Homework 4.6 Problem 12

Wednesday, January 25, 2017 7:26 PM

Set 4.6: 12, 16, 28 Prove by Contradiction.

12. If a and b are rational numbers, $b \neq 0$, and r is an irrational number, then a + br is irrational.

$P(x) =$ a and b are rational, $b \neq 0$ and r is an irrational number.

$Q(x) =$ a + br is irrational $\qquad P(x) \rightarrow Q(x)$

Assume for the sake of contradiction.

$P(x) \wedge \neg Q(x) \equiv$ a and b are rational, $b \neq 0$ and r is rational, a + br is rational

We know that r is irrational and a and b are rational with $b \neq 0$. Let s be the solution to a + br.

$$a + br = s$$

$$r = \frac{s-a}{b}$$

$\frac{s-a}{b}$ is a difference and quotient of rational numbers

$\boxed{\text{so } r = \frac{s-a}{b} \text{ is rational, as is } a - br = s \text{ is also rational}}$

this is a contradiction $\square$

Homework 4.6 Problem 16

Friday, January 27, 2017 8:18 PM

16. For all odd integers a, b, and c, if z is a solution of $ax^2 + bx + c = 0$ then z is irrational.

$P(x) =$ if z is a solution of $ax^2 + bx + c = 0$

$Q(x) =$ then z is irrational

Assume that for the sake of contradiction, z is rational

we know that $z = x = \frac{r}{s}$, with $s \neq 0$

$$a\left(\frac{r}{s}\right)^2 + b\left(\frac{r}{s}\right) + c = 0$$

we know that $\left(\frac{r}{s}\right)^2$ is rational as is $\frac{r}{s}$ as it is a quotient of two integers. Since $\left[a\left(\frac{r}{s}\right)^2 + b\left(\frac{r}{s}\right) + c\right]$ are a product and sum of integers, then it is rational. Thus.

$z = x = \frac{r}{s}$, $ax^2 + bx + c = 0$ is rational.

$\boxed{z \text{ is rational}}$

This is a contradiction to the original statement $\square$

Homework 4.6 Problem 28

Friday, January 27, 2017 8:19 PM

28. For all integers m and n, if mn is even, then m and n are both even or m and n are both odd.

P(x): If mn is even

Q(x): then m and n are both even or m and n are both odd.

Assume for the sake of contradiction that either m or n are even or either m or n are odd

If m is some integer that is even and n is some integer that is odd

$m = 2k$, $n = 2k+1$

$$mn = (2k)(2k+1)$$
$$4k^2 + 2k$$
$$= 2(2k^2+k)$$

Since we know by the definition of products and sums are integers, $(2k^2+k)$ is an integer. Because of this, let $2k^2+k = t$. Thus

$$mn = 2(t)$$

m is even and n is odd and results are even

this is a contradiction. $\square$