Exam 1 Review

- Exam through MyLab
- Available 10 minutes before (11:50 am) and 5 minutes after (1:20 pm)
- 6 questions (4 normal + 2 multiple choice)
- 1 attempt
- Open notes & browser

  No outside help from other people, tutors, websites

* If you would like to upload work, you can do it in Canvas under the assignment "Exam 1"

  ↳ This is optional, but it can be useful if you're petitioning your score.

▷ There will be a Zoom meeting for the exam to answer questions, but it's <u>optional</u>.

Material (Sections 1.1-1.3, 2.1-2.3)

$\Rightarrow$ HW #1, #2, #3

(Not HW #4!)

1) ODE Classification

- Ordinary vs. partial
- Order of the ODE
- Linear vs. nonlinear
- Separable ODEs
- Dependent vs. Independent Variables

2) Direction Fields & Isoclines

3) Quantitative Analysis

- Critical Points
- long-time limits $\left(\lim_{t\to\infty} x(t)\right)$
- Stablity

4) Solving Separable ODEs

5) Solving Linear ODEs
(using Integrating Factors)

Direction Fields

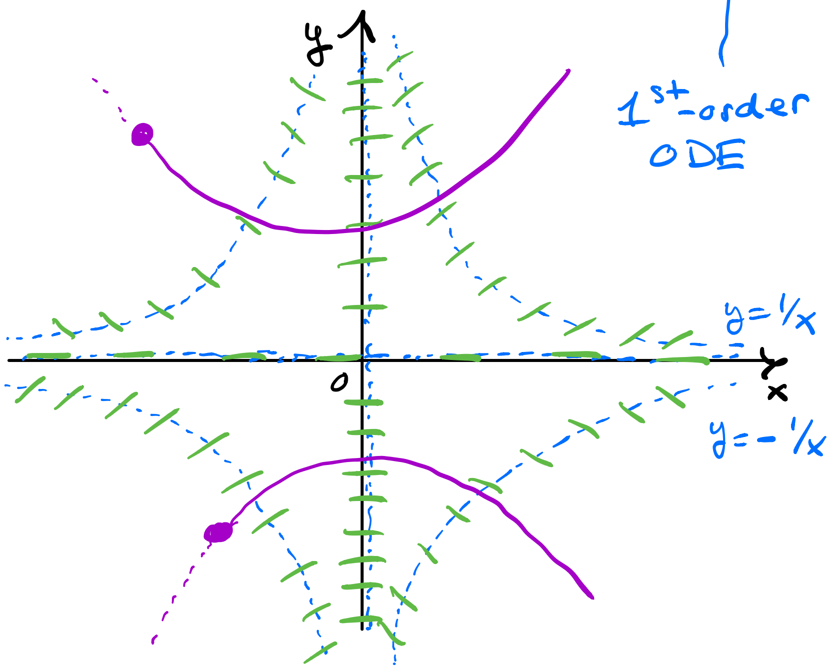
Dependent variable $y(x)$ shows up in powers of 1
$\Rightarrow$ Linear

$$\frac{dy}{dx} = xy$$

1^{st}-order ODE

can be written as $g(x)p(y)$
- $g(x) = x$
- $p(y) = y$

$\Rightarrow$ Separable



Isoclines are curves of constant slope.

Ex. Set $\frac{dy}{dx} = 1 \Rightarrow 1 = xy \Rightarrow y = 1/x$

Set $\frac{dy}{dx} = -1 \Rightarrow -1 = xy \Rightarrow y = -1/x$

Set $\frac{dy}{dx} = 0 \Rightarrow 0 = xy \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$

Critical Points

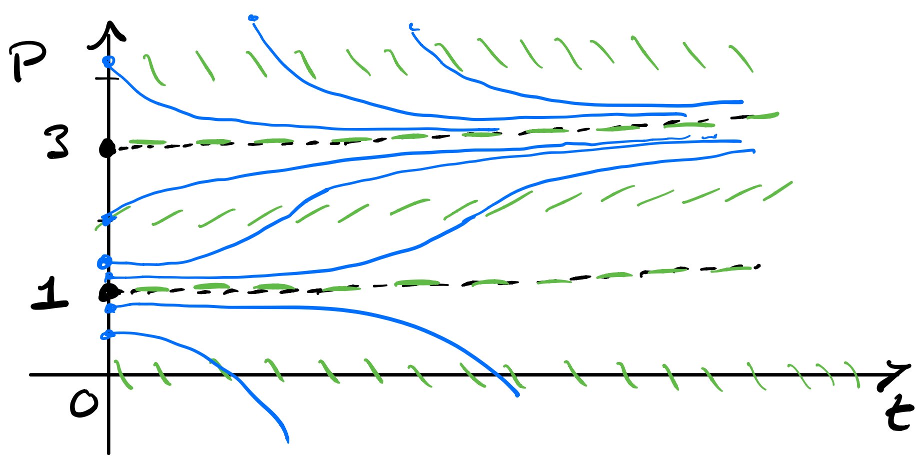
Example: $\dfrac{dP}{dt} = -3 + 4P - P^2 = f(P)$

1) Find critical points: $\dfrac{dP}{dt} = 0$

Set $0 = -3 + 4P - P^2$

$$P = \frac{-4 \pm \sqrt{16-12}}{-2} = \frac{-4 \pm \sqrt{4}}{-2}$$

$$= \frac{-4 \pm 2}{-2} = \{1, 3\}$$



$P=0$: $\dfrac{dP}{dt} = -3 + 4 \cdot 0 - 0^2 = -3$

$P=2$: $\dfrac{dP}{dt} = -3 + 4 \cdot 2 - 2^2 = 1$

$P=4$: $\dfrac{dP}{dt} = -3 + 4 \cdot 4 - 4^2 = -3$

If $P(0)=2$, what happens as

$t \to \infty$? $\Rightarrow$ Increase and level off at $P=3$

If $P(0)=1.5$, will $P(t)$ get to 4?

$\Rightarrow$ No, since solutions can't cross (i.e., "existence and uniqueness")

Separable ODEs

$$\frac{dy}{dx} = \underbrace{\left(y+\frac{1}{y}\right)}_{p(y)}\underbrace{\left(x+\frac{1}{x}\right)}_{g(x)}, \quad y(1)=0$$

$$\Rightarrow \frac{1}{y+1/y}\,dy = \left(x+1/x\right)dx$$

$$\frac{1}{y+1/y} = \frac{y}{y}\,\frac{1}{y+1/y} = \frac{y}{y^2+1}$$

$$\Rightarrow \int \frac{y}{y^2+1}\,dy = \int \left(x+\frac{1}{x}\right)dx$$

Let $u = y^2+1$

$du = 2y\,dy$

$\Rightarrow \frac{1}{2}du = y\,dy$

$$\int \frac{y}{y^2+1}\,dy = \int \frac{1}{2}\,\frac{1}{u}\,du = \frac{1}{2}\ln u$$

$$= \frac{1}{2}\ln|y^2+1|$$

OR use WolframAlpha

$$\Rightarrow \frac{1}{2}\ln|y^2+1| = \frac{1}{2}x^2 + \ln|x| + C$$

Implicit Form:

$$\boxed{\frac{1}{2}\ln|y^2+1| - \frac{1}{2}x^2 - \ln|x| = C}$$

Apply Initial Condition: $y(1) = 0$

$$\Rightarrow \frac{1}{2}\ln|0^2+1| - \frac{1}{2}1^2 - \ln|1| = C$$

$$0 - \frac{1}{2} - 0 = C$$

$$\Rightarrow C = -1/2$$

$$\boxed{\frac{1}{2}\ln|1+y^2| - \frac{1}{2}x^2 - \ln|x| = -1/2}$$

Linear ODEs

$$x\frac{dy}{dx} = e^x - 2y, \quad y(1) = 7$$

A) Put in form: $\frac{dy}{dx} + p(x)y = g(x)$

$$\Rightarrow x\frac{dy}{dx} + 2y = e^x$$

$$\Rightarrow \frac{dy}{dx} + \frac{2}{x}y = \frac{1}{x}e^x$$

$$\begin{cases} p(x) = 2/x \\ g(x) = \frac{1}{x}e^x \end{cases}$$

B) Find Integrating Factor

$$\mu(x) = e^{\int p(x)dx} = e^{\int \frac{2}{x}dx} = e^{2\ln x}$$

$$= e^{\ln x^2} = \underline{\underline{x^2}}$$

c) Either multiply through, rewrite and integrate, OR use this formula:

$$y(x) = \frac{1}{\mu(x)} \int \mu(x) g(x) dx$$

$$= \frac{1}{x^2} \int x^2 \left(\frac{1}{x} e^x\right) dx$$

$$= \frac{1}{x^2} \int x e^x dx$$

Integration by parts

$$= \frac{1}{x^2} \left(x e^x - e^x + c \right)$$

$$= \frac{1}{x} e^x - \frac{1}{x^2} e^x + \frac{c}{x^2}$$

Finally, apply the initial condition

$$y(1) = 7$$

$$\Rightarrow \quad 7 = \cancel{\frac{1}{1}e^{1}} - \cancel{\frac{1}{1^2}e^{1}} + \frac{c}{1^2} = c$$

$$\Rightarrow \quad c = 7$$

$$\boxed{y(x) = \frac{1}{x}e^{x} - \frac{1}{x^2}e^{x} + \frac{7}{x^2}}$$