

5. Indicate which of the following sentences are statements.

- a. 1,024 is the smallest four-digit number that is a perfect square.
- b. She is a mathematics major.
- c. $128 = 2^6$
- d. $x = 2^6$

Answer:

- a. Yes as the statement is true.
- b. No she can be true or false.
- c. No, mathematically false.
- d. No, x can be true or false.

Write the statements in 6–9 in symbolic form using the symbols \sim , \vee , and \wedge and the indicated letters to represent component statements.

10. Let p be the statement “DATAENDFLAG is off,” q the statement “ERROR equals 0,” and r the statement “SUM is less than 1,000.” Express the following sentences in symbolic notation.

- a. DATAENDFLAG is off, ERROR equals 0, and SUM is less than 1,000.
- b. DATAENDFLAG is off but ERROR is not equal to 0.
- c. DATAENDFLAG is off; however, ERROR is not 0 or SUM is greater than or equal to 1,000.
- d. DATAENDFLAG is on and ERROR equals 0 but SUM is greater than or equal to 1,000.
- e. Either DATAENDFLAG is on or it is the case that both ERROR equals 0 and SUM is less than 1,000.

Answer:

- a. $p \wedge q \wedge r$
- b. $p \wedge \sim q$
- c. $p \wedge (\sim q \vee \sim r)$
- d. $\sim p \wedge q \wedge \sim r$
- e. $\sim p \vee (q \wedge r)$

Use De Morgan's laws to write negations for the statements in 25–31.

25. Hal is a math major and Hal's sister is a computer science major.

Answer:

Hal is not a math major or his sister is not a computer science major.

26. Sam is an orange belt and Kate is a red belt.

Answer:

Sam is not an orange belt or Kate is not a red belt.

27. The connector is loose or the machine is unplugged.

Answer:

The connector is not loose and the machine is not unplugged.

28. The units digit of 4^{67} is 4 or it is 6.

Answer:

The units digit of 4^{67} is not 4 and not 6.

29. This computer program has a logical error in the first ten lines or it is being run with an incomplete data set.

Answer:

This computer program does not have a logical error in the first ten lines and is not being run with an incomplete data set.

30. The dollar is at an all-time high and the stock market is at a record low.

Answer:

The dollar is not at an all-time high or the stock market is not at a record low.

31. The train is late or my watch is fast.

Answer:

The train is not late and my watch is not fast.

Use Theorem 2.1.1 to verify the logical equivalences in 50–54.
Supply a reason for each step.

$$52. \sim(p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv \sim p$$

Answer:

$$\equiv (\sim p \wedge \sim(\sim q)) \vee (\sim p \wedge \sim q)$$

$$\equiv (\sim p \wedge q) \vee (\sim p \wedge \sim q)$$

$$\equiv \sim p \wedge (q \vee \sim q)$$

$$\equiv \sim p \wedge t$$

$$\sim p \equiv \sim p$$

by De Morgan's law

by Double negative law

by Distributive law

by Negation law

by Identity law

$$54. (p \wedge (\sim(\sim p \vee q))) \vee (p \wedge q) \equiv p$$

Answer:

$$\equiv (p \wedge (\sim(\sim p) \wedge \sim q)) \vee (p \wedge q)$$

$$\equiv (p \wedge (p \wedge \sim q)) \vee (p \wedge q)$$

$$\equiv p \wedge ((p \wedge \sim q) \vee q)$$

$$\equiv p \wedge (p \wedge (\sim q \vee q))$$

$$\equiv p \wedge (p \wedge t)$$

$$\equiv p \wedge p$$

$$p \equiv p$$

by De Morgan's law

by Double negative law

by Distributive law

by Associative law

by Negation law

by Identity law

by Idempotent law