CS 225 – Discrete Structures in CS
Khoa Phan
Week 1
Assignment 1 – Part 1
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Determine whether the statement forms in 16–24 are logically equivalent. In each case, construct a truth table and include a sentence justifying your answer. Your sentence should show that you understand the meaning of logical equivalence.

22. $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$

Answer:

p	q	r	$p \wedge q$	$p \wedge r$	$q \vee r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	T	F	F	T	T	T
T	F	F	F	F	F	F	F
T	F	F	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	F	F	F	T	F	F
F	T	F	F	F	F	F	F
F	F	F	F	F	F	F	F

$p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ have the same truth values, therefore they are logically equivalent, or distributive law.

Use truth tables to establish which of the statement forms in 40–43 are tautologies and which are contradictions.

42. $((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$

Answer: Contradiction

p	q	r	$\sim p$	$\sim p \wedge q$	$q \wedge r$	$(\sim p \wedge q) \wedge (q \wedge r)$	$\sim q$	$((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$
T	T	T	F	F	T	F	F	F
T	T	F	F	F	F	F	F	F
T	F	F	F	F	F	F	T	F
T	F	T	F	F	F	F	T	F
F	T	T	T	T	T	T	F	F
F	T	F	T	T	F	F	F	F
F	F	T	T	F	F	F	T	F
F	F	F	T	F	F	F	T	F

In 44 and 45, determine whether the statements in (a) and (b) are logically equivalent.

45.

a. Bob is a double math and computer science major and Ann is a math major, but Ann is not a double math and computer science major.

b. It is not the case *that both Bob* and Ann are double math and computer science majors, but it is the case that Ann is a math major and Bob is a double math and computer science major.

p = Bob is a double math and computer science major
q = Ann is a math major
r = Ann is NOT a double math and computer science major

Answer: a. $p \wedge q \wedge r$
b. $(p \wedge r) \wedge (p \wedge q)$

p	q	r	$(p \wedge r)$	$(p \wedge q)$	$p \wedge q \wedge r$	$(p \wedge r) \wedge (p \wedge q)$
T	T	T	T	T	**T**	**T**
T	T	F	F	T	F	F
T	F	F	F	F	F	F
T	F	T	T	F	F	F
F	T	F	F	F	F	F
F	T	T	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

They are logically equivalent as truth values do match.

Construct truth tables for the statement forms in 5–11.

11. $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$

Answer:

p	q	r	$q \to r$	$p \wedge q$	$p \to (q \to r)$	$(p \wedge q) \to r$	$(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
T	F	T	T	F	T	T	T
F	T	T	T	F	T	T	T
F	F	T	T	F	T	T	T
F	T	F	F	F	T	T	T
F	F	F	T	F	T	T	T

13. Use truth tables to verify the following logical equivalences. Include a few words of explanation with your answers.

b. $\sim(p \to q) \equiv p \wedge \sim q$.

Answer:

p	q	$\sim q$	$\sim(p \to q)$	$p \wedge \sim q$.
T	T	F	**F**	**F**
T	F	T	**T**	**T**
F	T	F	**F**	**F**
F	F	T	**F**	**F**

$\sim(p \to q)$ and $p \wedge \sim q$ have the same truth values, therefore they are logically equivalent

15. Determine whether the following statement forms are logically equivalent: $p \to (q \to r)$ and $(p \to q) \to r$

Answer:

p	q	r	$q \to r$	$p \to q$	$p \to (q \to r)$	$(p \to q) \to r$
T	T	T	T	T	T	T
T	T	F	F	T	F	F
T	F	F	T	F	T	T
T	F	T	T	F	T	T
F	T	T	T	T	T	T
F	F	T	T	T	T	T
F	T	F	F	T	**T**	**F**
F	F	F	T	T	**T**	**F**

$p \to (q \to r)$ and $(p \to q) \to r$ are not logically equivalent because their truth values do not match.

20. Write negations for each of the following statements. (Assume that all variables represent fixed quantities or entities, as appropriate.)

a. If P is a square, then P is a rectangle.
Answer: P is a square and P is not a rectangle.

b. If today is New Year's Eve, then tomorrow is January.
Answer: Today is New Year's Eve and tomorrow is not January.

c. If the decimal expansion of r is terminating, then r is rational.
Answer: The decimal expansion of r is terminating and r is not rational.

d. If n is prime, then n is odd or n is 2.
Answer: n is prime and n is not odd and not 2.

e. If x is nonnegative, then x is positive or x is 0.
Answer: x is nonnegative and x is not positive and not 0.

f. If Tom is Ann's father, then Jim is her uncle and Sue is her aunt.
Answer: Tom is Ann's father and Jim is not her uncle and Sue is not her aunt.

g. If n is divisible by 6, then n is divisible by 2 and n is divisible by 3.
Answer: n is divisible by 6 and n is not divisible by 2 and not divisible by 3.

Some programming languages use statements of the form "r unless sn" to mean that as long as s does not happen, then r will happen. More formally:

Definition: If r and s are statements, r unless s means if ∼s then r.

In 37–39, rewrite the statements in if-then form.

38. Ann will go unless it rains.
Answer: If it does not rain, Ann will go.

Rewrite the statements in 40 and 41 in if-then form.

41. Having two 45◦ angles is a sufficient condition for this triangle to be a right triangle.
Answer: If a triangle has two 45◦ angles, then this triangle is a right triangle.

Use the contrapositive to rewrite the statements in 42 and 43 in if-then form in two ways.

43. Doing homework regularly is a necessary condition for Jim to pass the course.

Answer: If Jim does not pass the course, then he did not do homework regularly.

Note that "a sufficient condition for s is r" means r is a sufficient condition for s and that "a necessary condition for s is r" means r is a necessary condition for s. Rewrite the statements in 44 and 45 in if-then form.

45. A necessary condition for this computer program to be correct is that it not produce error messages during translation.

Answer: If this computer program is to be correct then it will not produce error messages during translation.