Assignment 6: Part 2: *5.9: 6, 10, 13b, 16*

6. Define a set S recursively as follows:

I. BASE: $a \in S$

II. RECURSION: If $s \in S$, then,

a. $sa \in S$ b. $sb \in S$

III. RESTRICTION: Nothing in S other than objects defined in I and II above.

Use structual Induction to prove that every string in S begins with an a.

$P(s) \coloneqq \forall S,\ s = ax$ for some string $x \in \{a,b\}$

1. $a \in S$, $s = a$,

2. Assume $t \in S$ is a string. t begins with an a. By II, $ta \in S$ begins with a since t begins with a. Conversely, tb would also begin with a since t begins with a. Thus, it is true.

3. By structural induction we can conclude every string in S begins with an a.

Assignment 6: Part 2: *5.9: 6, 10, 13b, 16*

10. Define a set S recursively as follows:

I. BASE: $0 \in S$, $5 \in S$

II. RECURSION: If $s \in S$ then $t \in S$ then,

a. $s + t \in S$ b. $s - t \in S$

III. RESTRICTION: Nothing is in S other than objects defined in I and II above.

Use structural induction to prove that every integer in S is divisible by 5.

$P(s) \equiv \forall S,\ s = 5x$ for all integers.

i. 0 and 5 are divisible by 5.

ii. Assume that s and t are divisible by 5. there are integers a and b, such that $s = 5 \cdot a$, $t = 5 \cdot b$.

$$s + t = (5 \cdot a) + (5 \cdot b) = 5(a + b)$$

$$s - t = (5 \cdot a) - (5 \cdot b) = 5(a - b)$$

both $s+t$ and $s-t$ are divisible by 5.

By structural induction, we can conclude that every integer in S is divisible by 5.

Assignment 6: Part 2: *5.9: 6, 10, 13b, 16*

13b. Consider the set P of parentesis structures defined in Example 5.9.4. Give derivations showing each of the following is in P.

b. (())(())

I. BASE: () is in P

II. RECURSION:

a. If E is in P, so is (E)

b. If E and F is in P, so is EF.

III. RESTRICTION: No configurations of parenthesis are in P other than those derived from I and II above.

1. () is in P
2. By 1 and II, (()) is in P
3. By 2, 1, and II, (())(())

$E \rightarrow (E)$ $\quad EF \rightarrow (E)(F))$

$F \rightarrow (F)$ $\quad$ (())(())

if $E+F = ()$

Hence, by given recursions, (())(()) is proved.

Assignment 6: Part 2: *5.9: 6, 10, 13b, 16*

16. Give a recursive definition for the set of all strings of 0's and 1's for which all the 0's precede all the 1's.

Let S be the set defined recursively by these rules:

i. ϵ in S

ii. If s is in S then so are 0's and 1's

iii. No other string than those that are infered by the first 2 rules are in S.

In all the strings of S the 0's precede 1's.

It is easy to construct any string verifying that propiety w/ rule ii