Homework 1: Probability

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September 15, 2021

1.
Events:
A - have disease
B - test positive

P(A) = P(have disease) = 0.05
P($\bar{A}$) = P(not have disease) = 0.95
P(B|A) = P(test positive|have disease) = 0.98
P(B|$\bar{A}$) = P(test positive|not have disease) = 0.03
P(B) = P(test positive) = $0.98 \times 0.05 + 0.03 \times 0.95 = 0.0775$
P(A|B) = P(have disease|test positive) = $\frac{P(B|A)P(A)}{P(B)} = \frac{0.98 \times 0.05}{0.0775}$
$= 0.632$

2.1. **Univariate Normal Distribution**

PDF: $f(x) = \frac{1}{\sqrt{2\pi}\sigma}\ e^{\frac{-(x-\mu)^2}{2\sigma^2}}$
Z = $\frac{X-\mu}{\sigma}$
E[X] = $\int_{-\infty}^{\infty} xf(x)\,dx$
E[Z] = $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} te^{\frac{-t^2}{2}}\,dt$
$= -\frac{1}{2\pi}\ e^{\frac{-t^2}{2}}|_{-\infty}^{\infty}$
$= 0$
E[X] = μ+ σE[Z] = μ

Var(Z) = E[Z^2] - $E[Z]^2$ = E[Z^2] - 0 = E[Z^2]
$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} t^2 e^{\frac{-t^2}{2}}\,dt$
$= \frac{1}{\sqrt{2\pi}}(-te^{\frac{-x^2}{2}}|_{-\infty}^{\infty} + \int_{-\infty}^{\infty} e^{\frac{-t^2}{2}}\,dt)$
$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{\frac{-t^2}{2}}\,dt$
$= 1$
Var(X) = σ^2Var(Z) = σ^2