CS-225: Quiz 3 Solution keys

Question 1:

Suppose a,b $\in$ R. Use proof by contradiction to show that if a is rational and ab is irrational, then b is irrational.

Answer:

Let "P = a is rational ", "Q= ab is irrational " and "R = b is irrational". Suppose that $(P \wedge Q) \rightarrow R$, by contradiction let's assume that $(P \wedge Q) \rightarrow \neg R$ also holds. As a and b are rational numbers then they can be written as the ratio of two integers, a = m/n and b = r/s, where $n \neq 0$ and $s \neq 0$. Replacing a and b with their values in ab we get,
ab = m/n *(r/s)= mr /ns

As m, n, r, s all are integers and mr and ns are also integers. It shows that ab can be represented as a ratio of two integers, which means ab is rational. This leads to a contradiction of our original assumption of ab being irrational. That means the assumption $(P \wedge Q) \rightarrow \neg R$ is false and the original assumption is true.

Question 2:

Suppose a $\in$ Z. If a^2 is even, then a is even. Use proof by contradiction.

Answer:

For the sake of contradiction, suppose a^2 is even and a is not even. Then a^2 is even, and a is odd. Since a is odd, there is an integer c for which a = 2c +1. Then $a^2 = (2c+1)^2 = 4c^2 + 4c + 1 = 2(2c^2 + 2c) + 1$, so a^2 is odd. Thus a 2 is even and a^2 is not even, a contradiction.

Question 3:

Let $A = \{x \in Z \mid x = 5a + 2$ for some integer a $\}$ and $B = \{y \in Z \mid y = 10b - 3$ for some integer b$\}$.
Prove or disprove that $B \subseteq A$.

By definition of subset, $B \subseteq A$ if every element of B is also an element of A. We say that δ is an arbitrary element of B. Thus we know that $\delta=10b-3$ where b is an integer. We want to check if δ is also be an element of A. That is, we want to see if $\delta= 5a+2$.

So we set the two equations equal to each other:

$10b-3=5a+2$

Solve for a:

$5a=10b-3-2$

$5a=10b-5$

$a=2b-1$

Now we can see that a is an integer as it is the result of subtracting integers. Now we need to see if we substitute in the expression for a (a member of A), if it is still a member of B.

Take the equation for A and substitute in the expression for a.

$\delta==5(2b-1)+2$

$\delta=10b-5+2$

$\delta=10b-3$

This is our equation for B, so we know that δ is a member of both B and A.

We may say that B is a subset of A.

Thus, $B \subseteq A$. Q.E.D.

Question 4:

True or false: $\{\varphi\} = \varphi$

Answer: False, the first one is a set containing φ , which is different from φ.

Question 5:

True or false: $Z^+ \cap Z^- = Z$

Answer: False, The intersection of Z^+ and Z^- is φ.

Question 6:

True or false: $\{8\} \in \{6, 8, 10\}$

Answer: False, the set {8} is not an element of the set containing the numbers 6, 8 and 10.

Question 7:

True or false: $\{2, 4\} \subseteq \{x \in N \mid x \text{ is even}\}$

Answer: True, every element in the set {2, 4} is an element of the set $\{x \in N \mid x \text{ is even}\}$.

Question 8:

Find the power set of A, where $A = \{x \in Z \text{ (is an integer) } \mid -3 < x < 3\}$.

Answer:

P(A) = { ∅, {-2}, {-1}, {0}, {1}, {2}, {-2 ,-1 },{-2 ,0},{-2 ,1},{-2 ,2},{-1 ,0},{-1 ,1},{-1 ,2},{0,1},{0,2},

{1,2},{-2 ,-1 ,0},{-2 ,-1 ,1},{-2 ,-1 ,2},{-2 ,0,1},{-2 ,0,2},{-2 ,1,2},{-1 ,0,1},{-1 ,0,2},{-1 ,1,2},{0,1,2},

{-2 ,-1 ,0,1},{-2 ,-1 ,0,2},{-2 ,-1 ,1,2},{-2 ,0,1,2},{-1 ,0,1,2},{-2 ,-1 ,0,1,2} }

Question 9:

Let, $A=\{3n \in Z \mid -1 \leq n \leq 2, n \in Z\}$, $B=\{1, 2\}$ and $C = \{\{1,2\}\}$. Find A X (B X C) .

Answer:

$A=\{3n \in Z \mid -1 \leq n \leq 2, n \in Z\}$... n = -1, 0, 1, 2...$A=\{-3, 0, 3, 6\}$

$(B \times C) = \{(1, \{1,2\}), (2, \{1,2\})\}$

$A \times (B \times C) = \{(-3, (1, \{1,2\})), (-3, (2, \{1,2\})), (0, (1, \{1,2\})), (0, (2, \{1,2\})), (3, (1, \{1,2\})), (3, (2, \{1,2\})), (6, (1, \{1,2\})), (6, (2, \{1,2\}))\}$