CS225: Quiz 7

Question 1 (10 pts)

Suppose that an engineer is trying to model airline routes between airports. How can the engineer use a graph to encode whether pairs of airports have direct flights in between them?
Clearly describe the meaning of the vertices and the edges. Also, are multiple or parallel edges and loops allowed ? Explain why or why not .

Answer.
We can model airline networks by representing each airport by a vertex. In particular, we can model all the flights by a particular airline each day using a directed edge to represent each flight, going from the vertex representing the departure airport to the vertex representing the destination airport. The resulting graph will generally be a directed multigraph,as there may be multiple flights from one airport to some other airport during the same day. Loops are not allowed since a direct flight from one airport to itself is not an option.

Question 2 (10 pts)

Consider a graph where vertices are individuals and edges are drawn between vertices of individuals who are mutual friends. Argue that the sum across all individuals of their mutual friends is an even number.

Answer: The sum across all individuals of their mutual friends is the sum of the degree of all vertices, namely, $\sum_{v \in V} \deg(v)$. By the handshaking theorem, we know that $\sum_{v \in V} \deg(v) = 2e$, where *e* is the number of edges. Hence, the sum across all individuals of their mutual friends is an even number.

Question 3 (10 pts)

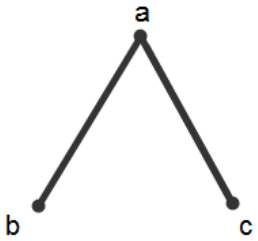
Draw a graph with the specified properties or show that no such graph exists.
Graph with four vertices of degrees 1, 2, 3, and 3.

Answer: No such graph is possible. As the total degree of a graph is even.
But a graph with four vertices of degrees 1, 2, 3, and 3 would have a
total degree of 1 + 2 + 3 + 3 = 9, which is odd .

Question 4 (10 pts)

Consider a graph G such that at least one vertex v is connected to all other vertices. Prove that G is not bipartite.

Answer: I doubt this question. Consider the right graph. The vertex a is connected to all the other vertices b and c. However, the graph can be partitioned in to {a} and {b,c} without any vertices connecting to other



vertices in the same set.

Question 5 (10 ptó

Determine whether the following graphs have an Euler circuit , if the graph does not have an Euler circuit, explain why not. If it does have an Euler circuit, describe one.

a) This does not have an Euler circuit. It is not a connected graph. There is no walk from v_0 to v_1.

b) This graph has an Euler circuit. One is ABCDAFCEFGA.