Assignment 5 *Part 2:* 5.4: 2, 10

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2. Suppose $b_1, b_2, b_3, \ldots$ is a sequence defined as follows:

$b_1 = 4$ $b_2 = 12$

$b_k = b_{k-2} + b_{k-1}$ for all integers $k \geq 3$

Prove that b_n is divisible by 4 for all integers $n \geq 1$.

Base case:

We know that b_1 and b_2 are divisble by 4. This means

$n=1$ and $n=2$

Induction case: $n=k$

Since $n=1$, $n=2$ are true, k must be greater then 2.

$k>2$ there exists an integer m that $1<m<k$.

$b_k = b_{k-2} + b_{k-1}$ by the original definition of $b_1, b_2, b_3 \ldots$

Since $n=k$, $n>2$ and since $n \geq 1$, this is true.

Show that $n=k+1$

$b_k = b_{(k+1)-2} + b_{(k+1)-1} = b_{k-1} + b_k$

$1 \leq k-1 < k$ and $1 \leq k$, which are divisble by 4.

$(k-1)$ is some integer $4r$, and (k) is some integer $4s$

$b_k = 4r + 4s = b_k = 4(r+s)$

this is divisble by 4

$b_k = b_n$

By mathematical induction,

b_n is divisible by 4 for all $n \geq 1$ $\square$

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10. Let $p(n)$ be " <u>n ¢</u> can be obtained using a combination of 3¢ and 5¢ coins. Use strong mathematical induction to prove that $P(n)$ is true for all integers $n \geq$ <u>8.</u> *(Question was changed based off instructor response on Piazza)

$P(8), P(9), P(10), \ldots P(n)$ for all integers $n \geq 8$

Base Case: $n = 8$

$P(8) = 8$ ¢ which can be obtained by $1(5¢) + 1(3¢)$ ✓
$P(9) = 9$ ¢ which can be obtained by $0(5¢) + 3(3¢)$ ✓
$P(10) = 10$ ¢ which can be obtained by $2(5¢) + 0(3¢)$ ✓

Induction step:

i cents can be made with 5¢ and 3¢ coins or $P(n) = 3a + 5b$

Some integer i exist between 8 and n such that
$n \leq i < k$

Show that $n = k+1$

Assume $n \geq 11$. $k+1 \geq 11$, $\underline{(k+1) - 3 \geq 8}$

$[(k+1)-3] + 3 = k+1$ by adding 3¢, we can get $k+1$

$[(11+1)-3] + 3 = 11+1$

$12 = 12$ ✓

if we assume $n = k+1$, $(k+1) - 3 = n - 3$

$n - 3 = 3a + 5b$

$n = 3a + 5b + 3$

$n = 3(a+1) + 5b$

we added the 3¢ here

$\square$