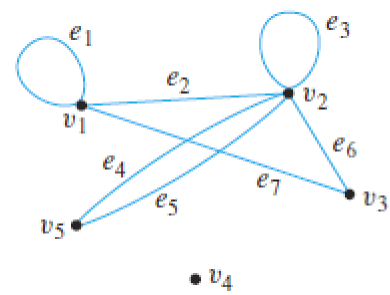
Assignment 9 Part 1: Set 10.1 : 9, 27.b, 44

(i) Find all edges that are incident on v_1.
(ii) Find all vertices that are adjacent to v_3.
(iii) Find all edges that are adjacent to e_1.
(iv) Find all loops.
(v) Find all parallel edges.
(vi) Find all isolated vertices.
(vii) Find the degree of v_3.
(viii) Find the total degree of the graph.

9.



i) $\{e_1, e_2, e_7\}$

ii) $\{v_2, v_3\}$

iii) $\{e_2, e_7\}$

iv) $\{e_1, e_3\}$

v) $\{e_4, e_5\}$

vi) v_4

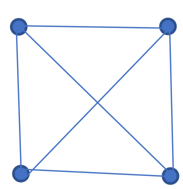
vii) 2

viii) $v_1 = 4, v_2 = 6, v_3 = 2, v_4 = 0, v_5 = 2$

$4 + 6 + 2 + 0 + 2 = 14$

27.
b. In a group of 4 people, is it possible for each person to have exactly 3 friends? Why?

Yes, imagine a scenario in which each person is represented by a vertex. Put them in in a corner forming a square. Each person will be able to connect to every other vertex with an edge. This means that there are 3 degrees for each vertex. This means there are a total of 12 degrees in this graph. Since there are a total of 6 edges, it is true by corollary 10.1.2, which states the total number of degrees is twice the amount of edges. 2(6) = 12.



44.
a. In a simple graph, must every vertex have degree that is less than the number of vertices in the graph? Why?

Since in a simple graph, there are 0 loops, each edge can only make 1 degree at each vertex.

b. Can there be a simple graph that has four vertices each of different degrees?

A simple graph with 4 vertices the only options of degrees are 0, 1, 2, 3. Since in a simple graph, there must be at least one edge, 0 degrees is not possible.

c. Can there be a simple graph that has *n* vertices all of different degrees?

Again, identical to the answer in b, instead with n vertices the possible degree options are 0, 1, 2, 3…n-1. Since there must be at least one edge, n-1 must connect to the vertex with a degree of 0, which means it shares degrees with at least one other vertex.