Demo Quiz over Week 4 Materials – Solutions:

- **Question 1:**

Let A = {0, 2, 4, 6, 7}, B = {1, 2, 3, 4, 6}, and C = {0, 3, 6, 8, 9} And U be the set of all integers. What are $(A - C) \cap B$, $B \cap C^c$ and $A \cup (B \cup C)$?

Answer to Question 1:

$(A – C) \cap B = \{2, 4, 7\} \cap \{1, 2, 3, 4, 6\} = \{2, 4\}$

$B \cap C^c = \{1, 2, 4\}$

A U (B U C) = {0, 2, 4, 6, 7} U {0, 1, 2, 3, 4, 6, 8, 9} = {0, 1, 2, 3, 4, 6, 7, 8, 9}

- **Question 2:**

For any sets A and B, prove that

$(A - B) - C \subseteq A – C$

Answer to Question 2:

Suppose $x \in (A - B) – C$. By definition of difference, $x \in A$ and $x \in B^c$ and $x \in C^c$. This states that x is an element of set A and not an element of set B and set C. On the other hand, $x \in (A - C)$ can be written as $x \in A$ and $x \in C^c$. This states that x is an element of set A and not an element of set C. Therefore x is in (A - B) – C follows that x is in (A-C). Hence, we can conclude that $(A - B) - C \subseteq A – C$.

- **Question 4:**

Write the first four terms of the following sequences -

a. $a_i = \frac{(-1)^i}{3^i}$, for all integers i >=0

b. $b_j = \frac{4-j}{4+j}$, for all integers j >=0

Answer to Question 4:

a. $a_0 = \frac{(-1)^0}{3^0} = 1$

$a_1 = \frac{(-1)^1}{3^1} = \frac{-1}{3} = -\frac{1}{3}$

$a_2 = \frac{(-1)^2}{3^2} = \frac{1}{9}$

$a_3 = \frac{(-1)^3}{3^3} = \frac{-1}{27} = -\frac{1}{27}$

b. $\mathbf{b_0} = \frac{\mathbf{4-0}}{\mathbf{4+0}} = \mathbf{1}$

$\mathbf{b_1} = \frac{\mathbf{4-1}}{\mathbf{4+1}} = \frac{\mathbf{3}}{\mathbf{5}}$

$\mathbf{b_1} = \frac{\mathbf{4-2}}{\mathbf{4+2}} = \frac{\mathbf{2}}{\mathbf{6}} = \frac{\mathbf{1}}{\mathbf{3}}$

$\mathbf{b_1} = \frac{\mathbf{4-3}}{\mathbf{4+3}} = \frac{\mathbf{1}}{\mathbf{7}}$

- **Question 5:**

a. Write the following as a single summation -

$4 * \sum{}^{n}{}_{i=1}\left(2^i - 3\right) + \sum_{i=1}{}^{n}\left(2^i + 9\,i\right)$

Answer to question 5 a:

$4 * \sum_{i=1}^{n}\left(2^i - 3\right) + \sum_{i=1}^{n}\left(2^i + 9\,i\right)$

$= \sum_{i=1}^{n} 4 * \left(2^i - 3\right) + \sum_{i=1}^{n}\left(2^i + 9\,i\right)$ [Theorem 5.1.1: Generalized distributive law]

$= \sum_{i=1}^{n} (4 * 2^i - 12) + \sum_{i=1}^{n}\left(2^i + 9\,i\right)$

$= \sum_{i=1}^{n} (4 * 2^i - 12 + 2^i + 9\,i)$ [Theorem 5.1.1: $\sum_{i=1}^{n} a_k + \sum_{i=1}^{n} b_k = \sum_{i=1}^{n} (a_k + b_k)$]

$= \sum_{i=1}^{n} (5 * 2^i + 9\,i - 12)$

b. Express the following sequence using summation or notation -

(n-1) + (n-2) + (n-3) + ... +1

Answer to question 5 b:

(n-1) + (n-2) + (n-3) + ... +1

= (n-1) + (n-2) + (n -3) + ...+ (n- (n-1))

$= \sum_{i=1}^{(n-1)} (n - i)$

- **Question 6:**

Compute the value of the following sums. (Instructions: Showing your work is necessary and you must use the formula from the attached notes (

); an intermediate form will be acceptable. You don't need to calculate the final result.)

<u>Answer to Question 6</u>:

a) $\sum_{j=0}^{10} 3 * 2^j = 3 * \sum_{j=0}^{10} 2^j = 3 * \frac{2^{10+1} - 1}{2-1} = 3 * (2^{11} - 1) = 3 * 2047 = 6141$ (Ans)

b) $\sum_{j=1}^{10} (4 * j + 1) = \sum_{j=1}^{10} 4j + \sum_{j=1}^{10} 1 = 4 * \sum_{j=1}^{10} j + \sum_{j=1}^{10} 1 = 4 * \frac{10*(10+1)}{2} + 10 * 1 = 220+10$
$= 230$ (Ans)

c) $\sum_{j=5}^{20} j = \sum_{j=1}^{20} j - \sum_{j=1}^{4} j = \frac{20*(20+1)}{2} - \frac{4*(4+1)}{2} = 10 * 21 - 2 * 5 = 200$ (Ans)

d) $\sum_{j=1}^{10} 3^{j+1} = \sum_{j=1}^{10} 3 * 3^j = 3 * \sum_{j=1}^{10} 3^j = 3 * \frac{3^{10+1} - 3}{3-1} = \frac{3}{2} * 177144 = 265716$ (Ans)

e) $\sum_{j=1}^{n} \frac{1}{j(j+1)}$ page 232-233 of the book (example 5.1.10)

Please remember that, $\sum_{i}^{n} {}^{=j} a * r^k = a * \frac{r^{n+1} - r^j}{r-1}$ if, j is > 0

- **<u>Question 3:</u>**

Construct an algebraic proof that for all sets A and B,

A – (A ∩ B) = A – B.

Cite a property from Theorem 6.2.2 for every step of the proof.

<u>Answer to Question 3:</u>

Suppose A and B are any sets. Then

$A - (A \cap B) = A \cap (A \cap B)^c$	by the set difference law
$= A \cap (A^c \cup B^c)$	by De Morgan's laws
$= (A \cap A^c) \cup (A \cap B^c)$	by the distributive law
$= \emptyset \cup (A \cap B^c)$	by the complement law
$= (A \cap B^c) \cup \emptyset$	by the commutative law for ∪
$= A \cap B^c$	by the identity law for ∪
$= A - B$	by the set difference law.