Estimating Emergency Room Wait Times Using Hazard Model with Change Points

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Abstract

This paper uses duration analysis to estimate emergency room wait times. The analysis time is fitted in a piecewise model that allows the parameters of the hazard function to take on a different value after a change point. Results from fitting such model in the 2010 National Hospital Ambulatory Medical Survey data point to two major conclusions. First, patients at the extreme ends of urgency receive attention quicker than those in the middle. Second, wait times are influenced unequally by visit, patient, and hospital characteristics. Even after clinical differences are accounted for, there are still significant wait gaps among patients of different demographic background. In a hypothetical scenario where patients are segregated into two groups of opposite traits, those who possess the disadvantage characteristics are expected to have double to triple wait lengths compared to their counterpart.

1. Introduction

The hospital emergency room has an important role in the US health care system besides providing urgent care; it serves as a safety net for the uninsured, underinsured, and people who lack access to other care providers (Tang et al, 2010). Emergency room wait times have been increasing over the past few years: during 1999-2009 average wait increased from 46.5 to 58.1, and median wait grew from 27 to 33 (CDC, 2012). Studies show that long dwelling times correlate with increased mortality, medical errors, and patient frustration (Bernstein et al, 2009; Plunkett et al, 2011; Soremekun et al, 2011; James et al, 2005). Longer waits also induce patients to abandon treatment, which leads to disutility and subsequent care demands (Russell et al, 2013; Levsky et al, 2008; Ding et al, 2006).

This paper discusses the use of hazard model in estimating wait times and applies the model to analyze wait patterns using data from the 2010 National Hospital Ambulatory Medical Survey. More attention is emphasized on functional hazard form and random effects. This goal is made possible by applying a hazard model with change points and a frailty component shared at hospital level. I focus on patients aged 15 to 64. Results point to several conclusions. First, the hazard for a patient to be seen by a physician in the next instance follows a non-monotonic path, with modest variances across hospitals. Second, the longer is the wait, the more likely it becomes extreme. Third, there are some wait patterns that can be predicted from patient profile and visit characteristics. Generally, a patient's waiting time is reduced heavily by ambulance arrival, high urgency, timing, and private insurance.

Preliminary to the data set

The National Hospital Ambulatory Medical Care Survey provides a wealth of information regarding emergency utilization. Participants include emergency rooms and outpatient departments of general hospitals. After re-sampling and excluding observations with no wait time, the data set for analysis consists of 20,782 observations[1]. Each observation represents one unique patient whose recorded wait time is equal to or more than zero.

Table 1 reports time differences among groups. For patient characteristics, only gender, expected pay types, and household income appear to be significant. Females, patients

[1] This account for two third of the original dataset provided by the CDC.

who do not hold private insurance, residents of low income areas[2] appear to have longer waits. Wait times are shorter for visits made on ambulance and visit with higher or very low urgency. Counter-intuitively, non-urgent visits have shorter waits than moderately-urgent visits.

Figure 1 plots the histogram of wait times. Wait times distribution is left-skewed with a long thin right tail. This suggest the use of survival analysis over traditional Ordinary Least Squares method. Figure 2 plots the survival curves computed by the Kaplan-Meier procedure (Kaplan and Meier, 1958). A patient survives up to t minute is the one who has *not* seen by a care provider by that time. The duration at which half of the patients are seen is 30 minutes. However as a patient wait longer in the emergency room, his wait time is likely to converge to the extreme, which is almost 24 hours[3] as recorded. This trend is implied in the flattening concavity toward the extreme values[4].

Figure 3 plots the hazard curve computed from the Kaplan-Meier procedure. The hazard rate is the instantaneous risk of having the event of interest, conditional on survivorship up to t. Accordingly, the likelihood of being seen follows a non-monotonically decreasing path: the risk reaches its first maximum at the 100th minute, then declines sharply before picking up the upward trajectory at the 900th minute. The former occurrence peaks at 0.015, and the latter at 0.010. This implies that patients who are waiting longer in the emergency room are marginally more likely to have extreme wait for each instance they remain in the emergency ward.

2. The Model

I use parametric duration analysis to estimate the length of wait time. Similar consideration of this application includes modeling ER frequent usage (Mandelberg, 2000), identifying the relationship between a continuant care and utilization by pediatric patients (Christakis, 2001), and analyzing factors of emergency room crowding (Arkun, 2000).

Following findings in the non-parametric procedure, I apply a change point hazard model and compare with other traditional alternatives using the Cox-Snell residuals (Cox-Snell,

[2] Household income less than $32,793/year.
[3] The maximum recorded wait time is 1,415 minutes. And such length is not an outlier in the dataset.
[4] Most graphs in the paper is set to display at the maximum 150 minutes. The extreme values are beyond the 1,400 minute mark.

1986) and the Akaike Information Criterion (Akaike, 1974)[5]. The inspiration for the change point hazard is drawn from a work by Noura and Read (1990).

Apart from fitting multi-phrasal hazard, I seek to account for potential intra-correlation among hospitals. Factors pertaining to patient traits, facility specifics, and information could be responsible for variability in dwelling time among hospitals. Neglecting the heterogeneity could result in biased conclusions if the heterogeneity is an important force of survivorship. If hazard is mis-specified, estimated residuals may cloak themselves as random effects across identities and leads to incorrect frailty estimation. My model therefore includes a frailty component that is shared among patients who entered the same hospital. The inspiration for this solution comes from a comprehensive work by Gutierrez (2002) and similar considerations of frailties in studies by Hougard (1986), Whitmore and Lee (1991), and Sahu et al (1997).

2.1. Changep oint hazard model

Start with a simple parametric model where hazard is monotonic, and I assume framed wait time follows a Weibull distribution[6]. The survival functions in Weibull form are as follows[7]:

$$S(t) = e^{-(\lambda t)^p e^{\beta X}}$$

where λ, p are the Weibull position and direction parameters; X and β are vectors of explanatory variables and its coefficients.

To allow the risk contribution to be piecewise defined, divide time axis by k changepoints, which gives $k+1$ intervals. Let j denote the interval (a_{j-1}, a_j) where a_j is the end point and a_{j-1} is the starting point[8]. Let $g(t) = \ln\left[(\lambda t)^p\right]$ denote the progression of cumulative hazard at time t $(a_{j-1} < t < a_j)$. The survival equation is rewritten as:

$$S(t) = e^{-e^{g(t_i)+\beta X}} \quad (1)$$

Within a time frame, cumulative hazard is growing at a stable rate, therefore:

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[5] The Cox-Snell residuals compare hazard fitness, while the AIC criterion compare information loss.
[6] Exponential is a special case of Weibull.
[7] See Appendix for details.
[8] Zero is the starting point of the first interval.

$$g(a_j) = g(a_{j+1}) \rightarrow \ln\left[(\lambda_j a_j)^{p_j}\right] = \ln\left[(\lambda_{j+1} a_j)^{p_{j+1}}\right]$$

$$\rightarrow p_{j+1} = p_j \frac{\ln(\lambda_j a_j)}{\ln(\lambda_{j+1} a_j)} \qquad j = 1,2, \dots, k+1$$

From this, derive a general function of p_j dependent on initial p_1 as follows,

$$p_j = p_1 \prod_{q=1}^{j-1} \frac{\ln[\lambda_q a_q]}{\ln[\lambda_{q+1} a_q]} \quad , \quad j = 1,2, \dots, k+1$$

Substitute to $g(t) = \ln\,[(\lambda t)^p]$ give,

$$\mathrm{g(t)} = \ln[(\lambda_j a_j)^{p_j}] = \ln\,(\lambda_j t) p_1 \prod_{q=1}^{j-1} \frac{\ln[\lambda_q a_q]}{\ln[\lambda_{q+1} a_q]}$$

For $i-th$ individual, let c_{ij} be a variable indicating the position of wait instance for the interval (a_{j-1}, a_j). c_{ij} takes on the value "1" if $a_{j-1} < t_j < a_j$ and "0" if otherwise. The piecewise function is now formed by summation,

$$g(t) = \sum_{j=1}^{k+1} c_{ij} \ln(\lambda_j t)\, p_1 \prod_{q=1}^{j-1} \frac{\ln[\lambda_q a_q]}{\ln[\lambda_{q+1} a_q]}$$

$$j = 1,2, \dots, k+1 \qquad i = 1,2, \dots, N$$

2.2. Shared frailty component

Suppose there exists heterogeneity among hospitals that generates different frailty among hospitals. This shared frailty is a time-variant latent multiplier to the survival function. For simplification, let Z denotes the unobserved frailty associating with the hospital where patient i seek care ($i = 1,2, \dots, N$). Rewrite the survival function gives:

$$S(t; X, Z) = e^{-(\lambda t)^p Z e^{\beta X}}$$

And the equation (1) becomes:

$$S(t_i|X) = e^{-e^{g(t_i) + \ln\,(Z) + \beta X}} \quad (2)$$

which is the changepoint model with integrated frailty component shared at hospital level.

2.3. Explanatory variables

The explanatory variables of interest are integrated in the component X of model 2, and each variable in X has an associated coefficient in matrix β. By estimating β, the model allows us to investigate the correlation between the explanatory variables and survival probability[9]. The variables in X are grouped into three categories: (a) *patient characteristics* include gender, age categories, insurance[10], income[11], and race-ethnicity, (b) *visit characteristics* include time of arrival, methods of arrival, and immediacy to be seen, and (c) *hospital characteristics* include boarding and urban/rural setting. The variables for geography and hospital ownership are not included in the analysis because they fail the $Log - Rank$ test, as described in the bottom section of Table 1[12]. Note that the immediacy in (b) is measured through nursing triage scale from 1 to 5 for very urgent to non-urgent. The triage scale does not signifies true urgency level on a clinical standpoint but it is assumed to indicate whether a patient has a significantly more pressing need than another patient. It is also assumed to be purely clinical base, and that nurses adhere to strict categorization rules[13].

3. Results and Discussion

Patients who eventually experienced the event "seen by a physician" provide full analysis times, while patients who "left against medical advice" provide *right-censored* time[14]. In the latter group, patients are known to have stayed at least until the recorded duration, but their events have never been observed. Since the number of these censored observations is small[15], the problem of informative censoring can be relieved. To fit the change point model, analysis time is divided into 14 grids of 100 minutes. The choice of 100 minute length is arbitrary after multiple trials: smaller window size does not always yield not better estimation but fitting difficulties due to insufficient data points. To account for potential heterogeneous times-to-failure among hospitals, two candidates for frailty distribution are Gamma or inverse Gaussian. The Gamma distribution assumes that the relative differences among patients are constant

[9] β is assumed to be non time-variant.
[10] The expected payment is coded with a hierarchy such that due-eligible Medicare and Medicaid patients are considered as Medicaid.
[11] Median house hold income of in patient's zip code.
[12] Further considerations of these variables also give insignificant results.
[13] This erogeneity assumption is a good research question.
[14] Patients who voluntarily left without noticing medical staff are not considered in this group. Such patients provide no analysis time, and therefore are not included the study.
[15] Less than 5 percent of the sample.

while in the inverse Gaussian distribution these differences are assumed to be diminishing overtime (Gutierrez, 2002).

5.1. Marginal improvement from fitting hazard model with changing points

To justify the implement of a non-monotone model over competing model, I first compare their predicted hazard curves. According to Figure 4, the change point model mimics hazard with high accuracy while the other models miss all hazard fluctuations after the first 60 minutes. This suggests that the non-monotone model provides better hazard prediction, where waiting time is less than 500 minutes. Beyond this point, the change point model is still better than the other options, but its proximity to true hazard is increasingly distorted.

Results from the AIC test point to a similar conclusion. The change point model is marginally preferred by the AIC procedure due to its small AIC value[16] compared to the monotone and one-time increasing models. Given the substantial differences in hazard behavior, it is seemingly unintuitive that the improvement from fitting the change point model is not substantial. However, one should be aware that hazard, as computed in Figure 4, traverses within a minuscule scale (between 0 and 1.5 percent). In addition, most events happened within the first few hours (95 percent patients were seen by the 500th minute), making distortion towards the extreme lengths less significant.

Hazard function	Frailty	AIC value
Exponential	None	50340
	Gamma	45624
	Inverse Gaussian	45609
Weibull	None	50082
	Gamma	45363
	Inverse Gaussian	45342
Log-normal	None	48307
	Gamma	44959
	Inverse Gaussian	44906
Weibull with change points	Inverse Gaussian	44693*

*: model preferred by the AIC

The Cox-Snell residual plots in Figure 5a indicates that the change point model may not be the most appropriate model when juxtaposed alongside the Log-normal model. Specifically, the log-normal distribution is marginally more accurate in fitting hazard, signified by lesser

[16] Smaller AIC values indicate better fit to the dataset, in the sense that the model suffers from less information loss.

deviations from the 45 degree line starting from the medium residual values. This implies that the change point model seems to be less efficient than the log-normal model for larger wait times. However, judging on the better hazard movement, lower AIC measurement, and the fact that distortions toward the extreme lengths are less common, the change point model can be preferred over the log-normal model.

5.2. Wait patterns and shared frailty

Wait patterns are inferred from the estimated coefficients, which are tabulated in Table 2. Positive coefficients indicate that the variable correlates with longer wait times, while negative coefficients imply the opposite[17]. According to the change point estimates, there are several patients of wait among patient, visit, and hospital characteristics. For patient characteristics, females appear to receive care slower than males, and this finding is consistent across all specifications. Such discrepancy might result from gender difference in tolerance thresholds and in communicating for attention: males are assumed to be more verbally open for care attention, thus they acquire care faster than females. Young adults age 15 to 24 receive care quicker than adults who are over 24 years old. Among insurance groups, Medicaid and Medicare beneficiaries appear to receive care slower than private insurance holders, possibly because of implicit rejections. Affluent patients tend to acquire care sooner than patients from lower income areas. The payment and income discrepancies could have resulted from network effect: high income patients and patients with private insurance may have been recommended by family doctors to hospitals so as to receive favorable treatments.

Among visit characteristics, arrival time appears to have a non-linear relationship with wait length: visits in afternoon and night shifts are executed slower than visits made in the morning. Such difference may result from increased crowding towards the afternoon and staffing shortage at night. For modality, arriving on an ambulance helps acquire care faster. One potential explanation for this situation is that visits made on ambulance may possess characteristics that merit immediate attention. Note that this effect is still valid after urgency level is accounted for, because recorded urgency levels are not comprehensive enough to take into consideration finer clinical details.

[17] Since the proposed model is parametric with accelerated failure-form, the coefficients have reverse signs compared to the traditional proportional hazard.

Urgency levels appear to have a non-uniformly progressive influence on reception rate. Specifically, patients on the most urgent and second-most urgent levels[18] have higher "seen" likelihood than do patients with trivial complaints. However, a patient who is categorized as having medium urgency appears to have lower "seen" rate than those who have trivial complaints. In fact, these medium urgent patients are the ones who wait the longest time in the emergency room. This difference might come from hospital procedures to handle crowding: non-urgent patients are diverted from the waiting flow to get treatment and to be discharged quickly. In reality, the implementation of this method can be as implicit as patients being discharged by nurses during initial assessments or as explicit as having a separated non-urgent lane[19]. Finally, hospitals which boarded inpatients in the previous year appear to have longer average dwell time.

The reported θ is an estimate of frailty variance[20]. According to the regression results, there is trivial unobserved heterogeneity among hospitals and finding is consistent between different frailty specifications. This means that in addition to the explanatory variables discussed above, wait times are influenced by unobserved random effects among hospitals. The inclusion of frailty and change points into duration models appear to result in a marginal improvement in overall fit of the model, signs and magnitude of coefficients. Generally, there are more similarities than dissimilarities among models: factors that are not significant in the non-frailty specifications are found insignificant in the frailty models and vice versa.

5.3. Estimated survival curves

To visualize how patients absorb care over time, Figures 5b shows estimated survival function calculated from the change point model with inverse-Gaussian frailties. The vertical axis represents the estimated probability that a patient's wait time reaches t minute, unconditional on frailty. The probability of waiting decreases from 100 percent to 40 percent within the first 50 minutes of waiting. However, the longer a patient stays in the emergency room, the more likely is his wait time to be extreme. After 150 minutes, the waiting likelihood is reduced to approximately 10 percent, but for these remaining 10 percent patients to be seen, wait times

[18] Nursing triage 1 and 2, respectively.
[19] R.A.P.I.D procedure.
[20] Deviation from 1.

need to be extended to 1,400 minutes. This estimated survival curve mimicked the survival function computed by the Kaplan-Meier method appropriately.

To illustrate the extent of influence on wait times by variable groups, Figure 6 plots the survival curve adjusted each for variable while holding other variables at their mean values. After 50 minutes, about 60 percent of males are seen versus 58 percent of females. Similar discrepancy is also found between young adults and adults over 25 years old. Patients on private insurance tend to receive attention earlier than those who depend on social insurance and other pay types. Among income groups, shorter wait times seem to cluster in highest income levels. Generally, there are discrepancies in term of wait among patient's characteristics, but not as many shades of variation are observed.

Factors pertaining to visit characteristics appear to have distinguishable influence on wait times, signified by obvious gaps among categories within each variable. The urgency plot depicts a phenomenon discussed earlier in this paper: beyond the most and second-most urgency levels, higher urgency does not promise faster seen rates. Accordingly, patients triaged as having moderate but not severe conditions will be seen the last. Visits made to hospitals with a boarding history show lower likelihood of being seen at any given time.

To illustrate the collective effects of the explanatory variables, I create two patient groups: the "advantage" and the "disadvantage". Patients in the former group have all the characteristics that reduce wait times, while patients in the latter group possess all characteristics that increase wait times. The construction of these groups is regression-based, and in reality patients are not expected to have all the characteristics defined in either group. However, such method allows us to envision the worst case scenario where patients are highly segregated. The traits of each group are described in the Figure 7. Using the hypothetical information as input data, I estimate in the survival curves for both group according to the change point model. According to Figure 7, patients in the advantage group enjoy shorter wait times compared to patients in the disadvantage group. Such gaping occurs early and widens over time. At the 50^{th} minute, 90 percent of patient in the first group has been seen, while the corresponding figure for the second group is around 50 percent. At the 100^{th} minute, almost 100 percent patients in the first group are seen versus 70 percent in the latter group. Generally, patients in the "less advantage" group may wait for double to triple the time patients in the "more advantage" group receive.

The excruciating discrepancies are unjustified because the disadvantaged patients, by structure, have legitimate reasons to visit the emergency room. These patients bear moderate urgency, but through segregating by patient and visit factors, they end up being the last to receive care. Since segregation could happen through patient and visit factors, policy attention can be paid to identifying risk groups and their behavior when it comes to choosing an emergency room to land. Hypothetically, if most patients in an area attend one particular hospital for trivial symptoms, it could have been due to the lack of other care providers in the area. On the other hand, policy makers should be aware that hospitals themselves have procedures to handle crowding, which may induce some initial levels of segregation.

4. Conclusions

Estimating emergency room wait times using change point hazard model and shared frailty is an appropriate alternative to traditional non-parametric models. Results from fitting such model to the 2010 NHAMCS dataset confirm most of the current understanding of wait patterns and add some new insights: almost two third of waits are less than one hour, but the longer a patient stay in the emergency room, the more likely his wait times converge to extremes; visit characteristics differentiate wait times more profoundly than do patient and hospital characteristics; should a patient has several "disadvantage" traits, his wait times could be disturbingly long. Although there is no evidence of the segregation of patients into distinguished groups as discussed in the section above, the analysis suggests potential gaps in wait times caused by patient and visit factors. Therefore, a policy aiming at solving prolonged wait times should take into consideration demographic behaviors and access to care. Should there be any situation that induces a strong segregation, remarkable discrepancies may manifest. At the end of the day, vulnerable patients, as usual, suffer more from long waits and other adverse outcomes[21].

[21] An interesting question arises: Do patients actually know their triage to justify for continuant waits in the emergency room?

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Table 1: Sample means of wait times by patient characteristics

	Average wait times, in minutes	Standard deviation
Gender † *		
Female	55.18	84.85
Male	52.27	81.85
Age groups		
15-24	53.18	82.47
25-44	54.71	85.08
45-64	53.43	82.4
Expected Payment † *		
Private Insurance	47.68	72.19
Medicare/Medicaid	55.11	82.11
Workers' compensation	45.27	77.52
Others (self-pay, no-charge, etc.)	60.97	98.39
Household Income † *		
Quartile 1 (Below $32,793)	57.68	84.34
Quartile 2 ($32,794-$40,626)	53.47	84.24
Quartile 3 ($40,627-$52,387)	52.8	87.36
Quartile 4 ($52,388 or more)	47.82	73.64
Race and Ethnicity †		
White, non-Hispanic	48.41	74.57
Black, non-Hispanic	65.55	96.01
Others	53.45	1.16
Urban settings †		
Medium to Large Central Metro	56.65	87.22
Small and non-core	43.88	67.57
Boarded patients in 2009 †		
Yes	58.19	88.22
No	37.69	60.35
Time of Arrival †		
6 a.m. - 12 p.m.	50.04	84.68
12 p.m. - 6 p.m.	55.1	75.51
6 p.m. - 6 a.m.	55.29	89.3
Means of Arrival † *		
On Ambulance	44.52	86.57
Others	55.54	83.05
Urgency Level † *		
Immediate	30.39	52.02
Emergent	51.29	80.97
Urgent	60.36	97.53
Semi-urge	52.29	70.42
Non-urgent	46.01	63.99

†: t-test or f-test for difference in wait times among groups, significant at 95% confidence level.

*: Log-rank test of equality of survival function among groups, significant at 95% confidence level.

Table 2: Regression results

	Hazard Distribution Shared Frailty Distribution	Exponential None	Weibull None	Log-normal none	Log-normal Gamma	Log-normal Inv. Gauss.	Change point Weibull Inv. Gauss.
		Accelerated Failure-Time Coefficients					
Gender	Female	0.052	0.051	0.053	0.058	0.058	0.049
	Male						
Age groups	15-24	-0.037 °	-0.041 °	-0.050	-0.039 °	-0.039 °	-0.041
	25-44	-0.001 °	-0.004 °	-0.010 °	-0.005 °	-0.005 °	-0.004 °
	45-64						
Expected payment sources	Private Insurance						
	Medicare/Medicaid	0.093	0.094	0.093	0.068	0.068	0.055
	Worker compensations	-0.019 °	-0.032 °	-0.130 °	-0.147	-0.147	-0.062 °
	Others	0.182	0.177	0.130	0.067	0.066	0.071
Household income	Quartile 1						
	Quartile 2	-0.059	-0.060	-0.034 °	0.013 °	0.012 °	-0.004 °
	Quartile 3	-0.129	-0.132	-0.132	-0.091	-0.089	-0.110
	Quartile 4	-0.200	-0.200	-0.170	-0.061	-0.059	-0.070
Race and Ethnicity	White, non-Hispanic						
	Black, non-Hispanic	0.161	0.160	0.160	0.051	0.051	0.001 °
	Others	0.014 °	0.010 °	-0.041 °	-0.026 °	-0.024 °	-0.007 °
Time of arrival	6 a.m. - 12 p.m.						
	12 p.m. - 6 p.m.	0.123	0.131	0.150	0.167	0.165	0.180
	6 p.m. - 6 a.m.	0.146	0.147	0.099	0.103	0.101	0.143
Mode of arrival	On Ambulance	-0.220	-0.239	-0.382	-0.382	-0.382	-0.256
	Others						
Emergency level	Immediate	-0.138 °	-0.139 °	-0.201	-0.321	-0.326	-0.161
	Emergent	0.157	0.154	0.113	-0.143	-0.146	-0.148
	Urgent	0.293	0.286	0.225	0.097	0.096	0.094
	Semi-urgent	0.134	0.136	0.172	0.074	0.073	0.019 °
	Non-urgent						
Urban settings	Medium to Large	0.238	0.232	0.143	0.002 °	0.005 °	0.005 °
	Small and non-core						
Boarded patients?	Yes	0.341	0.343	0.309	0.112	0.107	0.301
	No						
Shared Frailty	ϴ				0.546	0.454	0.597

Note:
° Not significant at 95 percent confidence level. Other coefficients without (°) are significant.

Figure 1: Wait times histogram

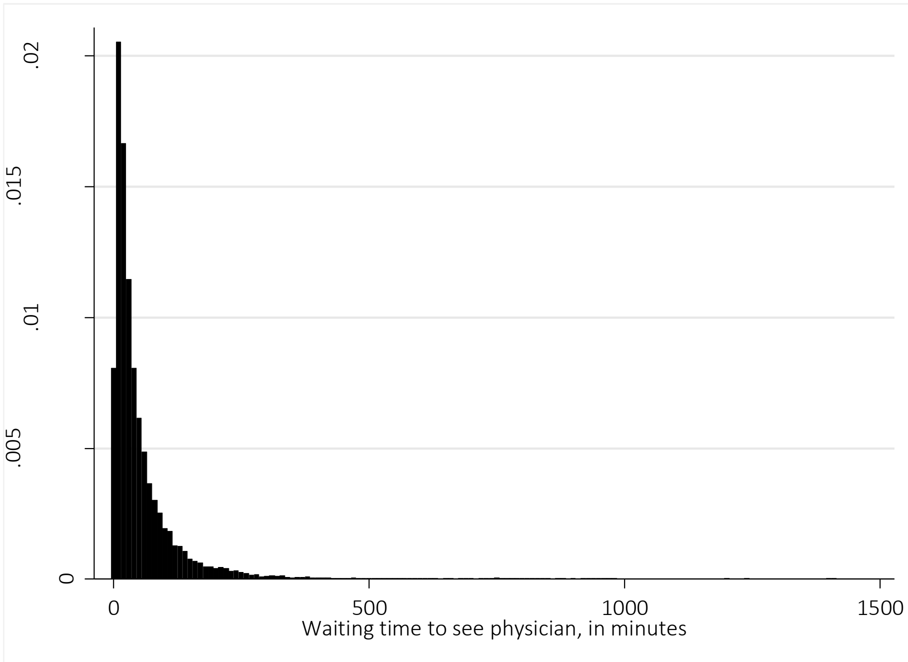


Figure 2: Probability of waiting up to time *t* (Survival curve)

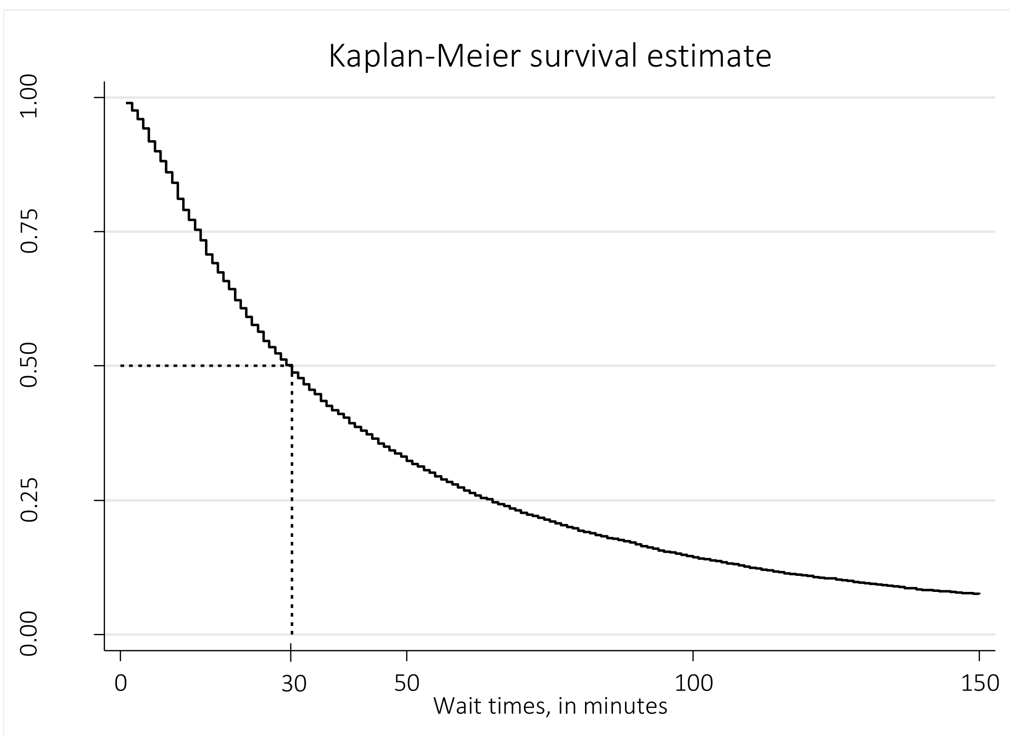


Figure 3: Hazard curve

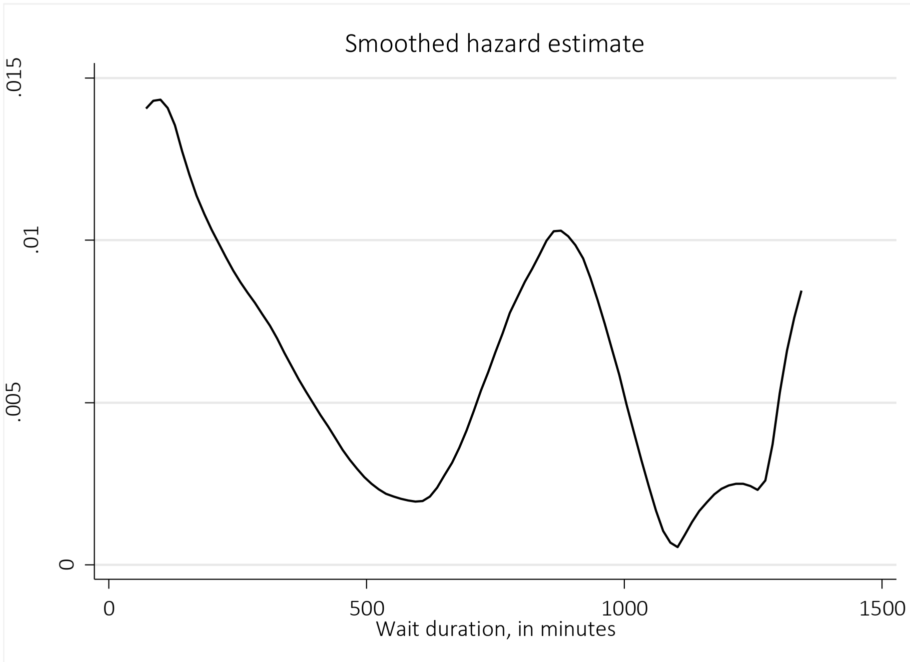


Figure 4: Smoothed hazard estimate from 3 models by polynomial smoothing

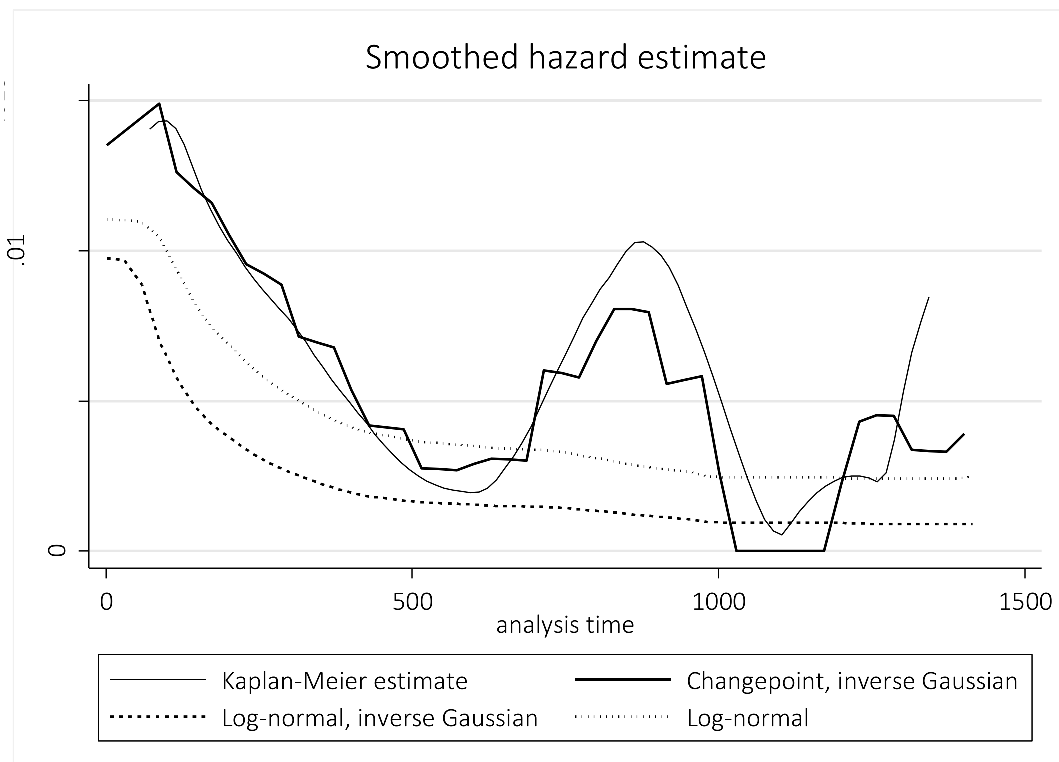


Figure 5a: Cox-Snell Residuals Plot

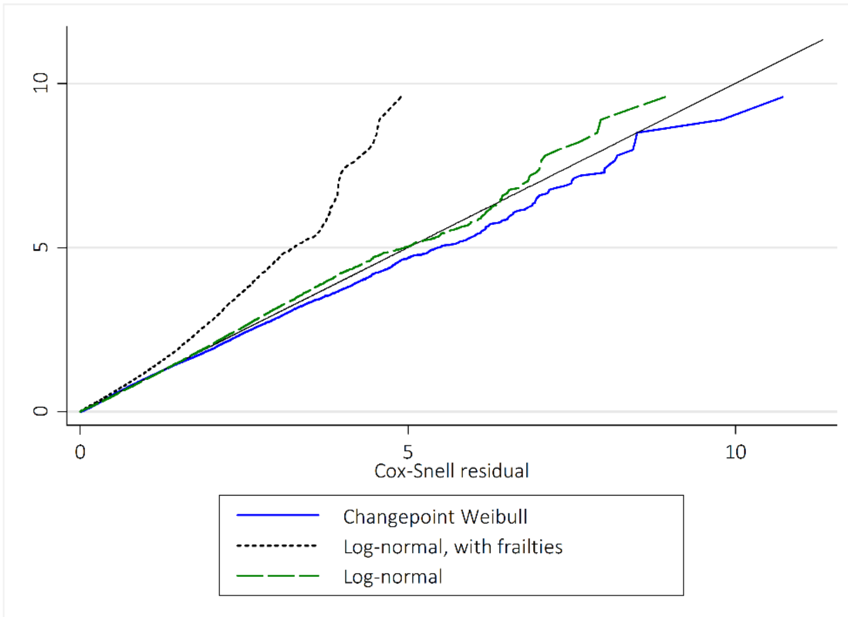


Figure 5b: Probability of waiting up to time *t*, estimated from the changepoint model

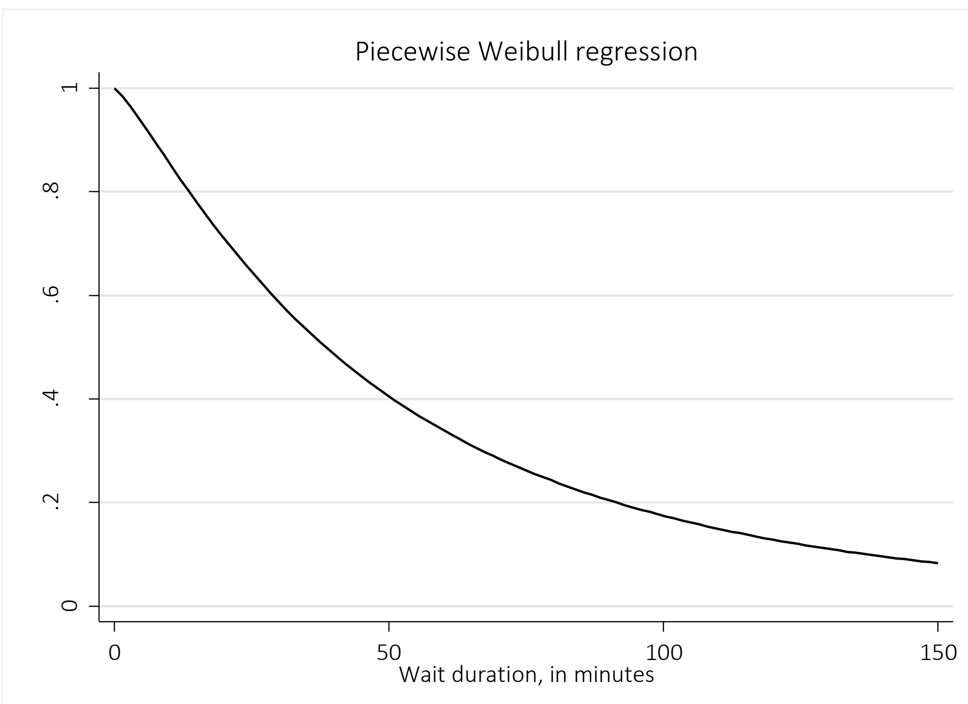


Figure 6a: Probability of waiting up to time *t* by patient characteristics

Estimated survival function from changepoint model
All other explanatory variables set to mean values, unconditional on frailty

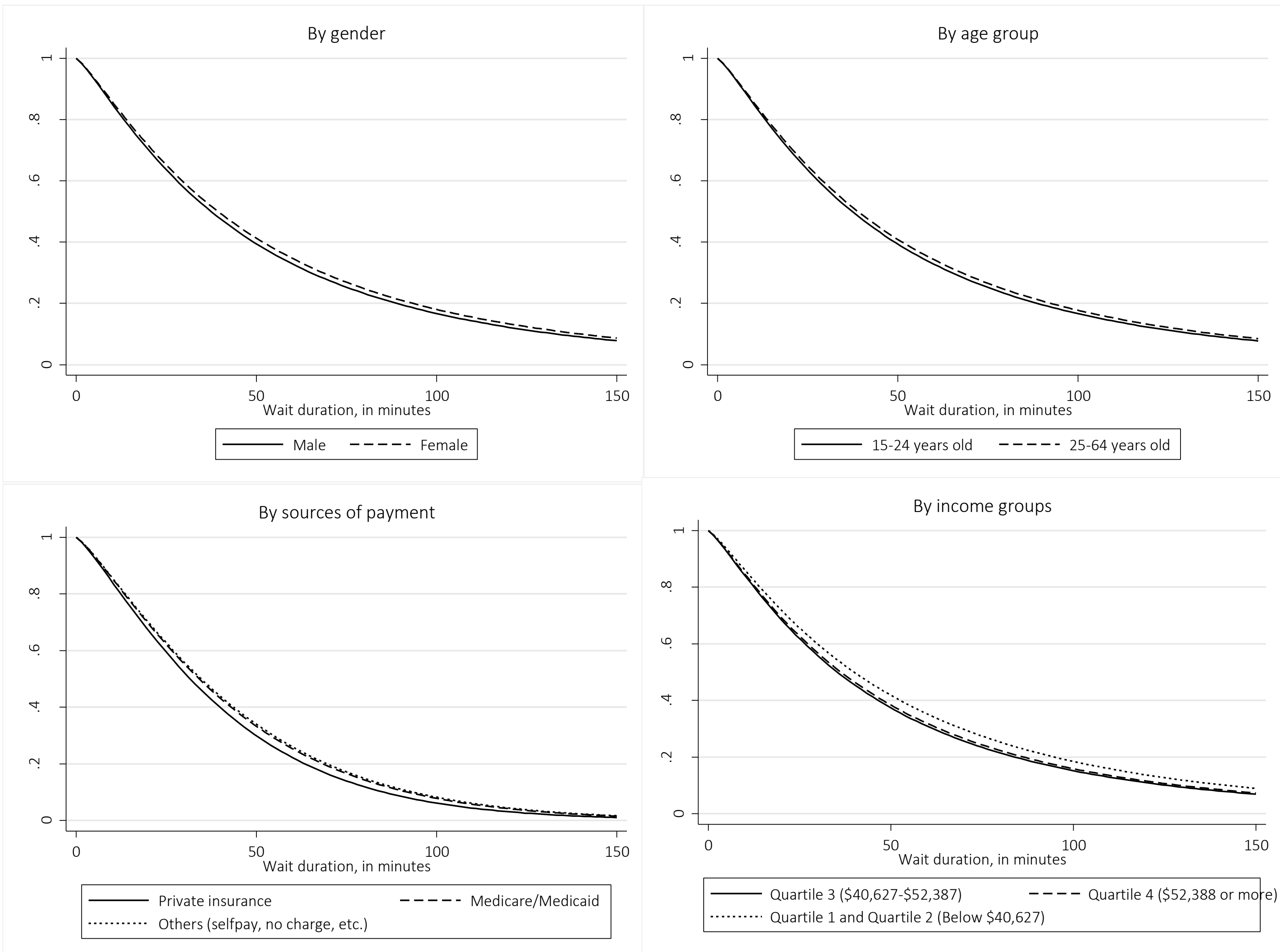


Figure 6b: Probability of waiting up to time *t* by visit and hospital characteristics

Estimated survival function from change point model

All other explanatory variables set to mean values, unconditional on frailty

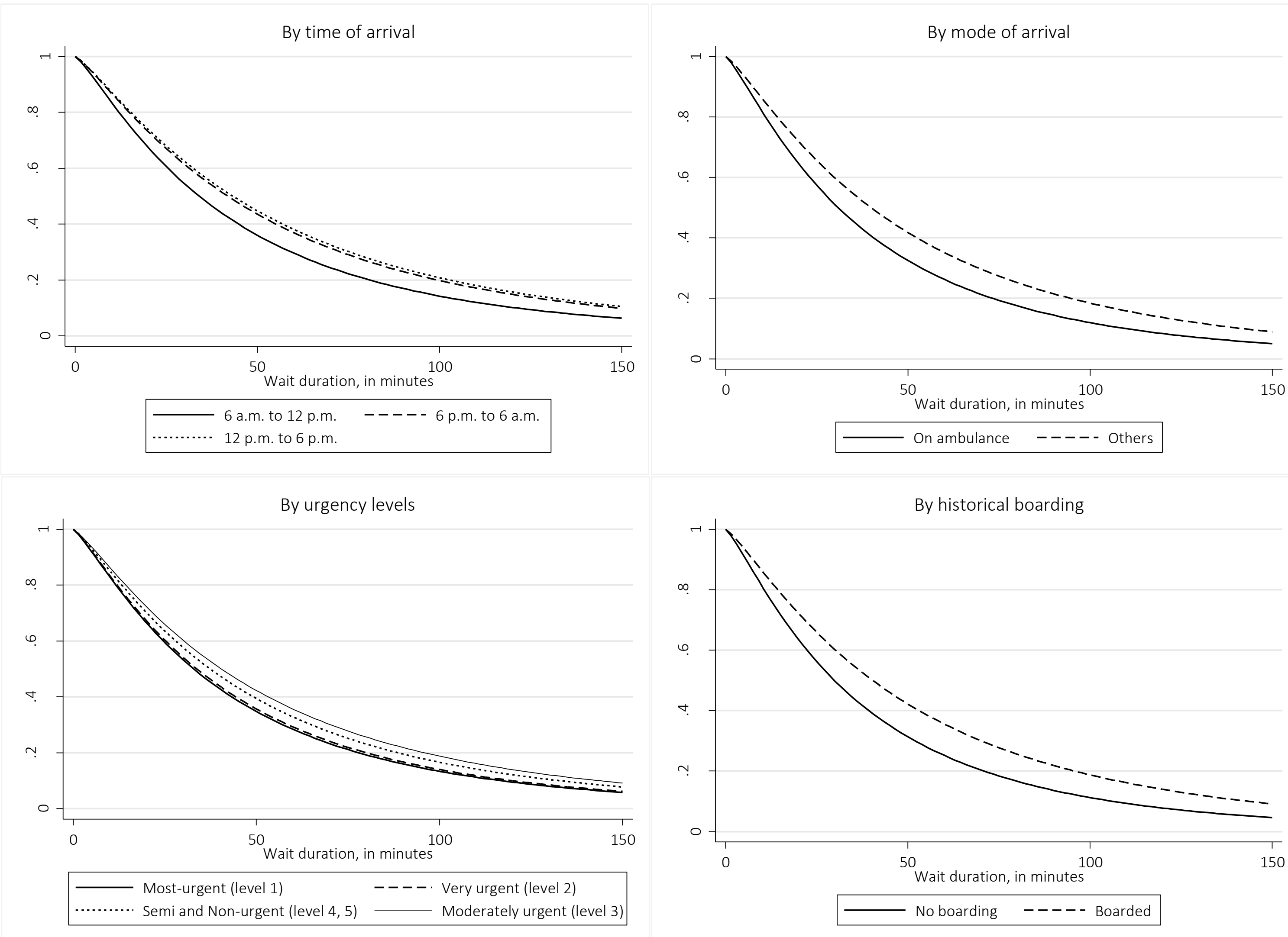
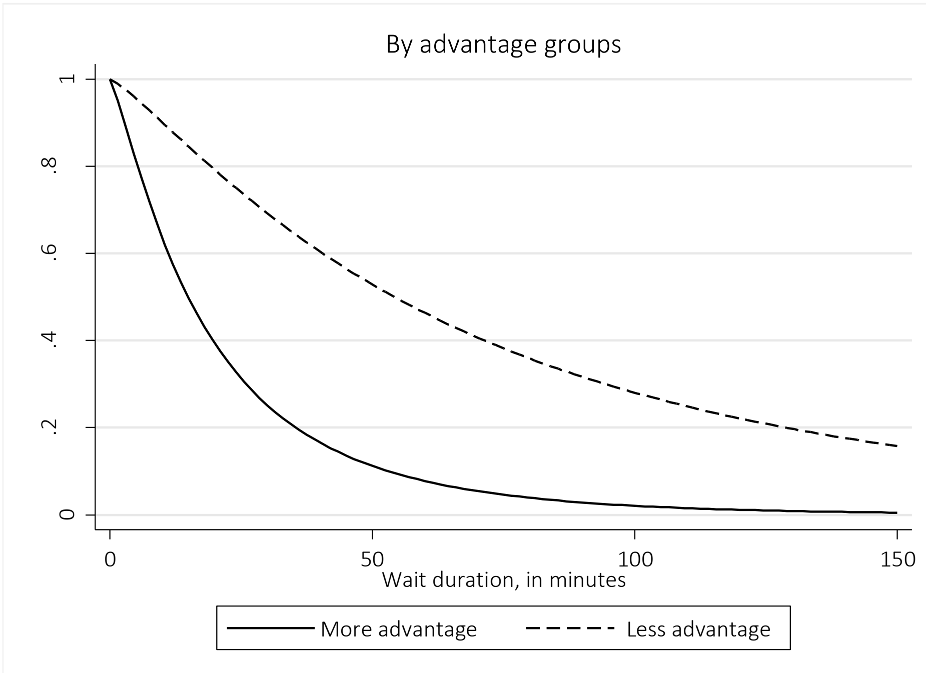


Figure 7: Estimated wait times by "More advantage" and "Less advantage" groups

(Unconditional on frailty)



Characteristics of hypothetical groups

	More Advantage	Less Advantage
Gender	Male	Female
Age	15-24	25-64
Expected payment	Private insurance	Medicare/Medicaid
Household income	Quartile 3	Quartile 1 and 2
Arrival time	6 a.m. – 12 p.m.	12 p.m. – 6 p.m.
Mode of arrival	Ambulance	Other methods
Urgency level	Very urgent	Moderate
ER boarded patients in 2009	No	Yes

Appendix

Deriving Weibull Hazard Function

1. Survival density function

Let T be a non-negative random variable for survival time until an event occurs. T has a probability density function $f(t)$ and an cumulative density function $F(t)$

$$F(t) = \Pr\{T < t\}$$

Let $S(t)$ be the cumulative density function of survival to time t and beyond, have:

$$S(t) = 1 - F(t) = \int_t^{\infty} f(t) \ (1)$$

2. Hazard function

Let $\lambda(t)$ denotes the hazard function. As hazard function is the instantaneous rate of occurrence of event, have:

$$\lambda(t) = \lim_{dt \to 0} \frac{\Pr\{t \leq T < t + dt \mid T \geq t\}}{dt} \ (2)$$

The numerator represents the likelihood of T to fall between the interval $[t, t + dt]$ conditional on $T \geq t$. Dividing this numerator for dt gives the rate of occurrence. Taking the limit as the interval converges to zero give us the instantaneous rate of occurrence.

The expression (1) can be rewritten as the ratio between the (a) likelihood of T being in the interval $[t, t + dt]$ and (b) the likelihood of $T \geq t$.

(a) can be rewritten as $f(t)dt$, with dt being a very small progression in time.

(b) is $S(t)$ by definition.

Integrating (a) and (b) into (2) gives:

$$\lambda(t) = \lim_{dt \to 0} \frac{f(t)dt}{S(t)dt} = \frac{f(t)}{S(t)} \ (3)$$

Since $S(t) = \int_t^{\infty} f(t)$, (2) can now be rewritten as

$$\lambda(t) = -\frac{d}{dt} \log S(t) \ (4)$$

Derive (4) for $S(t)$, with a restriction for lower bound zero gives

$$S(t) = \exp\left\{-\int_0^t \lambda(x)\, dx\right\} \ (5)$$

In the simplest form where hazard is assumed to be constant, have:

$$S(t) = \exp\{-\lambda t\}, (6)$$

Where λ is a hazard parameter.

3. Weibull hazard

Assuming risk can be either increasing or decreasing over time, we can rewrite (6) as:

$$S(t) = \exp\{-(\lambda t)^p\} \ (7)$$

And the hazard rate function is:

$$\lambda(t) = \lambda p(\lambda t)^{p-1} \ (8)$$

Where $\lambda > 0$ and $p > 0$. When $p = 1$, the risk is constant, when $p > 1$ the risk is increasing, and when $0 < p < 1$ the risk is decreasing.