Homework 6.1 Problem 3

Wednesday, January 25, 2017 10:04 PM

3. Let sets R, S, and T be defined as follows:

$R = \{x \in Z \mid x \text{ is divisible by } 2\}$ $x = \frac{a}{2}$

$S = \{y \in Z \mid y \text{ is divisible by } 3\}$ $y = \frac{a}{3}$

$T = \{z \in Z \mid z \text{ is divisible by } 6\}$ $z = \frac{a}{6}$

a) Is $R \subseteq T$? Explain.
b) Is $T \subseteq R$? Explain.
c) Is $T \subseteq S$? Explain.

a. No because not every number divisible by 2 is divisible by 6.
b. Yes because every number divisible by 6 is divisible by 2.
c. Yes because every number divisible by 6 is divisible by 3.

Homework 6.1 Problem 7

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7. Let $A = \{x \in \mathbb{Z} \mid x = 6a + 4$ for some integer $a\}$,

$B = \{y \in \mathbb{Z} \mid y = 18b - 2$ for some integer $b\}$ and

$C = \{z \in \mathbb{Z} \mid z = 18c + 16$ for some integer $c\}$

Prove or disprove each of the following statements.

a) $A \subseteq B$ b) $B \subseteq A$ c) $B = C$

a. False. Reason is, 10 is in A because setting $a=1$,

$6(1)+4=10$.

If A is a proper subset B, 10 should be true in B aswell.

$10 = 18(1) - 2$

$10 \neq 16$

Thus, A is not a proper subset of B.

b. Suppose that x is a particular but arbitrarily chosen element of B.

[We must show that $x \in A$. By definition, we must show that $x = 6(\text{some integer}) + 4$]

By definition of B, there is an integer b such that $x = 18b - 2$.

[Given that $x = 18b-2$ can we express x as $6(\text{some integer})+4$?]

solving for a

$$6a+4 = 18b-2$$
$$6a+6 = 18b$$
$$a+1 = 3b$$
$$a = 3b-1$$

(a is an integer as a sum of integers)

Let $a = 3b+1$

$$6(3b-1)+4 = x$$
$$18b-6+4 = x$$
$$\underline{18b-2 = x}$$

Thus, $B \subseteq A$.

c. Suppose x is b particular but arbitrarily chosen element of B
[we must prove $x \in C$. By definition, we must show that
$x = 18(\text{some integer}) - 2$]

By definition of B, there is an integer b such that $x = 18b - 2$.

Solving for c:

$$18c + 16 = 18b - 2$$
$$18c = 18b - 18$$
$$c = b - 1$$

Letting $c = b - 1$

$$18(b-1) + 16 = x$$
$$18b - 18 + 16 = x$$
$$\underline{18b - 2 = x}$$

Thus, $B \subseteq C$

$\boxed{\text{True}}$

Suppose x is c particular but arbitrarily chosen element of C
[we must prove $x \in B$. By definition, we must show that
$x = 18(\text{some integer}) + 16$]

By definition of C, there is an integer c such that $x = 18c + 16$

Solving for b

$$18c + 16 = 18b - 2$$
$$18c + 18 = 18b$$
$$c + 1 = b$$

Let $b = c+1$

$$18(c+1)-2 = x$$

$$18c+18-2 = x$$

$$\underline{18c+16 = x}$$

Thus $C \subseteq B$ True

This means $B \subseteq C$ and $C \subseteq B$

$$\underline{B = C}$$

Homework 6.1 Problem 13

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13. Indicate which of the following relationships are true and which are false:

a) $Z^+ \subseteq Q$

True, any integer can be written in form $\frac{n}{1}$

b) $R^- \subseteq Q$

False, there are some negative real numbers that are not rational

c) $Q \subseteq Z$

false, not every rational number is an integer.

d) $Z^- \cup Z^+ = Z$

false, 0 is not included with Z^- and Z^+ but it is in Z

e) $Z^- \cap Z^+ = \phi$

True as $Z^- \cap Z^+$ never include 0 in their sets.

f) $Q \cap R = Q$

True because Q is a subset of R

g) $Q \cup Z = Q$

True because Z is a subset of Q

h) $Z^+ \cap R = Z^+$

True because all Real numbers include Z^+, Z^+ is a subset of R.

i) $Z \cup Q = Z$

false, Z is a subset of Q
no intersection

Homework 4.6 Problem 18

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18. a) is the number 0 in $\emptyset$? why?.

No, the empty set contains nothing, not even 0.

b) is $\emptyset = \{\emptyset\}$?. Why?.

No, because $\{\emptyset\}$ contains an element. It is $|1|$.
an empty set cannot contain anything.

c) is $\emptyset \in \{\emptyset\}$? why?

Yes because $\{\emptyset\}$ is a set containing $\emptyset$ only.

d) is $\emptyset \in \emptyset$? why?.

No because an empty set contains no elements,
not even itself.

Homework 4.6 Problem 33

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33. a) Find $\mathcal{P}(\emptyset)$

$$\mathcal{P}(\emptyset) = \{\emptyset\}$$

b) Find $\mathcal{P}(\mathcal{P}(\emptyset))$

$$\mathcal{P}(\mathcal{P}(\emptyset) = \mathcal{P}\{\emptyset\} = \{\emptyset, \{\emptyset\}\}$$

c) Find $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset)))$

$$\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset) = \mathcal{P}(\{\emptyset, \{\emptyset\}\}) =$$

$$\{\underline{\emptyset}, \underline{\{\emptyset\}}, \underline{\{\{\emptyset\}\}}, \underline{\{\emptyset, \{\emptyset\}\}}\}$$

Homework 4.6 Problem 34

Cartesian Products

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34. Let $A_1 = \{1,2,3\}$, $A_2\ \{u,v\}$, and $A_3\ \{m,n\}$

Find each of the following sets:

a) $A_1 \times (A_2 \times A_3)$

$A_1 \times (A_2 \times A_3) = \{(s,t) \mid s \in A_1 \text{ and } t \in A_2 \times A_3\}$

$A_2 \times A_3 = \{(u,m), (u,n), (v,m), (v,n)\}$

$A_1 \times (A_2 \times A_3)\ \{(1,(u,m)), (1,(u,n)), (1,(v,m)), (1,(v,n)),$
$(2,(u,m)), (2,(u,n)), (2,(v,m)), (2,(v,n)),$
$(3,(u,m)), (3,(u,n)), (3,(v,m)), (3,(v,n))\}$

b) $(A_1 \times A_2) \times A_3$

$(A_1 \times A_2) \times A_3 = \{(s,t) \mid s \in A_1 \times A_2 \text{ and } t \in A_3\}$

$A_1 \times A_2 = \{(1,u), (1,v), (2,u), (2,v), (3,u), (3,v)\}$

$(A_1 \times A_2) \times A_3 = \{((1,u),m), ((1,u),n), ((1,v),m), ((1,v),n), ((2,u),m), ((2,u),n),$
$((2,u),m), ((2,u),n), ((2,v),m), ((2,v),n), ((3,u),m), ((3,u),n),$
$((3,v),m), ((3,v),n)\}$

c) $A_1 \times A_2 \times A_3$

$A_1 \times A_2 \times A_3 = \{(s,t,r) \mid s \in A_1\ t \in A_2 \text{ and } r \in A_3\}$

$A_1 \times A_2 \times A_3 = \{(1,u,m), (1,u,n), (1,v,m), (1,v,n),$
$(2,u,m), (2,u,n), (2,v,m), (2,v,n),$
$(3,u,m), (3,u,n), (3,v,m), (3,v,n)\}$