Homework 6: Logistic Regression

Trần Minh Khoa

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1.

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

$$\Rightarrow \sigma'(x) = \frac{e^{-x}}{(1+e^{-x})^2} = \frac{\frac{1-\sigma(x)}{\sigma(x)}}{\frac{1}{\sigma(x)^2}} = \sigma(x)(1-\sigma(x))$$

$p(C_1|\phi) = y(\phi) = \sigma(w^T\phi)$

$p(C_2|\phi) = 1 - p(C_1|\phi)$

Likelihood function:

$$p(t|w) = \prod_{n=1}^{N} y_n^{t_n}(1-y_n)^{1-t_n}$$

Maximize Likelihood $\Leftrightarrow$ Minimize Cross-entropy function:

$$L = -log\ p(t|w) = -\sum_{n=1}^{N}\Big(t_n\ log\ y_n + (1-t_n)\ log(1-y_n)\Big)$$

$$y = \sigma(z)\ ;\ z = w_0 + w_1\phi_1 + ... + w_n\phi_n$$

$$L = -\big(t\ log\ y + (1-t)\ log(1-y)\big)$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial y}.\frac{\partial y}{\partial z}.\frac{\partial z}{\partial w_1}$$

$$= -\left(\frac{t}{y} - \frac{1-t}{1-y}\right).y(1-y).\phi_1$$

$$= -\left(\frac{t(1-y) - y(1-t)}{y(1-y)}\right).y(1-y).\phi_1$$

$$= (y-t).\phi_1$$

$$\frac{\partial L}{\partial w_0} = y - t$$

$$\Rightarrow \frac{\partial L}{\partial w} = \begin{bmatrix} (y-t).1 \\ (y-t).\phi_1 \\ ... \\ (y-t).\phi_n \end{bmatrix} = (y-t).\phi$$

2.

$$f'(x) = f(x)(1-f(x))$$

$$\frac{df(x)}{dx} = f(x)(1-f(x))$$

$$\frac{df(x)}{f(x)(1-f(x))} = dx$$

$$\int \frac{df(x)}{f(x)(1-f(x))} = \int 1 \ dx$$

$$\int \left(\frac{1}{f(x)} + \frac{1}{1-f(x)}\right) df(x) = x + C$$

$$\int \frac{1}{f(x)} \ df(x) + \int \frac{1}{1-f(x)} \ df(x) = x + C$$

$$ln(f(x)) - ln(1-f(x)) = x + C$$

$$\frac{f(x)}{1-f(x)} = e^{x+C}$$

$$\frac{1-f(x)}{f(x)} = \frac{1}{e^{x+C}}$$

$$\frac{1}{f(x)} - 1 = \frac{1}{e^{x+C}}$$

$$\frac{1}{f(x)} = \frac{1+e^{x+C}}{e^{x+C}}$$

$$f(x) = \frac{e^{x+C}}{1+e^{x+C}} = \frac{1}{\frac{1}{e^{x+C}}+1} = \frac{1}{1+e^{-x-C}} = \frac{1}{1+e^{-x+C}}$$