Modeling the growth and competition of *Rhyzopertha dominica* and *Ryzaephilus surinamensis*

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1 Abstract

Competition for limited resources is an important factor that influences population dynamics in animal species. Competition comes in two types: intraspecies, in which animals compete with members of their own species, and interspecies, in which animals compete with members of their own species and of another species. We investigate both types of competition for the pests Rhyzopertha dominica (lesser grain borer) and Oryzaephilus surinamensis (saw-tooth grain beetle) to better understand their population dynamics and how competition affects their population dynamics to better improve pest control in stored grain.

2 Introduction

The lesser grain borer (*Rhyzopertha dominica*) and the sawtoothed grain beetle (*Oryzaephilus surinamensis*) are two graminivorous pests that infest stored grain products [1]. *Rhyzopertha dominica* is a pest known for boring into whole, undamaged kernels in agricultural storage areas and in packaged grain products [3]. *Oryzaephilus surinamensis* is a pest that often targets damaged or processed grain products, and its populations are highly influenced by temperature and substrate type [4]. Both species has demonstrated resistance to commonly used insecticides, making them prime targets targets for improved pest management strategies in agriculture and storage environments [5].

We model the competition between the interspecies interaction between the two beetles using logistic growth, and their competition using the Lotka-Volterra model [2]. Using data obtained from three separate experiments (one raising Rhyzopertha dominica alone, one raising Oryzaephilus surinamensis alone, and one raising both species together), we estimate the species' intrinsic growth rate, their carrying capacity in the experimental environment and the competition coefficient that shows how competition affects the growth of each species [2].

3 Methodology

3.1 Data

Data was obtained through conducting three experiments, Oryzaephilus surina- mensis was raised alone, one where Rhyzopertha dominica was raised alone, and one where the two species were raised together. The experiments were conducted under the same controlled conditions: in dark, contained jars at 30 degrees celcius, 70% humidity and containing 10g of cracked wheat as the food source. The wheat was replenished daily to maintain a renewable but limited food supply, and adult beetle counts were taken approximately every 14 days [1].

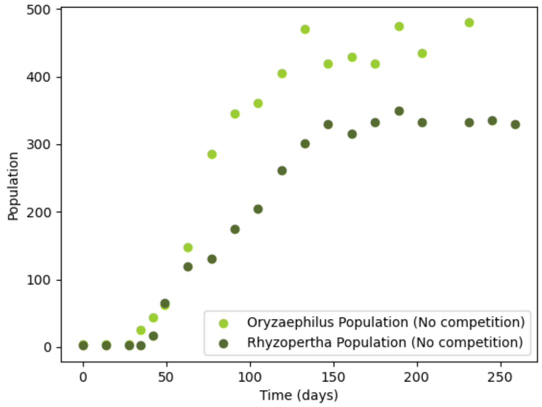


Figure 1: The population of the two species of beetles kept separately over approximately 250 days. The population growth of the two species appears to be logistic, where each population levels out after reaching the carrying capacity of the environment.

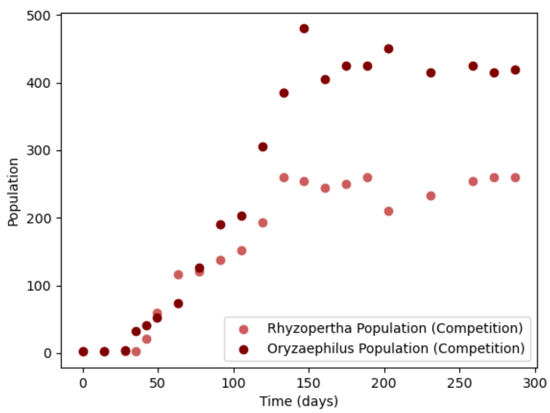


Figure 2: The population of the two species of beetles kept together over approximately 250 days. When competition between the two species occur in a controlled environment, their population also appears to be logistic.

3.2 Logistic Model

We observe that population data in the experiments with no competition look to be logistic [2]. This can be expressed with the following logistic model:

$$\frac{dX}{dt} = r_R X \left(1 - \frac{X}{K_R}\right)$$
$$\frac{dY}{dt} = r_O Y \left(1 - \frac{Y}{K_O}\right)$$

where X is the *Rhyzopertha dominica* population (beetles), Y is the *Oryzaephilus surinamensis* population (beetles), and t is time (days). The parameter r_R and r_O is the intrinsic growth rate of *Rhyzopertha dominica* (beetles per day) and *Oryzaephilus surinamensis* (beetles per day) respectively, and K_R and K_O are the carrying capacity of *Rhyzopertha dominica* (beetles), and *Oryzaephilus surinamensis* (beetles) respectively.

This model describes the logistic growth of each beetle population when raised alone and captures intraspecies competition as each population grows toward its carrying capacity. This serves as a baseline before introducing interspecies competition through the Lotka–Volterra model.

Due to the controlled conditions of the experiment described in the introduction, we assume that the environment both population is kept in is controlled for both models. From this, we can assume that the differences in the carrying capacity and the growth rate in the two beetles are caused by inherent differences between the two species. We also assume that in both experiments, there are no new beetles introduced, and that all individuals come from the original founding population.

3.3 Competitive Lotka-Volterra Model

In the Lotka-Volterra model, we add a competition coefficient to the logistic models to represent interspecies competition, modeling how one species may negatively impact the growth of the other through shared resource use and resource competition [2].

$$\frac{dX}{dt} = r_R X \left(1 - \frac{X}{K_R}\right) - a_R XY$$
$$\frac{dY}{dt} = r_O Y \left(1 - \frac{Y}{K_O}\right) - a_O XY,$$

We alter the previous logistic model to include competition coefficients. a_R describes the effect of competition with Oryzaephilus on the Rhyzopertha population, and a_O describes the effect of Rhyzopertha on the Oryzaephilus population.

In the Lotka-Volterra model, we maintain all assumptions about the environment from the logistic models. Because the two species of beetle are graminivorous, we assume that interaction between the species is strictly competitive (i.e no mutualistic or predatory relationships).

3.4 Logistic Model Parameter Fitting

We first want to fit the logistic models to their respective data set to find the intrinsic rate of growth (r_R and r_O) and carrying capacity (K_R and K_O) of the two beetle species. We do this because estimating the intrinsic growth rates and carrying capacities from the experiments where beetles were kept separately allows us to analyze intraspecies interactions without the influence of interspecies interactions. By fitting the logistic models first, we ensure that any changes in population behavior observed in the competitive model is due to interaction effects, not differences in baseline growth. Because we assumed that environments are the same, the parameters r_R, r_O, K_R, and K_O should remain constant across all experiments, allowing us to use them in the Lotka–Volterra model and focus solely on estimating the effects of interspecies competition.

For each individual differential equation in the logistic model, we first numerically solve the equation to obtain a model that would give the population of each species of beetle at a specific time, given the inputs time in days, the species' intrinsic rate of growth and the carrying capacity of the environment for that specific species. We then estimate the parameters of interest (intrinsic growth rate and carrying capacity) by minimizing the sum of squared errors between this model and the observed data from each experiment where beetles were raised alone using numerical methods.

3.5 Competitive Lotka-Volterra Model Parameter Fitting

As stated in the section above, we assume intrinsic rate of growth (r_R and r_O) and carrying capacity (K_R and K_O) of the two beetle species to be the same across experiments. Therefore, we substitute the parameters previously found from fitting the logistic model into the Lotka-Volterra Model and use data from the experiment where competition was involved to fit the competition coefficients (a_R and a_O).

This process is similar to the process described to fit parameters in the logistic function. However, because the population of the two species are interdependent in a competitive model, we consider the two differential equations as a pair. When fitting the Lotka-Volterra Model, we try and find the best estimates for the competition coefficients (a_R and a_O).

3.6 Sensitivity and Identifiability Analysis

We perform a local sensitivity analysis to see how changes in the growth rate (r_R), carrying capacity (K_R), and competition coefficient (a_R) affect the model fit of the Rhyzopertha dominica population in the competitive Lotka–Volterra model. We do this by perturbing each parameter by 1% above and below its estimated value while keeping other parameters fixed and observing the resulting change in the model output.

We then perform a parameter identifiability analysis using the Pearson correlation. A high correlation (i.e., $R^2 > 0.85$) between two sensitivities suggests the parameters are not identifiable, and that the two variables should not be estimated simultaneously. This gives us ground to justify the need for conducting three experiments, rather than using one experiment to estimate all six parameters.

4 Results

We first fit the logistic function to the dataset using the methodology described above. We then use the `solve_ivp` function in python's `scipy` library to numerically solve each differential equation, and use the `minimize` function to estimate r_R, r_O, K_R and K_O that minimize the sum of squared errors between our fitted model and the observations. We then graphed the two models alongside their observed data to evaluate the accuracy of our estimates.

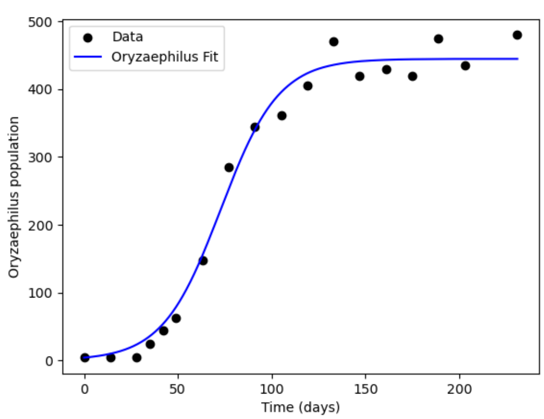


Figure 3: Logistic fit vs Observed Data for the non-competitive Oryzaephilus surinamensis experiment

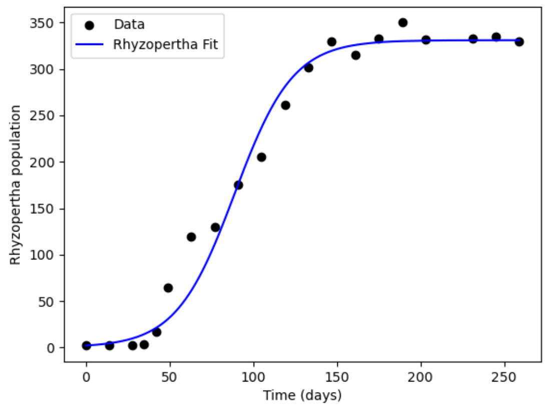


Figure 4: Logistic fit vs Observed Data for the non-competitive Rhyzopertha dominica experiment

The model fits for both populations look to be accurate, being able to portray both the drastic increase in population between day 50 to around day 100, and the population leveling off into a steady state afterwards. This corresponds to our high R^2 values, 0.9807 for the logistics model of the Rhyzopertha dominica population and 0.9871 for the logistics model of the Oryzaephilus surinamensis population. From this fitting, we obtain the following values of r_R, r_O, K_R and K_O.

$$r_O = 0.0646 \text{ beetles/day}, \quad K_O = 444.741 \text{ beetles}$$
$$r_R = 0.0573 \text{ beetles/day}, \quad K_R = 330.992 \text{ beetles}$$

Because the intrinsic rate of growth and carrying capacity are inherent to each species and does not change when competition is introduced, we substitute the fitted coefficients from the logistic model into our Lotka-Volterra model. We then follow the same procedures in python stated above to find the values of a_R and a_O that would best fit our data. We obtain the following values of a_R and a_O.

$$a_R = 0.00003556$$
$$a_O = 0.00002098$$

With all coefficients fitted, we graph the Lotka-Volterra model against the observed data to see how well our model fits.

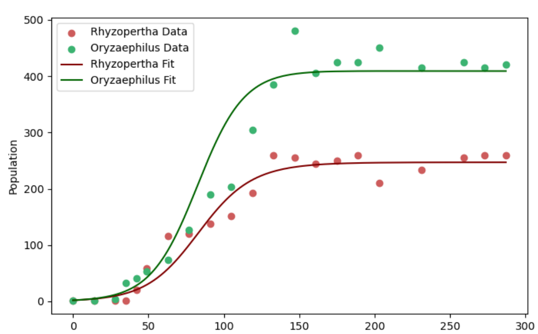


Figure 5: Fit vs Observed Data for the competitive experiment between two species

From Figure 5, we are able to see that the data fits the data well, also being able to describe the overall trend of the two data. We see that the model fits the population of Rhyzopertha dominica better than it fits the population of Oryzaephilus surinamensis, with the curve slightly overestimating the population growth of the latter between approximately days 50 and 100. Overall, however, the model fits look to be accurately representing the trend of a drastically increasing population and the subsequent leveling off. This corresponds with high R^2 values, 0.9529 for the Rhyzopertha dominica population and 0.9440 for the Oryzaephilus surinamensis population.

We then perform a sensitivity analysis for the models. As described in the methodology section, we changed the variables by 1% and observed how the changes affect the model over time.

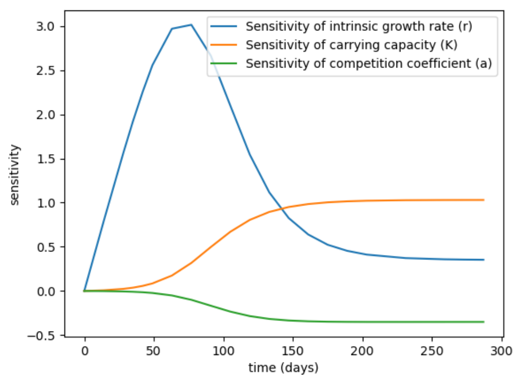
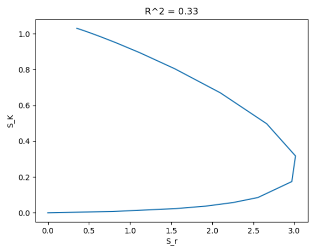


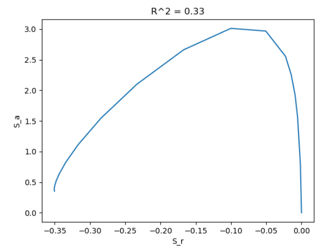
Figure 6: Sensitivity analysis for the intrinsic growth rate, carrying capacity and competition coefficient of the Rhyzopertha dominica population in the competive experiment

The sensitivity analysis shows how the Rhyzopertha dominica population responds to 1% changes in model parameters over time. The intrinsic growth rate (r_R) shows the highest sensitivity early, peaking around day 75. This shows that the intrinsic growth rate has a strong influence on the initial growth in the population. The carrying capacity (K_R) becomes important later on, as its sensitivity rises from 0 near the first 50 days to about 1, and ending higher than the intrinsic growth rate. This makes sense contextually as well, as as the effects of the carrying capacity can be felt strongly once the population has reached a certain population level. The competition coefficient (a_R) turns from 0 to a small negative number at around day 100, suggesting that interspecies competition has a relatively minor negative effect on the population dynamics of the Rhyzopertha dominica population.

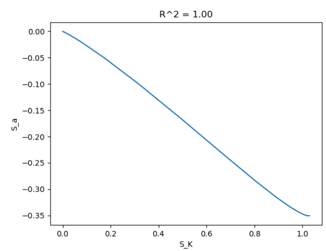
We then performed a parameter identifiability analysis on each of the parameters in this experiment.



(a) S_r vs S_K



(b) S_r vs S_a



(c) S_K vs S_a

Figure 7: Identifiability Analysis on each pair of variables

We see a slight correlation equal to $R^2 = 0.33$ between S_r and both S_K and S_a. However, there is a perfect correlation $R^2 = 1.00$ between S_K and S_a. This means that the variables are collinear, or that an increase in one parameter will cause an increase in the other. As addressed in the methodology section, this means that we cannot estimate all six parameters using one experiment. This is because we will not be able to know whether observed changes in the model output are due to variations in K_R, a_R or a combination of both, meaning that there is no way of interpreting their individual effects on the outcome of the data.

We also want to see whether adding complexity to our model is worth it to get better data. We can capture this using the adjusted Akaike Information Criterion (AICc), which accounts for model accuracy along with the number of parameters used in the model. A lower AICc means that the model has better descriptive power, and an AICc difference of more than 2 indicates a significantly better model (i.e adding the extra parameter is worth the complexity tradeoff for better results).

When calculating the AICc for this model, we are concerned with whether or not the non-competitive logistic model can capture data as well as the competitive model in a competitive setting (i.e seeing if there is a need for the competition coefficient). Thus, we used the competition data to calculate the AICc of the logistic models and the competitive Lotka-Volterra model. The calculated AICcs were as following.

$$\begin{aligned}\mathrm{AAIC}_{\text{Lotka-Volterra}} &= 103.23\\ \mathrm{AAIC}_{\text{Oryzaephilus (Logistic)}} &= 168.70\\ \mathrm{AAIC}_{\text{Rhyzopertha (Logistic)}} &= 178.93\end{aligned}$$

From this result, we can see that the descriptive power of the model is significantly better when adding the competitive coefficient, meaning that a model that captures the two species sharing a habitat should not omit the competition coefficient.

5 Discussion

This model performs well in modeling the processes in this specific experiment and shows that the population dynamics of these insects can be modeled by logistic functions, with a competition coefficient if the environment contains both species. These models have high R^2 values which indicates a strong model fit.

Research about these two species of beetles, as well as more generally on Graminivorous insects is used for pest control, both in agriculture and in manufactured goods. The sawtooth grain beetle and the lesser grain borer are both known for infesting packaged grains and oats products [3][4]. A further study could possibly suggest temperatures that would reduce the insects' intrinsic rate of growth or the insects' carrying capacity to reduce chances of an infestation. This can also have implications for agriculture as well, as the lesser grain borer has shown resistance to insecticide [5]. Wheat fields, where lesser grain borers are found, are vast and therefore hard to control, so further studies on how changing field conditions (e.g crop rotation practices, soil type, soil moisture) affect the intrinsic growth rate of these insects can help reduce their effects on crop yield.

Implicating this model for the real world has some downsides. The model assumes that there is a carrying capacity. However, a wheat field is usually large and thus would have an abundance of food for the two species. There would be a carrying capacity reached, but the time it takes to reach that carrying capacity may be much longer than the crop cycle itself. The experiment also assumed that beetles are not migratory, and no new beetles were introduced throughout the experiment. The saw-toothed grain beetle has "high emigration activity", specifically around climates suitable for wheat growth (28 degrees to 30 degrees celcius) [6]. Models that acknowledge these facts may better predict the population dynamics of these two insects.

6 References

[1] Crombie (1945). On Competition between Different Species of Graminivorous Insects. Proceedings of the Royal Society B: Biological Sciences.
[2] Sanft, Walter (2020). Exploring Mathematical Modeling in Biology Through Case Studies and Experimental Activities. Elsevier Inc.
[3] Doug Johnson, *Insect Pests of Stored Grain: Lesser Grain Borer*, ENTFACT-137, University of Kentucky College of Agriculture, Revised July 2009. `https://entomology.ca.uky.edu/ef137`
[4] Ahmed S. Hashem, Raul Narciso C. Guedes, and Hagar S. Awadalla, *Feeding substrate and temperature interplay determining infestations and losses by the sawtoothed grain beetle (Oryzaephilus surinamensis)*, Journal of Stored Products Research, Volume 94, 2021, Article 101887. `https://doi.org/10.1016/j.jspr.2021.101887`
[5] David K. Weaver and A. Reeves Petroff, *Lesser Grain Borer*, High Plains IPM Guide, a cooperative effort of the University of Wyoming, University of Nebraska, Colorado State University, and Montana State University, 2004. `https://wiki.bugwood.org/HPIPM:Lesser_Grain_Borer`
[6] Kłyś, M. (2012). Emigration activity of the saw-toothed grain beetle OryzaePhilus surinamensis L. (Coleoptera: Silvanidae) in various environmental temperatures. Journal of Plant Protection Research, 52(4), 397-400. https://doi.org/10.2478/v10045-012-0064-x