CS-225: Discrete Mathematics
Quiz 1 Solution

1) Is the following a proposition?

Adding 3 to both sides of x−3 = 37 gives x =42.

Yes, it's a proposition with truth value false.

2) Is the following a proposition? $4^{n+1} \geq 100$

No, as n is unknown.

3) Is the following a proposition? Call me Abraham.

No. It's an imperative sentence.

4) Is the following a proposition? If it does not walk like a duck and it does not talk like a duck, then it is not a duck.

Yes, it's a complex proposition.

5) A conditional statement is logically equivalent to its converse. –

Answer: False.

Question 6

Give the converse, the contrapositive, and the inverse of the following statements-

1) If the number is 64 then it is both even and a power of 4.
2) Having a microscope is a necessary condition for (our) seeing viruses.
3) John will break the world's record for the mile run only if he runs the mile in under four minutes.

Answer:

1) "If the number is 64, then it is both even and a power of 4."

Let p = "The number is 64"
q = "The number 64 is even "
r = " The number 64 is a power of 4"

The statement can be written as $p \rightarrow (q \wedge r)$

- Converse $(q \wedge r) \rightarrow p$: If the number is both even and a power of 4, then it is 64.
- Contrapositive $\sim (q \wedge r) \rightarrow \sim p$: If the number is not even **or** not a power of 4, then it is not 64. (By De Morgan's Law)
- Inverse $\sim p \rightarrow \sim (q \wedge r)$: If the number is not 64, then it is **not** even **or** a **not** power of 4. (By De Morgan's Law)

2) "Having a microscope is a necessary condition for (our) seeing viruses."

Let p = “We can see viruses”
q = “We have a microscope”

The statement can be expressed as $p \rightarrow q$

- Converse $q \rightarrow p$: If we have a microscope, then we can see viruses.
- Contrapositive $\sim q \rightarrow \sim p$: If we do not have a microscope, then we can’t see viruses.
- Inverse $\sim p \rightarrow \sim q$: If we can’t see viruses, then we don’t have a microscope.

3) John will break the world’s record for the mile run only if he runs the mile in under four minutes.

Let p = “John will break the world’s record for the mile run”
q= “John runs the mile in under four minutes”
The statement can be expressed as $p \rightarrow q$

- Converse $q \rightarrow p$: If John runs the mile in under four minutes, then he will break the world’s record for the mile run.
- Contrapositive $\sim q \rightarrow \sim p$: If John does not run the mile in under four minutes, then he will not break the world’s record for the mile run.
- Inverse $\sim p \rightarrow \sim q$:" If John will not break the world’s record for the mile run, then he does not run the mile in under four minutes "

Question 7:

Negate the following statements
i) In order for it to rain it is sufficient that there be clouds.
ii) A necessary condition for this computer program to be correct is that it not produce error messages during translation.
iii) It is neither raining nor sleeting.
iv) If the security code is not entered, then the door will not open.
v) Only if Marc studies will he pass the test.

Answer:

i) In order for it is sufficient that there be clouds.
Let p = “There are clouds” and
q = “It will rain”

If-else: “If there are clouds, then it will rain.”

By De Morgan's Law, its negation is equal to: $(p \wedge \sim q)$

This translates to: “**There are clouds and it will not rain**.”

ii) A necessary condition for this computer program to be correct is that it not produce error messages during translation.

Let p = “The computer program is correct” and
q = “The computer program produces error messages during translation.”

If–then form: If the computer program is correct then it does not produce error messages during translation.

By De Morgan's Law, its negation is equal to: $\sim (p \rightarrow \sim q) \equiv \sim ((\sim p) \vee \sim q) \equiv \equiv p \wedge q$

This translates to: “**the computer program is correct and it produces error messages during translation”.**

iii)It is neither raining nor sleeting.

Let p = “It is raining” and
q = “it is sleeting”

Restated: “It is neither raining nor sleeting” $\sim p \wedge \sim q$

By De Morgan’s laws and the double negation law, this is equivalent to: $\sim(\sim p) \vee \sim(\sim q) \equiv p \vee q$

This translates to: “**It is either raining or sleeting**”

iv) If the security code is not entered, then the door will not open.

Let p = “The security code is entered” and
q= " The door will open"

By De Morgan's Law, its negation is equal to: $\sim (\sim p \rightarrow \sim q) \equiv \sim (\sim(\sim p) \vee \sim q) \equiv \sim (p \vee \sim q) \equiv \sim p \wedge q$

This translates to: “The security code is not entered and the door will open.**”**

v) Only if Marc studies will he pass the test.

Let p = “Marc passes the test” and
q = “Marc studies well”.

In if -else form: If Marc passes the test, then he studies well.

By De Morgan's Law, its negation is equal to: $\sim(p \rightarrow q) \equiv \sim((\sim p) \vee q)) \equiv (p \wedge \sim q)$

This translates to: “**Marc passes the test and he does not study well**”

<u>Question 8:</u>

Determine whether the following statements are true or false -

1) 1 + 1 = 3 if and only if 3 + 4 = 9.
2) 1 + 1 = 2 only if 3 + 4= 9.

<u>Answer:</u>

1) 1 + 1 = 3 if and only if 3 + 4 = 9.

Let p="1 + 1 = 3" and q = "3 + 4 = 9." And is saying $p \leftrightarrow q$

Here, both p is not true and q is not true so, which is **true.**

2) 1 + 1 = 2 only if 3 + 4 = 9. Here p="1+1=2" and q="3+4=9" so, $p \rightarrow q$ and p is true, q is false, so $p \rightarrow q$ is **false**.

Question 9:

The following statements are not equivalent -

The following statements are not equivalent (You may need to apply a rule here . Use [Tables.png]) -

"Alice is smart, or she is smart but honest.

and

"Alice is smart."

Answer: False. By the first absorption law, $(p \vee (p \wedge q)) \equiv p$ they are equivalent.

Question 10:

Let P, Q, and R be the propositions

P: Grizzly bears have been seen in the area.
Q: Hiking is safe on the trail.
R: Berries are ripe along the trail.

Translate the following English sentences into compound logical propositions.

a) It is necessary that berries are ripe for the fact that grizzly bears have seen in the area.

b) Hiking is safe if and only if berries are ripe along the trail or grizzly bears have not been seen in the area.

c) Neither hiking on the trail is safe nor are the berries ripe along the trail.

Answer:

a) It is necessary that berries are ripe for the fact that grizzly bears have been seen in the area.

$P \rightarrow R$ (**not R → P**)

b) Hiking is safe if and only if berries are ripe along the trail or grizzly bears have not been seen in the area.

$Q \leftrightarrow (R \vee \sim P)$

c) Neither hiking on the trail is safe nor the berries are ripe along the trail.

~Q ∧ ~R

Question 11:

(P --> Q) ∨ R   and ~ ((P∧ ~ Q) ∧ ~ R) are logically equivalent using truth table method -

Answer:

P	Q	R	P →	P ∧	(P ∧ ~Q) ∧	~ ((P ∧ ~Q) ∧R	(P → Q) ∨
T	T	T	T	F	F	T	T
T	T	F	T	F	F	T	T
T	F	T	F	T	F	T	T
T	F	F	F	T	T	F	F
F	T	T	T	F	F	T	T
F	T	F	T	F	F	T	T
F	F	T	T	F	F	T	T
F	F	F	T	F	F	T	T

The highlighted columns show that the expressions have the same truth values. So, they are equivalent.

Question 12:

Simplify the following equation (use the tables attached herewith Tables.png) -

~ ((P ∧ ~ Q) ∨ ~ (P ∨ Q))

Answer:

~ ((P ∧ ~ Q) ∨ ~ (P ∨ Q))
≡ ~ ((P ∧~Q) ∨ (~P ∧ ~Q)) (De Morgan's law)
≡ ~ ((~Q ∧P) ∨ (~Q ∧ ~P)) (Commutative law)
≡ ~ ((~Q ∧ (P ∨ ~P)) (Distributive law)
≡ ~ (~Q ∧ T) (Negation law)
≡ ~ (~Q) (Identity law)
≡ Q (Double negation law)
