DESIGN AND IMPLEMENTATION OF MULTIPLE ADAPTIVE CONTROLLERS FOR THE QUANSER QUBE SERVO

Prepared by:
Khodor M. Safa
Danny G. Wehbe
Nasser J. Zein

24/5/2017
EECE 665

INTRODUCTION

Why the Quanser Qube

- One of the most used educational platforms
- It models an inverted pendulum
- The inverted pendulum can be present in many cases in real life
- Those cases range from a human being, balancing brooms or meter sticks.

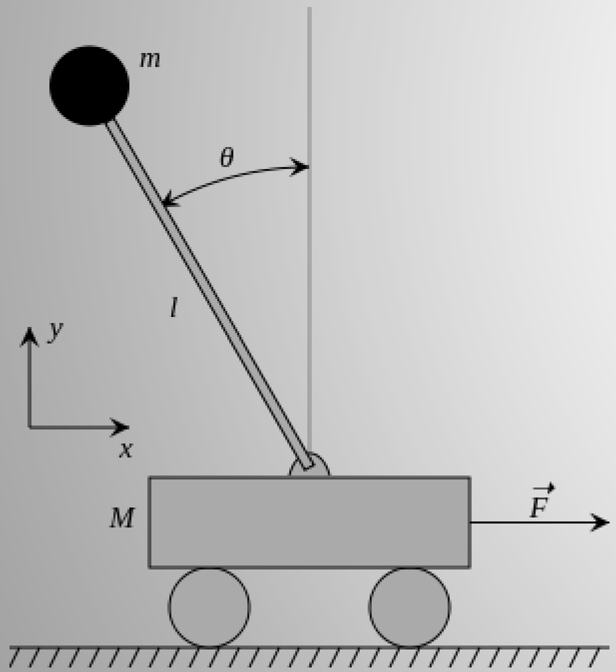
Why adaptive control

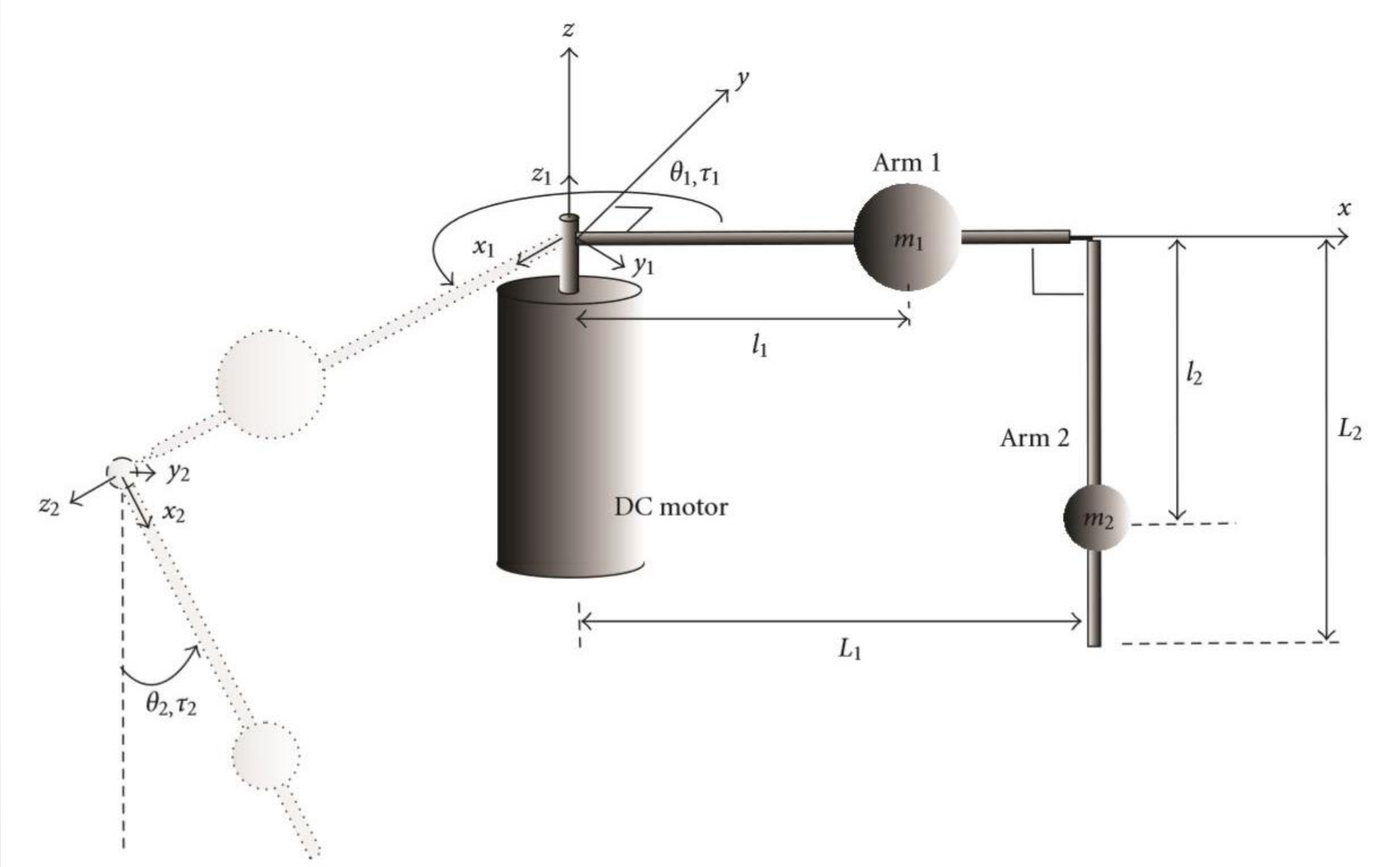
- Real life presents many disturbances
- Those disturbances need to be handled by adaptive control
- Though the Quanser Qube can be controlled by traditional controllers, adaptive control is very efficient when an external disturbance is present

Previous adaptation attempts

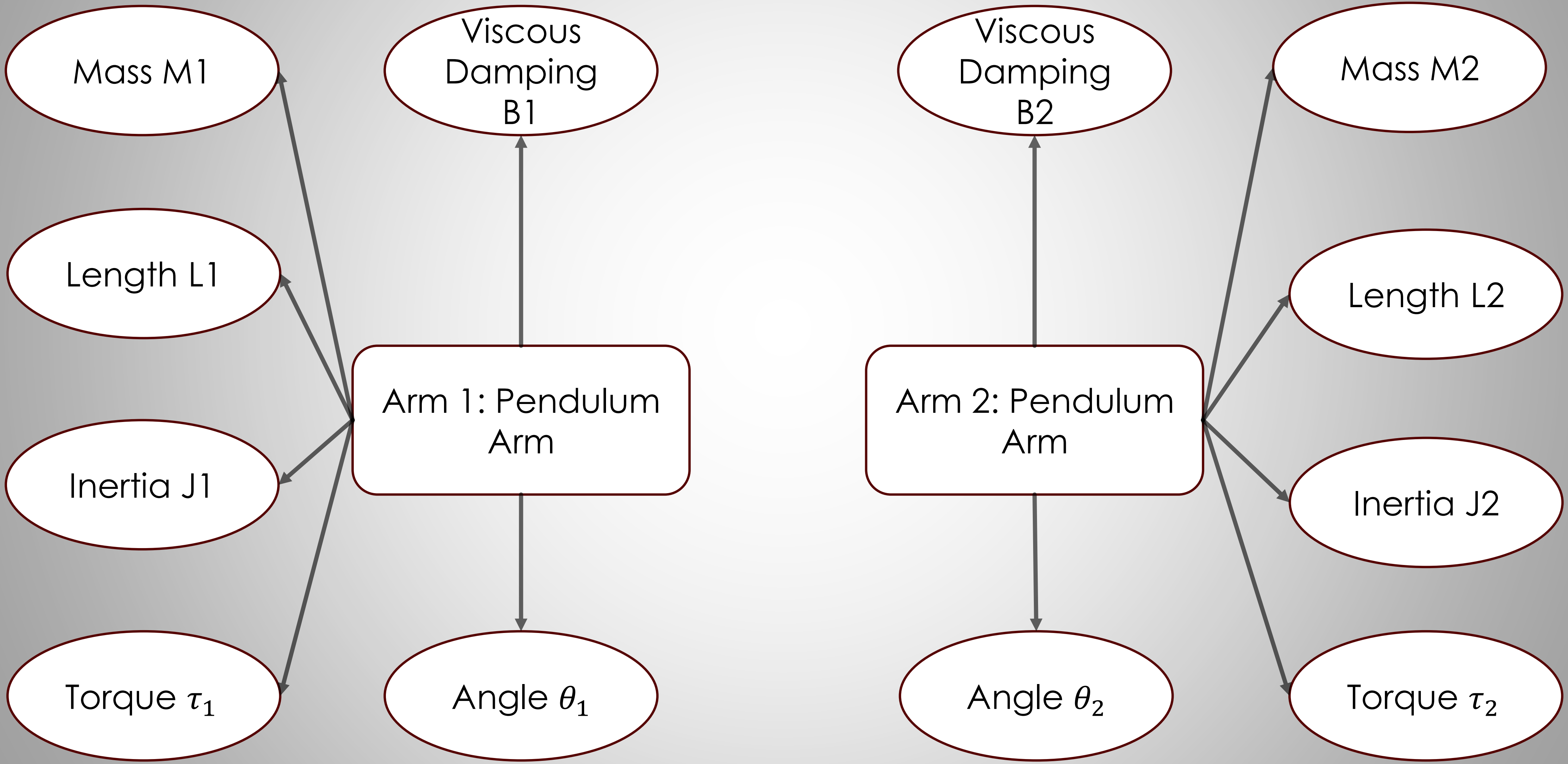
- Adaptive control is not a very old subject
- However, several adaptive controllers were proposed ranging from variable structure adaptive control to adaptive robust control

DYNAMIC MODELING OF THE INVERTED PENDULUM





THE PENDULUM



Deriving the Rotation Matrices of both Arm

Linear and Angular velocities of both arms

Kinetic and Potential Energy

Inserting the obtained values in the Euler - Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) + b_i\dot{q}_i - \left(\frac{\partial L}{\partial q_i}\right) = Q_i$$

Non linear system

$$\ddot{\theta}_1 = \frac{\begin{bmatrix} -\hat{J}_2 b_1 \\ m_2 L_1 l_2 \cos(\theta_2) b_2 \\ -\hat{J}_2{}^2 \sin(2\theta_2) \\ -\frac{1}{2}\hat{J}_2 m_2 L_1 l_2 \cos(\theta_2)\sin(2\theta_2) \\ \hat{J}_2 m_2 L_1 l_2 \sin(\theta_2) \end{bmatrix}^T \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_1\dot{\theta}_2 \\ \dot{\theta}_1{}^2 \\ \dot{\theta}_2{}^2 \end{bmatrix} + \begin{bmatrix} \hat{J}_2 \\ -m_2 L_1 l_2 \cos(\theta_2) \\ -\frac{1}{2} m_2{}^2 l_2{}^2 L_1 \sin(2\theta_2) \end{bmatrix}^T \begin{bmatrix} \tau_1 \\ \tau_2 \\ g \end{bmatrix}}{\hat{J}_0\hat{J}_2 + \hat{J}_2{}^2 \sin^2(\theta_2) - m_2{}^2 L_2{}^2 l_2{}^2 \cos^2(\theta_2)}$$

$$\ddot{\theta}_2 = \frac{\begin{bmatrix} m_2 L_1 l_2 \cos(\theta_2) b_1 \\ -b_2(\hat{J}_0 + \hat{J}_2 \sin^2(\theta_2)) \\ m_2 L_1 l_2 \hat{J}_2 \cos(\theta_2)\sin(2\theta_2) \\ -\frac{1}{2}\sin(2\theta_2)\,(\hat{J}_0\hat{J}_2 + \hat{J}_2{}^2 \sin^2(\theta_2)) \\ -\frac{1}{2} m_2{}^2 l_2{}^2 L_1{}^2 \sin(2\theta_2) \end{bmatrix}^T \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_1\dot{\theta}_2 \\ \dot{\theta}_1{}^2 \\ \dot{\theta}_2{}^2 \end{bmatrix} + \begin{bmatrix} -m_2 L_1 l_2 \cos(\theta_2) \\ \hat{J}_0 + \hat{J}_2 \sin^2(\theta_2) \\ -m_2 l_2 \sin(2\theta_2)\,(\hat{J}_0 + \hat{J}_2 \sin^2(\theta_2)) \end{bmatrix}^T \begin{bmatrix} \tau_1 \\ \tau_2 \\ g \end{bmatrix}}{\hat{J}_0\hat{J}_2 + \hat{J}_2{}^2 \sin^2(\theta_2) - m_2{}^2 L_2{}^2 l_2{}^2 \cos^2(\theta_2)}$$

ASSUMPTIONS

1) All the disturbances at the base of arm 2 are considered negligible hence $\tau_2 = 0$
2) The damping coefficient at arm 2 is considered 0
3) Assume that θ_2 variations are small, then the above nonlinear system can be linearized about the upright equilibrium position of the pendulum i.e. $\theta_1 = 0 \; and \; \theta_2 = 0$
4) The inductance of the DC Motor is assumed to be very small (Next Slide)

LINEARIZING AND ADDING THE MODEL OF THE DC MOTOR

- The differential equation of the DC motor producing the torque τ_1: $Lm\dot{i} + Rmi + Km\dot{\theta}_1 = V$

$\rightarrow \tau_1 = k_m i = k_t \frac{[v(t) - k_m \dot{\theta}_1]}{R_m}$

- Km is the electromotive torque of the servomotor
- Rm is the electrical resistance
- Lm is the electrical inductance it is assumed to be equal to 0.

Final Linearized model

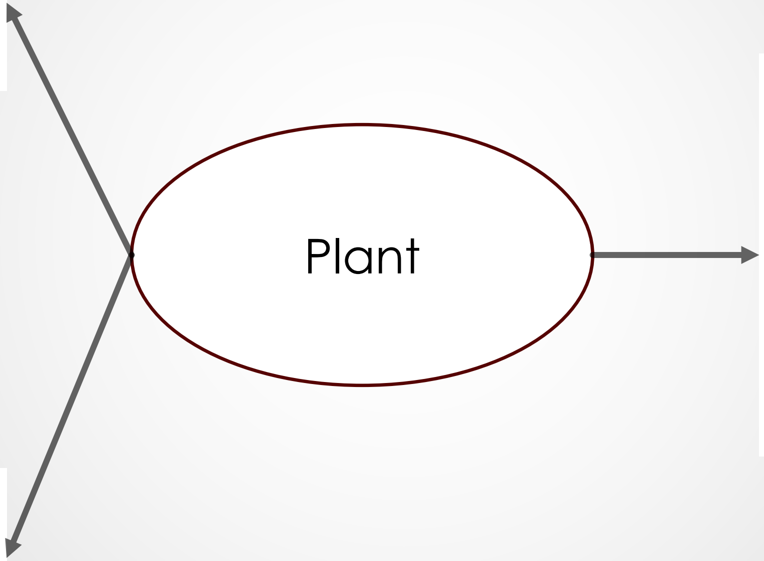
$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & A_{32} & -B_{31}\frac{k_t k_m}{R_m} & 0\\ 0 & A_{42} & -B_{41}\frac{k_t k_m}{R_m} & 0\end{bmatrix}\begin{bmatrix}\theta_1\\ \theta_2\\ \dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix} + \begin{bmatrix}0\\ 0\\ B_{31}\frac{k_t}{R_m}\\ B_{41}\frac{k_t}{R_m}\end{bmatrix}[v(t)]$$

$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & a & b & 0\\ 0 & c & d & 0\end{bmatrix}\begin{bmatrix}\theta_1\\ \theta_2\\ \dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix} + \begin{bmatrix}0\\ 0\\ e\\ f\end{bmatrix}[v(t)] \qquad (16)$$

FINAL OBTAINED MODEL

$$\frac{\theta_1(s)}{V(s)} = \frac{49.71s^2 - 5669}{s^4 + 2.088s^3 - 261.5s^2 - 238.2s}$$

$$\frac{\theta_2(s)}{V(s)} = \frac{49.13s}{s^3 + 2.088s^2 - 261.5s - 238.2}$$



$$\begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 149.226 & -2.08815 & 0 \\ 0 & 261.525 & -2.06359 & 0 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 49.7178 \\ 49.1331 \end{pmatrix} u$$

CONTROLLER DESIGN

1. A TRADITIONAL CONTROLLER: THE PID

- The Quanser Qube works perfectly with traditional controllers, and a PID will do a perfect job in controlling the pendulum without any disturbances present.
- A PID is of the form : $u(t) = k_p e(t) + k_i \int_0^t e(t)dt + k_d \frac{de}{dt}$
- The transfer function used are that of the pendulum and the arm :

$$\frac{\theta_2}{V} = \frac{49.13s}{s^3 + 2.088s^2 - 261.5s - 238.2}$$

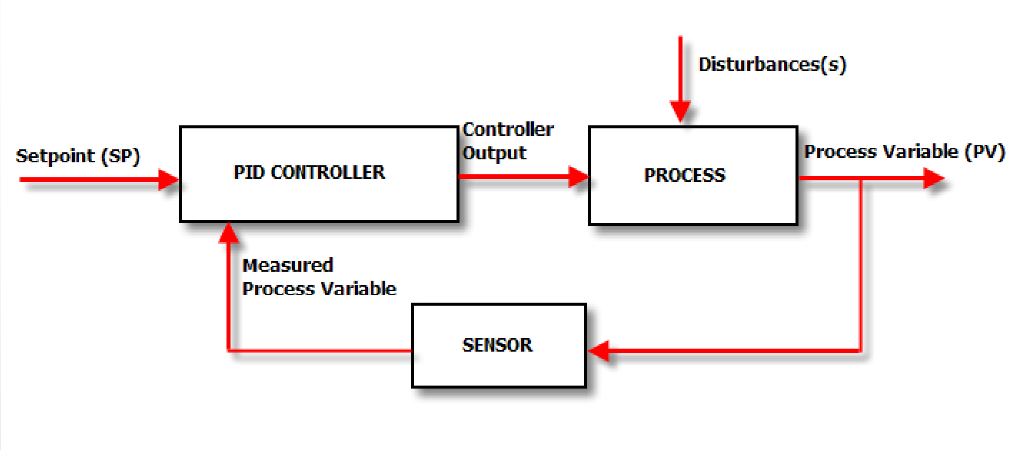
$$\frac{\theta_1}{V} = \frac{49.72s - 5671}{s^{43} + 2.088s^3 - 261.5s^2 - 238.2s}$$

- And the gains were designed to be:

$$k_p = 50$$

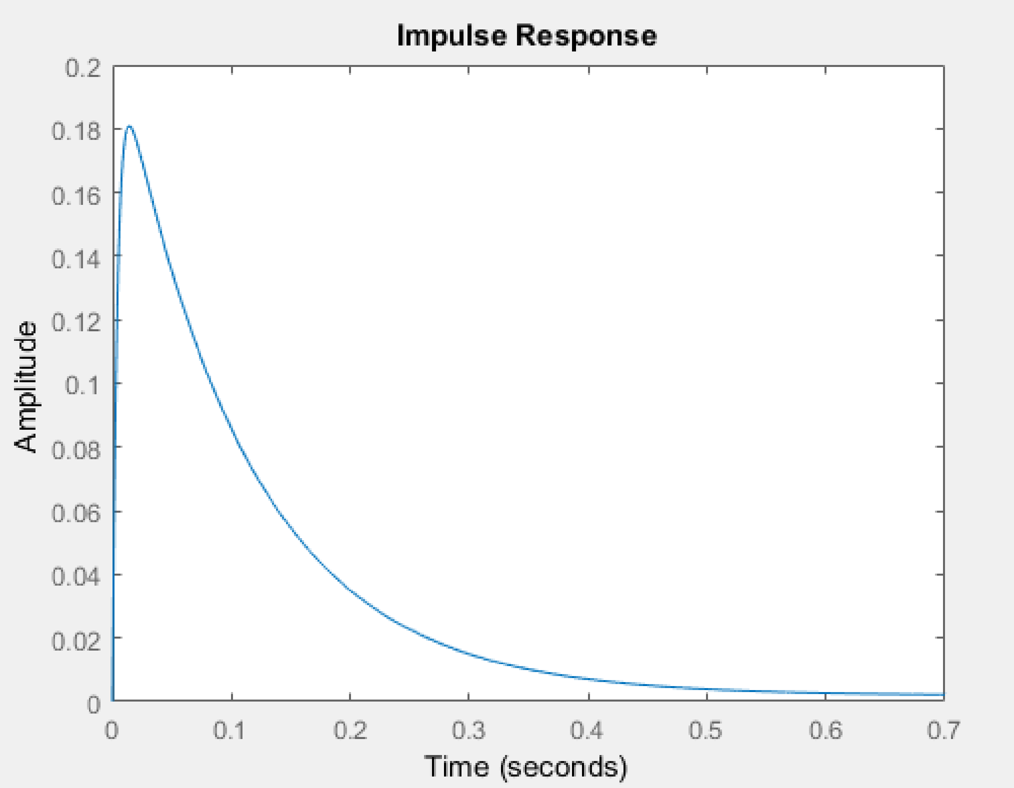
$$k_i = 1$$

$$k_d = 5$$



PID SIMULATION RESULTS

Dynamics of the pendulum

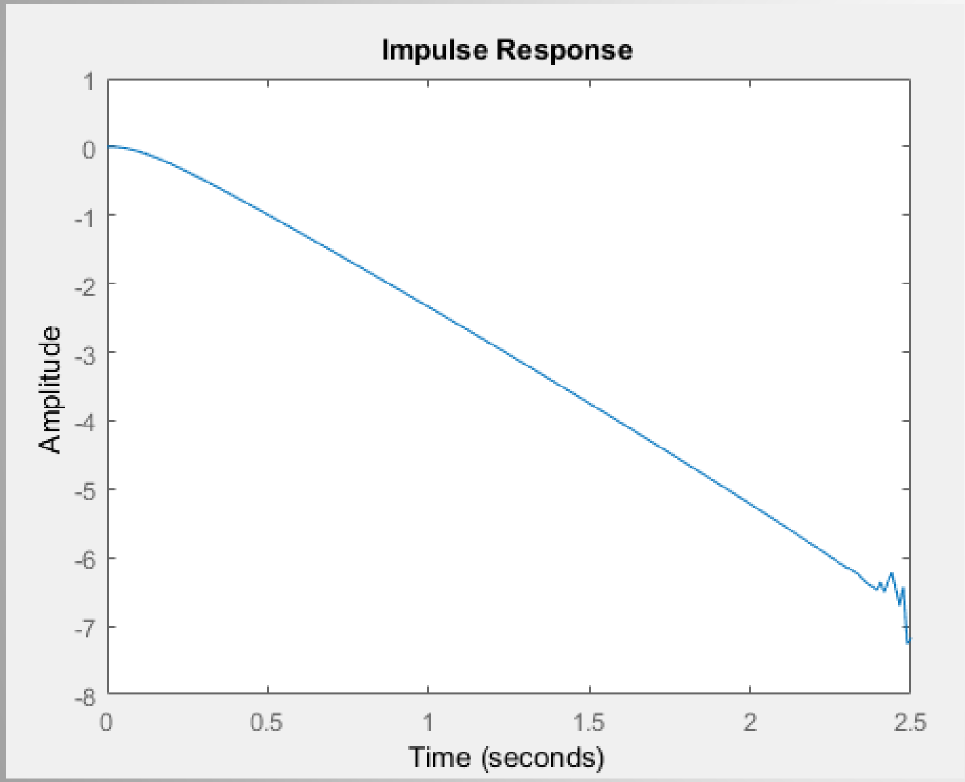


Observations

- The pendulum does not move more than 0.18 radians before it settles to zero
- Thus the PID proves effective in stabilizing the pendulum
- Next we observe the dynamics of the arm

PID SIMULATION RESULTS (CONT'D)

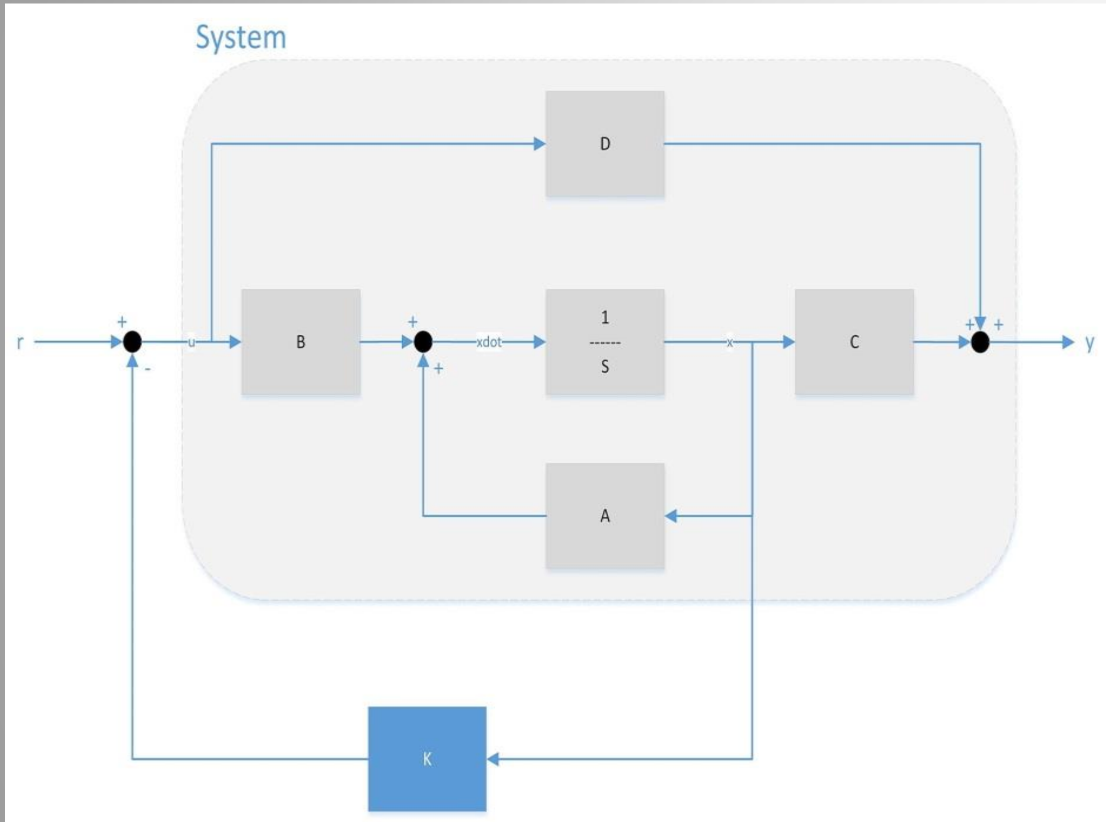
Dynamics of the arm



Observations

- The arm increases in the negative direction at a constant speed
- The arm reaches a point where it cannot rotate anymore (do a full rotation)
- The PID implementation is infeasible

2. FULL STATE FEEDBACK



- The controller was designed via pole placement technique
- The desired poles were designed to be at -25,-25,-10 and 10: far enough from the origin
- LabVIEW's pole placement block aided in obtaining new 4 gains for each state in order to drive the poles to the desired positions.
- The gains were as follows: -11.02, 51.58, -3.13 and 4.55
- The new a matrix to be simulated is then obtained

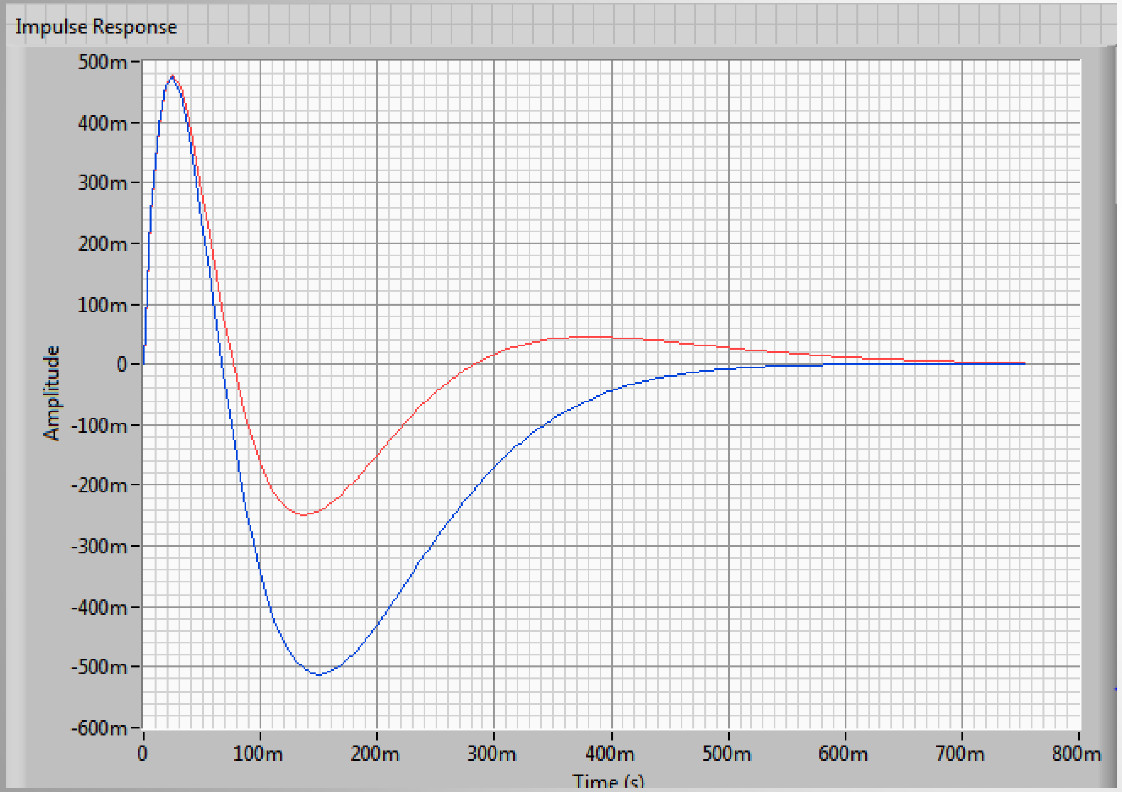
FULL STATE FEEDBACK (CONT'D)

```
A =

   1.0e+03 *

         0         0    0.0010         0
         0         0         0    0.0010
    0.5479   -2.4148    0.1535   -0.2262
    0.5413   -2.2720    0.1517   -0.2236

>> eig(A)

ans =

  -28.1972
  -20.9866
  -12.2478
   -8.6026
```

- The obtained a matrix is shown
- It can be readily noted that the gains provided are effective
- The desired poles are very close to the ones designed.
- The discrepancies are at a maximum of 5 which is not bad
- Those errors are mainly due systematic errors and truncations while performing the calculations

FULL STATE FEEDBACK SIMULATION RESULTS

The Arm Positions

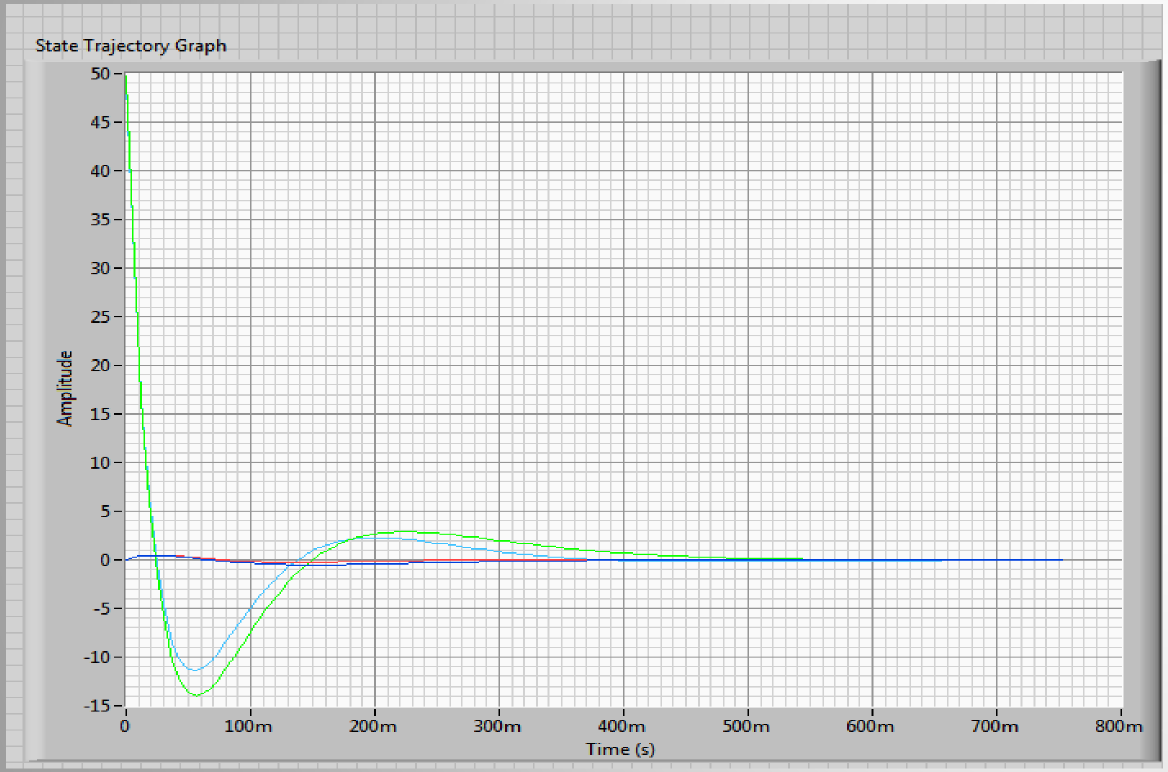


Observation

- Both arms (the rotary arm and the pendulum) are synchronized in their motion, though the magnitude of the position may differ.
- As expected, the position will eventually settle down and go to zero.

FULL STATE FEEDBACK SIMULATION RESULTS (CONT'D)

The Arm Velocities

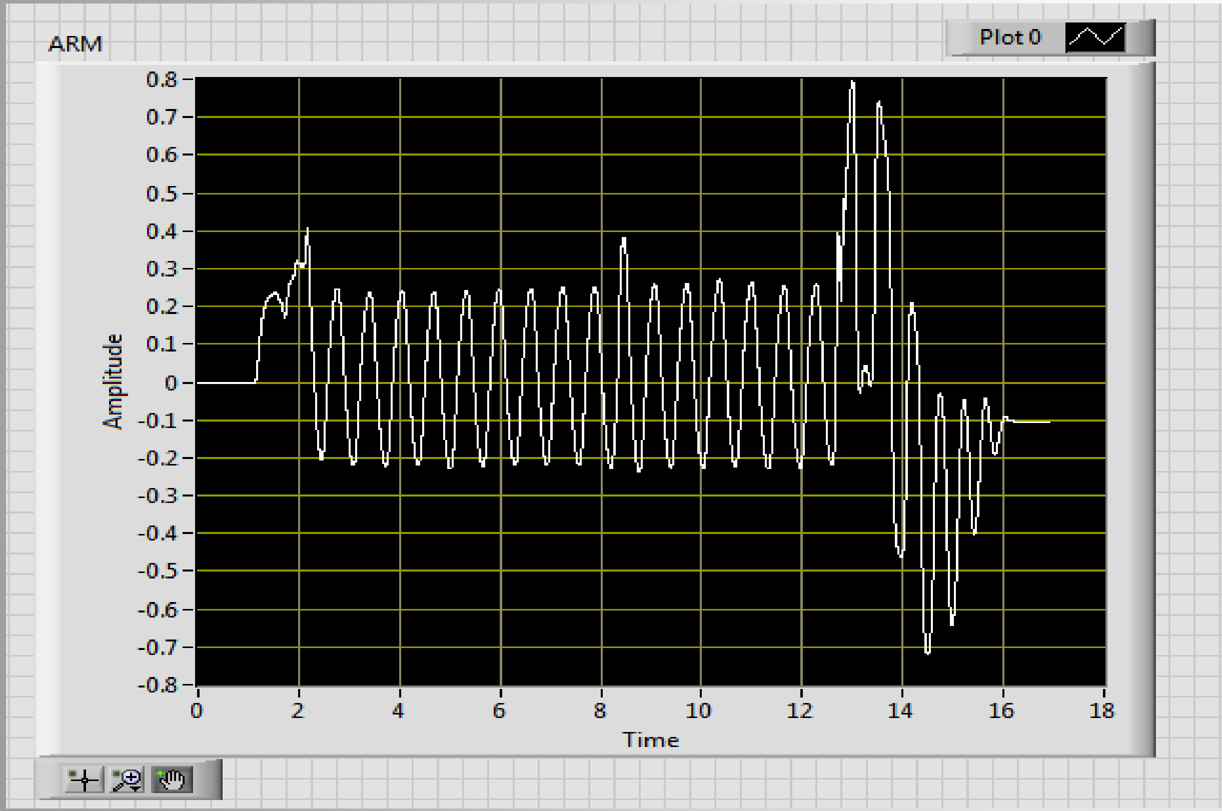


Observation

- As the speeds decrease, the arm positions increase until they reach their maximum extension at zero velocity and vice versa.
- As expected, the position will eventually settle down and go to zero as the speeds also go to zero.

FULL STATE FEEDBACK EXPERIMENTAL RESULTS

The arm angle position

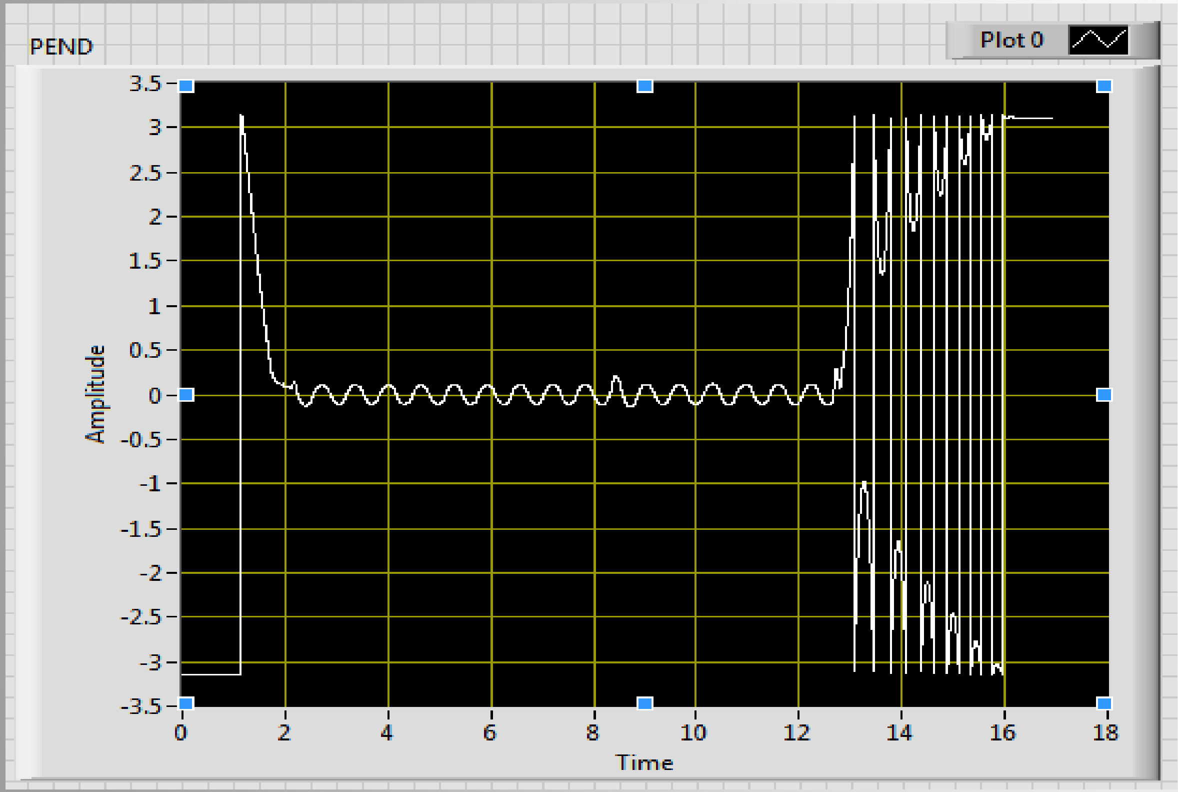


Observation

- Around 8 seconds, a disturbance (hand push) is applied
- It can be readily noticed that the angle position increases positively (since the push was applied when the rotation was to the right side).
- The system recovered and maintained the original position

FULL STATE FEEDBACK EXPERIMENTAL RESULTS (CONT'D 1)

The pendulum angle position

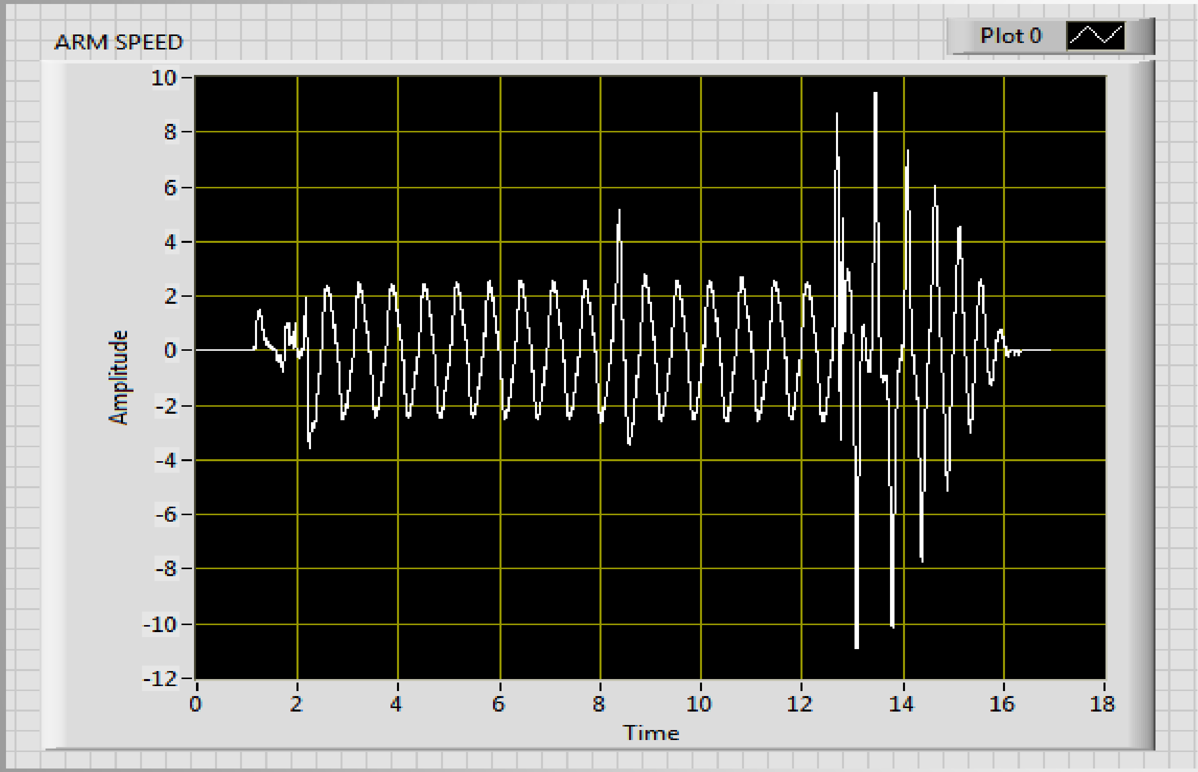


Observation

- The arm and pendulum are interrelated
- The same disturbance caused a fluctuation but at a different magnitude
- Another disturbance (a stronger push) was applied at 13 seconds which pushed the system to an unstable region that could not be handled by the controller and that triggered the system to stop.

FULL STATE FEEDBACK EXPERIMENTAL RESULTS (CONT'D 2)

The Arm Speed

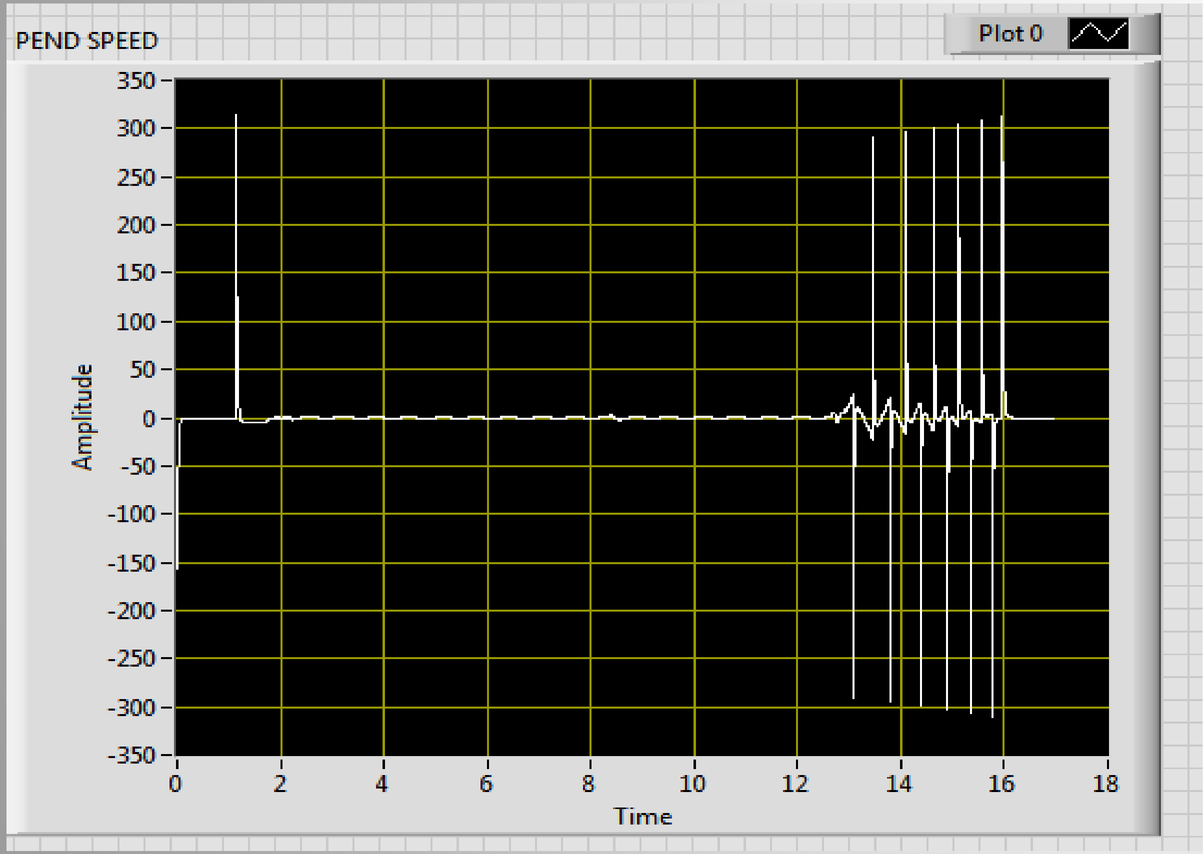


Observation

- When the disturbance is applied, an increase of the arm speed is observed.
- This speed then goes back to original value after the system stabilizes (adapts).
- After the second, bigger disturbance, the speed exceeds the limit and causes the system to stop (speed goes to zero)

FULL STATE FEEDBACK EXPERIMENTAL RESULTS (CONT'D 3)

The pendulum speed



Observation

- When the disturbance is applied, an increase of the arm speed is observed.
- This speed then goes back to original value after the system stabilizes (adapts).
- After the second, bigger disturbance, the speed exceeds the limit and causes the system to stop (speed goes to zero)

3. INDIRECT SELF TUNING REGULATOR

Data:

- Discretized system: $G(z) = \frac{9.814\times10^{-5} - 1.365\times10^{-7}z - 9.8\times10^{-5}}{z^3 - 2.997z^2 + 2.993z - 0.9958}$

- Reference Model: $H(z) = \frac{b_{m0}z^2 + b_{m1}z + b_{m2}}{z^3 + a_{m1}z^2 + am_2 z + am_3} = \frac{B_m}{A_m}$

- Observer Polynomial:

- No zero Cancelation

Step 1: The parameters of A and B are estimated using RLS block in Simulink

Step 2: Applying the MDPP we get that: deg (R)=deg (S) = deg (T)= 2

- Solving the Diophantine equation ($AR + BS = A_O A_m$) we get the following 5 equations with 5 unknowns:

$$r_1 + b_0 s_0 = a_{m1} - a_1$$
$$a_1 r_1 + r_2 + b_1 s_0 + b_0 s_1 = a_{m2} - a_2$$
$$a_2 r_1 + a_1 r_2 + b_1 s_1 + b_0 s_2 = a_{m3} - a_3$$
$$a_3 r_1 + a_2 r_2 + b_2 s_1 + b_1 s_2 = 0$$
$$a_3 r_2 + b_2 s_2 = 0$$

- Using MATLAB the expressing of s_0,s_1, s_2, r_1 and r_2 are obtained. The obtained expressions are very bulky.

- Hence the final control Law:

$$U(t) = -r_1 \mathrm{u}(\mathrm{t}-1) - r_2\, u(t-2) - s_0 y(t) - s_1 y(t-1) - s_2 y(t-2)$$

- Note that the expression of T was not calculated since the reference input is 0.

REFERENCES

[1] Astrom, K. J., & Wittenmark, B. (1995). *Adaptive Control.* Massachusetts: Dover.
[2] *Quanser - QUBE-Servo*. (2017). Retrieved from Quanser Innovate.Educate: http://www.quanser.com/products/qube_servo

[3] Parakash, B., Roy, B. K., & Biswas, R. K. (2016). *IEEE XPLORE*. Retrieved from http://ieeexplore.ieee.org.ezproxy.aub.edu.lb/stamp/stamp.jsp?arnumber=7853165

[4] Wikipedia. (2017). *Inverted Pendulum*. Retrieved from Wikipedia: https://en.wikipedia.org/wiki/Inverted_pendulum#Types_of_inverted_pendulums

[5] Chen, Y.-F., & Huang, A. C. (2013, November 6). *SpringerLink*. Retrieved from http://download.springer.com/static/pdf/652/art%253A10.1007%252Fs11071-013-1112-4.pdf?originUrl=http%3A%2F%2Flink.springer.com%2Farticle%2F10.1007%2Fs11071-013-1112-4&token2=exp=1491237158~acl=%2Fstatic%2Fpdf%2F652%2Fart%25253A10.1007%25252Fs11071-013-111

[6] Quanser Inc,. (2017). *USER MANUAL QUBE-Servo Rotary Servo Experiment Set Up and Configuration*. (Markham, Ontario. Retrieved from: http://www.quanser.com/Content/CoursewareNavigators/qubeservo_matlab/Courseware/Setup/QUBE-Servo%20User%20Manual.pdf

[7] EECE 460L Lab Manual Fall 2016-2017: Experiment 10

[8] Astrom, K. & Wittenmark, B. (2008). *Adaptive Control* (2nd ed.). New York: Dover Publications, INC.

[9] Cazzolato, B. & Prime, Z. (2011). On the Dynamics of the Furuta Pendulum. *Journal of Control Science and Engineering, 2011*, 1-8