Quiz over Week 4 Materials – Solutions:

- **Question 1:**

Let $A = \{x \mid -2 < x < 3\}$, $B = \{x \mid -9 \le x \le 1\}$ and $C = \{x \mid 2 \le x < 4$, where x represents an integer number. Determine the sets $(A - C) \cup A$, $(A \cap B) - C$ and $B \cap C^c$.

Answer to Question 1:

$(A - C) \cup A = \{ -1, 0, 1\} \cup \{-1, 0, 1, 2\} = \{-1, 0, 1, 2\} = \{x \mid -1 \le x \le 2\}$

$(A \cap B) - C = \{ -1, 0, 1\} - \{2, 3\} = \{-1, 0, 1\} = \{ x \mid -1 \le x \le 1\}$

$B \cap C^c = \{-9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1\} \cap \{x \mid x<2 \text{ and } x>=4\} = B = \{ x \mid -9 \le x \le 1\}$

- **Question 2:**

Use an element argument to prove the statement:

For all sets A and B, $(A \cap B)^c \subseteq A^c \cap B^c$

Answer to Question 2:

[We must show that $\forall x$, if $x \in (A \cup B)^c$ then $x \in A^c \cap B^c$.]
Suppose $x \in (A \cup B)^c$. [We must show that $x \in A^c \cap B^c$.] By definition of complement, $x \notin (A \cup B)$.
But to say that $x \notin A \cup B$ means that it is false that (x is in A or x is in B).
By De Morgan's laws of logic, this implies that x is not in A and x is not in B,which can be written $x \notin A$ and $x \notin B$.
Hence, $x \in A^c$ and $x \in B^c$ by definition of complement. It follows, by definition of intersection, that x that $x \in A^c \cap B^c$ [as was to be shown]. So $(A \cup B)^c \subseteq A^c \cap B^c$ by definition of subset.

- **Question 3:**

For all sets A and B, simplify the given expression,

$A - (A \cap B)$

Cite a property from Theorem 6.2.2 for every step of the proof.

Answer to Question 3:

$A - (A \cap B)$

$= (A \cap (A \cap B)^c)$ By Set Difference laws,

$= (A \cap (A^c \cup B^c)$ by De Morgan's law,

$= ((A \cap A^c) \cup (A \cap B^c))$ by Distributive laws,

$= (\emptyset \cup (A \cap B^c))$ by Complement laws,

$= (A \cap B^c)$ by Identity laws, (they can keep upto this step)

$= A - B$ Set difference laws

- **Question 4:**

What are the terms a_0, a_1, a_2 and a_3 of the sequence $\{a_n\}$, where a_n equals:

- 1)$a_n = (-1)^{n+1} * n^2$:
- 2)$a_n = 5$

Answer to Question 4:

1) $a_n = (-1)^{n+1} * n^2$: $a_0 = 0$, $a_1 = 1$, $a_2 = -4$, $a_3 = 9$.

2) $a_n = 5$: $a_0 = 5$, $a_1 = 5$ $a_2 = 5$, $a_3 = 5$

- **Question 5:**

Given that, $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$ Use this identity to find a simple expression for $\sum_{k=1}^{n-1} \frac{1}{k(k+1)}$

Answer to question 5:

$$\sum_{k=1}^{n-1} \frac{1}{k(k+1)} = \sum_{k=1}^{n-1} (\frac{1}{k} - \frac{1}{k+1})$$

$= (\frac{1}{1} - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + \quad ----+ (\frac{1}{n-2} - \frac{1}{n-1}) + (\frac{1}{n-1} - \frac{1}{n})$

$= \left(\frac{1}{1} - \frac{1}{n}\right)$

$= \frac{n-1}{n}$

- **Question 6:**

Compute the value of the following sums. (Instructions: Showing your work is necessary and you must use the formula from the attached notes (

TABLE 2 Some Useful Summation Formulae.	
Sum	*Closed Form*
$\sum_{k=0}^{n} ar^k \ (r \neq 0)$	$\frac{ar^{n+1} - a}{r - 1}, r \neq 1$
$\sum_{k=1}^{n} k$	$\frac{n(n+1)}{2}$
$\sum_{k=1}^{n} k^2$	$\frac{n(n+1)(2n+1)}{6}$
$\sum_{k=1}^{n} k^3$	$\frac{n^2(n+1)^2}{4}$
$\sum_{k=0}^{\infty} x^k, \|x\| < 1$	$\frac{1}{1-x}$
$\sum_{k=1}^{\infty} kx^{k-1}, \|x\| < 1$	$\frac{1}{(1-x)^2}$

); an intermediate form will be acceptable. You don't need to calculate the final result.)

Answer to Question 6:

Please remember that, $\sum_{i=j}^{n} a * r^k = a * (\frac{r^{n+1} - r^j}{r-1})$ if, j is > 0

$$a. \sum_{i=3}^{5}(7i+6) = \sum_{i=3}^{5} 7i + \sum_{i=3}^{5} 6 = 7 * \left(\sum_{i=1}^{5} i - \sum_{i=1}^{2} i\right) + 6 * 3 = 7 * \left(\frac{5(5+1)}{2} - 3\right) + 18$$
$$= 7 * 12 + 18 = 102 \ (Ans)$$

$$b. \sum_{j=0}^{3} 4^{j+3} = \sum_{j=0}^{3} 4^3 * 4^j = \frac{4^3 * (4)^{3+1} - 4^3 * 1}{(4) - 1} = \frac{4^7 - 4^3}{3} = 5440 \ (Ans)$$

$$c. \quad \sum_{j=2}^{5} 2 * (-2)^j = \frac{2*(-2)^{5+1} - 2*(-2)^2}{(-2)-1} = \frac{2*(-2)^6 - 2*(-2)^2}{-3} == \frac{2^7 - 2^3}{-3} = -40 \ (Ans)$$