From Structured Prediction *to* Inverse Reinforcement Learning

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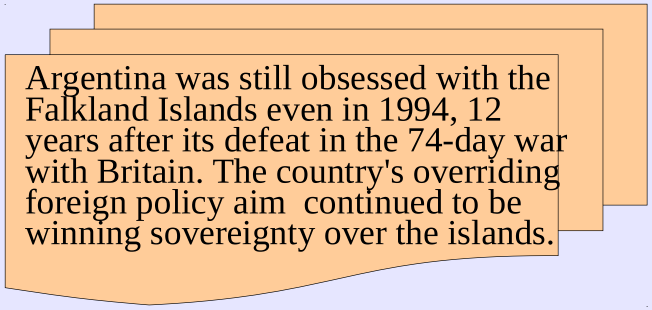
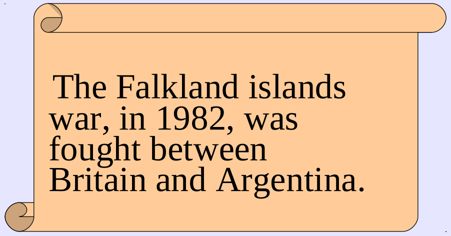
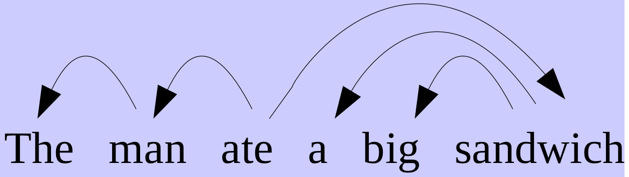
Nathan Ratliff

Discussions/Feedback:
MLRG Spring 2010

NLP as transduction

Task	Input	Output
Machine Translation	Ces deux principes se tiennent à la croisée de la philosophie, de la politique, de l’économie, de la sociologie et du droit.	Both principles lie at the crossroads of philosophy, politics, economics, sociology, and law.
Document Summarization	 Argentina was still obsessed with the Falkland Islands even in 1994, 12 years after its defeat in the 74-day war with Britain. The country's overriding foreign policy aim continued to be winning sovereignty over the islands.	 The Falkland islands war, in 1982, was fought between Britain and Argentina.
Syntactic Analysis	The man ate a big sandwich.	 The man ate a big sandwich
...many more...		

Structured prediction 101

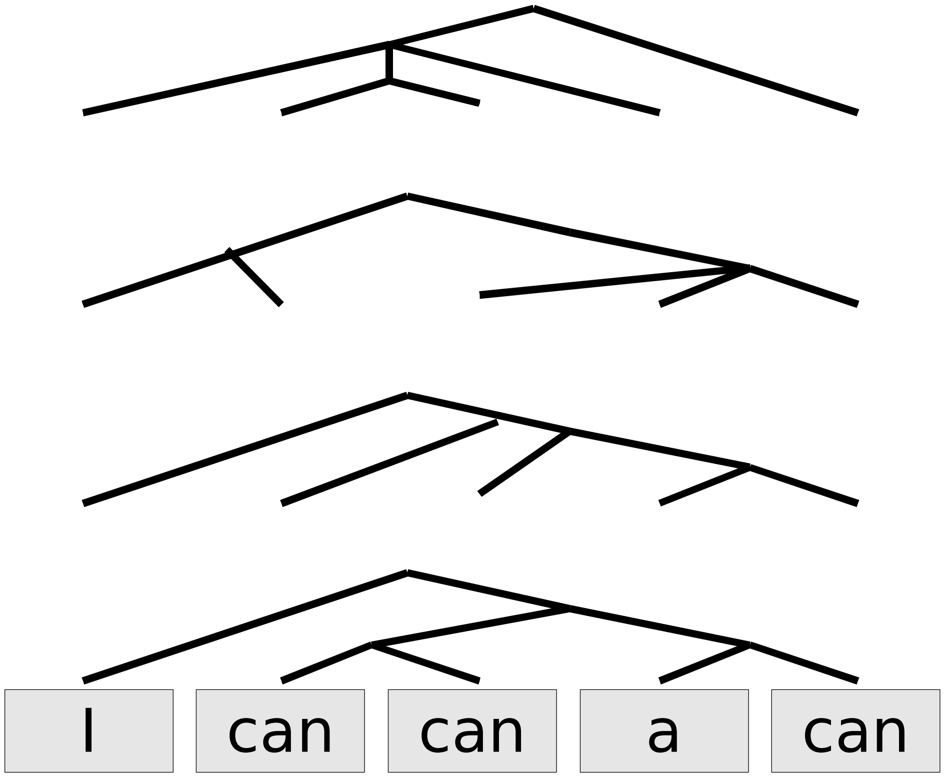
Learn a function mapping inputs to complex outputs:

$$f : X \rightarrow Y$$

Input Space

Decoding

Output Space



I can can a can

Why is structure important?

- Correlations among outputs
  - Determiners often precede nouns
  - Sentences usually have verbs
- Global coherence
  - It just *doesn't make sense* to have three determiners next to each other
- My objective (aka “loss function”) forces it
  - Translations should have good sequences of words
  - Summaries should be coherent

Outline: Part I

- What is Structured Prediction?
- Refresher on Binary Classification
  - What does it mean to learn?
  - Linear models for classification
  - Batch versus stochastic optimization
- From Perceptron to Structured Perceptron
  - Linear models for Structured Prediction
  - The “argmax” problem
  - From Perceptron to margins
- Learning to Search
  - Incremental Parsing
  - Search-based Structured Prediction

Outline: Part II

- Refresher on Reinforcement Learning
  - Markov Decision Processes
  - Q learning
- Apprenticeship Learning
  - Inverse RL
  - Apprenticeship Learning via IRL
- Inverse Optimal Control and A* Search
  - Maximum Margin Planning
  - Learning to Search
- Discussion

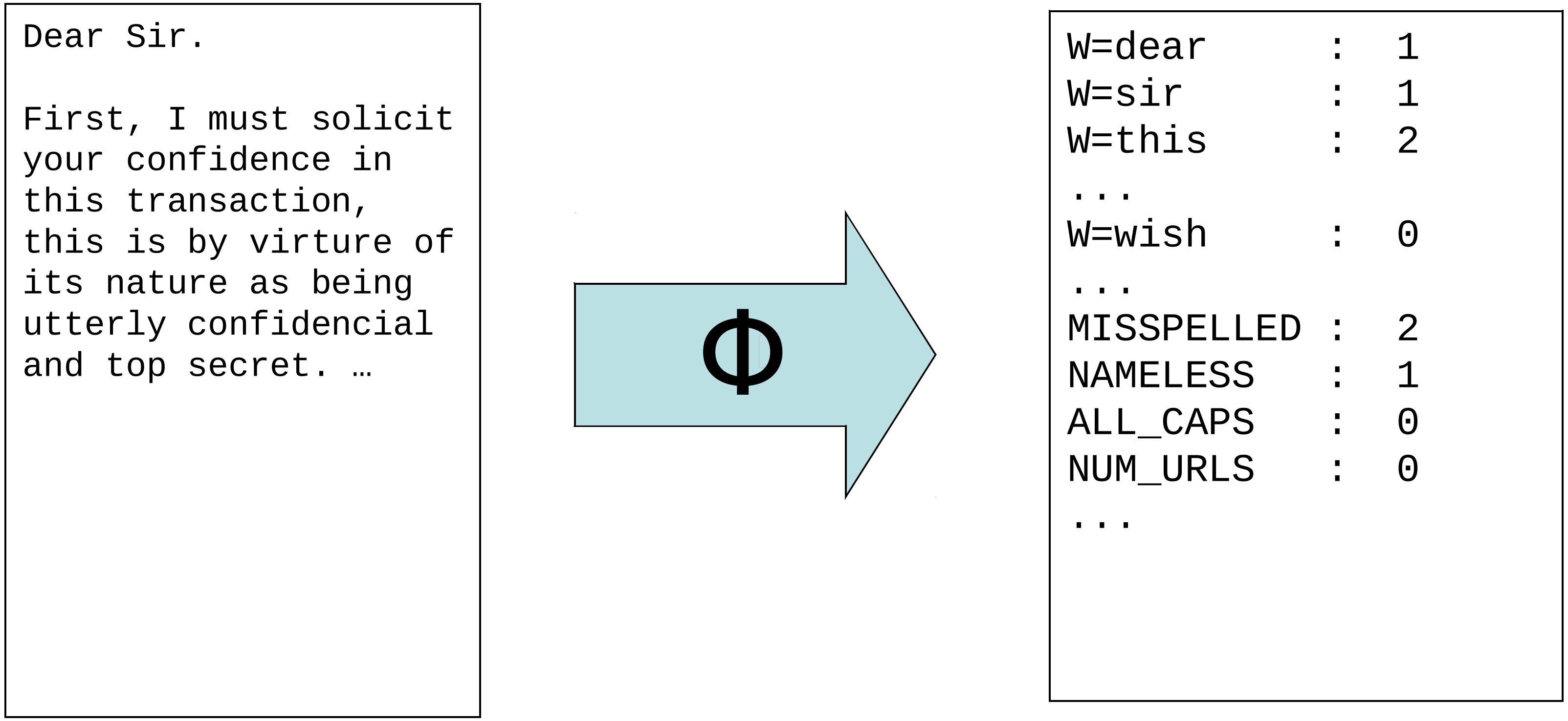
Refresher on Binary Classification

What does it mean to learn?

- Informally:
  - to predict the future based on the past
- Slightly-less-informally:
  - to take *labeled examples* and construct a function that will label them as a human would
- Formally:
  - Given:
    - A fixed unknown distribution D over X*Y
    - A loss function over Y*Y
    - A finite sample of (x,y) pairs drawn i.i.d. from D
  - Construct a function f that has low expected loss with respect to D

Feature extractors

- A feature extractor Φ maps examples to vectors



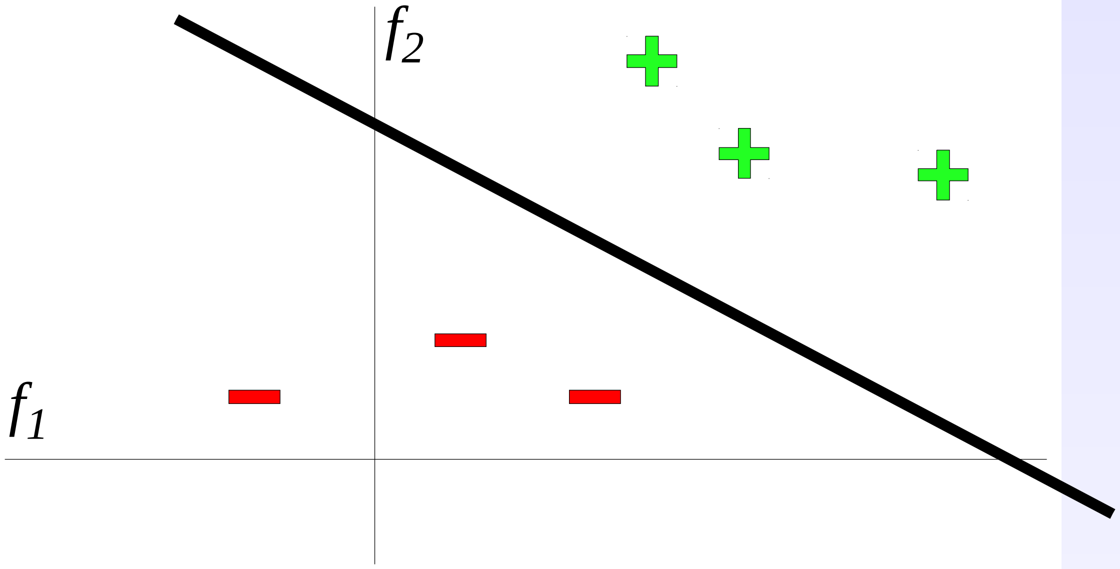
```
Dear Sir.

First, I must solicit
your confidence in
this transaction,
this is by virture of
its nature as being
utterly confidencial
and top secret. …
```

```
W=dear      :  1
W=sir       :  1
W=this      :  2
...
W=wish      :  0
...
MISSPELLED  :  2
NAMELESS    :  1
ALL_CAPS    :  0
NUM_URLS    :  0
...
```

- Feature vectors in NLP are frequently sparse

Linear models for binary classification

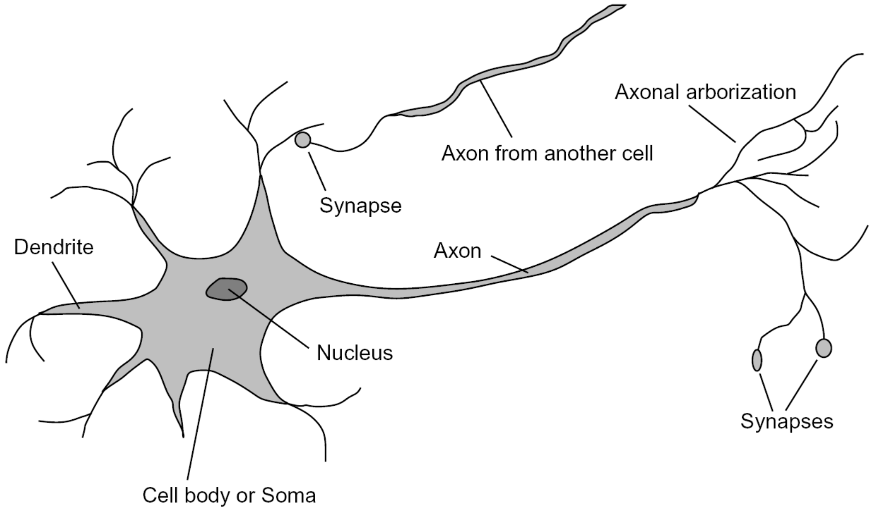
- Decision boundary is the set of "uncertain"points
- Linear decision boundaries are characterized by weight vectors



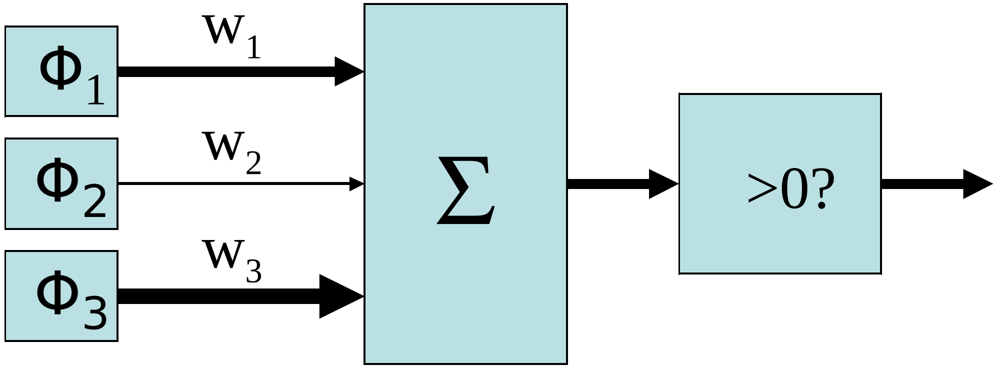
x	$\Phi(x)$	w	$\sum_i w_i \Phi_i(x)$
"free money"	BIAS : 1 free : 1 money : 1 the : 0 ...	BIAS : -3 free : 4 money : 2 the : 0 ...	[redacted]

The perceptron

- Inputs = feature values
- Params = weights
- Sum is the response



- If the response is:
  - Positive, output +1
  - Negative, output -1



- When training, update on errors:

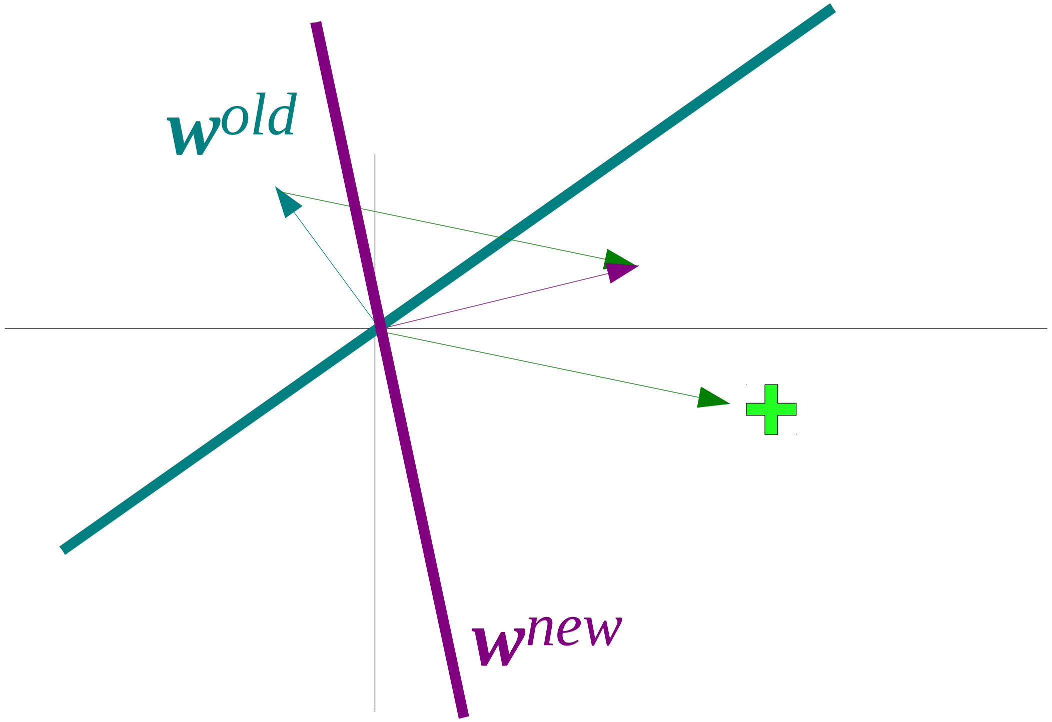
$$\mathbf{w} = \mathbf{w} + y\,\phi(x)$$

“Error” when:

$$y\,\mathbf{w}\cdot\phi(x) \le 0$$

Why does that update work?

- When $y\,\mathbf{w}^{old}\cdot\phi(x)\leq 0$, update: $\mathbf{w}^{new}=\mathbf{w}^{old}+y\,\phi(x)$

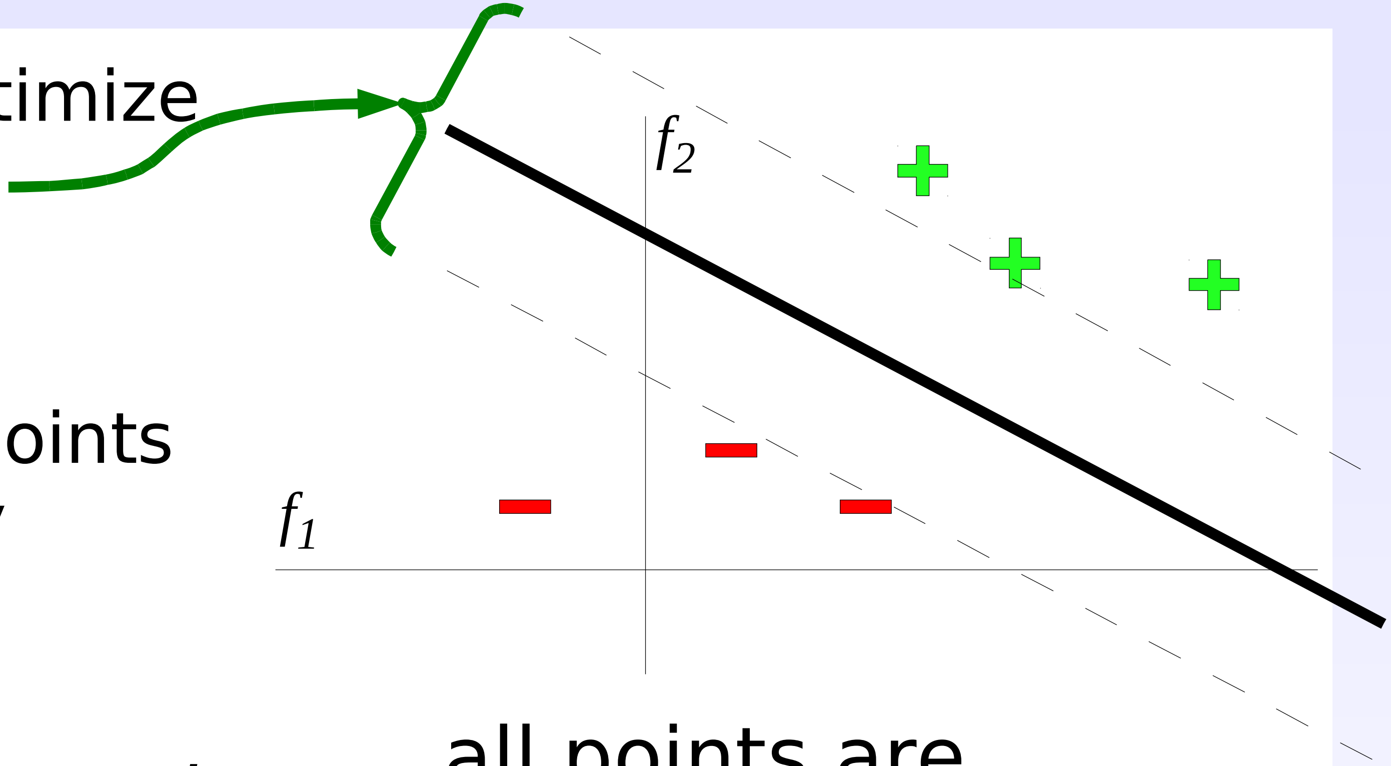


$$y\,\mathbf{w}^{new}\phi(x)=y\left(\mathbf{w}^{old}+y\,\phi(x)\right)\phi(x)$$

$$=\underbrace{y\,\mathbf{w}^{old}\phi(x)}_{<0}+\underbrace{yy\,\phi(x)\phi(x)}_{>0}$$

Support vector machines

- Explicitly optimize the ***margin***
- Enforce that all training points are correctly classified



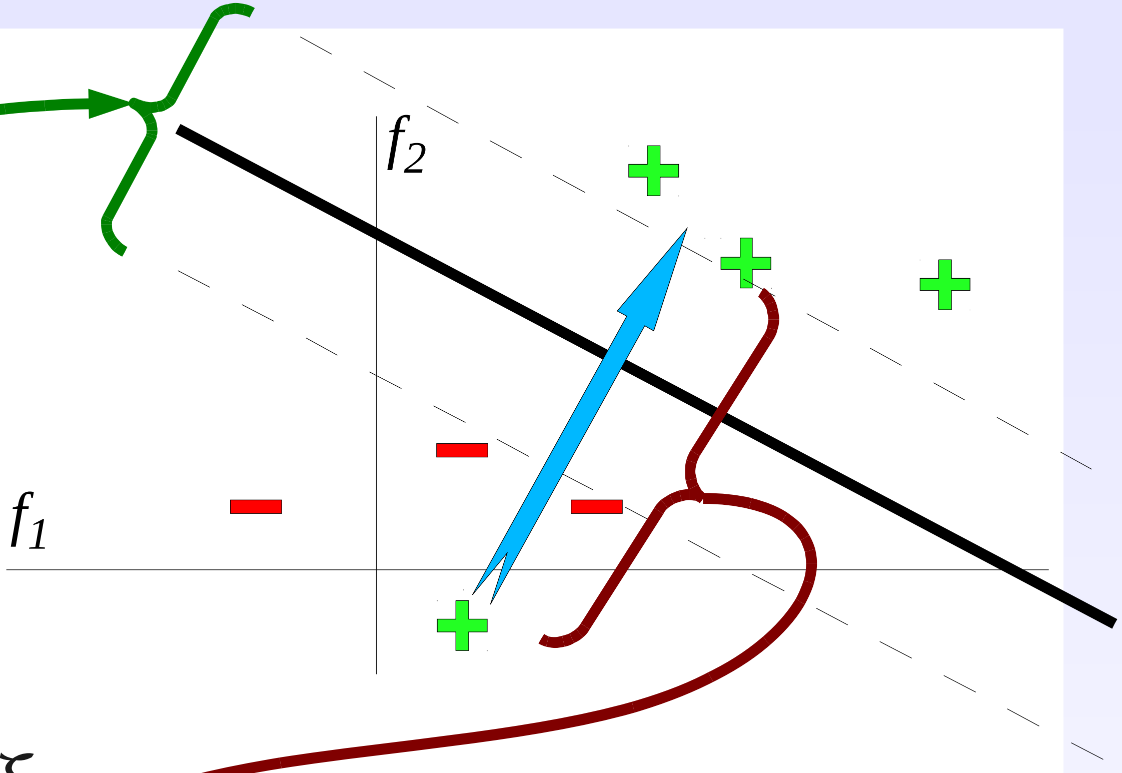
$$\max_{\mathbf{w}} \ \text{margin} \quad s.t. \quad \text{all points are correctly classified}$$

$$\max_{\mathbf{w}} \ \text{margin} \quad s.t. \quad y_n \mathbf{w} \cdot \phi(x_n) \geq 1 \ , \ \forall n$$

$$\min_{\mathbf{w}} \ \|\mathbf{w}\|^2 \quad s.t. \quad y_n \mathbf{w} \cdot \phi(x_n) \geq 1 \ , \ \forall n$$

Support vector machines with *slack*

- Explicitly optimize the ***margin***
- Allow some “noisy” points to be misclassified



$$\min_{\mathbf{w},\xi} \quad \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_n \xi_n$$

$$s.t. \quad y_n \mathbf{w}\cdot\phi(x_n) + \boxed{\xi_n} \geq 1 \;, \; \forall n$$

$$\xi_n \geq 0 \;, \; \forall n$$

Batch versus stochastic optimization

- Batch = read in all the data, then process it
- Stochastic = (roughly) process a bit at a time

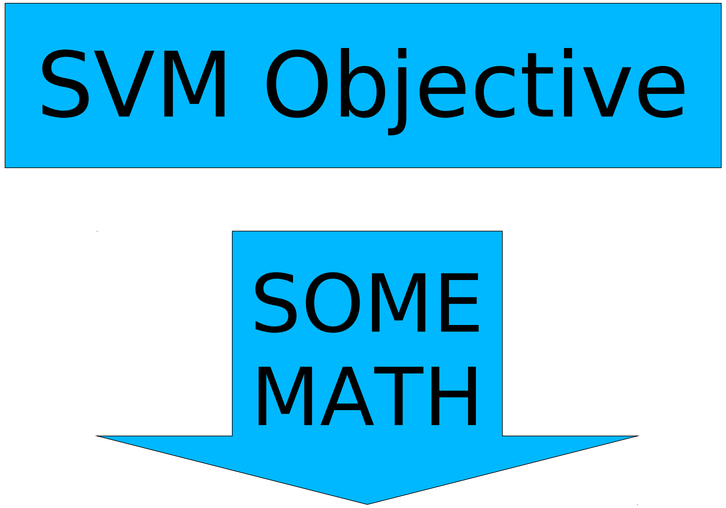
$$\min_{\mathbf{w},\xi} \quad \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_n \xi_n$$

$$s.t. \quad y_n \mathbf{w}\cdot\phi(x_n) + \xi_n \geq 1 \quad , \quad \forall n$$

$$\xi_n \geq 0 \quad , \quad \forall n$$

- For n=1..N:
  - If $y_n \mathbf{w}\cdot\phi(x_n) \leq 0$
    - $\mathbf{w} = \mathbf{w} + y_n \phi(x_n)$

Stochastically optimized SVMs



SVM Objective

SOME MATH

Implementation Note:

Weight shrinkage is *SLOW*. Implement it lazily, at the cost of double storage.

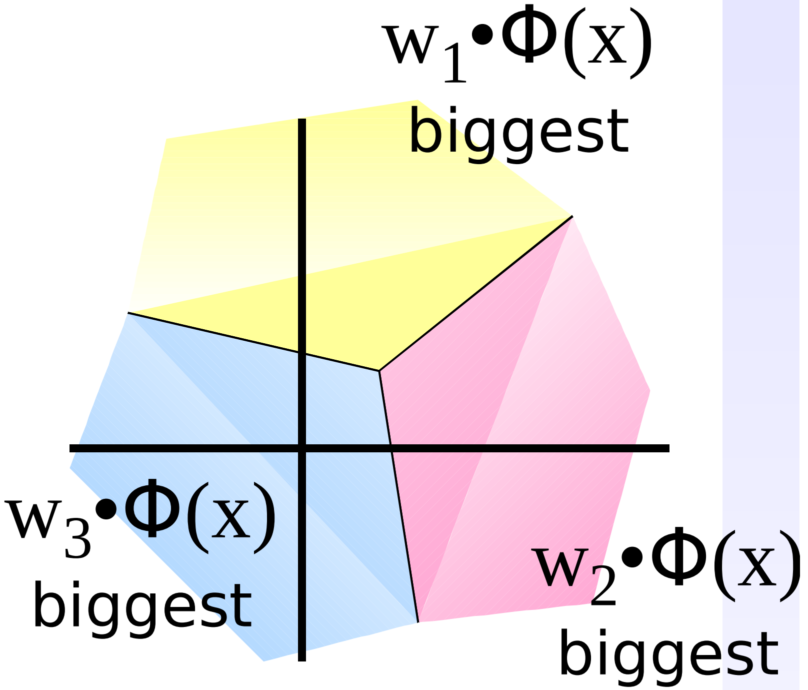
```
For n=1..N:
    If $y_n \mathbf{w} \cdot \phi(x_n) \le 1$
        $\mathbf{w} = \mathbf{w} + y_n \phi(x_n)$
    $\mathbf{w} = \left(1 - \frac{1}{CN}\right) \mathbf{w}$
```

```
For n=1..N:
    If $y_n \mathbf{w} \cdot \phi(x_n) \le 0$
        $\mathbf{w} = \mathbf{w} + y_n \phi(x_n)$
```

From Perceptron to Structured Perceptron

Perceptron with multiple classes

- Store separate weight vector for each class $w_1, w_2, \dots, w_K$


$w_1 \bullet \Phi(x)$ biggest
$w_3 \bullet \Phi(x)$ biggest
$w_2 \bullet \Phi(x)$ biggest

- For n=1..N:
  - Predict:

$$\hat{y} = argmax_k \boldsymbol{w_k} \cdot \phi(x_n)$$

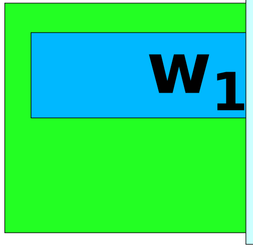
  - If $\hat{y} \neq y_n$:

$$\boldsymbol{w}_{\hat{\boldsymbol{y}}} = \boldsymbol{w}_{\hat{\boldsymbol{y}}} - \phi(x_n)$$
$$\boldsymbol{w}_{\boldsymbol{y_n}} = \boldsymbol{w}_{\boldsymbol{y_n}} + \phi(x_n)$$

?! Why does this do the right thing?

Perceptron

- Originally:



x	Φ(x,1)	Φ(x,2)
“free money”	spam_BIAS : 1 spam_free : 1 spam_money : 1 spam_the : 0 ...	ham_BIAS : 1 ham_free : 1 ham_money : 1 ham_the : 0 ...

- For n=1..N:
  - Predict:

$$\hat{y} = \arg\max_k \mathbf{w_k} \cdot \phi(x_n)$$

  - If $\hat{y} \neq y_n$:

$$\mathbf{w}_{\hat{\mathbf{y}}} = \mathbf{w}_{\hat{\mathbf{y}}} - \phi(x_n)$$
$$\mathbf{w}_{\mathbf{y_n}} = \mathbf{w}_{\mathbf{y_n}} + \phi(x_n)$$

- For n=1..N:
  - Predict:

$$\hat{y} = \arg\max_k \mathbf{w} \cdot \phi(x_n, k)$$

  - If $\hat{y} \neq y_n$:

$$\mathbf{w} = \mathbf{w} - \phi(x_n, \hat{y}) + \phi(x_n, y_n)$$

Features for structured prediction

- Allowed to encode *anything* you want

Pro	Md	Vb	Dt	Nn
I	can	can	a	can

$$\phi(\boldsymbol{x}, y) =$$

```
I_Pro      : 1   <s>-Pro   :  1   has_verb    :  1
can_Md     : 1   Pro-Md    :  1   has_nn_lft  :  0
can_Vb     : 1   Md-Vb     :  1   has_n_lft   :  1
a_Dt       : 1   Vb-Dt     :  1   has_nn_rgt  :  1
can_Nn     : 1   Dt-Nn     :  1   has_n_rgt   :  1
...              Nn-</s>   :  1   ...
                 ...
```

- Output features, Markov features, other features

Structured perceptron

Enumeration over 1..K

- For n=1..N:
  - Predict:

    $$\hat{y} = arg\,max_k\, \mathbf{w} \cdot \phi(x_n, k)$$

  - If $\hat{y} \neq y_n$:

    $$\mathbf{w} = \mathbf{w} - \phi(x_n, \hat{y}) + \phi(x_n, y_n)$$

Enumeration over all outputs

- For n=1..N:
  - Predict:

    $$\hat{y} = arg\,max_k\, \mathbf{w} \cdot \phi(x_n, k)$$

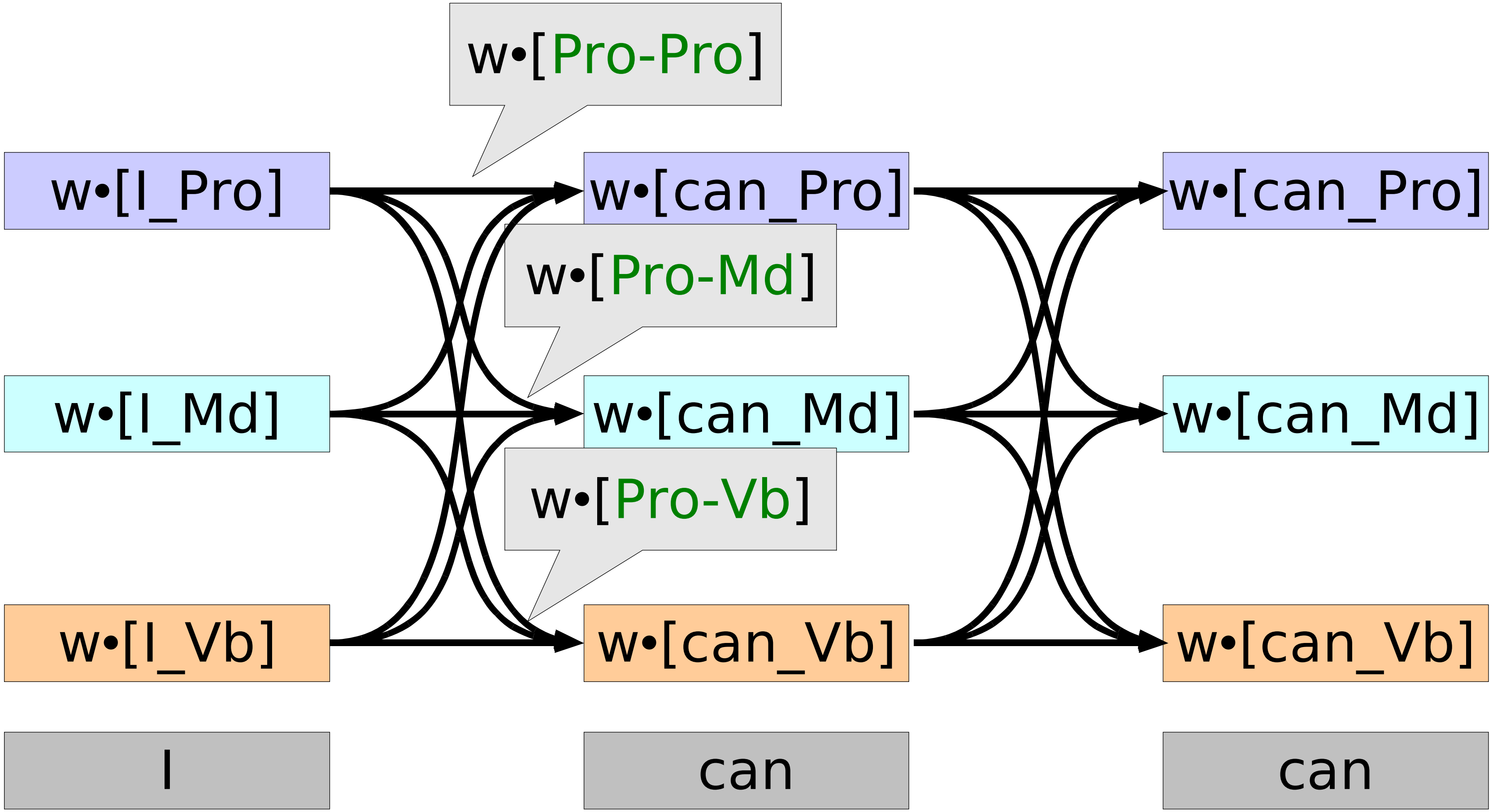
  - If $\hat{y} \neq y_n$:

    $$\mathbf{w} = \mathbf{w} - \phi(x_n, \hat{y}) + \phi(x_n, y_n)$$

[Collins, EMNLP02]

Argmax for sequences

- If we only have output and Markov features, we can use Viterbi algorithm:



w•[Pro-Pro]

w•[I_Pro]

w•[can_Pro]

w•[can_Pro]

w•[Pro-Md]

w•[I_Md]

w•[can_Md]

w•[can_Md]

w•[Pro-Vb]

w•[I_Vb]

w•[can_Vb]

w•[can_Vb]

I

can

can

(plus some work to account for boundary conditions)

Structured perceptron as ranking

- For n=1..N:
  - Run Viterbi: $\hat{y} = arg\,max_k \mathbf{w} \cdot \phi(x_n, k)$
  - If $\hat{y} \neq y_n$: $\mathbf{w} = \mathbf{w} - \phi(x_n, \hat{y}) + \phi(x_n, y_n)$

- When does this make an update?

Pro	Md	Vb	Dt	Nn
Pro	Md	Md	Dt	Vb
Pro	Md	Md	Dt	Nn
Pro	Md	Nn	Dt	Md
Pro	Md	Nn	Dt	Nn
Pro	Md	Vb	Dt	Md
Pro	Md	Vb	Dt	Vb
I	can	can	a	can

From perceptron to margins

Maximize Margin

Minimize Errors

$$\min_{\mathbf{w},\xi} \; \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_n \xi_n$$

$$\text{s.t.} \quad y_n \mathbf{w}\cdot\phi(x_n) + \xi_n \geq 1 \;, \; \forall n$$

Each point is correctly classified, modulo ξ

$$\min_{\mathbf{w},\xi} \; \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_n \xi_{n,\hat{y}}$$

Response for truth

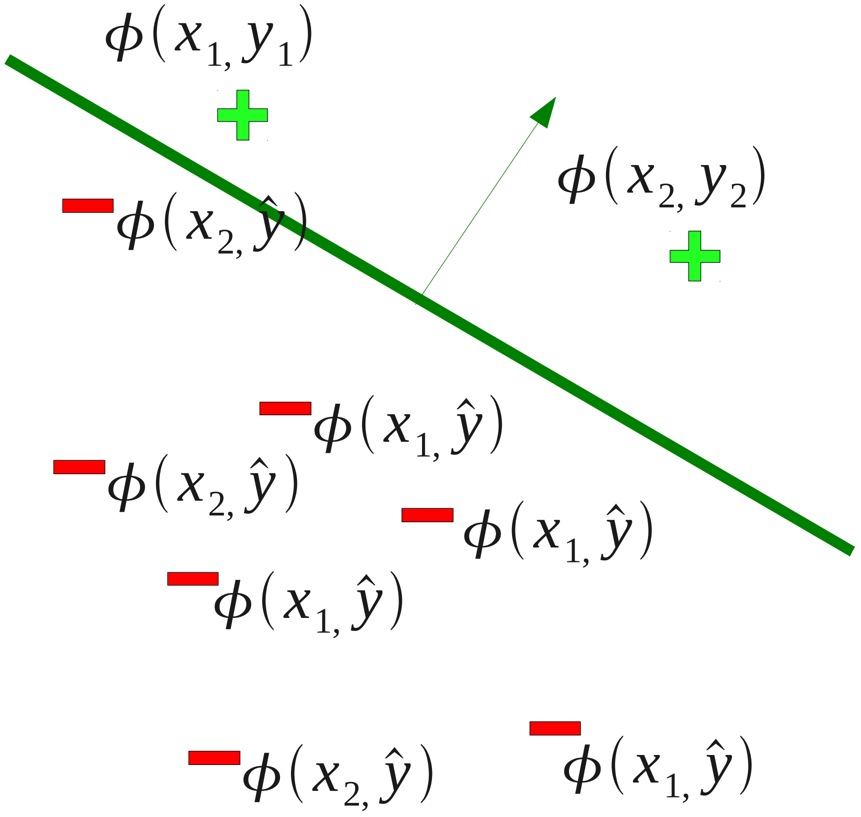
Response for other

$$\text{s.t.} \quad \mathbf{w}\cdot\phi(x_n, y_n) - \mathbf{w}\cdot\phi(x_n, \hat{y}) + \xi_n \geq 1, \forall n, \hat{y} \neq y_n$$

Each true output is more highly ranked, modulo ξ

[Taskar+al, JMLR05; Tshochandaritis, JMLR05]

From perceptron to margins



$$\min_{\mathbf{w},\xi} \quad \frac{1}{2}\|\boldsymbol{w}\|^2 + C\sum_n \xi_{n,\hat{y}}$$

Response for truth

Response for other

$$\text{s.t.} \quad \boldsymbol{w}\cdot\phi(x_n,y_n) - \boldsymbol{w}\cdot\phi(x_n,\hat{y}) + \xi_n \geq 1, \forall n, \hat{y} \neq y_n$$

Each true output is more highly ranked, modulo ξ

[Taskar+al, JMLR05; Tsochandaritis, JMLR05]

Ranking margins

- Some errors are worse than others...

Pro	Md	Vb	Dt	Nn

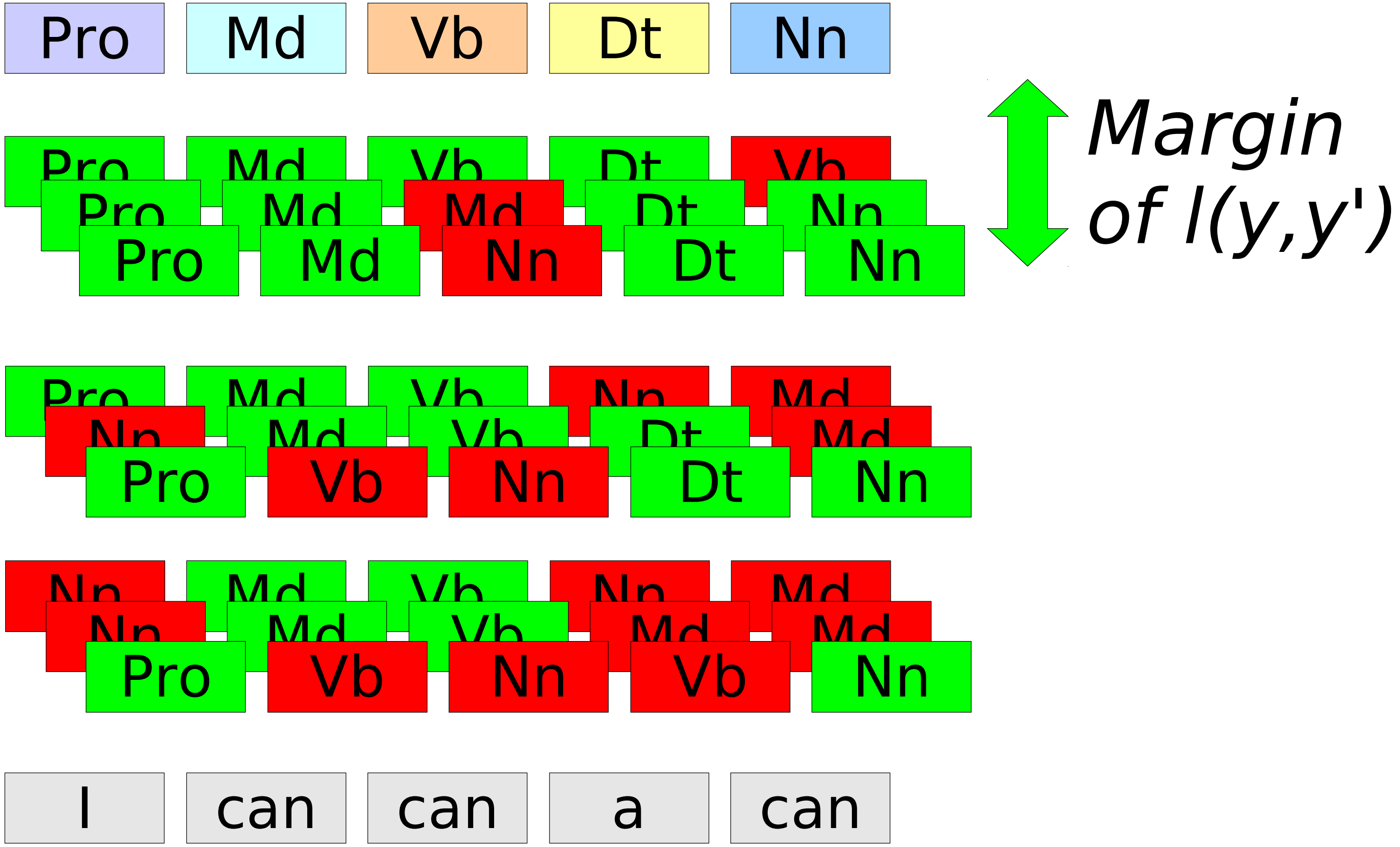
Margin of one

Pro	Md	Md	Dt	Vb
Pro	Md	Md	Dt	Nn
Pro	Md	Nn	Dt	Md
Pro	Md	Nn	Dt	Nn
Pro	Md	Vb	Dt	Md
Pro	Md	Vb	Dt	Vb
I	can	can	a	can

[Taskar+al, JMLR05; Tshochandaritis, JMLR05]

Accounting for a loss function

- Some errors are worse than others...



Pro Md Vb Dt Nn

Pro Md Vb Dt Vb
Pro Md Md Dt Nn
Pro Md Nn Dt Nn

Pro Md Vb Nn Md
Nn Md Vb Dt Md
Pro Vb Nn Dt Nn

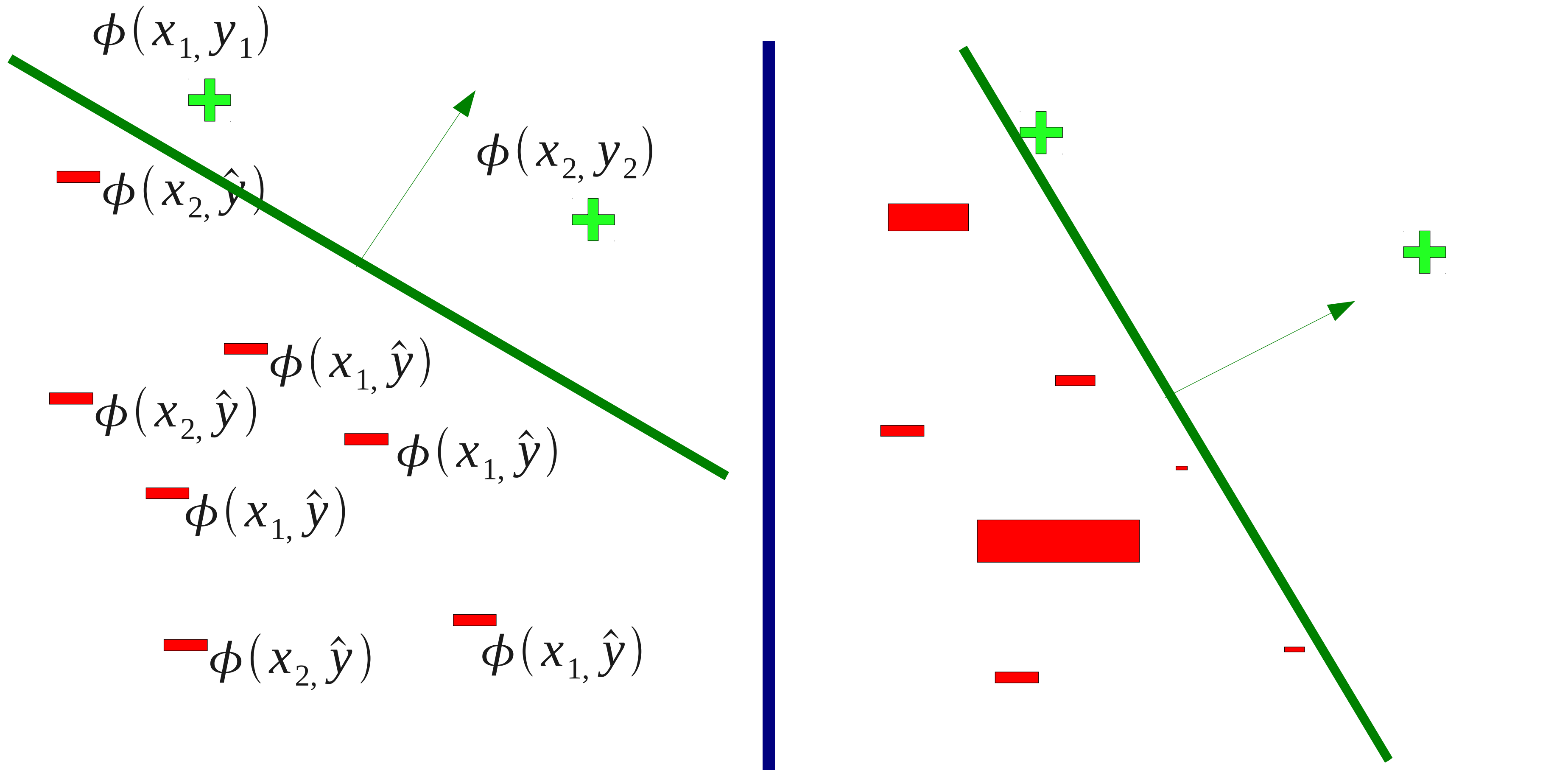
Nn Md Vb Nn Md
Nn Md Vb Md Md
Pro Vb Nn Vb Nn

I can can a can

Margin of l(y,y')

[Taskar+al, JMLR05; Tshochandaritis, JMLR05]

Accounting for a loss function



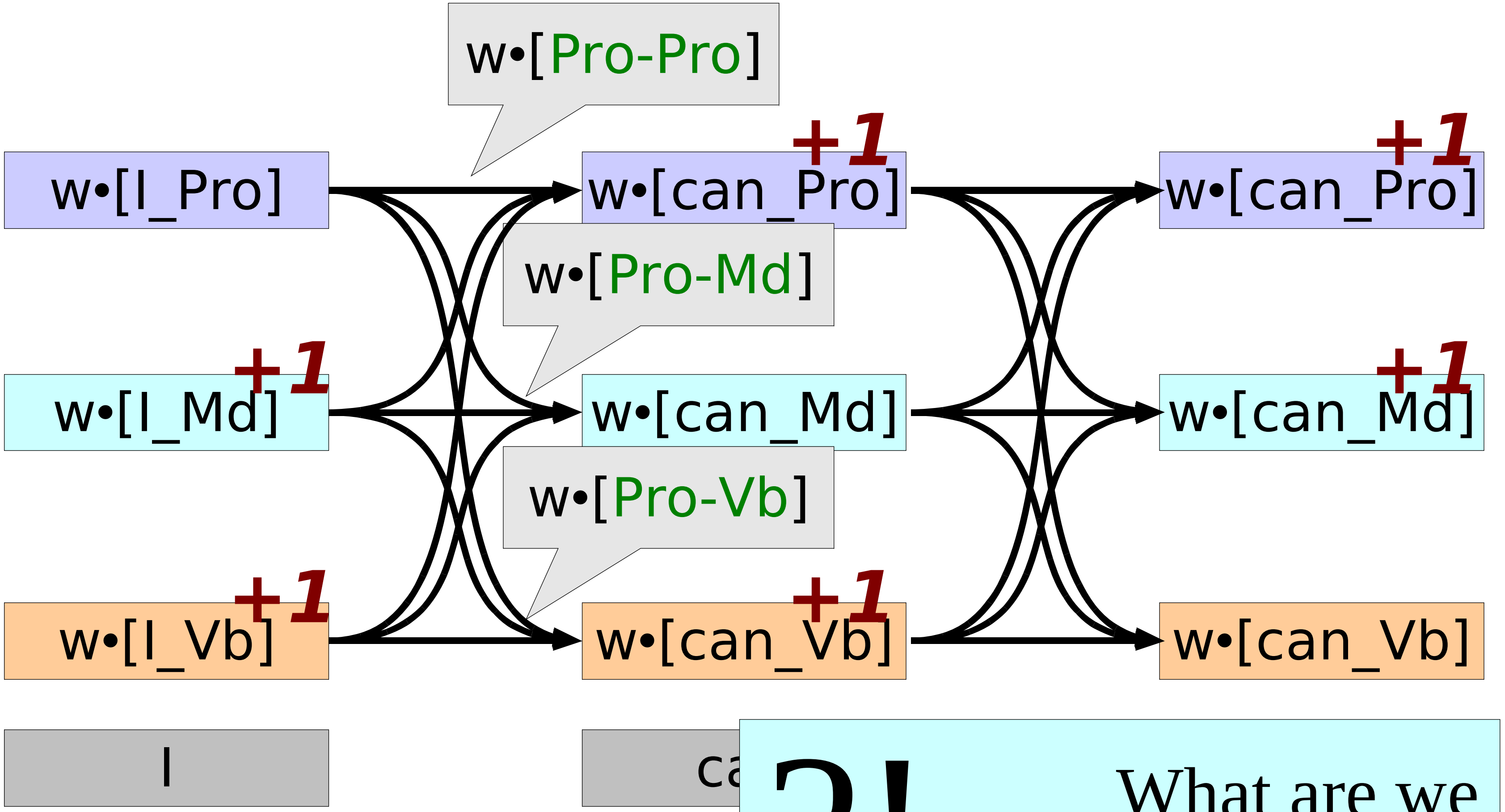
$$\mathbf{w}\cdot\phi(x_n, y_n) - \mathbf{w}\cdot\phi(x_n, \hat{y}) + \xi_n \geq 1$$

$$\mathbf{w}\cdot\phi(x_n, y_n) - \mathbf{w}\cdot\phi(x_n, \hat{y}) + \xi_n \geq l(y_n, \hat{y})$$

[Taskar+al, JMLR05; Tsochandaritis, JMLR05]

Augmented argmax for sequences

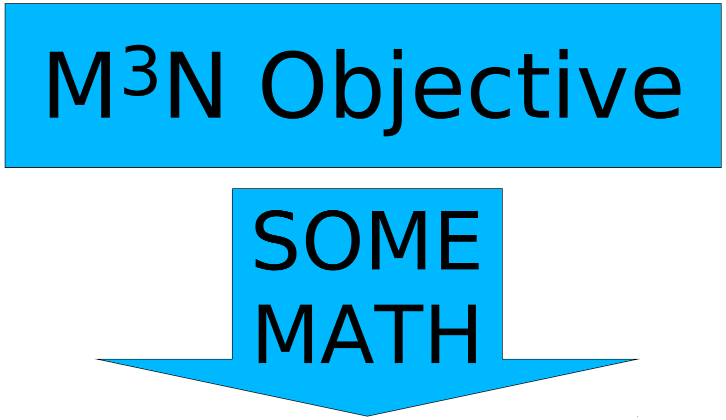
- Add “loss” to each wrong node!



?! What are we assuming here?

[Taskar+al, JMLR05; Tsochandaritis, JMLR05]

Stochastically optimizing Markov nets



M3N Objective

SOME MATH

- For n=1..N:
  - Augmented Viterbi:
    $$\hat{y} = arg\,max_k\, \mathbf{w} \cdot \phi(x_n, k) + l(y_n, k)$$
  - If $\hat{y} \neq y_n$:
    $$\mathbf{w} = \mathbf{w} - \phi(x_n, \hat{y}) + \phi(x_n, y_n)$$
  - $$\mathbf{w} = \left(1 - \frac{1}{CN}\right)\mathbf{w}$$

- For n=1..N:
  - Viterbi:
    $$\hat{y} = arg\,max_k\, \mathbf{w} \cdot \phi(x_n, k)$$
  - If $\hat{y} \neq y_n$:
    $$\mathbf{w} = \mathbf{w} - \phi(x_n, \hat{y}) + \phi(x_n, y_n)$$

[Ratliff+al, AIStats07]

Learning to Search

Argmax is *hard!*

- Classic formulation of structured prediction:

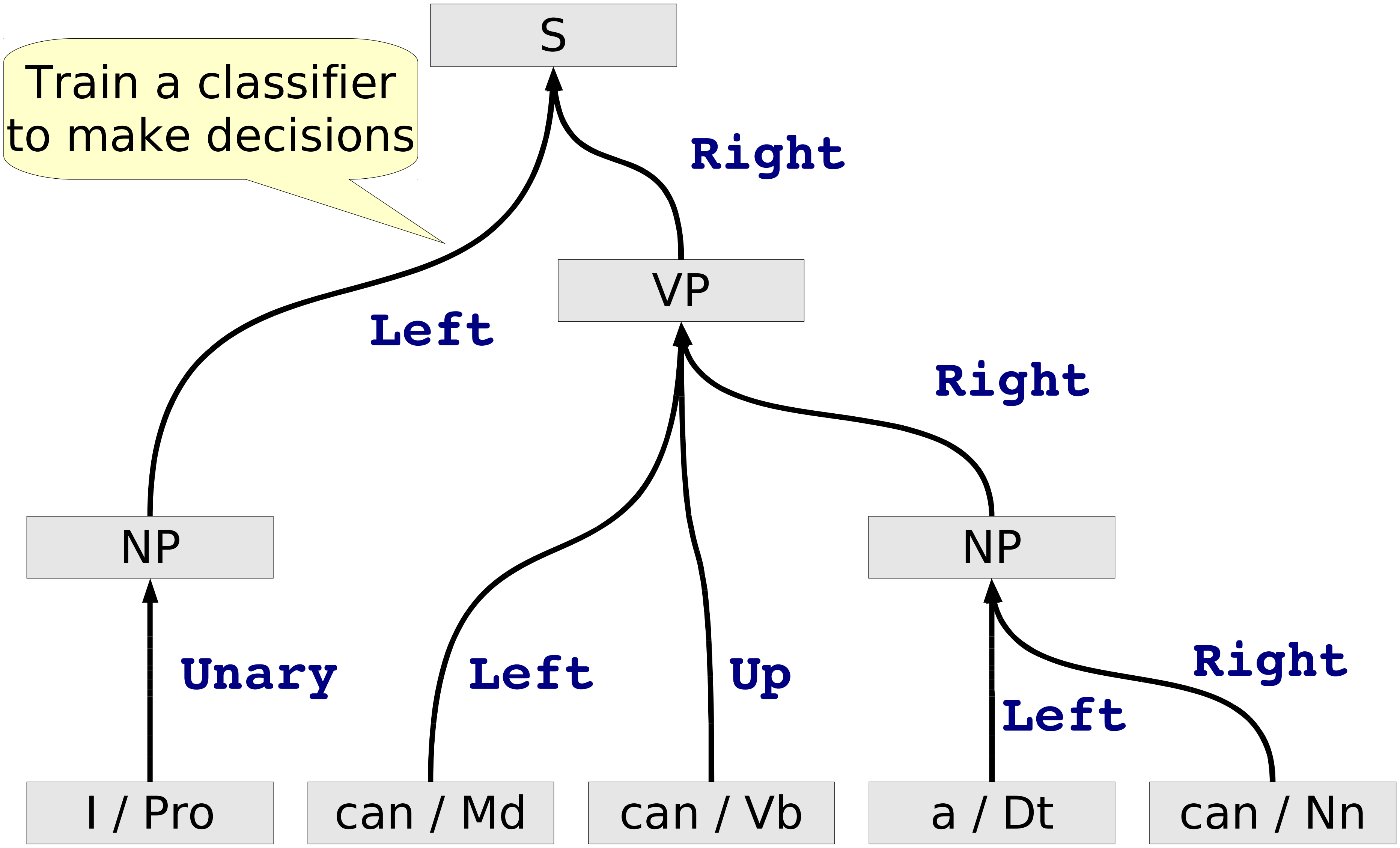
$$score(x,y) \;=\; \textit{something we learn to make “good” x,y pairs score highly}$$

- At test time:

$$f(x) = argmax_{y \in Y} score(x,y)$$

- Combinatorial optimization problem
  - Efficient only in very limiting cases
  - Solved by heuristic search: beam + A* + local search

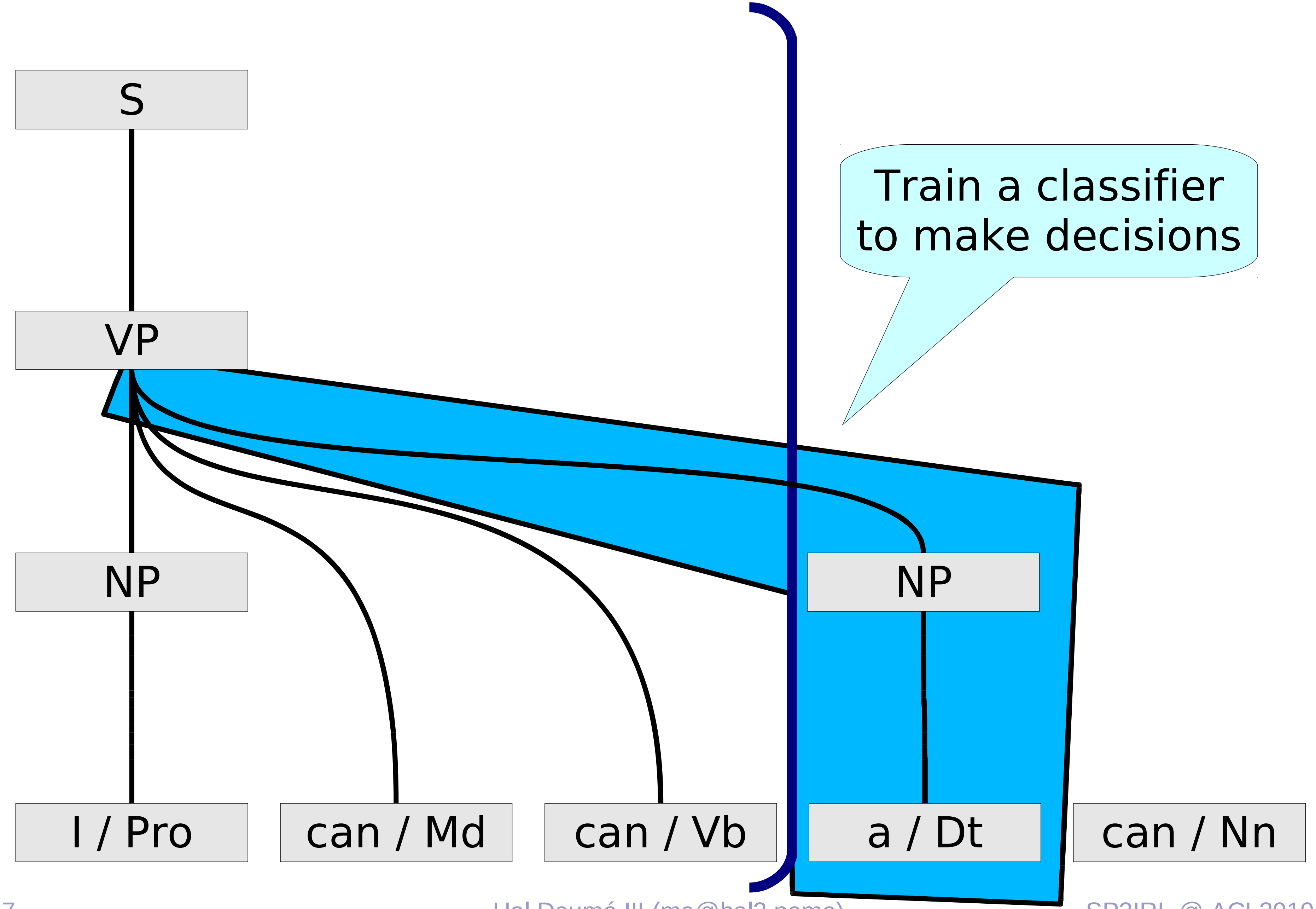
Incremental parsing, early 90s style



Train a classifier to make decisions

[Magerman, ACL95]

Incremental parsing, mid 2000s style

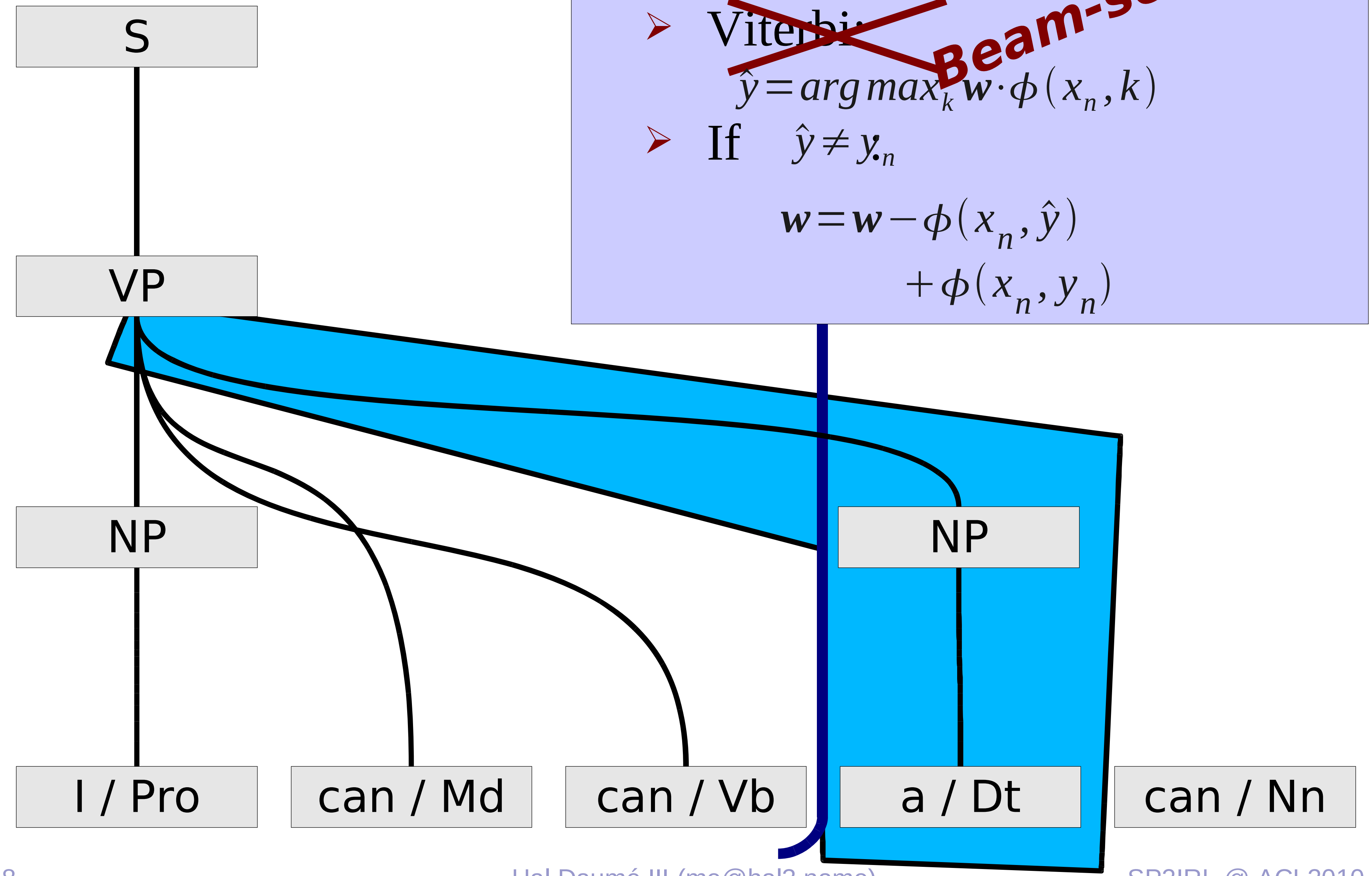


Train a classifier to make decisions

[Collsns+Roark, ACL04]

Learning to beam-search

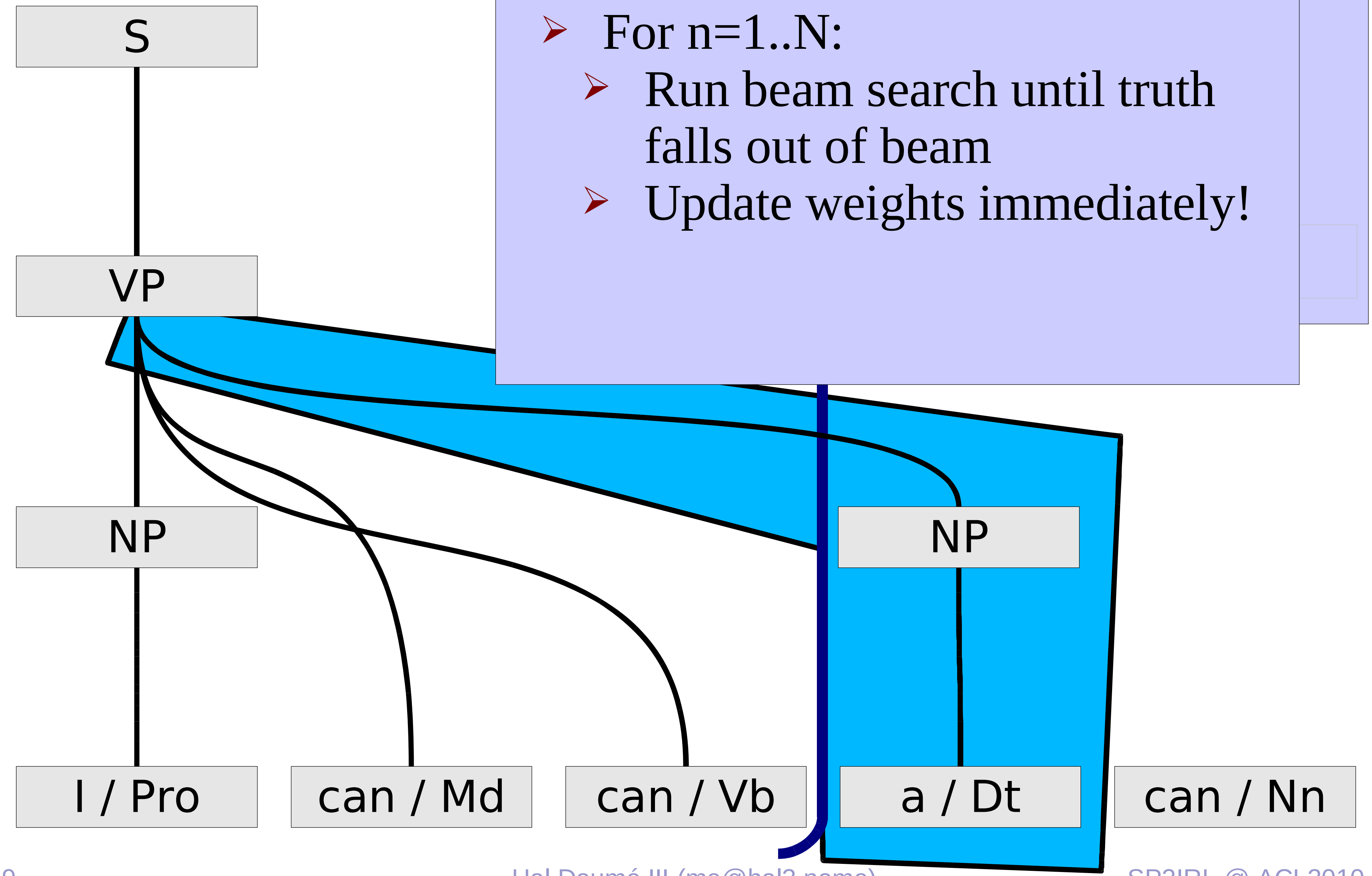
- For n=1..N:
  - ~~Viterbi:~~ Beam-search
    $\hat{y} = \arg\max_k \mathbf{w} \cdot \phi(x_n, k)$
  - If $\hat{y} \neq y_n$:
    $\mathbf{w} = \mathbf{w} - \phi(x_n, \hat{y})$
    $+ \phi(x_n, y_n)$



[Collins+Roark, ACL04]

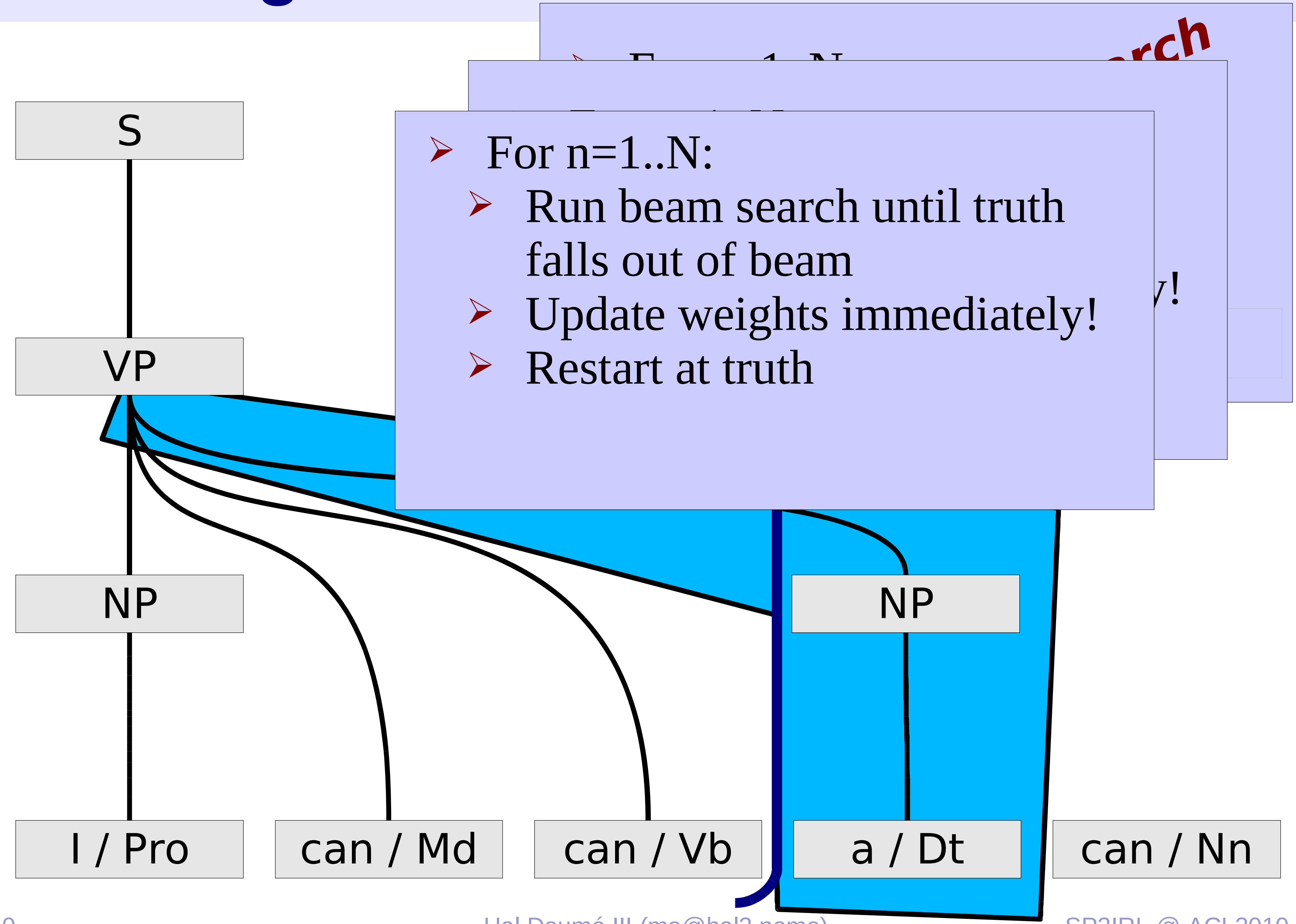
Learning to beam-search

- For n=1..N:
  - Run beam search until truth falls out of beam
  - Update weights immediately!



[Collins+Roark, ACL04]

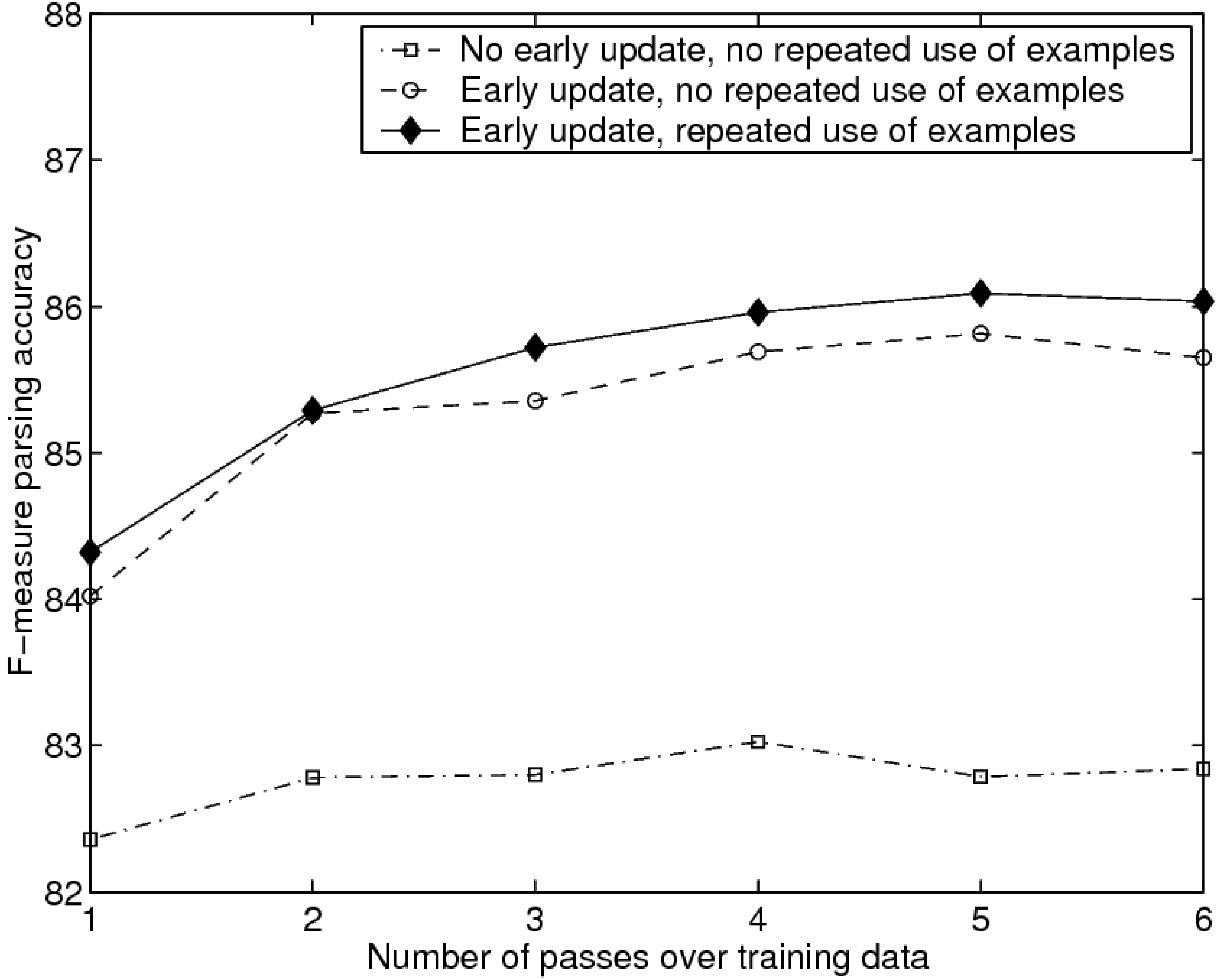
Learning to beam-search



- For n=1..N:
  - Run beam search until truth falls out of beam
  - Update weights immediately!
  - Restart at truth

[D+Marcu, ICML05; Xu+al, JMLR09]

Incremental parsing results



[Collins+Roark, ACL04]

Generic Search Formulation

- Search Problem:
  - Search space
  - Operators
  - Goal-test function
  - Path-cost function
- Search Variable:
  - Enqueue function

Varying the **Enqueue** function can give us DFS, BFS, beam search, A* search, etc...

```
nodes := MakeQueue(S0)
while nodes is not empty
    node := RemoveFront(nodes)
    if node is a goal state return node
    next := Operators(node)
    nodes := Enqueue(nodes, next)
fail
```

[D+Marcu, ICML05; Xu+al, JMLR09]

Online Learning Framework (LaSO)

```
nodes := MakeQueue(S0)
while nodes is not empty
    node := RemoveFront(nodes)
    if none of {node} ∪ nodes is y-good or node is a goal & not y-good
        sibs := siblings(node, y)
        w := update(w, x, sibs, {node} ∪ nodes)
        nodes := MakeQueue(sibs)
    else
        if node is a goal state return w
        next := Operators(node)
        nodes := Enqueue(nodes, next)
```

Monotonicity: for any node, we can tell if it can lead to the correct solution or not

If we erred...

Where should we have gone?

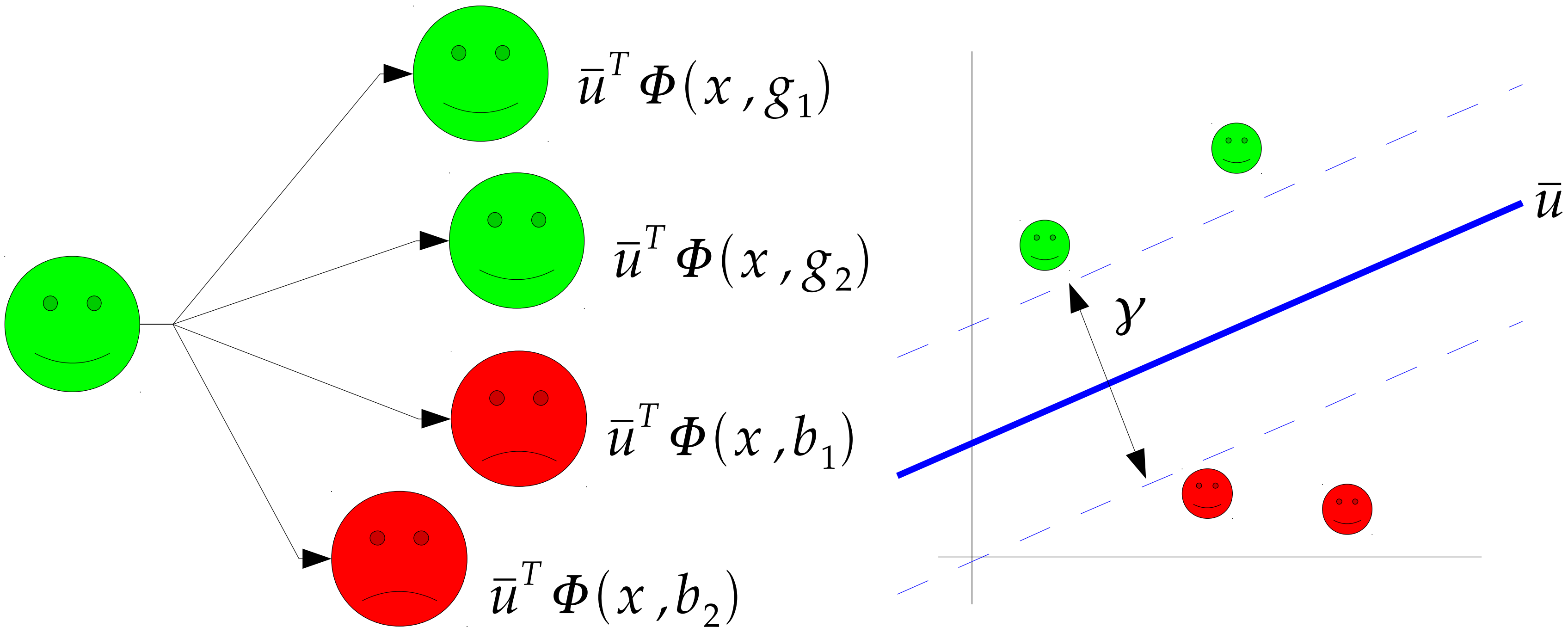
Update our weights based on the good and the bad choices

Continue search...

[D+Marcu, ICML05; Xu+al, JMLR09]

Search-based Margin

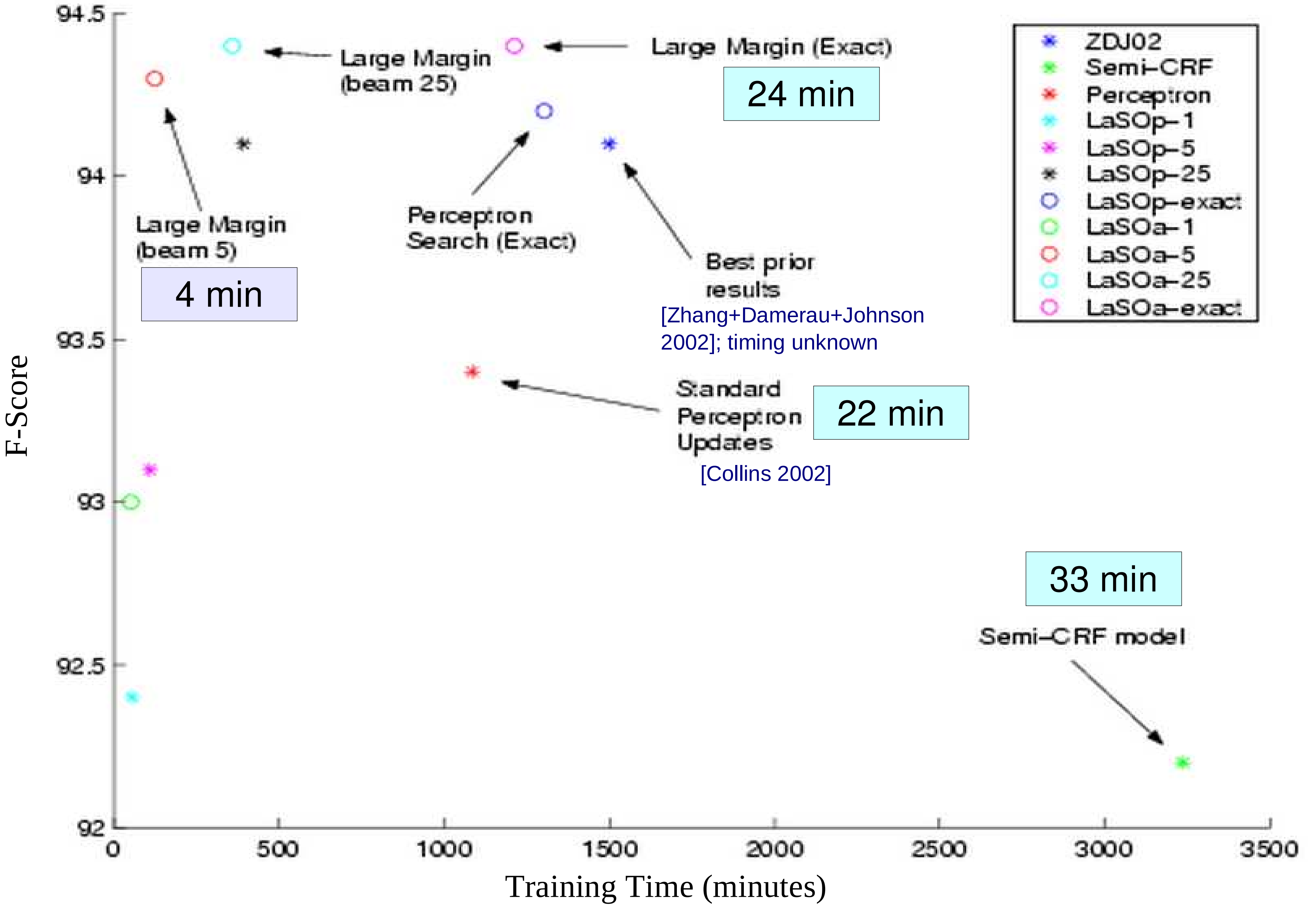
- The *margin* is the amount by which we are correct:



- Note that the *margin* and hence *linear separability* is also a function of the *search algorithm!*

[D+Marcu, ICML05; Xu+al, JMLR09]

Syntactic chunking Results



24 min

4 min

[Zhang+Damerau+Johnson 2002]; timing unknown

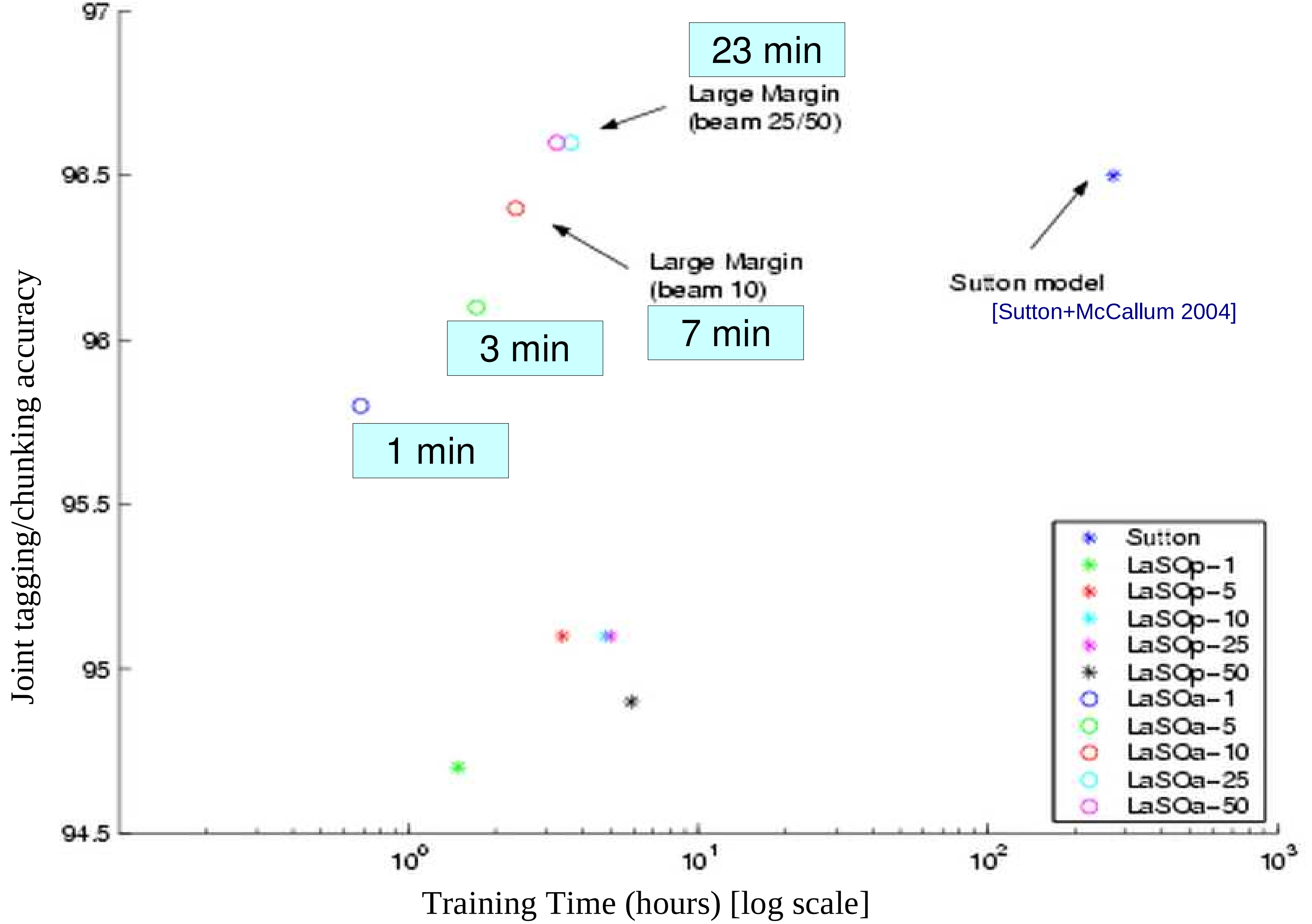
22 min

[Collins 2002]

33 min

[D+Marcu, ICML05; Xu+al, JMLR09]

Tagging+chunking results



23 min

7 min

3 min

1 min

[Sutton+McCallum 2004]

[D+Marcu, ICML05; Xu+al, JMLR09]

Variations on a beam

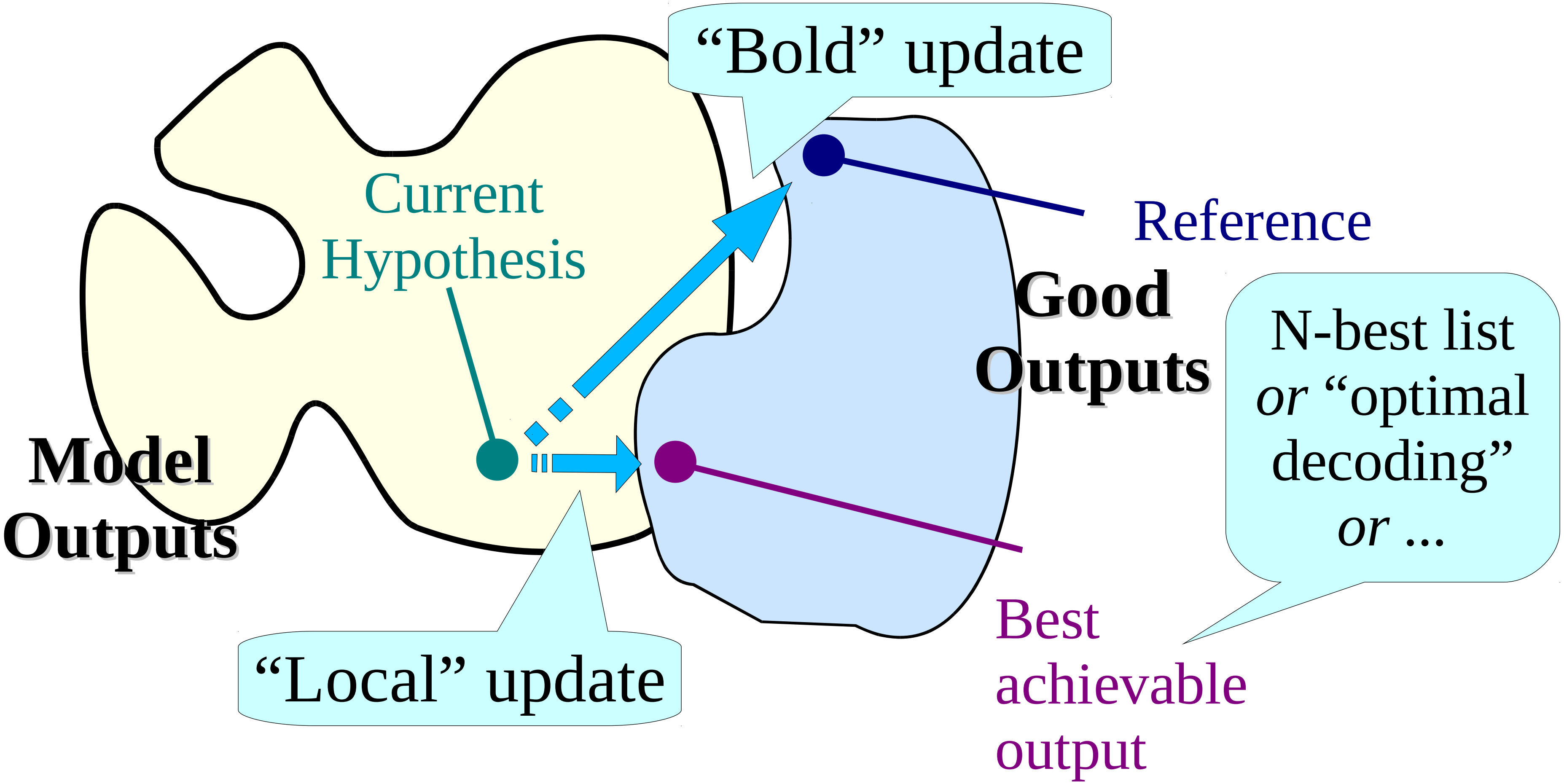
- Observation:
  - We needn't use the same beam size for training and decoding
  - Varying these values independently yields:

		Decoding Beam				
		1	5	10	25	50
Training Beam	1	93.9	92.8	91.9	91.3	90.9
	5	90.5	94.3	94.4	94.1	94.1
	10	89.5	94.3	94.4	94.2	94.2
	25	88.7	94.2	94.5	94.3	94.3
	50	88.4	94.2	94.4	94.2	94.4

[D+Marcu, ICML05; Xu+al, JMLR09]

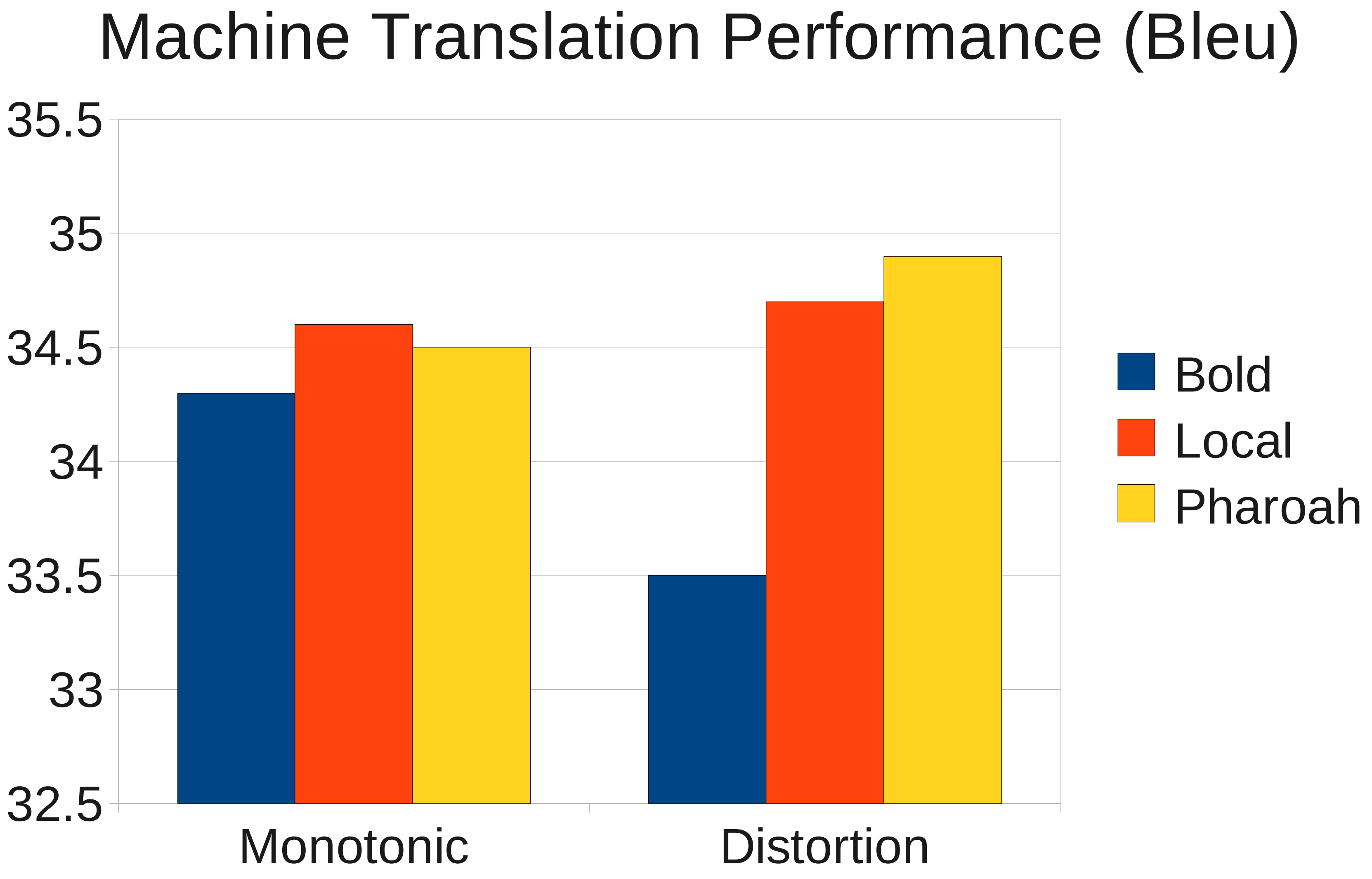
What if our model sucks?

- Sometimes our model *cannot* produce the "correct" output
  - canonical example: machine translation



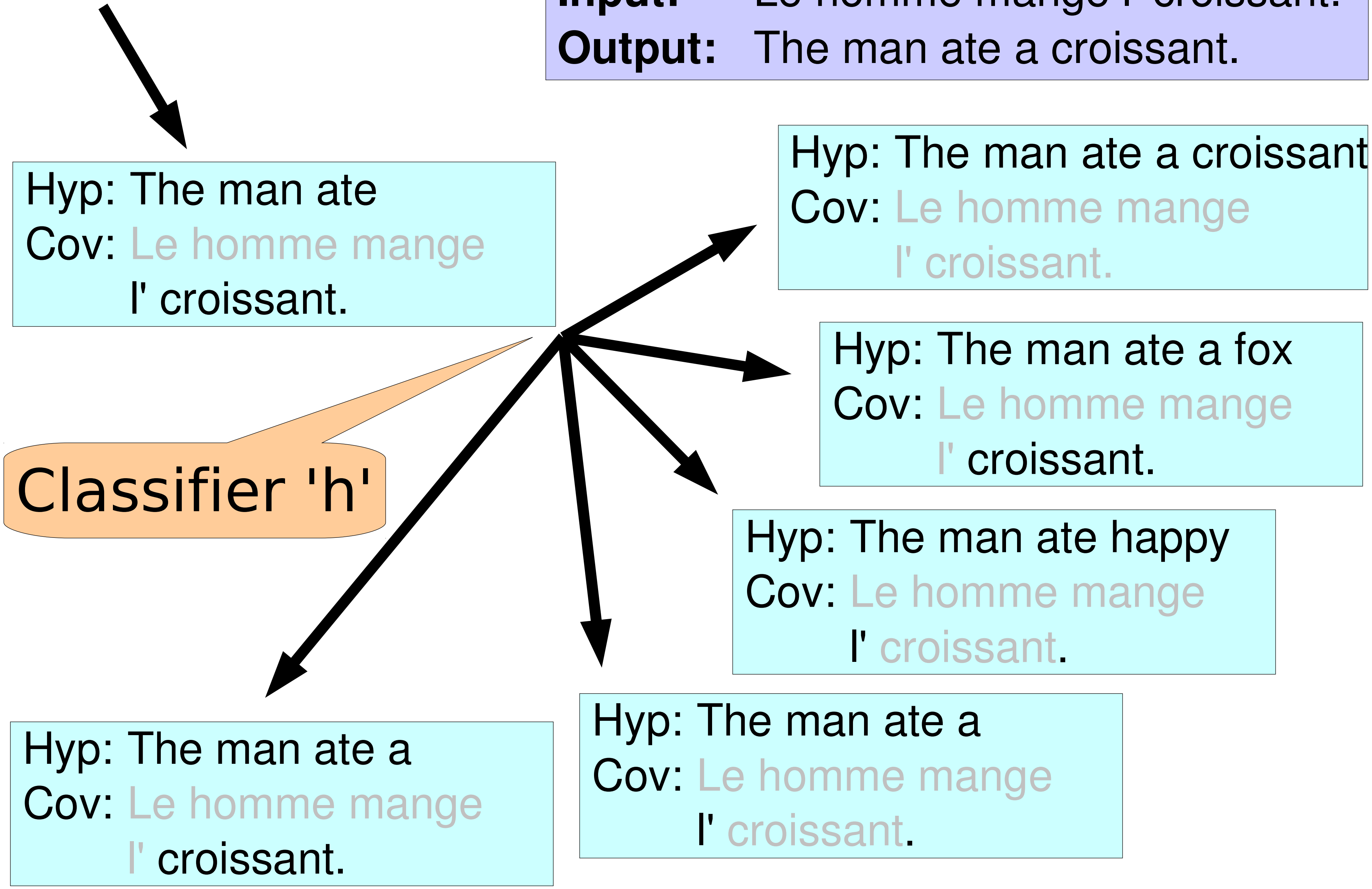
[Och, ACL03; Liang+al, ACL06]

Local versus bold updating...



Integrating search and learning

Input: Le homme mange l' croissant.
Output: The man ate a croissant.



Hyp: The man ate
Cov: Le homme mange l' croissant.

Classifier 'h'

Hyp: The man ate a croissant
Cov: Le homme mange l' croissant.

Hyp: The man ate a fox
Cov: Le homme mange l' croissant.

Hyp: The man ate happy
Cov: Le homme mange l' croissant.

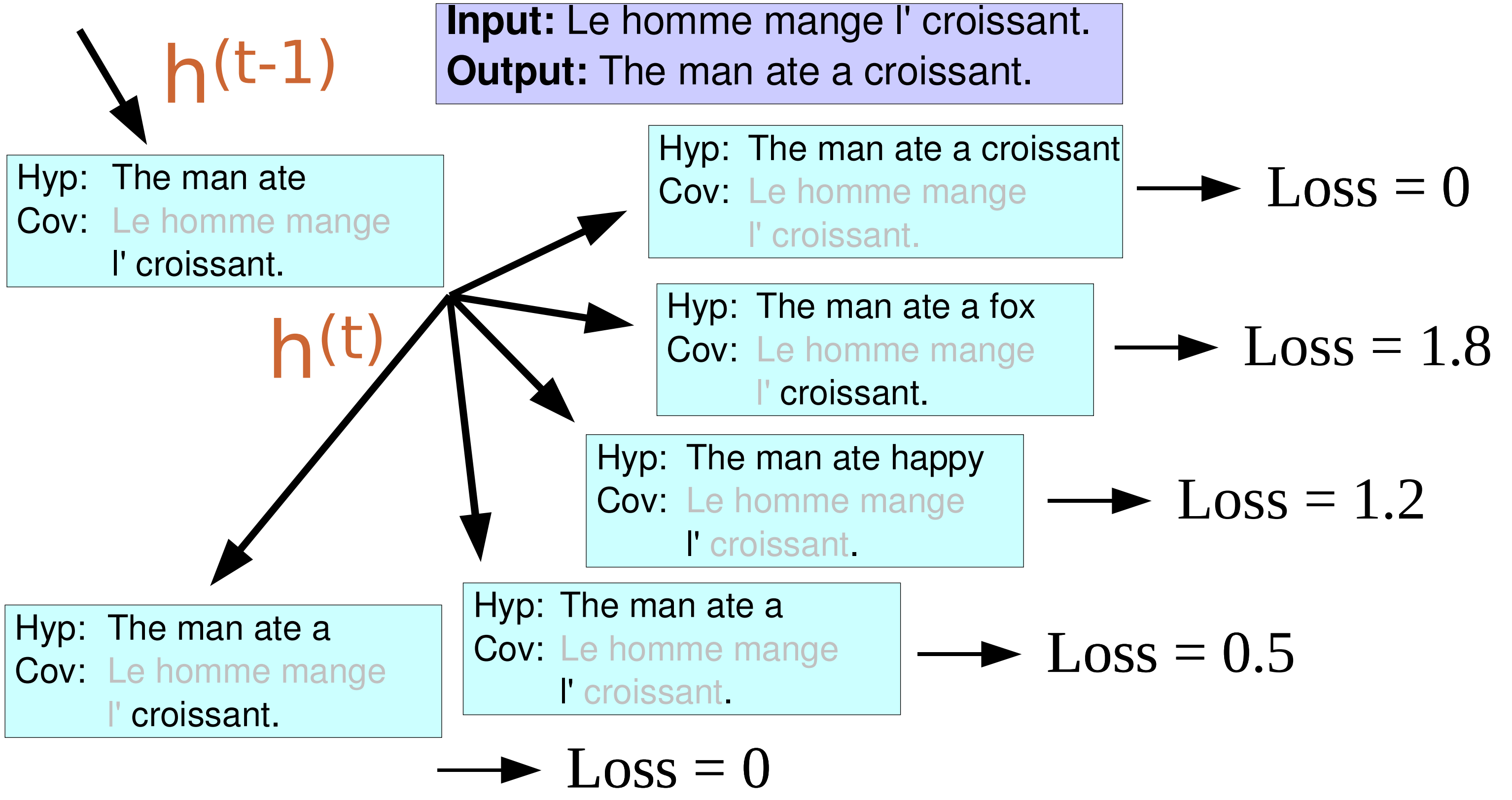
Hyp: The man ate a
Cov: Le homme mange l' croissant.

Hyp: The man ate a
Cov: Le homme mange l' croissant.

[D+Marcu, ICML05; D+Langford+Marcu, MLJ09]

Reducing search to classification

- Natural chicken and egg problem:
  - Want *h* to get low expected future loss
  - ... on future decisions made by *h*
  - ... and starting from states visited by *h*
- Iterative solution



Input: Le homme mange l' croissant.
Output: The man ate a croissant.

Hyp: The man ate
Cov: Le homme mange l' croissant.

Hyp: The man ate a croissant
Cov: Le homme mange l' croissant.
→ Loss = 0

Hyp: The man ate a fox
Cov: Le homme mange l' croissant.
→ Loss = 1.8

Hyp: The man ate happy
Cov: Le homme mange l' croissant.
→ Loss = 1.2

Hyp: The man ate a
Cov: Le homme mange l' croissant.
→ Loss = 0.5

Hyp: The man ate a
Cov: Le homme mange l' croissant.
→ Loss = 0

[D+Langford+Marcu, MLJ09]

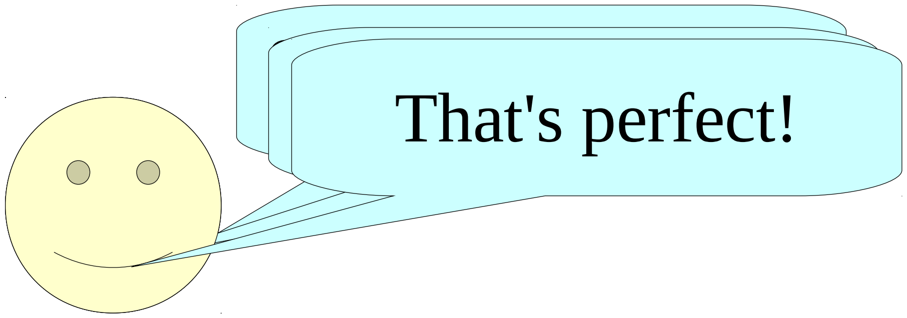
Theoretical results

Theorem: After *2T*^{3} ln *T* iterations,
the loss of the learned policy
is bounded as follows:

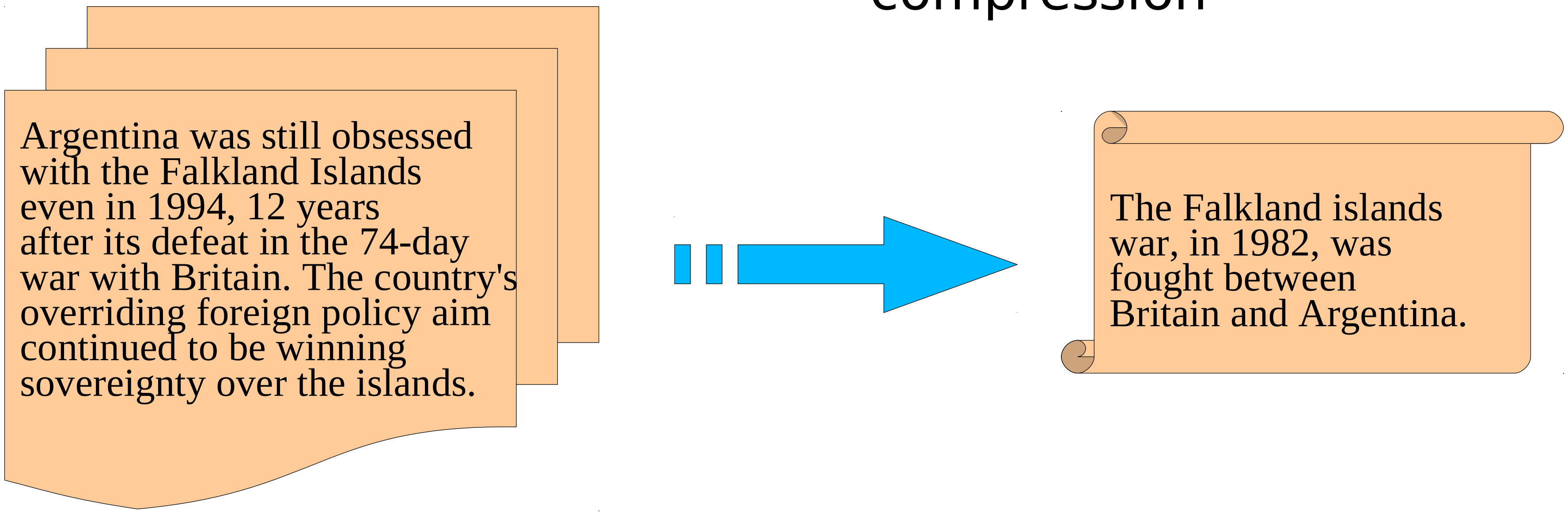
$$L(h) \leq \underbrace{L(h_0)}_{\text{Loss of the optimal policy}} + 2T \ln T \underbrace{l_{avg}}_{\text{Average multiclass classification loss}} + (1 + \ln T) \underbrace{\frac{c_{max}}{T}}_{\text{Worst case per-step loss}}$$

[D+Langford+Marcu, MLJ09]

Example task: summarization



Standard approach is sentence extraction, but that is often deemed to “coarse” to produce good, very short summaries. We wish to also drop words and phrases => document compression



Argentina was still obsessed with the Falkland Islands even in 1994, 12 years after its defeat in the 74-day war with Britain. The country's overriding foreign policy aim continued to be winning sovereignty over the islands.

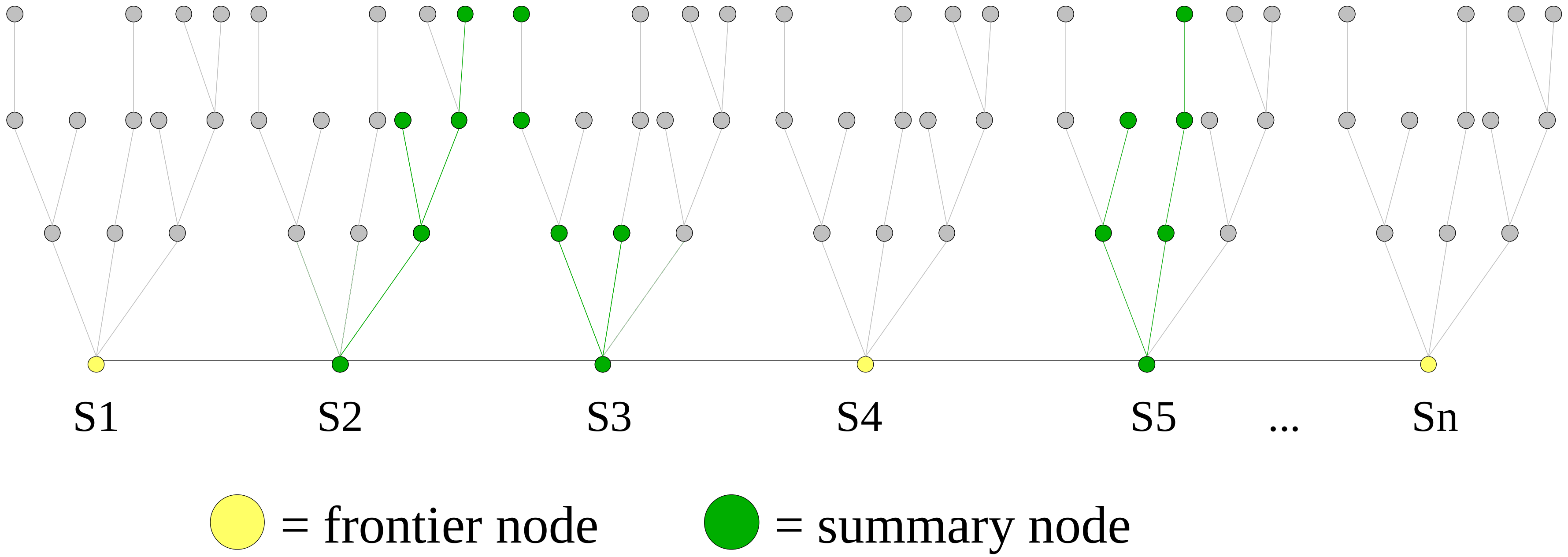
The Falkland islands war, in 1982, was fought between Britain and Argentina.

[D+Langford+Marcu, MLJ09]

Structure of search

Argentina was still obsessed with the Falkland Islands even in 1994, 12 years after its defeat in the 74-day war with Britain. The country's overriding foreign policy aim continued to be winning sovereignty over the islands.

- Lay sentences out sequentially
- Generate a dependency parse of each sentence
- Mark each root as a frontier node
- Repeat:
  - Choose a frontier node node to add to the summary
  - Add all its children to the frontier
  - Finish when we have enough words



[D+Langford+Marcu, MLJ09]

Example output (40 word limit)

Sentence Extraction + Compression:

+13 Argentina and Britain announced an agreement, nearly eight years after they fought a 74-day war a populated archipelago off Argentina's coast. Argentina gets out the red carpet, official royal visitor since the end of the Falklands war in 1982.

Vine Growth (Searn):

+24 Argentina and Britain announced to restore full ties, eight years after they fought a 74-day war over the Falkland islands. Britain invited Argentina's minister Cavallo to London in 1992 in the first official visit since the Falklands war in 1982.

6	Diplomatic ties restored	3	Falkland war was in 1982
5	Major cabinet member visits	3	Cavallo visited UK
5	Exchanges were in 1992	2	War was 74-days long
3	War between Britain and Argentina		

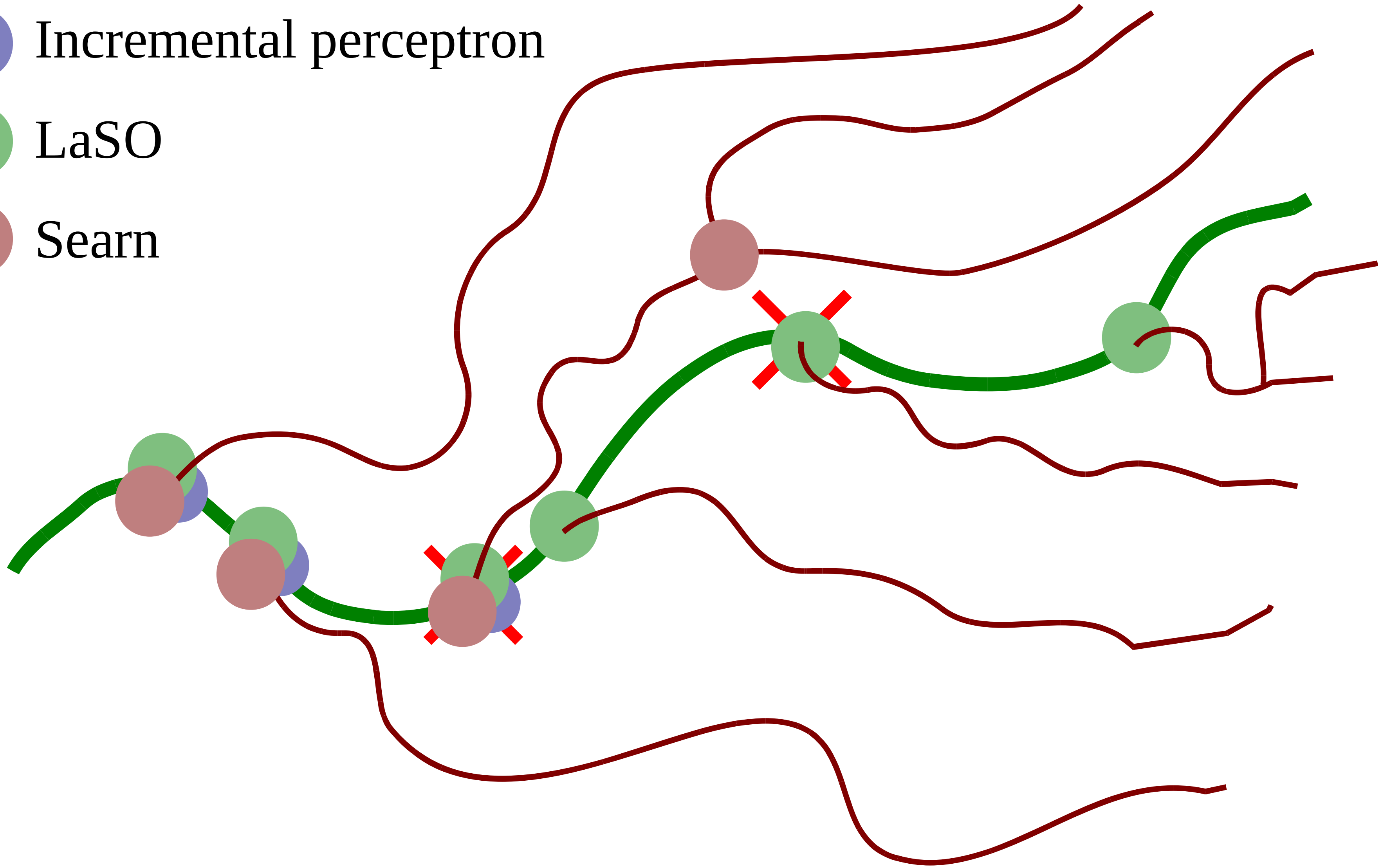
[D+Langford+Marcu, MLJ09]

Perceptron vs. LaSO vs. Searn

● Incremental perceptron

● LaSO

● Searn



✗ Un-learnable decision

Take-home messages

If not, this can be a *really* bad idea! [Kulesza+Pereira, NIPS07]

- If you can predict (ie., solve argmax) you can learn (use structured perceptron)
- If you can do loss-augmented search, you can do max margin (add two lines of code to perceptron)
- If you can do beam search, you can learn using LaSO (with no loss function)
- If you can do beam search, you can learn using Searn (with any loss function)

Coffee Break!!!

Refresher on Reinforcement Learning

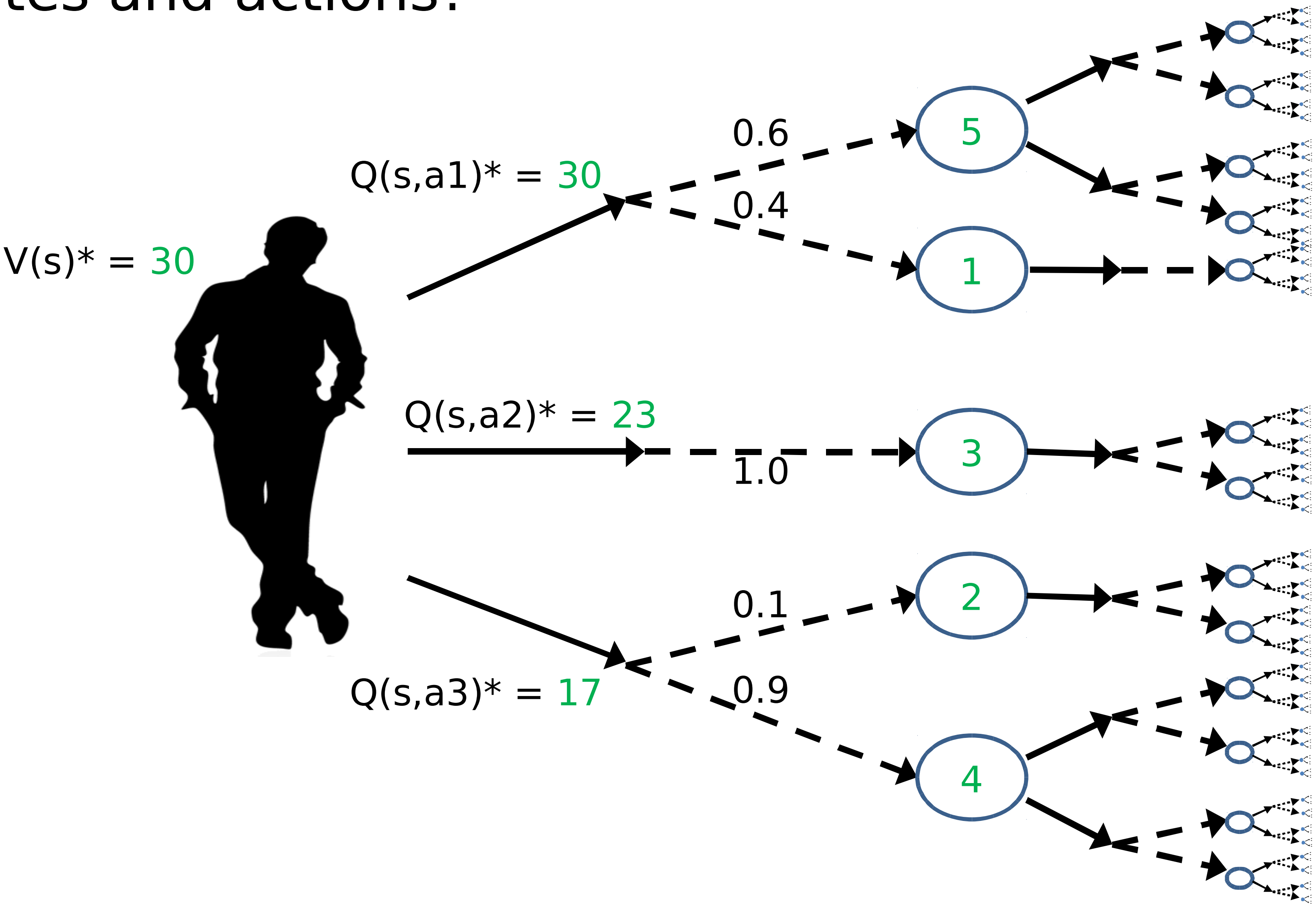
Reinforcement learning

- Basic idea:
  - Receive feedback in the form of rewards
  - Agent’s utility is defined by the reward function
  - Must learn to act to maximize expected rewards
  - Change the rewards, change the learned behavior

- Examples:
  - Playing a game, reward at the end for outcome
  - Vacuuming, reward for each piece of dirt picked up
  - Driving a taxi, reward for each passenger delivered

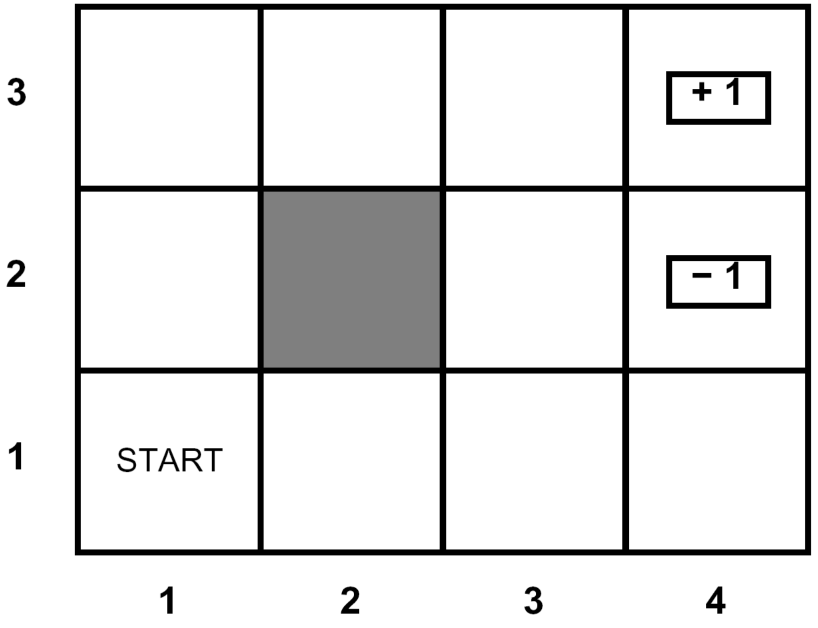
Markov decision processes

What are the values (expected future rewards) of states and actions?



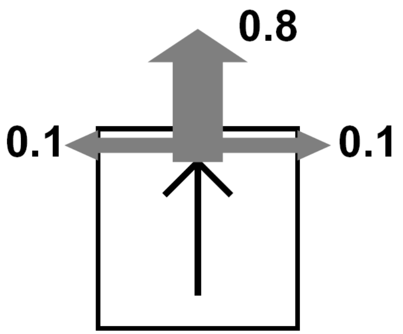
Markov Decision Processes

- An MDP is defined by:
  - A set of states s ∈ S
  - A set of actions a ∈ A
  - A transition function T(s,a,s')
    - Prob that a from s leads to s
    - i.e., P(s' | s,a)
    - Also called the model
  - A reward function R(s, a, s')
    - Sometimes just R(s) or R(s')
  - A start state (or distribution)
  - Maybe a terminal state



- MDPs are a family of non-deterministic search problems
- Total utility is one of:

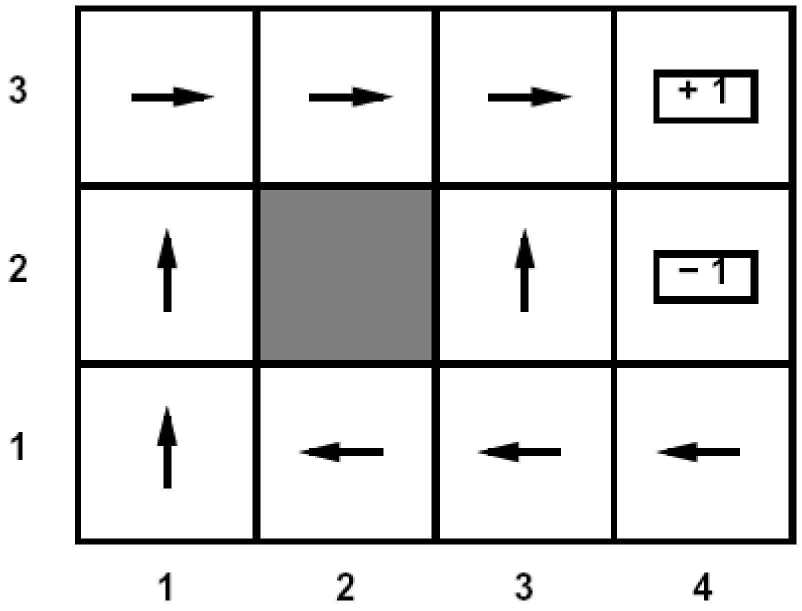
$$\sum_t r_t \quad \text{or} \quad \sum_t \gamma^t r_t$$



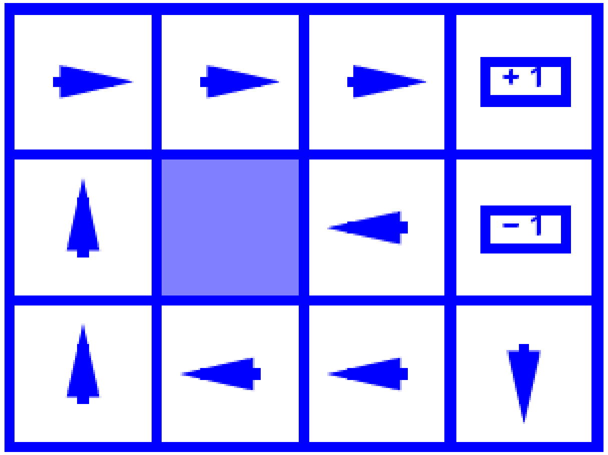
Solving MDPs

- In deterministic single-agent search problem, want an optimal plan, or sequence of actions, from start to a goal
- In an MDP, we want an optimal policy π(s)
  - A policy gives an action for each state
  - Optimal policy maximizes expected if followed
  - Defines a reflex agent

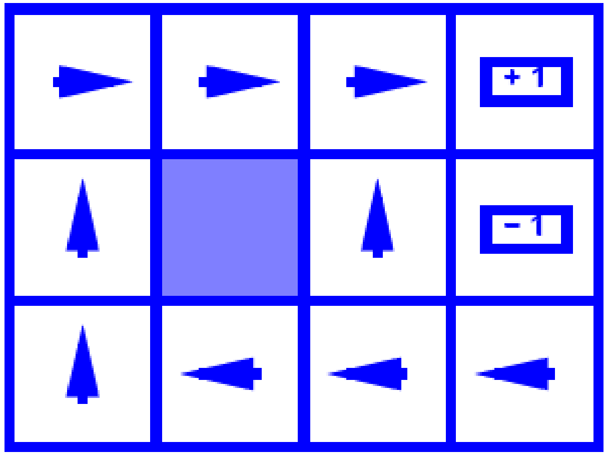
Optimal policy when R(s, a, s') = -0.04 for all non-terminals s



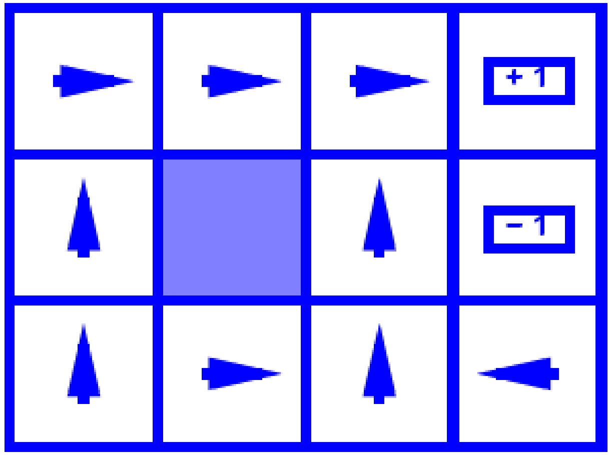
Example Optimal Policies



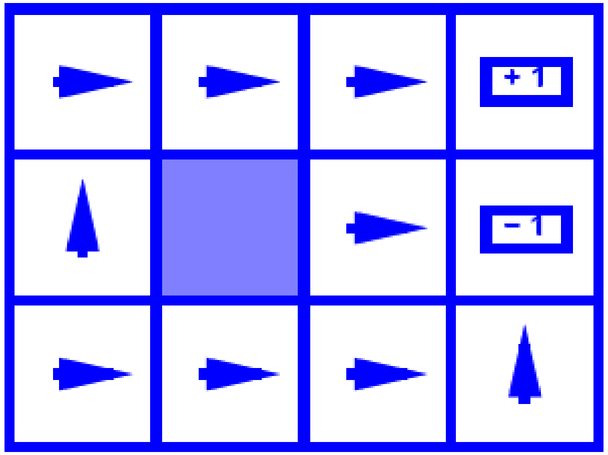
R(s) = -0.01



R(s) = -0.03

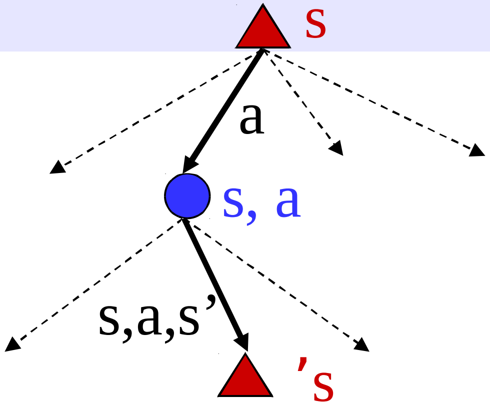


R(s) = -0.4



R(s) = -2.0

Optimal Utilities



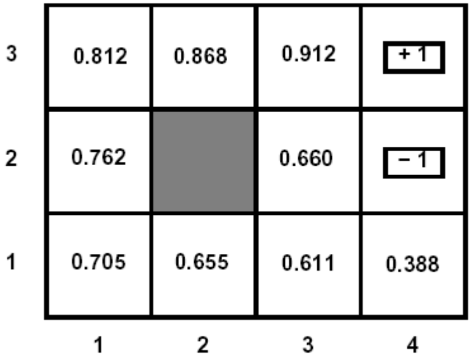
- Fundamental operation: compute the optimal utilities of states s (all at once)

- Why? Optimal values define optimal policies!

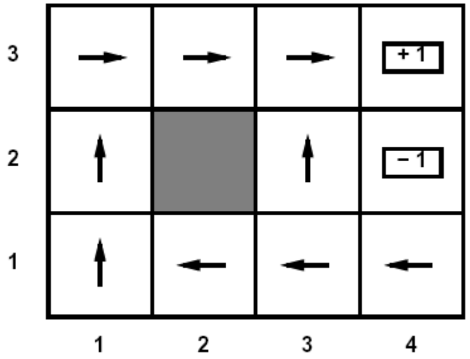
- Define the utility of a state s:
  V^*(s) = expected return starting in s and acting optimally

- Define the utility of a q-state (s,a):
  Q^*(s,a) = expected return starting in s, taking action a and thereafter acting optimally

- Define the optimal policy:
  π^*(s) = optimal action from state s



3	0.812	0.868	0.912	+ 1
2	0.762	▨	0.660	− 1
1	0.705	0.655	0.611	0.388
	1	2	3	4

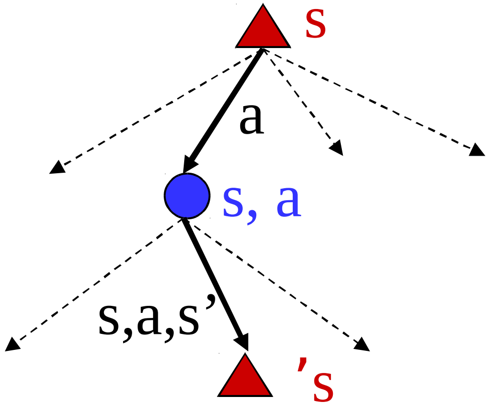


3	→	→	→	+ 1
2	↑	▨	↑	− 1
1	↑	←	←	←
	1	2	3	4

The Bellman Equations

- Definition of utility leads to a simple one-step lookahead relationship amongst optimal utility values:

  Optimal rewards = maximize over first action and then follow optimal policy



- Formally:

$$V^*(s) = \max_a Q^*(s,a)$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s') \right]$$

$$V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s') \right]$$

Solving MDPs / memoized recursion

- Recurrences:

$$V_0^*(s) = 0$$

$$V_i^*(s) = \max_a Q_i^*(s,a)$$

$$Q_i^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_{i-1}^*(s') \right]$$

$$\pi_i(s) = \arg\max_a Q_i^*(s,a)$$

- Cache all function call results so you never repeat work
- What happened to the evaluation function?

Q-Value Iteration

- Value iteration: iterate approx optimal values
  - Start with $V_0^*(s) = 0$, which we know is right (why?)
  - Given V_i^*, calculate the values for all states for depth i+1:

$$V_{i+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \, V_i(s') \right]$$

- But Q-values are more useful!
  - Start with $Q_0^*(s,a) = 0$, which we know is right (why?)
  - Given Q_i^*, calculate the q-values for all q-states for depth i+1:

$$Q_{i+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \, \max_{a'} Q_i(s',a') \right]$$

RL = Unknown MDPs

- If we *knew* the MDP (i.e., the reward function and transition function):
  - Value iteration leads to optimal values
  - Q-value iteration leads to optimal Q-values
  - Will always converge to the truth

- Reinforcement learning is what we do when we *do not know* the MDP
  - All we observe is a *trajectory*
  - $(s_1,a_1,r_1, \quad s_2,a_2,r_2, \quad s_3,a_3,r_3, \quad \dots)$

Q-Learning

- Learn Q*(s,a) values
  - Receive a sample **(s,a,s',r)**
  - Consider your old estimate: $Q(s,a)$
  - Consider your new sample estimate:

$$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q^*(s',a') \right]$$

  - Incorporate the new estimate into a running average:

$$sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha) \left[sample \right]$$

Exploration / Exploitation

- Several schemes for forcing exploration
  - Simplest: random actions (ε greedy)
    - Every time step, flip a coin
    - With probability ε, act randomly
    - With probability 1-ε, act according to current policy
  - Problems with random actions?
    - You do explore the space, but keep thrashing around once learning is done
    - One solution: lower ε over time
    - Another solution: exploration functions

Q-Learning

- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory

- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar states:

[redacted]

  - Very simple stochastic updates:

$$Q(s,a) \leftarrow Q(s,a) + \alpha\,[error]$$

$$w_i \leftarrow w_i + \alpha\,[error]\,f_i(s,a)$$

Inverse RL *and* Apprenticeship Learning

Inverse RL: Task

- Given:
  - measurements of an agent's behavior over time, in a variety of circumstances
  - if needed, measurements of the sensory inputs to that agent
  - if available, a model of the environment.
- Determine: the reward function being optimized
- Proposed by [Kalman68]
- First solution, by [Boyd94]

Why inverse RL?

- Computational models for animal learning
  - “In examining animal and human behavior we must consider the reward function as an unknown to be ascertained through empirical investigation.”
- Agent construction
  - “An agent designer […] may only have a very rough idea of the reward function whose optimization would generate 'desirable' behavior.”
  - eg., “Driving well”
- Multi-agent systems and mechanism design
  - learning opponents’ reward functions that guide their actions to devise strategies against them

IRL from Sample Trajecto

Warning: need to be careful to avoid trivial solutions!

- Optimal policy available throug trajectories (eg., driving a car)
- Want to find *Reward* function that makes this policy look *as good as possible*
- Write $R_{\mathbf{w}}(s) = \mathbf{w}\phi(s)$ so the reward is linear

  and $V_{\mathbf{w}}^{\pi}(s_0)$ be the value of the starting state

$$\max_{\mathbf{w}} \sum_{k=1}^{K} f\left(V_{\mathbf{w}}^{\pi^*}(s_0) - V_{\mathbf{w}}^{\pi_k}(s_0)\right)$$

How good does the "optimal policy" look?

How good does the some other policy look?

[Ng+Russell, ICML00]

Apprenticeship Learning via IRL

- For $t = 1,2,\ldots$
  - Inverse RL step:
    Estimate expert's reward function $R(s)= w^T\phi(s)$ such that under $R(s)$ the expert performs better than all previously found policies $\{\pi_i\}$.
  - RL step:
    Compute optimal policy π_t for the estimated reward w

[Abbeel+Ng, ICML04]

Car Driving Experiment

- No explicit reward function at all!
- Expert demonstrates proper policy via 2 min. of driving time on simulator (1200 data points).
- 5 different “driver types” tried.
- Features: which lane the car is in, distance to closest car in current lane.
- Algorithm run for 30 iterations, policy hand-picked.
- Movie Time! (Expert left, IRL right)

[Abbeel+Ng, ICML04]

“Nice” driver

“Evil” driver

Maxent IRL

Distribution over trajectories:

$$\mathbf{P(\zeta)}$$

Match the reward of observed behavior:

$$\sum_{\zeta} P(\zeta)\, f_{\zeta} = f_{\mathbf{dem}}$$

Maximizing the **causal entropy** over trajectories given stochastic outcomes:

$$\max \mathbf{H(P(\zeta)||O)}$$

(Condition on random uncontrolled outcomes, but only **after** they happen)

As uniform as possible

[Ziebart+al, AAAI08]

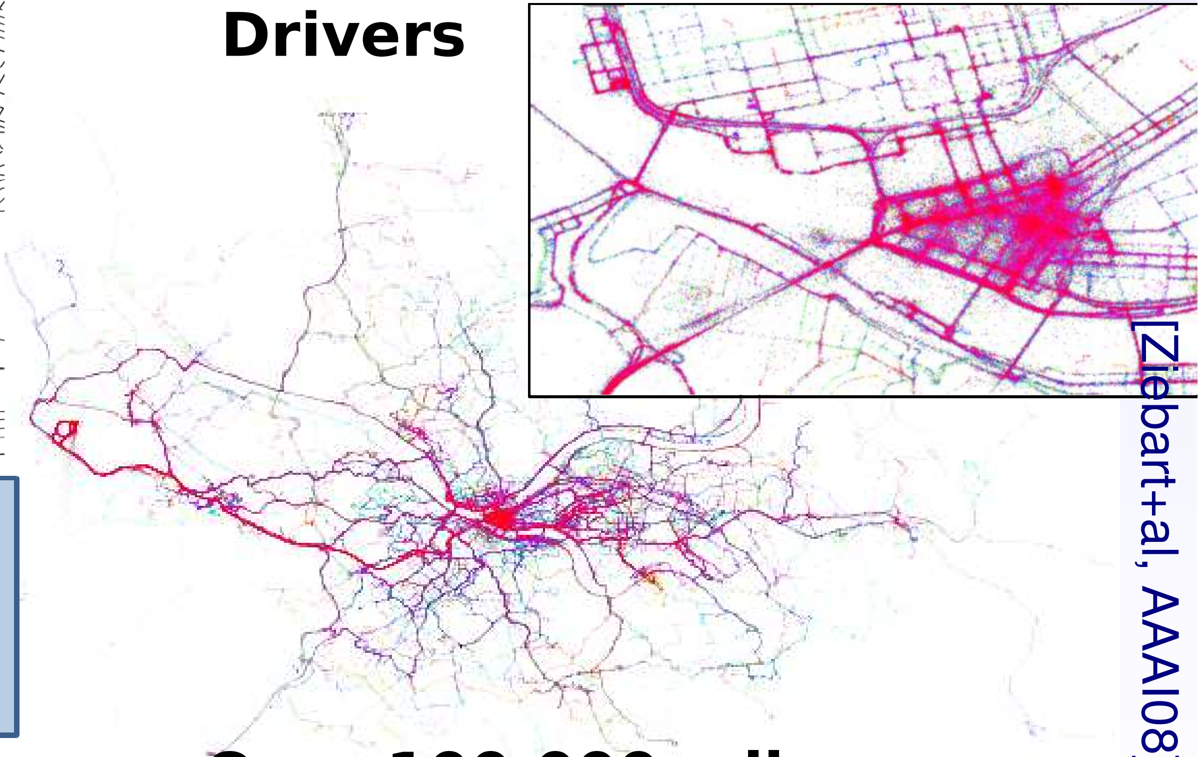
Data collection

CAUTION!
CAUTION!

Length
Speed
Road
Type
Lanes

Accidents
Construction
Congestion
Time of day

25 Taxi
Drivers



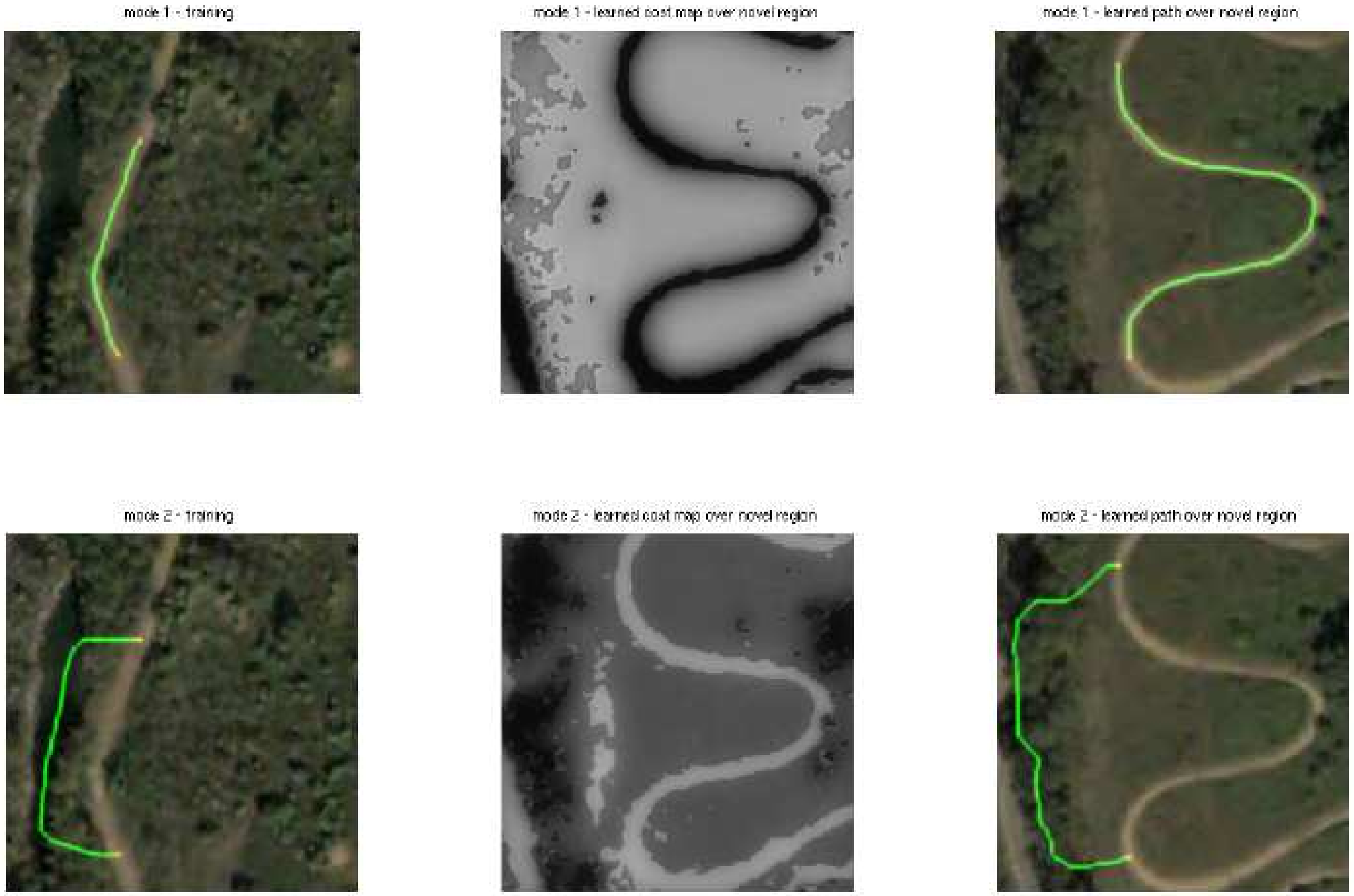
Over 100,000 miles

[Ziebart+al, AAAI08]

Predicting destinations....

Inverse Optimal Control

Planning as structured prediction


mode 1 - training
mode 1 - learned cost map over novel region
mode 1 - learned path over novel region
mode 2 - training
mode 2 - learned cost map over novel region
mode 2 - learned path over novel region

[Ratliff+al, NIPS05]

Maximum margin planning

- Let μ(s,a) denote the probability of reaching q-state (s,a) under current model w

$$\max_{\mathbf{w}} \; \text{margin} \quad \textit{s.t.} \quad \text{planner run with w yields human output}$$

$$\min_{\mathbf{w}} \; \frac{1}{2}\|\mathbf{w}\|^2 \quad \textit{s.t.} \quad \mu(s,a)\,\mathbf{w}\cdot\phi(x_n,s,a) - \hat{\mu}(s,a)\,\mathbf{w}\cdot\phi(x_n,s,a) \geq 1 \;, \quad \forall n,s,a$$

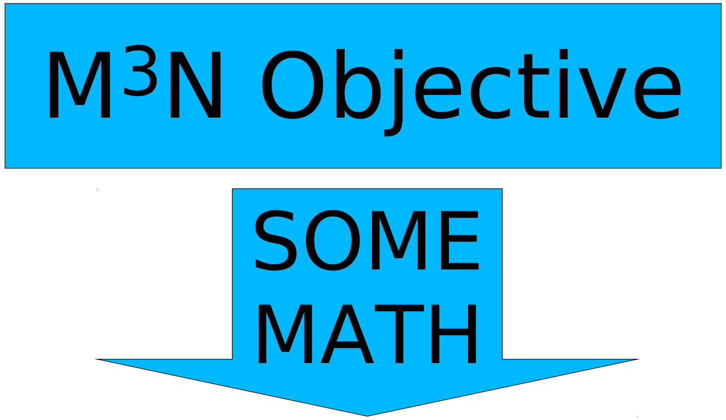
Q-state visitation frequency by human

Q-state visitation frequency by planner

All trajectories, and all q-states

[Ratliff+al, NIPS05]

Optimizing MMP



M3N Objective

SOME MATH

- For n=1..N:
  - Augmented planning:
    Run A* on current (augmented) cost map
    to get q-state visitation frequencies $\mu(s,a)$
  - Update: $\mathbf{w} = \mathbf{w} + \sum_s \sum_a \left[\hat{\mu}(s,a) - \mu(s,a) \right] \phi(x_n, s, a)$
  - Shrink: $\mathbf{w} = \left(1 - \frac{1}{CN} \right) \mathbf{w}$

[Ratliff+al, NIPS05]

Maximum margin planning movies

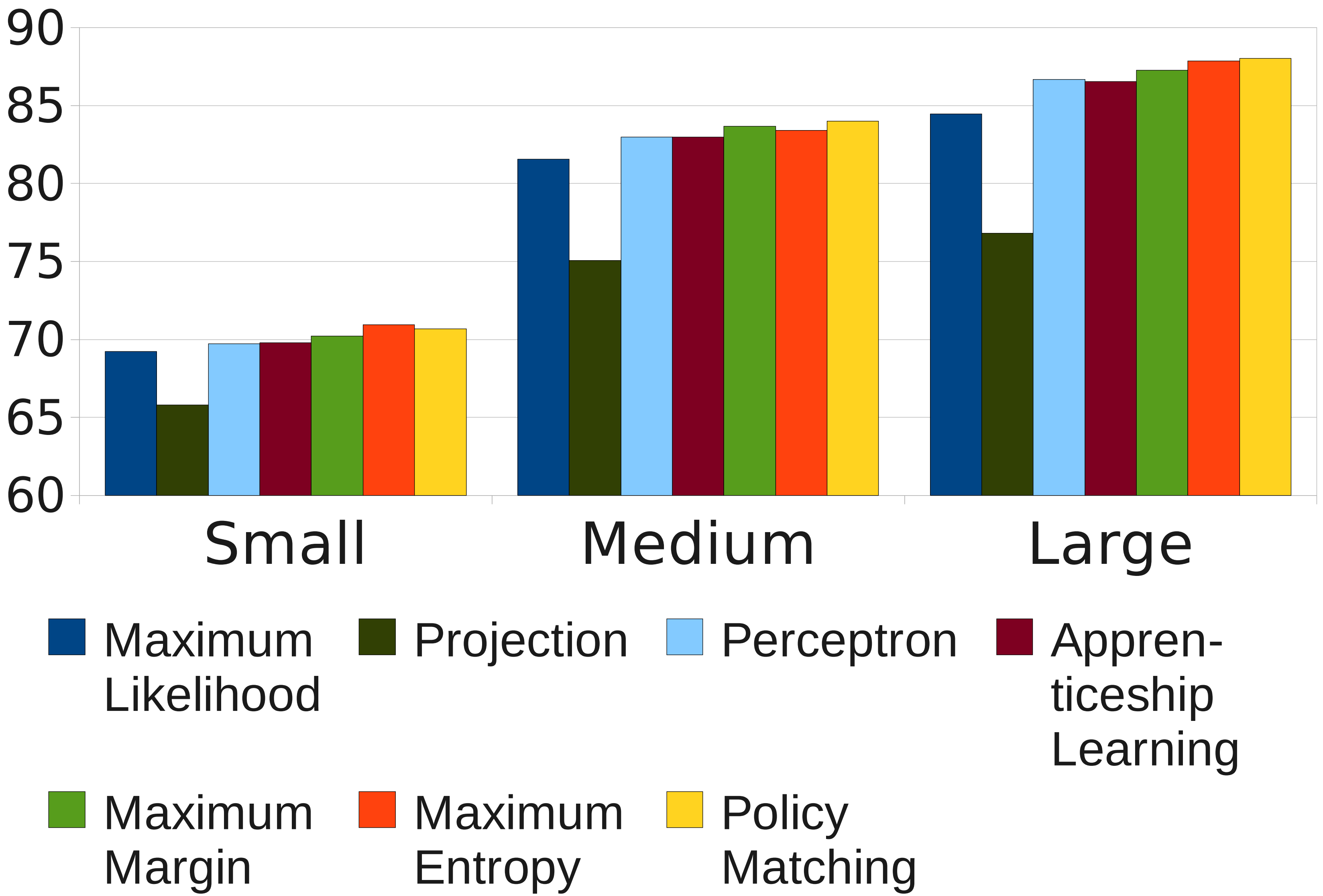
[Ratliff+al, NIPS05]

Parsing via inverse optimal control

- State space = all partial parse trees over the full sentence labeled “S”
- Actions: take a partial parse and split it anywhere in the middle
- Transitions: obvious
- Terminal states: when there are no actions left
- Reward: parse score at completion

[Neu+Szepevari, MLJ09]

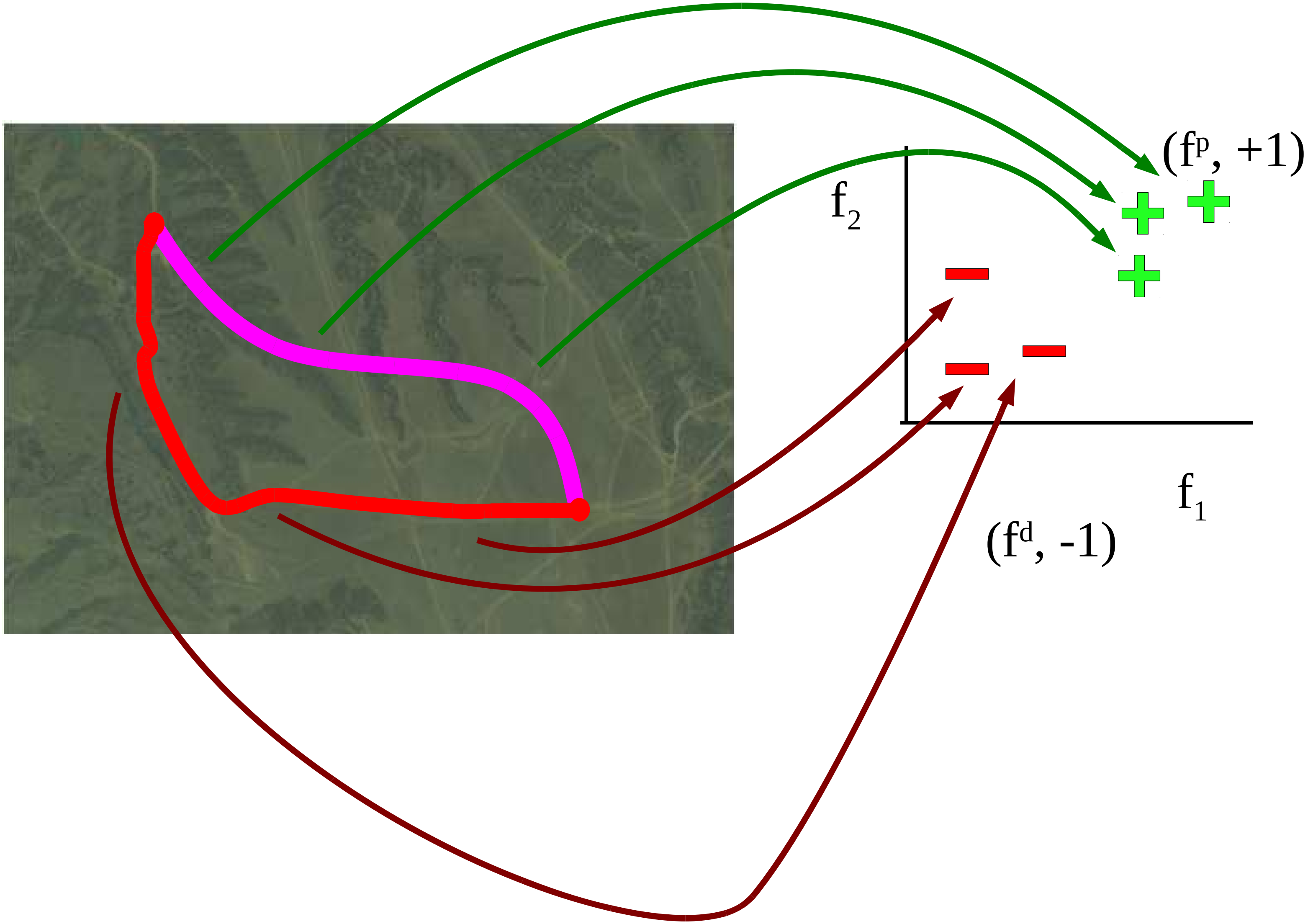
Parsing via inverse optimal control



[Neu+Szepevari, MLJ09]

Learning to Search

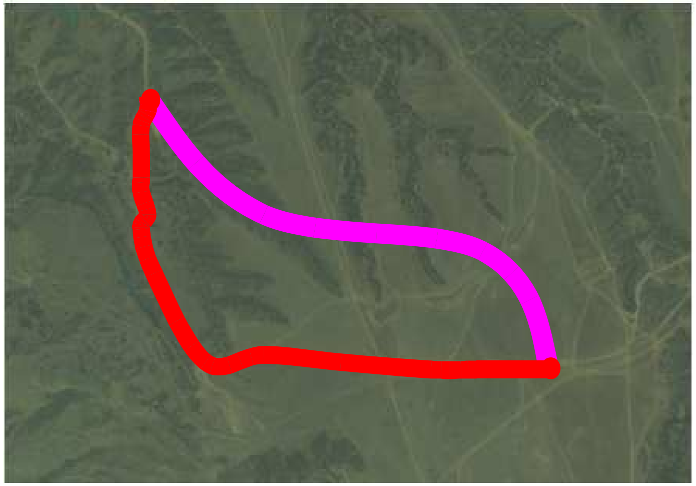
Learning to search



[Ratliff+al, AutRobots09]

Learch

```
Until converged do:
    Initialize modification set to empty
    For each example, add cost function modifications:
        Make loss-augmented prediction using current cost
        Update data set: Label desired feature vector as -1
            and predicted feature vector as +1
    Generalize using a least-squares regression
    Add it to the current cost function
```



[Ratliff+al, AutRobots09]

Discussion

Relationship between SP and IRL

- Formally, they're (nearly) the same problem
  - See humans performing some task
  - Define some loss function
  - Try to mimic the humans
- Difference is in philosophy:
  - (I)RL has little notion of beam search or dynamic programming
  - SP doesn't think about separating reward estimation from solving the prediction problem
  - (I)RL has to deal with stochastiticity in MDPs

Important Concepts

- Search and loss-augmented search for margin-based methods
- Bold versus local updates for approximate search
- Training on-path versus off-path
- Stochastic versus deterministic worlds
- Q-states / values
- Learning reward functions vs. matching behavior

Hal's Wager

- Give me a structured prediction problem where:
  - Annotations are at the lexical level
  - Humans can do the annotation with reasonable agreement
  - You give me a few thousand labeled sentences
- Then I can learn reasonably well...
  - ...using one of the algorithms we talked about
- Why do I say this?
  - Lots of positive experience
  - I'm an optimist
  - I want your *counter-examples!*

Open problems

- How to do SP when argmax is intractable….
  - Bad: simple algorithms diverge [Kulesza+Pereira, NIPS07]
  - Good: some work well [Finley+Joachims, ICML08]
  - And you can make it fast! [Meshi+al, ICML10]
- How to do SP with delayed feedback (credit assignment)
  - Kinda just works sometimes [D, ICML09; Chang+al, ICML10]
  - Generic RL also works [Branavan+al, ACL09; Liang+al, ACL09]
- What role does structure actually play?
  - Little: only constraints outputs [Punyakanok+al, IJCAI05]
  - Little: only introduces non-linearities [Liang+al, ICML08]
  - Lots: ???

Things I have no idea how to solve...

all : (a → Bool) → [a] → Bool

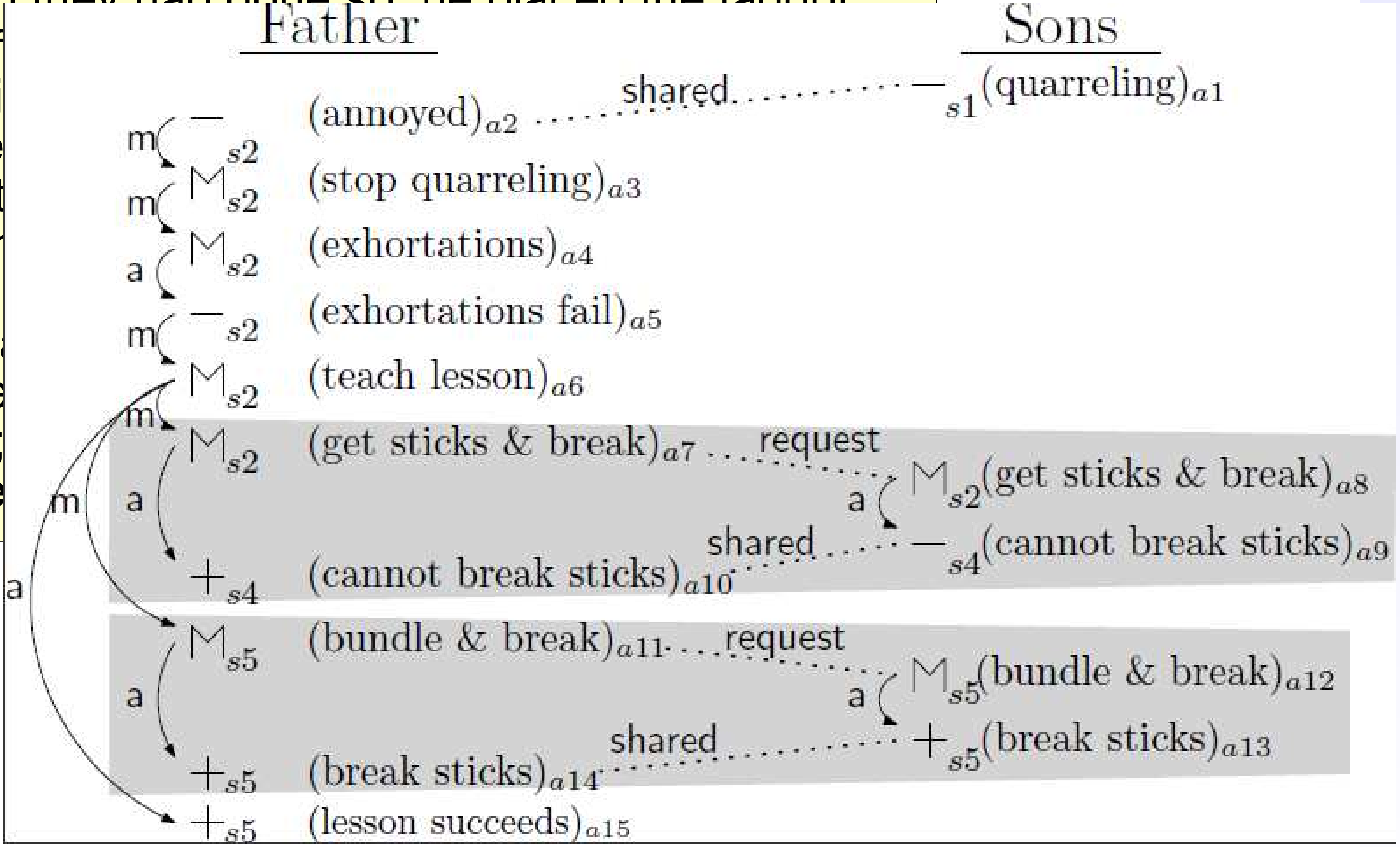
Applied to a predicate and a list, returns `` `True' `` if all elements of the list satisfy the predicate, and `` `False' `` otherwise.

```
%module main:MyPrelude
  %data main:MyPrelude.MyList aadj =
    {main:MyPrelude.Nil;
     main:MyPrelude.Cons aadj ((main:MyPrelude.MyList aadj))};
  %rec
  {main:MyPrelude.myzuall :: %forall tadA . (tadA ->
                                             ghczmprim:GHCziBool.Bool)
                                            ->
                                            (main:MyPrelude.MyList tadA) ->
                                            ghczmprim:GHCziBool.Bool =
     \ @ tadA
       (padk::tadA -> ghczmprim:GHCziBool.Bool)
       (dsddE::(main:MyPrelude.MyList tadA)) ->
         %case ghczmprim:GHCziBool.Bool dsddE
         %of (wildB1::(main:MyPrelude.MyList tadA))
           {main:MyPrelude.Nil ->
              ghczmprim:GHCziBool.True;
            main:MyPrelude.Cons
            (xadm::tadA) (xsadn::(main:MyPrelude.MyList tadA)) ->
              %case ghczmprim:GHCziBool.Bool (padk xadm)
              %of (wild1Xc::ghczmprim:GHCziBool.Bool)
                {ghczmprim:GHCziBool.False ->
                   ghczmprim:GHCziBool.False;
                 ghczmprim:GHCziBool.True ->
                   main:MyPrelude.myzuall @ tadA padk xsadn}}};
```

```
all p
all p
  if p
    the
    els
```

Things I have no idea how to solve...

(s1) A father had a family of sons who were perpetually quarreling among themselves. (s2) When he failed to heal their disputes by his exhortations, he determined to give them a practical illustration of the evils of disunion; and for this purpose he one day told them to bring him a bundle of sticks. (s3) When they had done so, he placed the faggot
into the hands of
them to break it i
strength, and we
the faggot, took t
again put them in
them easily. (s6)
“My sons, if you a
other, you will be
of your enemies;
you will be broke


Father
Sons
$(\text{annoyed})_{a2}$
shared
$-_{s1}(\text{quarreling})_{a1}$
m
$-_{s2}$
M_{s2}
$(\text{stop quarreling})_{a3}$
m
M_{s2}
$(\text{exhortations})_{a4}$
a
$-_{s2}$
$(\text{exhortations fail})_{a5}$
m
M_{s2}
$(\text{teach lesson})_{a6}$
m
M_{s2}
$(\text{get sticks \& break})_{a7}$
request
$\text{M}_{s2}(\text{get sticks \& break})_{a8}$
a
m
a
shared
$-_{s4}(\text{cannot break sticks})_{a9}$
a
$+_{s4}$
$(\text{cannot break sticks})_{a10}$
M_{s5}
$(\text{bundle \& break})_{a11}$
request
$\text{M}_{s5}(\text{bundle \& break})_{a12}$
a
a
shared
$+_{s5}(\text{break sticks})_{a13}$
$+_{s5}$
$(\text{break sticks})_{a14}$
$+_{s5}$
$(\text{lesson succeeds})_{a15}$

Software

- Sequence labeling

Mallet	http://mallet.cs.umass.edu
CRF++	http://crfpp.sourceforge.net

- Search-based structured prediction

LaSO	http://hal3.name/TagChunk
Searn	http://hal3.name/searn

- Higher-level “feature template” approaches

Alchemy	http://alchemy.cs.washington.edu
Factorie	http://code.google.com/p/factorie

Summary

- Structured prediction is *easy* if you can do argmax search (esp. loss-augmented!)
- Label-bias can kill you, so iterate (Searn)
- Stochastic worlds modeled by MDPs
- IRL is all about learning reward functions
- IRL has fewer assumptions
  - More general
  - Less likely to work on easy problems
- We're a long way from a complete solution
- Hal's wager: we can learn pretty much anything

Thanks! Questions?

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See also:

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