

## Demo Quiz over week 9 materials – Solutions

### Question 1:

Suppose that a biologist is studying a fixed population of gorillas. How can the biologist use a graph to encode whether pairs of gorillas have exhibited hostility toward each other? Clearly describe the meaning of the vertices and the edges. Mention whether loops and multiple edges are possible.

### Answer:

The biologist should use a directed graph to represent the gorillas' information. In the graph, each a vertex represents a gorilla and an edge starts at the gorilla a and ends at the gorilla b if there a has exhibited hostility to b. There could not be multiple directed edges connecting  $\{a, b\}$ . Loops are not allowed as a gorilla can't be hostile to itself.

### Question 2:

Draw a graph with the specified properties or show that no such graph exists.

Graph with four vertices of degrees 1, 2, 3, and 4.

### Answer:

Suppose there were a simple graph with four vertices of degrees 1, 2, 3, and 4. Then the vertex of degree 4 would have to be connected by edges to four distinct vertices other than itself because of the assumption that the graph is simple (and hence has no loops or parallel edges.) This contradicts the assumption that the graph has four vertices in total. Hence there is no simple graph with four vertices of degrees 1, 2, 3, and 4.

### Question 3:

- a) A graph has vertices of degrees 1, 1, 4, 4, and 6. How many edges does the graph have?
- b) Is there a graph with ten vertices of degrees 1, 1, 2, 2, 3, 3, 3, 3, 4, and 5 ?

### Answer:

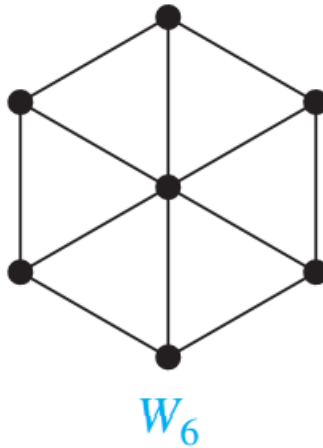
- a) The total degree of the graph is  $1 + 1 + 4 + 4 + 6 = 16$ , so by Theorem (If  $G$  is any graph, then the sum of the degrees of all the vertices of  $G$  equals twice the number of edges of  $G$ ) , the number of edges is  $16/2 = 8$ .
- b) No. Such a graph would have seven vertices of odd degree, which is impossible by Proposition “ In any graph there are an even number of vertices of odd degree”.

**Question 4:**

- a) Draw a graph with 6 vertices that is bipartite. Explain why it is bipartite.
- b) Draw a graph with 6 vertices that is not bipartite. Explain why it is not bipartite.

**Answer:**

- a) Many possible answers. One possible answer is  $K_{3,3}$ . Use the definition on Page: 633 to explain.



- b) Many possible answers.  
Use the same definition to explain why not.

$W_6$  is not a bipartite graph.

**Question 5:**

Prove that If a graph has an Euler circuit, then every vertex of the graph has positive even degree.

**Answer:**

Theorem 10.2.2 , page 648-649