

## Demo Quiz 6 Solution

### Answer to Question 1:

- a) How many 16-bit strings have at most 4 zero's?
- b) How many 16-bit strings have more than 13 one's?
- c) How many 6-bit strings have zero's than one's?
- d) How many 6-bit strings have at least 3 one's?

a) The number of 16-bit strings that contain at most 4 zero's is the same as the number which contain zero zero, plus the number that contain one zero, plus the number that contain two zero's, plus the number that contain three zero's, plus the number that contain four zero's. That is,

$$C(16,0) + C(16,1) + C(16,2) + C(16,3) + C(16,4)$$

- b) More than 13 one's means 14, 15 and 16 one's.

$$C(16,14) + C(16,15) + C(16,16)$$

- c) More zeros than ones means 4, 5, 6 zeros .

$$C(6,4) + C(6,5) + C(6,6)$$

- d) T least 3 one's meaning 3, 4, 5, 6 ones

$$C(6,3) + C(6,4) + C(6,5) + C(6,6)$$

### Answer to Question 2:

- a) From among a group of 6 men and 9 women, how many four-member committees contain only men or only women?

The number of four-member committees containing only men is  $C(6,4)$  and the number of three-member committees containing only women is  $C(9,3)$  . Since the set of committees containing only men is disjoint from the set of committees containing only women, the number of four-member committees only men or only women is-

$$C(6,4) + C(9,3) = 15 + 84 = 99$$

- b) From among a group of 6 men and 9 women, how many four-members committees contain 2 men and 2 women ?

The committee has 4 people, 2 women and 2 men must be chosen from 9 women and 6 men at the same time. So,

$$C(9,2) * C(6,2) = 36 * 15 = 540$$

### Answer to Question 3:

- 3) How many three digit numbers can be formed from the digits 1,2,3,4,5 and 6, if each digit can only be used once? How many of these are odd numbers? How many are

greater than 330?

There are 6 digits in total. The numbers with 3 digits, with all digits distinct from each other, are the permutations of the 6 digits taken 3 at a time, and therefore there are  $P(6, 3) = 120$  of them.

To be odd, one such number must end with 1, 3, or 5. We can construct all of the odd three digit numbers. For example, the odd numbers ending in 1 are the permutation of the remaining five digits taken two at a time. So the total number of odd numbers is  $3 * P(5, 2) = 60$ .

A number is larger than 330 if its first digit is 4, 5 or 6, or if its first digit is 3 and its second digit is 4, 5 or 6. In the first case, the number of possibilities is 3 times the number of permutations of the remaining 5 digits taken 2 at a time, i.e.  $3 * P(5, 2) = 60$ . In the second case, the number of possibilities is 3 times the number of permutations of the remaining 4 digits taken 1 at a time, that is  $3 * P(4, 1) = 12$ . In total, 72.

#### **Answer to Question 6:**

How many non-negative integer solutions are there to the equation  $x_1 + x_2 + x_3 + x_4 + x_5 = 21$ ? This is the same as the number of ways to choose  $r=21$  objects from a set with  $n=5$  distinct objects with repetition and order does not matter,  $C(21+5-1, 21) = C(25, 21) = C(25, 4) = 12650$ .

#### **Answer to Question 5:**

A bakery produces five different kinds of pastry, one of which is eclairs. Assume there are at least 30 pastries of each kind. How many different selections of 30 pastries contain at most five eclairs? Let  $D( \leq 5)$  is the selection of 30 pastries containing at most 5 eclairs

Then,  $D( \leq 5) = \text{Total number of different selections of 30 pastries from five different kinds} - \text{The selection of 30 pastries containing at least 6 eclairs}$   
$$= C(30+5-1, 30) - C(24+5-1, 24)$$