 October***, *2023*

1.1 Let p be the proposition I will do every exercise in this book and q be the proposition I will get an A in this course. Express each of these as a combination of p and q.

a) I will get an A in this course only if I do every exercise in this book.

b) I will get an A in this course and I will do every exercise in this book.

c) Either I will not get an A in this course or I will not do every exercise in this book.

d) For me to get an A in this course it is necessary and sufficient that I do every exercise in this book.

1.2 Find the truth table of the compound proposition

$$p \vee q \to p \wedge \neg r.$$

1.3 Show that these compound propositions are tautologies.

a) $(\neg q \wedge (p \to q)) \to \neg p$.

b) $((p \vee q) \wedge \neg p) \to q$.

1.4 Let $P(x)$ be the statement Student x knows calculus and let $Q(y)$ be the statement Class y contains a student who knows calculus. Express each of these as quantifications of $P(x)$ and $Q(y)$.

a) Some students know calculus.

b) Not every student knows calculus.

c) Every class has a student in it who knows calculus.

d) Every student in every class knows calculus.

1.5 Determine the truth value of each of these statements if the domain consists of all real numbers.

a) $\exists x, (x^3 = 1))$

b) $\exists x, (x^4 < x^2$

c) $\forall x\,((-x)2 = x2)$

d) $\forall x\,(2x > x)$

1.6 Express each of these statements using mathematical and logical operators, pred- icates, and quantifiers, where the domain consists of all integers.

a) The sum of two negative integers is negative.

b) The difference of two positive integers is not necessarily positive.

c) The sum of the squares of two integers is greater than or equal to the square of their sum.

2.1 Let A be the set of English words that contain the letter x, and let B be the set of English words that contain the letter q. Express each of these sets as a combination of A and B.

a) The set of English words that do not contain the letter x.

b) The set of English words that contain both an x and a q.

c) The set of English words that contain an x but not a q.

2.2 Let A and B be sets. Show that $A \subseteq B$ if and only if $A \cap B = A$.

2.3 Let f and g be functions from $\{1, 2, 3, 4\}$ to $\{a, b, c, d\}$ and from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$, respectively, with $f(1) = d, f(2) = c, f(3) = a$, and $f(4) = b$, and $g(a) = 2, g(b) = 1, g(c) = 3$, and $g(d) = 2$.

a) Is f one-to-one? Is g one-to-one?

b) Is f onto? Is g onto?

c) Does either f or g have an inverse? If so, find this inverse.

2.4 Find the value of each of these sums.

a) $\sum_{i=0}^{2023} (1 + (-1)^i)$

b) $\sum_{i=0}^{5} \sum_{j=0}^{5} (i + j)$

c) $\sum_{i=1}^{10} \frac{1}{i(i+1)(i+2)}$

2.5 Given the sequence $(a_n)_{n=1}^{\infty}$

$$1, 3, 3, 3, 5, 5, 5, 5, 5, \ldots$$

constructed by including the integer $2k + 1$ exactly $2k + 1$ times. Show that

$$a_n = 2\lceil \sqrt{n} \rceil - 1$$

for all $n \geq 1$.

2.6 Let S, T be two sets defined by

$$S = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \leq 100\}$$

and

$$T = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x - 1)^2 + (y + 2)^2 \leq 100\}.$$

Which one is larger, $|S|$ or $|T|$?

Remark. *Students must complete the assignment on paper and do as much as possible.*