Assignment 8: Permutations and Combinations PART 1

9.2: 32-c, 33, 36, 39b,d

9.5: 7-b, 14, 20

32. c.
How many ways can the letters of the word *ALGORITHM* be arranged in a row if the letters *GOR* must remain together (in order) as a unit?

A, L, GOR, I, T, H, M = 7! = 7 * 6 * 5 * 4 * 3 * 2 * 1 = **5,040**

33. Six people attend the theater together and sit in a row with exactly six seats.

a. How many ways can they be seated together in the row?

b. Suppose one of the six is a doctor who must sit on the aisle in case she is paged. How many ways can the people be seated together in the row with the doctor in an aisle seat?

c. Suppose the six people consist of three married couples and each couple wants to sit together with the husband on the left. How many ways can the six be seated together in the row?

a. 6! = **720**
b. 5! = **120**
c. 3! = **6**

36.
Write all the 3-permutations of {*s, t, u, v*}.

P(n,r)=$\frac{n!}{(n-r)!}$ r = 3 n = 4

$\frac{4!}{(4-3)!} = \frac{4!}{1!} = \frac{4*3*2*1}{1} =$ 24

stu, stv, suv, sut, svt, svu,
tsu, tsv, tuv, tvs, tvu, tus,
ust, usv, ust, utv, uvs, uvt,
vst, vsu, vts, vtu, vus, vut

39-b,d

b. How many ways can six of the letters of the word ALGORITHM be selected and written in a row?

d. How many ways can six of the letters of the word ALGORITHM be selected and written in a row if the first
two letters must be OR?

b. r = 6, n = 9

$\frac{n!}{r!(n-r)!} * r! = \frac{9!}{6!(9-6)!} * 6! = \frac{9*8*7}{3!} * 6! = \frac{9*8*7}{6} * 6! = 84 * 6! = 84 * 720 =$ **60480**

d. r = 4, n = 7

$\frac{n!}{r!(n-r)!} * r! = \frac{7!}{4!(7-4)!} *4! = \frac{7*6*5}{3!} 4! = \frac{7*6*5}{6} * 4! = 35 * 4! = 35 * 24 =$ **840**

9.5: 7-b, 14, 20

7b.

A computer programming team has 13 members.

b. Suppose seven team members are women and six are men.

(i) How many groups of seven can be chosen that contain four women and three men?

Total: 13
Women: 7
Men : 6

n groups if 7 with 4 women and 3 men.

$\binom{7}{4} * \binom{6}{3} =$

$\binom{7}{4} = \frac{n!}{r!(n-r)!} = \frac{7!}{4!(7-4)!} = \frac{7*6*5}{3!} = \frac{7*6*5}{6} = 35$

$\binom{6}{3} = \frac{n!}{r!(n-r)!} = \frac{6!}{3!(6-3)!} = \frac{6*5*4}{3!} = \frac{6*5*4}{6} = 20$

35 * 20 = **700**

14.

a. How many 16-bit strings contain exactly seven 1's?
b. How many 16-bit strings contain at least thirteen 1's?
c. How many 16-bit strings contain at least one 1?
d. How many 16-bit strings contain at most one 1?

a. $\binom{16}{7} = \frac{16!}{7!(16-7)!} = \frac{16!}{7!9!} = \frac{16*15*14*13*12*11*10}{7*6*5*4*3*2*1}$ = **11440**

b. $\binom{16}{13} + \binom{16}{14} + \binom{16}{15} + \binom{16}{16} =$

$\frac{16!}{13!(16-13)!} + \frac{16!}{14!(16-14)!} + \frac{16!}{15!(16-15)!} + \frac{16!}{16!(16-16)!} =$

$\frac{16*15*14}{6} + \frac{16*15}{2} + \frac{16}{1} + 1 =$ **697**

c. 16 total bits, 2 per bit, at least 1.
$2^{16} - 1 =$ **65535**

d. $\binom{16}{1} + \binom{16}{0} = \frac{16!}{1!(16-1)!} + \frac{16!}{0!(16-0)!} = \frac{16!}{15!} + 1 =$ **17**

20.
a. How many distinguishable ways can the letters of the word MILLIMICRON be arranged in order?

b. How many distinguishable orderings of the letters of MILLIMICRON begin with M and end with N?

c. How many distinguishable orderings of the letters of MILLIMICRON contain the letters CR next to each other in order and also the letters ON next to each other in order?

a. n = 11: M = 2, I = 3, L = 2, C = 1, R = 1, 0 = 1, N = 1

$\frac{11!}{2!*3!*2!} = \frac{11*10*9*8*7*6*5*4*3!}{3!*2*2} = \frac{11*10*9*8*7*6*5*4}{4} =$ **1663200**

b. n = 9: M = 1, I = 3, L = 2, C = 1, R = 1, 0 = 1

$\frac{9!}{3!*2!} = \frac{9*8*7*6*5*4*3!}{3!*2!} = \frac{9*8*7*6*5*4}{2} =$ **30240**

c. n = 9: M = 2, I = 3, L = 2
$\frac{9!}{3!*2!*2!} = \frac{9*8*7*6*5*4*3!}{3!*2!*2!} = \frac{9*8*7*6*5*4}{2*2} =$ **15120**