Demo Quiz 3 Solutions

Question 1:

The sum of any rational number and any irrational number is irrational. Use proof by contradiction.

Answer:

Suppose not. That is, suppose there is a rational number r and an irrational number s such that r + s is rational. [We must deduce a contradiction.] By definition of rational, r = a/b and r + s = c/d for some integers a, b, c, and d with b! = 0 and d! = 0.

By substitution, a/b + s = cd, and so

s = c/d – a/b (by subtracting a/b from both sides)

= bc – ad /bd (by the laws of algebra).

Now bc − ad and bd are both integers [since a, b, c, and d are integers and since products and differences of integers are integers], and bd ! = 0 [by the zero product property]. Hence s is a quotient of the two integers bc − ad and bd with bd != 0.

Thus, by definition of rational, s is rational, which contradicts the supposition that s is irrational. [Hence the supposition is false and the theorem is true.]

Question 2:

Use proof by contradiction to show that for any selection of 3 distinct integers between 0 and 6 that at least one of those numbers will be odd.

Answer:

Let P = "3 distinct integers between 0 and 6" and Q = "at least one of those numbers will be odd". Assume for the sake of contradiction ¬ (P → Q) or P ^ ¬Q meaning that at most none of the 3 distinct integers will be odd or all the numbers will be even. The distinct integers between 0 and 6 are 1, 2, 3, 4, and 5. The only even numbers in this rage are 2 and 4, so the third number should be odd. Hence, our assumption that all the 3 numbers should be even is false and P → Q is true.

Question 3:

Let A = {1, {1}, {1, 2}}. Label each of the following as true of false.
(a) 1 ∈ A.
(b) 1 ⊆ A.
(c) {1} ⊆ A.
(d) {1} ∈ A.
(e) {{1}} ⊆ A.
(f) 2 ∈ A.
(g) {2} ⊆ A.
(h){1, 2} ∈ A.
(i){1, 2} ⊆ A.
j){{1, 2}} ⊆ A.
(k) ∅ ∈ A.

(l) $\emptyset \subseteq A$.

Answer:
a. True
b. False
c. True
d. True
e. True
f. False
g. False
h. True
i. False
j. True
k. False
l. True

Question 4:

Suppose A = {1, 2}, B = {u, v} and C = {p, q}. Find the value of (A X C) X B.

Answer:

A X C = {(1, p), (1, q), (2, p), (2, q)}

(A X C) X B = {((1, p), u), ((1, p), v), ((1, q), u), ((1, q), v), ((2, p), u), ((2, p), v), ((2, q), u), ((2, q), v)}

Question 5:

The set A and B are defined as followed -

$A = \{m \in Z \mid m = 2a \text{ for some integer } a\}$

$B = \{n \in Z \mid n = 2b - 2 \text{ for some integer } b\}$

Show that, $A \subseteq B$.

Answer:

Suppose x is a particular but arbitrarily chosen element of A. [We must show that $x \in B$. By definition of B, this means we must show that $x = 2\cdot(\text{some integer}) - 2$.] By definition of A, there is an integer a such that $x = 2a$. [Given that $x = 2a$, can x also be expressed as $2\cdot(\text{some integer}) - 2$? I.e., is there an integer, say b, such that $2a = 2b - 2$? Solve for b to obtain $b = (2a + 2)/2 = a + 1$. Check to see if this works.] Let $b = a + 1$. [First check that b is an integer.] Then b is an integer because it is a sum of integers. [Then check that $x= 2b - 2$.] Also $2b - 2 = 2(a + 1) - 2 = 2a + 2 - 2 = 2a = x$, Thus, by definition of B, x is an element of B [which is what was to be shown]

Question 6:

Let $C = \{n \in Z^+ \mid 2^n + 1 \text{ is a prime integer}\}$. List the smallest 3 elements of C.

Answer: {1, 2, 4}

n is a positive integer such that 2^n+1 is a prime integer

n=1 → 2^1+1 = 2+1 = 3 (prime)

n=2 → 2^2+1 = 4+1 = 5 (prime)

n=3 → 2^3+1 = 8+1 = 9 (not prime)

n=4 → 2^4+1 = 16+1 = 17 (prime)

n=5 → 2^5+1 = 32+1 = 33 (not prime)

n=6 → 2^6+1 = 64+1 = 65 (not prime)

n=7 → 2^7+1 = 128+1 = 129 (not prime)

n=8 → 2^8+1 = 256+1 = 257 (prime)

C = {1, 2, 4}

Question 7:

Find the power set of {x, y, {x} } .

Answer:

P ({ x, y, {x} }) = { φ , {x} , {y} , {{x}} , {x, y} , {x, {x} } , {y, {x}} , { x, y, {x}} }