CS – 225: Discrete Math

Demo Quiz 1

1) $21x + 1 < 5x$ - is a proposition. False, it is not a proposition because it has an unknown variable x in it.
2) 1 divides every integer - The statement is not a proposition. False, it is a proposition.
3) "What time is it?" is a proposition. False, this is an interrogative statement. So, it’s not a proposition.
4) "Adam is a college student" - This statement is a proposition. True, it is a proposition.

5) Consider the propositions:
   p: Juan is a math major.
   q: Juan is a computer science major.
   Use symbolic connectives to represent the proposition “Juan is a math major but not a computer science major.”

Answer: $p \land \neg q$

6) Write each of these propositions in the form “p if and only if q” in English.

a) For you to get an A in this course, it is necessary and sufficient that you learn how to solve discrete mathematics problems.
Answer: You get an A in this course if and only if you learn how to solve discrete mathematics problems.
b) If you read the newspaper every day, you will be informed, and conversely.
Answer: You will be informed if and only if you read the newspaper every day.
c) It rains if it is a weekend day, and it is a weekend day if it rains.
Answer: It rains if and only it is a weekend day.
d) You can see the wizard only if the wizard is not in, and the wizard is not in only if you can see him.
Answer: You can see the wizard if and only if he is not in.

7) State the converse, contrapositive, and inverse of each of these implications.

1) When I stay up late, it is necessary that I sleep until noon.

Answer:

Converse: When I sleep until noon, it is necessary that I stay up late.

Contrapositive: When I don’t sleep until noon, it is necessary that I not stay up late.

Inverse: When I don’t stay up late, it is necessary that I not sleep until noon.

2) If it snows tonight, then I will stay at home.

Answer:

Converse: If I stay at home, it will have snowed tonight.

Contrapositive: If I don’t stay home tonight, it didn’t snow.

Inverse: If it doesn’t snow tonight, then I won’t stay home.

3. (Converting it to if -else form first) If a new hearing is not granted, payment will be made on the fifth.

Converse: If payment will be made on the fifth, a new hearing is not granted.

Contrapositive: If payment will not be made on the fifth, a new hearing is granted.

Inverse: If a new hearing is granted, payment will not be made on the fifth.

8) What is the negation of each of these propositions?

i) To get tenure as a professor, it is sufficient to be world famous.

Answer: p = You are world famous

q = You get a tenure as a professor

In if p then q form = if you are world famous, you get tenure as a professor.

We know that p ->q = $p \rightarrow q = \neg p \vee q$ so the negation is

$\neg(\neg p \vee q) = \neg(\neg p) \wedge \neg q = p \wedge \neg q$ so the answer is, you are world famous and you do not get a tenure as a professor.

ii) If I am lying, I am dying.

Answer:

r: I'm lying. s: I'm dying.

Original statement: $r \rightarrow s$

The negation of a conditional corresponds simply to the one case where the hypothesis is true and the conclusion is false: $r \wedge \neg s$

This translates to: I'm lying and I'm not dying.

iii) Tom’s smartphone has at least 32GB of memory.

Answer: Toms's smartphone has less than 32GB of memory.

iv)If the home team does not win, then it is not raining.

Answer: t: The home team wins. u: It is raining .

Original statement: $\neg r \rightarrow \neg s = \neg(\neg r) \vee \neg s = r \vee \neg s$

By DeMorgan's Law its negation is equal to: $\neg r \wedge s$

This translates to: The home team does not win and it is raining.

v) Being divisible by 2 is a necessary condition that a number is divisible by 10.

p = A number is divisible by 10 q= A number is divisible by 2

In if p then q form : If a number is divisible by 10, then it is divisible by 2.

We know that $p \text{->} q = \neg p \vee q$ so the negation is $\neg(\neg p \vee q) = \neg(\neg p) \wedge \neg q = p \wedge \neg q$

A number is divisible by 10 and a number is not divisible by 2.

9) Construct a truth table for the statement $(p \rightarrow q) \wedge (p \rightarrow r)$

Answer:

p	q	r	p->q	p->r	p → q) ∧ (p → r)
F	F	F	T	T	T
F	F	T	T	T	T
F	T	F	T	T	T
F	T	T	T	T	T
T	F	F	F	F	F
T	F	T	F	T	F
T	T	F	T	F	F
T	T	T	T	T	T

10) Determine whether $[p \wedge (p \rightarrow q)] \rightarrow q$ is a tautology using the tables attached herewith - Tables-1.png.

Answer:

$p \wedge (p \rightarrow q)] \rightarrow q$

$\equiv [p \wedge (\neg p \vee q)] \rightarrow q$ (Rule of Implication)

$\equiv [(p \wedge \neg p) \vee (p \wedge q)] \rightarrow q$ (Distributive Law)

$\equiv [F \vee (p \wedge q)] \rightarrow q$ (Negation Law)

$\equiv (p \wedge q) \rightarrow q$ (Identity)

$\equiv \neg(p \wedge q) \vee q$ (Rule of Implication)

$\equiv (\neg p \vee \neg q) \vee q$ (De Morgan's Law)

$\equiv (\neg p) \vee (\neg q \vee q)$ (Associative Law)

$\equiv (\neg p) \vee T$ (Negation Law)

$\equiv T$ (Domination Law)

It is a tautology .