Assignment 5 Part 1 <u>6.1:</u> 12, 16

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12. Let the universal set R of all Real Numbers and let
$A = \{x \in R \mid -3 \leq x \leq 0\}$, $B = \{x \in R \mid -1 < x < 2\}$, $C = \{x \in R \mid 3 \leq x < 9\}$

Find each of the following.

a) $A \cup B = \{x \in R \mid -3 \leq x < 2\}$

b) $A \cap B = \{x \in R \mid -1 < x \leq 0\}$

c) $A^c = \{x \in R \mid x < -3 \text{ or } x > 0\}$

d) $A \cup C = \{x \in R \mid -3 \leq x \leq 0 \text{ or } 3 \leq x < 9\}$

e) $A \cap C = \emptyset$

f) $B^c = \{x \in R \mid x \leq -1 \text{ or } x \geq 2\}$

g) $A^c \cap B^c = \{x \in R \mid x < -3 \text{ or } x \geq 2\}$

h) $A^c \cup B^c = \{x \in R \mid x \leq -1 \text{ or } x > 0\}$

i) $(A \cap B)^c = \{x \in R \mid x \leq -1 \text{ or } x > 0\}$

j) $(A \cup B)^c = \{x \in R \mid x < -3 \text{ or } x \geq 2\}$

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16. Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, and $C = \{b, c, e\}$

a) Find $\underbrace{A}_{a,b,c} \cup (\underbrace{B \cap C}_{b,c})$, $(\underbrace{A \cup B}_{a,b,c,d}) \cap \underbrace{C}_{b,c,e}$ and $(\underbrace{A \cup B}_{a,b,c,d}) \cap (\underbrace{A \cup C}_{a\ b,c,e})$

$\{a, b, c\}$ $\{b, c\}$ $\{a, b, c\}$

Which of these sets are equal?

$A \cup (B \cap C)$ and $(A \cup B) \cap (A \cup C)$

b) Find $\underbrace{A}_{a,b,c} \cap (\underbrace{B \cup C}_{b,c,d,e})$, $(\underbrace{A \cap B}_{b,c}) \cup \underbrace{C}_{b,c,e}$, and $(\underbrace{A \cap B}_{b,c}) \cup (\underbrace{A \cap C}_{b,c})$

$\{b, c\}$ $\{b, c, e\}$ $\{b, c\}$

Which of these sets are equal?

$A \cap (B \cup C)$ and $(A \cap B) \cup (A \cap C)$

c) Find $(\underbrace{A - B}_{a}) - \underbrace{C}_{b,c,e}$ and $\underbrace{A}_{a,b,c} - (\underbrace{B - C}_{d})$. Are these sets equal?

$\{a, b, c, e\}$ $\{a, b, c, d\}$

$\boxed{\text{NO}}$

Assignment 5 Part 1 6.2: 4, 10, 14

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4. Fill in blanks for proof.

a) $A \cup B \subseteq B$
b) $A \cup B$
c) $x \in B$
d) A
e) or
f) B
g) A
h) B
i) B

10. For all sets A, B, and C, (use Element arguement)

$$(A-B) \cup (C-B) = (A \cap C) - B$$

Show that

$(A-B) \cup (C-B) \subseteq (A \cap C) - B$ and $(A \cap C) - B \subseteq (A-B) \cup (C-B)$

Suppose that $x \in (A-B) \cup (C-B)$

By definition of intersection,

$x \in A$ and $x \notin B$ and $x \in C$ and $x \notin B$

$x \in A$ and $x \in C$ and $\underline{x \notin B}$

By definition of intersection,

$(A \cap C) - B$

Hence. $(A-B) \cup (C-B) \subseteq (A \cap C) - B$

Show that

$(A \cap C) - B \subseteq (A-B) \cup (C-B)$

Suppose that $x \in (A \cap C) - B$

By definition of set difference, $x \in (A \cap C)$ and $x \notin B$

By definition of intersection, $x \in A$ and $x \in C$ and $x \notin B$.

Thus we can write that since $x \notin B$,

$x \in A$ and $x \notin B$ and $x \in C$ and $x \in B$

$x \in (A-B)$ and $x \in (C-B)$

$(A-B) \cup (C-B)$

Hence $(A \cap C) - B \subseteq (A-B) \cup (C-B)$

Thus $\underline{(A-B) \cup (C-B) = (A \cap C) - B}$

Assignment 5 Part 1 6.2: 4, 10, 14

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14. For all sets A, B, and C

if $A \subseteq B$ then $A \cup C \subseteq B \cup C$

Case 1: $x \in A \cup C$ by definition of union,
$x \in A$ or $x \in C$
if A is a subset of B, then
$x \in A = x \in B$ thus
$x \in B$ or $x \in C$

Case 2: $x \in B \cup C$ by definition of Union,
$x \in B$ or $x \in C$

In both cases, B∪C is true which means

$A \cup C \subseteq B \cup C$ is a proper subset

Thus, If $A \subseteq B$ then $A \cup C \subseteq B \cup C$ is true $\square$

Assignment 5 Part 1 6.3: 12, 37, 42

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12. For all sets A, B, C

$$A \cap (B - C) = (A \cap B) - (A \cap C)$$

Case 1:

Suppose $A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$

$x \in A \cap (B - C)$

$x \in A$ and $x \in B$ and $x \notin C$ then

$x \in A$ and $x \in B$ and $x \in A$ and $x \notin C$

can be written as: $(A \cap B) \cap (A - C)$

by definition of set difference $(A \cap B) - (A \cap C)$

Hence $A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$

Case 2:

Suppose $(A \cap B) - (A \cap C) \subseteq A \cap (B - C)$

$x \in (A \cap B) - (A \cap C)$

by defintion of set differance.

$x \in (A \cap B) \cap (A \cap C)^c$

by definiton of distributive law,

$A \cap (B \cap C^c)$

by definition of set difference

$\underline{A \cap (B - C)}$

Hence $(A \cap B) - (A \cap C) \subseteq A \cap (B - C)$

Thus $A \cap (B - C) = (A \cap B) - (A \cap C)$

Assignment 5 Part 1 6.3: 12, 37, 42

37. For all sets A and B, $(B^c \cup (B^c - A))^c = B$

$B = (B^c \cup (B^c \cap A^c))^c$ By set difference law

$B = (B^c \cup (B \cup A)^c)^c$ By Demorgan's law.

$B = (B^c)^c \cap ((B \cup A)^c)^c$ By DeMorgan's Law

$B = B \cap (B \cup A)$ By double complement Law

$B = B$ By Absorption Law

42. $(A - (A \cap B)) \cap (B - (A \cap B))$ simplify.

$= (A \cap (A \cap B)^c) \cap (B \cap (A \cap B)^c)$ by set difference law

$= ((A \cap B)^c \cap A) \cap ((A \cap B)^c \cap B)$ by associative law

$= (A \cap B)^c \cap (A \cap B)$ by distributive law

$= (A \cap B) \cap (A \cap B)^c$ by associative law

$= \phi$ by complement law