Assignment 2 part 2

Thursday, January 19, 2017 10:04 PM

4.1: 32, 61		4.2: 20,25		4.6; 28 (by contraposition)

Prove the statements in 24–34. In each case use only the definitions of the terms and the Assumptions listed on page 146, not any previously established properties of odd and even integers. Follow the directions given in this section for writing proofs of universal statements.

32. If a is any odd integer and b is any even integer, then, 2a + 3b is even.

Proof:
Suppose that *a* is some *[particular but arbitrarily chosen]* integer and *b* is some [*particular but arbitrarily chosen]* integer. *[We must show that 2a + 3b is even.]* By definition of odd, $a = 2r + 1$ and by definition of even, $b = 2s$ for some integers *r* and *s*. Then

$$2a + 3b = 2(2r + 1) + 3(2s)$$

$= 4r + 6s + 2$		by substitution
$= 2(2r + 3s + 1)$		by factoring out 2

Let $t = (2r + 3s + 1)$. Note that *t* is an even integer because of the definition of even. Hence

$2a + 3b = 2t$			where *t* is an integer.

It follows by definition of even that $2a + 3b$ is even. *[This is what we needed to show.]* $\square$ QED

61. Suppose that integers m and n are perfect squares. Then $m + n + 2\sqrt{mn}$ is also a perfect square. Why?

Proof:
Suppose that *m* and *n [particular but arbitrarily chosen]* some integers and are perfect squares. *[We must show that $m + n + 2\sqrt{mn}$ is also a perfect square.]* Since *m* and *n* are perfect squares, $m = r^2$ and $n = s^2$ for some integers *r* and *s*.

$$\boldsymbol{m + n + 2\sqrt{mn}} = r^2 + s^2 + 2\sqrt{(r^2)(s^2)}$$

$= r^2 + 2rs + s^2$			by simplification
$= \boldsymbol{(r \cdot s)^2}$			by factoring

By definition, $(r \cdot s)^2$ is a perfect square. Thus $m + n + 2\sqrt{mn}$ is a perfect square. *[This is what we needed to show.]*

$\square$ QED

In case the statement is false, determine whether a small change would make it true. If so, make the change and prove the new statement. Follow the directions for writing proofs on page 154.

20. Given any two rational numbers r and s with r < s, there is another rational number between r and s. (Hint: Use the results of exercises 18 and 19.)

Proof:

Suppose that *r* and *s* are rational numbers. *[Must prove that there is another rational number between r and s.]* Using the results of problem 18, we know that:

$r < s = \frac{r+s}{2}$ is rational.

Using the results from problem 19, we know that:

$r < s = r < \frac{r+s}{2} < s$ is rational by the definition of transitive law

Since $\frac{r+s}{2}$ is rational, then $\frac{r+s}{2}$ is a rational number between *r* and *s*. *[This is what we needed to show].*

$\square$ QED

Derive the statements in 24–26 as corollaries of Theorems 4.2.1, 4.2.2, and the results of exercises 12, 13, 14, 15, and 17.

25. If r is any rational number, then 3r2 — 2r + 4 is rational.

Proof:

Suppose that *r* is a *[particular but arbitrarily chosen]* rational number. Since we know that r^2 is rational from problem 13, we know that $3r^2$ is also rational.

By definition of Theorem 4.2.3, the product of $-2r$ is rational. Because of this, by the definition of Theorem 4.2.2, $3r^2-2r$ is rational as they are the sum of two rational numbers.

By the definition of Theorem 4.2.1, 4 is rational. Again, Theorem 4.2.2 makes $(3r^2 - 2r) + 4$ rational. Thus,

$3r^2 - 2r + 4$ is rational.

Prove each of the statements in 23–29 in two ways: (a) by contraposition and (b) by contradiction.

28. For all integers m and n, if mn is even then m is even or n is even.

∀ all integers m and n, if mn is even then m is even or n is even.
∀ all integers m and n, if mn is odd then m is odd and n is odd. (contraposition)

Proof:

Suppose that *m* and *n* are odd integers. *[We must show that mn is odd and even.]* By definition of odd,

$m = 2k + 1$ and $n = 2k + 1$ for some integer k. By solution and algebra,

$$mn = (2k + 1)(2k + 1) = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$$

We know that $(2k^2 + 2k)$ is a product and sums of some integer are integers. So mn = 2(integer) + 1, and thus, by definition of odd, mn is odd. *[This is what we wanted to prove.]*