

## HW5

The deadline for submission is 11:59:59PM Tuesday, November 30.

## 0.1 Instructions

- Submit your assignment via the submit server, at <http://submit.cs.umd>.
- Please also **turn in a hardcopy** of your write-up. You may either turn it in during class, or slip it under Prof. Getoor's office door. The hardcopy should exactly match the electronic PDF submitted, and should be turned in within 12 hours of the electronic submission.
- Your electronic submission should be a single .PDF file, named `hw5_<last_name>_<first_name>.pdf`.
- Do *not* submit the .tex file.
- You may use LaTeX or Word to create your write-up, OR you may scan in a **clearly legible** handwritten write-up but **you must submit a single file in PDF format**.
- For your convenience, you can use the included .tex file. To do so,
  1. Edit the header information with your own personal information. Be sure to include your name and email address.
  2. Uncomment the `\begin{solution} ... \end{solution}` blocks and fill in your solution.  
(Note: you don't *need* to use the solution blocks if you find the formatting too restrictive. If you find that your solution spans multiple pages, you can break the solution block into multiple parts. Also, feel free to use `\begin{proof} ... \end{proof}` for proofs.)
- To add an image to this tex file, use the command `\includegraphics{filename}` (search the L<sup>A</sup>T<sub>E</sub>X documentation for additional arguments).
- Note: this homework is **optional**. You will not be penalized for not completing it. If you do complete it, then we will drop your lowest homework grade, and your homework grade will become the average of your four highest homeworks.

## 1 Calculating Conditional Independence from a Distribution

Exercise 8.3 from Bishop book.

## 2 Testing Conditional Independence from a DAG

Suppose we have the following DAG  $G$ . Which of the following independence statements are true with respect to any distribution for which  $G$  is an I-Map?

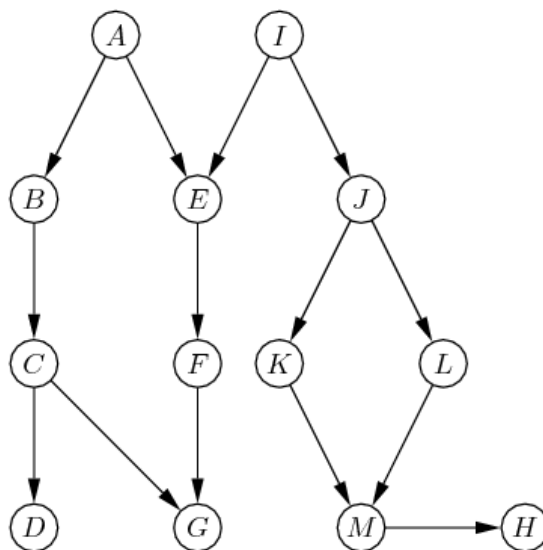


Figure 1: The DAG for a toy Bayes net

- (a)  $I(D, H \mid \emptyset)$
- (b)  $I(A, I \mid \emptyset)$
- (c)  $I(A, I \mid G)$
- (d)  $I(J, G \mid F)$
- (e)  $I(J, M \mid \{K, L\})$
- (f)  $I(E, C \mid \{A, G\})$
- (g)  $I(E, C \mid A)$

## 3 Variable Elimination

The Bayes net shown in (Figure 3) describes (a somewhat simplified model for) your chances of achieving fame upon completion of your CMSC726 class project. The variables of interest are: attending machine learning class ( $A$ ), your mastery of machine learning ( $M$ ), your time-management skills ( $T$ ), being late to

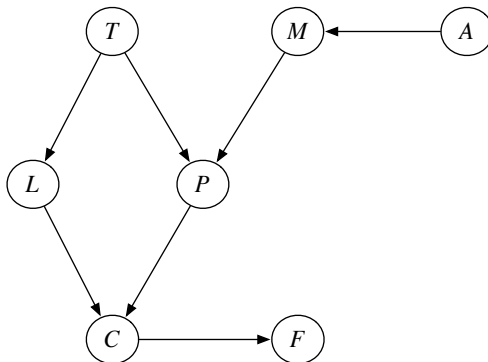


Figure 2: A Bayes net for modeling fame and fortune

the airport ( $L$ ), writing a good paper ( $P$ ), making it to the conference ( $C$ ), and achieving fame ( $F$ ). All of these variables are binary valued  $\{\text{True}, \text{False}\}$ . The conditional probability tables are:

$$\begin{aligned}
 \Pr(A = \text{True}) &= 0.7 & \Pr(T = \text{True}) &= 0.6 \\
 \Pr(M = \text{True} | A = \text{True}) &= 0.9, & \Pr(M = \text{True} | A = \text{False}) &= 0.3 \\
 \Pr(L = \text{True} | T = \text{True}) &= 0.1, & \Pr(L = \text{True} | T = \text{False}) &= 0.4 \\
 \Pr(F = \text{True} | C = \text{True}) &= 0.5, & \Pr(F = \text{True} | C = \text{False}) &= 0.1
 \end{aligned}$$

$$\begin{aligned}
 \Pr(P = \text{True} | T = \text{True}, M = \text{True}) &= 0.9, & \Pr(P = \text{True} | T = \text{True}, M = \text{False}) &= 0.5 \\
 \Pr(P = \text{True} | T = \text{False}, M = \text{True}) &= 0.7, & \Pr(P = \text{True} | T = \text{False}, M = \text{False}) &= 0.2
 \end{aligned}$$

$$\begin{aligned}
 \Pr(C = \text{True} | L = \text{True}, P = \text{True}) &= 0.2, & \Pr(C = \text{True} | L = \text{True}, P = \text{False}) &= 0.1 \\
 \Pr(C = \text{True} | L = \text{False}, P = \text{True}) &= 0.9, & \Pr(C = \text{True} | L = \text{False}, P = \text{False}) &= 0.2
 \end{aligned}$$

### 3.1 Joint Probability

**Exercise 3a:** Write down the formula for the joint probability distribution that makes the same conditional independence assumptions as the graph in Figure 3.

### 3.2 Variable ordering in Variable Elimination

**Exercise 3b:** Efficient variable elimination depends on the order in which we eliminate variables. Suppose we use the following elimination order  $\{P, M, C, L, T, A\}$  where we are trying to compute  $\Pr(F)$ . What is the size of the largest intermediate factor? Can you find a better ordering?

### 3.3 Variable Elimination for Fame

**Exercise 3c:** Using variable elimination, compute the probability of being famous ( $F$ ) given that you attended ( $A$ ) machine learning class.

$$\Pr[F = \text{True} | A = \text{True}] = ?$$

Please show your work, clearly indicating the elimination order that you choose and the calculation of the factors at each step.

### 3.4 Most Likely Explanation for Your Future

**Exercise 3d, Extra Credit:** We don't know your future but the graphical model does. Suppose you attend class and have good time management skills, what does your future hold? Specifically, will you be famous?