CS225 Final Exam Review

The final exam will cover the materials from week 2 to week 10. The exam questions will be very much in the spirit of the book examples, homework and quiz questions. This material summarizes some important contents and gives some example questions from homework assignments and quizzes as review exercise. The exam will cover only the **topics and the problems** listed below –

Good Luck!!

Unit 1: Non-Inductive Proof Techniques (10%)

- Direct Proof, Contrapositive & Contradiction:

Students should master the technique of direct proof, proof by contraposition and proof by

contradiction.

Example questions from the book and homework assignments - week 2 and 3 and

Demo and actual quiz - 2, 3 (Proof related questions)

Unit 2: Basic Discrete Structures (10%)

- Set Theory: Subset Relation and Set Equality

Students should master the techniques of proving subset relations and set equality. (Textbook - Example 6.1.2, 6.1.3, page 338 - 339.

Example questions from Assignments and Quizzes:

Homework Assignment: Set 6.1 – 3, 6, 7

Demo and actual quiz - 3 (Set related questions)

- Properties of Sets:

Students should master the techniques of proving set identities. (Textbook – Example 6.2.2, Theorem 6.2.2 (3)(a), Theorem 6.2.2(9)(a), Theorem 6.2.3)

Example questions from Assignments and Quizzes:

Exercise Set 6.2 4, 10, 13(Answer provided at the end of the book)

Demo and actual quiz - 4

Unit 3: Induction and Recursion (30%)

- Weak and Strong Induction (10%)

Student should master the technique of proof by mathematical induction. A proof by mathematical induction has two parts, a basis step, where we show that $P(1)$ is true, and an inductive step, where we show that for all positive integers k, if $P(k)$ is true, then $P(k + 1)$ is true. (Textbook Examples – Theorem 5.2.3, Example 5.2.4, Proposition 5.3.1, proposition 5.3.2)

Student should master the technique of proof by strong induction. Similar with a proof by mathematical induction, a strong induction also has two parts, a basis step, and an inductive step. The basis step of strong induction is the same with mathematical induction, where we show that $P(1)$ is true. Differently, in the inductive step, we show that if $P(j)$ is true for all positive integers not exceeding k, then $P(k + 1)$ is true. (Textbook Examples – Example 5.4.1, 5.4.2, Theorem 5.4.1)

Example questions from Assignments and Quizzes:

Homework examples of week 5 .

- Recursive Definitions and Structural Inductions (20%)

Student should master the recursive definitions and structural induction. We use two steps to define a function with the set of nonnegative integers as its domain:

BASIS STEP: Specify the value of the function at zero.

RECURSIVE STEP: Give a rule for finding its value at an integer from its values at smaller integers. Such a definition is called a recursive or inductive definition. (Textbook Example – Example 5.7.1, Example 5.7.3)

However, instead of using mathematical induction directly to prove results about recursively defined sets, we can use a more convenient form of induction known as structural induction. A proof by structural induction consists of two parts. These parts are:

However, instead of using mathematical induction directly to prove results about recursively defined sets, we can use a more convenient form of induction known as structural induction. A proof by structural induction consists of two parts. These parts are:

BASIS STEP: Show that the result holds for all elements specified in the basis step of the recursive definition to be in the set.

RECURSIVE STEP: Show that if the statement is true for each of the elements used to construct new elements in the recursive step of the definition, the result holds for these new elements. (Textbook Example- Example 5.9.1, Example 5.9.2, Example 5.9.4)

Example questions from Assignments and Quizzes:

Exercise set 5.7 – 6, set 5.9 – 6, 10, 15, 16

Demo and actual quiz over week 6 and 7 - all of them

** And this example -- Consider the following recursive definition of a set S of ordered pairs of integers.

Base Case: $(0,0) \in S$

Recursive Case: "If" $(a,b) \in S$,"then" $(a,b+1) \in S, (a+1,b+1) \in S$,"and" $(a+2,b+1) \in S$. Use structural induction to prove that for any $(a,b) \in S$, it is the case that $a \leq 2b$

Unit 4: Basic Counting Rules and the Pigeonhole Principles (10%)

- Basic Counting Rules and the Pigeonhole Principles (10%)

Student should master the two basic counting principles, the product rule and the addition rule and how they can be used to solve many different counting problems. The product rule (Textbook – Example 9.2.2, 9.2.3) applies when a procedure is made up of separate tasks. The addition rule (Textbook – Example 9.3.1, 9.3.2) applies If a procedure can be done either in one of n1 ways or in one of n2 ways, where none of the set of n1 ways is the same as any of the set of n2 ways, then there are n1 + n2 ways to do the task. The Pigeonhole principles states (Textbook – Example 9.4.1-9.4.3), If k is a positive integer and k + 1 or more objects are placed into k boxes, then there is at least one box containing two or more of the objects.

Example questions from Assignments and Quizzes:

Textbook Example -9.3.7

Examples from Homework Assignment – week 7

Demo and actual quiz week 6 and 7 – All of them

Unit 5: Permutations and Combinations (20%)

- Permutations and Combinations (20%)

Student should master the techniques of permutation (Textbook – example – 9.2.8- 9.2.11) and combinations (Textbook - Example 9.5.4 - 9.5.7, 9.5.9-9.5.11) with and without repetitions. In combination with repetition student should consider examples (Textbook – Examples 9.6.2-9.6.6) from the textbook also.

Example questions from Assignments and Quizzes:

Examples from Homework Assignment – Week 8

Demo and actual quiz over week 8 materials – all of them

Unit 6: Graph Theory (20%)

- Graph (20%)

Students should master graph definitions and properties, graph representations (Textbook Example 10.1.4, 10.1.6, 10.1.7), graph Degree (Theorem 10.1.1, Proposition 10.1.3), simple path , graph connectivity and Euler circuits (Theorem 10.2.2-10.2.3, Example 10.2.5-10.2.6*, Theorem 10.2.4). Example 10.7.5.

Homework – Week 9 and 10

Demo and actual quiz over week 9 materials – all of them

<u>Students should also try the examples from -</u>

- The midterm review exam
- The midterm exam
- The final review exam