Permutations and Combinations with Repetition PART 2

Set 9.6 - 4, 12, 18

4. A camera shop stocks eight different types of batteries, one of which is type A7b. Assume there are at least 30 batteries of each type.

a. How many ways can a total inventory of 30 batteries be distributed among the eight different types?

b. How many ways can a total inventory of 30 batteries be distributed among the eight different types if the inventory must include <u>at least</u> four A76 batteries?

c. How many ways can a total inventory of 30 batteries be distributed among the eight different types if the inventory includes <u>at most</u> three A7b batteries?

a. 8 types, 30 of each type

$\binom{r+n-1}{r} = \binom{30+8-1}{30} = \binom{37}{30} = \frac{37!}{30!(37-30)!} = \frac{37!}{30!7!} = \frac{37*36*35*34*33*32*31}{7*6*5*4*3*2}$

= **<u>10295472</u>**

b. 4 'A76', 8 types, 30 of each type

$\binom{r+n-1}{r} = \binom{26+8-1}{26} = \binom{33}{26} = \frac{33!}{26!(33-26)!} = \frac{33*32*31*30*29*28*27*26!}{26!7!}$

$= \frac{33*32*31*30*29*28*27}{5040}$ = **<u>4272048</u>**

c. 3 'A76', 8 types, 30 of each type

1 or 2 or 3

$\binom{r+n-1}{r} = \binom{27+8-1}{27} = \binom{34}{27} = \frac{34!}{27!(34-27)!} = \frac{34*\ 33*32*31*30*29*28*27!}{27!7!} =$

$\frac{34*\ 33*32*31*30*29*28}{5040}$ = **<u>5379616</u>**

In 10–14, find how many solutions there are to the given equation that satisfy the given condition.

12. $y_1 + y_2 + y_3 + y_4 = 30$, each y_i is a nonnegative integer.

$\binom{30+3}{30} = \binom{33}{30} = \frac{33!}{30!(33-30)!} = \frac{33!}{30!3!} = \frac{33*32*31}{6}$ = **<u>5456</u>**

18. A large pile of coins consists of pennies, nickels, dimes, and quarters.

a. How many different collections of 30 coins can be chosen if there are at least 30 of each kind of coin?

b. If the pile contains only 15 quarters but at least 30 of each other kind of coin, how many collections of 30 coins can be chosen?

c. If the pile contains only 20 dimes but at least 30 of each other kind of coin, how many collections of 30 coins can be chosen?

d. If the pile contains only 15 quarters and only 20 dimes but at least 30 of each other kind of coin, how many collections of 30 coins can be chosen?

30 of ea. kind of coin. 4 types of coins.

a. $\binom{r+n-1}{r} = \binom{30+4-1}{30} = \binom{\mathbf{33}}{\mathbf{30}} = \frac{\mathbf{33!}}{\mathbf{30!(33-30)!}} = \frac{\mathbf{33!}}{\mathbf{30!3!}} = \frac{\mathbf{33*32*31}}{\mathbf{6}} =$ **5456**

b. $\binom{r+n-1}{r} = \binom{30+4-1}{30} - \binom{15+3-1}{15} = \binom{33}{30} - \binom{17}{15} =$

$\frac{33!}{30!(33-30)!} - \frac{17!}{15!(17-15)!} = \frac{33!}{30!3!} - \frac{17!}{15!2!} = \frac{33*32*31}{6} - \frac{17*16}{2}$ = 5456 – 136 = **5320**

c. $\binom{r+n-1}{r} = \binom{30+4-1}{30} - \binom{20+3-1}{20} = \binom{33}{30} - \binom{22}{20} =$

$\frac{33!}{30!(33-30)!} - \frac{22!}{20!(22-20)!} = \frac{33!}{30!3!} - \frac{22!}{20!2!} = \frac{33*32*31}{6} - \frac{22*21}{2}$ = 5456 – 231 = **5225**

d. $\binom{r+n-1}{r} = \binom{30+4-1}{30} - \binom{20+3-1}{20} - \binom{15+3-1}{15} = \binom{33}{30} - \binom{22}{3} - \binom{17}{3}$

$= \frac{33!}{30!(33-30)!} - \frac{22!}{20!(22-20)!} - \frac{17!}{15!(17-15)!} = \frac{33!}{30!3!} - \frac{22!}{20!2!} - \frac{17!}{15!2!} = \frac{33*32*31}{6} - \frac{22*21}{2} - \frac{17*16}{2}$

= 5456 – 231 – 136

= **5089**