CS225: Quiz 2 Solutions

Question 1:

In the domain of all students, we define predicates
M(x): x is a math major
C(x): x is a computer science major
A(x): x is required to take CS 225.
Express each of the following English sentences in terms of M(x), C(x), A(x), quantifiers, and logical connectives.
(a) Some math majors are not required to take CS 225.
(b) No math majors are computer science majors.
(c) All computer science students who are math majors are not required to take CS 225.
(d) There is a student who is both math and computer major is not required to take CS 225.

Answer:

(a) $\exists x\, [M(x) \land \sim A(x)]$

(b) $\forall x\, [M(x) \longrightarrow \sim C(x)]$ or $\sim (\exists x\, [M(x) \land C(x)])$

(c) $\forall x\, [(C(x) \land M(x)) \longrightarrow \sim A(x)]$

(d) $\exists x\, [M(x) \land C(x) \land \sim A(x)]$

Question 2

Let B(x), S(x), and A(x) be the predicates
B(x): x is a good basketball player
S(x): x is a good soccer player
A(x): x is a good athlete
Translate each of the following quantified logic expressions (provided in the file) into English considering the domain to consist of all people.
i) $\forall x\, [A(x) \rightarrow (S(x) \lor B(x))]$
ii) $\sim\forall x[B(x)]$
iii) $\exists x[(S(x) \land \sim B(x)) \lor \sim A(x)]$
iv)$\exists x \sim[S(x) \land \sim B(x)]$

Answer:

i) All good athletes are either good soccer players or good basketball players.
ii) Not everyone is a good basketball player.
iii) There is a person who is a good soccer player but not a good basketball player, or not a good athlete.
iv) There exists a person who is not a good soccer player or is a good basketball player.

Question 3:

Negate each of the following statements:
1) Everything in that store is either overpriced or poorly made.
2) There is a horse that does not fly.
3) No exercises have answers.

Answer:

1) There exists one item in that store that is not overpriced and not poorly made. (Alternate solution: Some items in the store is not overpriced and not poorly made.)
2) All horses fly.
3) Some exercises have answers. (Alternate solution: There exists an exercise which has answer)

Question 4:
True or false: For the set of all integers, $\exists x\,(x + 1 < -x)$

Answer:
True. This applies to all negative integers.

Question 5:
Prove or disprove that $\forall x\,(P(x) \rightarrow Q(x))$ and $\sim\exists x \sim(\sim Q(x) \rightarrow \sim P(x))$ are logically equivalent.

Answer:

$\forall x\,(P(x) \rightarrow Q(x)) \equiv \sim(\sim\forall x\,(P(x) \rightarrow Q(x)))$ (By double negation)
$\sim(\sim\forall x\,(P(x) \rightarrow Q(x))) \equiv \sim\exists x\,(\sim(P(x) \rightarrow Q(x)))$ (De Morgan's law for Quantifiers)
$\sim\exists x\,(\sim(P(x) \rightarrow Q(x))) \equiv \sim\exists x\,(\sim(\sim Q(x) \rightarrow \sim P(x)))$ (By contrapositive)
Therefore, $\forall x\,(P(x) \rightarrow Q(x))$ and $\sim\exists x \sim(\sim Q(x) \rightarrow \sim P(x))$ are logically equivalent.

Question 6:

Use direct method to prove the following statement -

Suppose that integers m and n are perfect squares, then $4(m + n) + 8\sqrt{mn}$ is also a perfect square.

Answer:
Since integers m and n are perfect squares, there exist integers a and b such that $m=a^2$ and $n=b^2$. Thus, $4(a^2 + b^2) + 8\sqrt{a^2b^2} = (2a)^2 + (2b)^2 + 2.2a.2b = (2a+2b)^2$. Therefore, $4(m + n) + 8\sqrt{mn}$ is also a perfect square.

Question 7:
Prove that if x is irrational, then 1/x is irrational.

Answer:

We will use proof by contraposition. The contrapositive is “If 1/x is rational, then x is rational.” Suppose that 1/x is rational and $x \neq 0$. Then there exists integers p and q such that $1/x = p/q$ and $q \neq 0$. $1/x \neq 0$ because $1 \neq x \cdot 0$, this would mean that $p \neq 0$. Since $p \neq 0$, then $x = 1/(1/x) = 1/(p/q) = q/p$. Hence x can be written as a quotient of two integers with a nonzero denominator. Thus, x is rational.