CS 225: Demo Quiz 2

Question 3:

Let the following predicates be given. The domain is all living beings.

$B(x)$ = “x is one of my poultry”

$S(x)$ = “x is a duck”

$A(x)$ = “x is willing to waltz”

$M(x)$ = "x is an officer"

Express each of the following English sentences in terms of B(x), S(x), A(x), M(x) quantifiers, and logical connectives.

i. No ducks are willing to waltz.

ii. All my poultry are ducks.

iii. No officers ever declined to waltz.

iv.Some Poultry are ducks but not officers.

1. $\forall x\,(S(x) \rightarrow \neg A(x))$

ii. $\forall x\,(B(x) \rightarrow S(x))$

iii. $\forall x\,(M(x) \rightarrow A(x))$

iv. $\exists x(B(x) \wedge S(x) \wedge \neg M(x))$

Question 4:

Show that $\neg\forall x(P(x) \rightarrow Q(x))$ and $\exists x(P(x) \wedge \neg Q(x))$ are logically equivalent.

Answer:

By De Morgan’s law for universal quantifiers, we know that $\neg\forall x(P(x) \rightarrow Q(x))$ and $\exists x(\neg(P(x) \rightarrow Q(x)))$ are logically equivalent. By the fifth logical equivalence in Table 7 in Section 1.3, we know that $\neg(P(x) \rightarrow Q(x))$ and $P(x) \wedge \neg Q(x)$ are logically equivalent for every x. Because we can substitute one logically equivalent expression for another in a logical equivalence, it follows that $\neg\forall x(P(x) \rightarrow Q(x))$ and $\exists x(P(x) \wedge \neg Q(x))$ are logically equivalent.

Question 5:

Write the negation for each of the following statements.

a. If a computer program has more than 100,000 lines, then it contains a bug.

b. Some estimates are accurate.

c. The number 1,357 is divisible by some integer between 1 and 37.

Answer:

a. ~(If a computer program has more than 100,000 lines, then it contains a bug) ≡ There is at least one computer program that has more than 100,000 lines and does not contain a bug.

2. ~ (Some estimates are accurate) ≡ All estimates are not accurate.

3. ~ (The number 1,357 is divisible by some integer between 1 and 37) ≡
~ (There is an integer n between 1 and 37 such that the number 1,357 is divisible by n) ≡
For all integers n between 1 and 37 , the number 13,57 is not divisible by n .

Question 6

Using direct method, prove that If m is an even integer and n is an odd integer then mn is an even integer .

Answer:

Suppose that m and n are arbitrary odd integers. Then $m = 2a$ and $n = 2b + 1$; where a and b are integers. Then

$mn = (2a)(2b + 1)$ (substitution)

$= 4ab + 2a$ (associative, commutative, and distributive laws)

$= 2(2ab + a)$ (distributive law)

Since mn is twice an integer (namely, $2ab + a$) , mn is even. (Proved)

Question 7 **:**
Use direct proof method to show that $n^2 + n + 1$ is odd for all $n \in N$. Hint : Use cases.

Answer:
Case 1: Suppose n is odd. According to the definition of odd number, $n = 2k + 1$, for some $k \in Z$ .Then,
$\mathbf{n^2} + n + 1 = (2k+1)^2 + (2k+1) + 1 = 4k^2 + 4k + 1 + 2k + 1 + 1 = 4k^2 + 6k + 2 + 1 = 2(2k^2 + 3k + 1) + 1$.
Take , **k " = $\mathbf{2k^2+3k+1}$ and as k** $\in Z$, k"$\in Z$ (no need to show additional proof here) , then $\mathbf{n^2}$ $+ n + 1 = 2$ k " +1 , which is an odd number.

Case 2: Suppose n is even. According to the definition of even number, $n = 2k$, for some $k \in Z$ .Then,
$\mathbf{n^2} + n + 1 = (2k)^2 + (2k) + 1 = 4k^2 + 2k + 1 = 2(2k^2 + k) + 1$.
Take , k '= $2k^2+k$ and as k' $\in Z$, $k \in Z$, so, $\mathbf{n^2} + n + 1 = 2$ k '+1 , which is an odd number.

The above cases show that no matter whether n is an odd or even number , $\mathbf{n^2} + n + 1$ is odd for all $n \in N$. (Proved)

Question8:

Use a proof by contraposition to show that if $n^3 + 5$ is even then n is odd for natural numbers.

Answer: Let p be the proposition that $n^3 + 5$ is even and q is the proposition that n is an odd natural number. The contrapositive of the implication is "if n is even then $n^3 + 5$ is odd". By the definition of even integer, n= 2k for some k. Substituting the value of n , we find that $(2k)^3+5= 8k^3+4+1= 2(4k^3+2)+1= 2k'+1$ where k 'is a natural number, is an odd number. This satisfies the contrapositive proposition we wished to solve, and thus solves the original proposition that if $n^3 + 5$ is even then n is odd for natural numbers. This concludes our proof.