Rhymes to Music

Matching Algorithm

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Introduction

2.1 Motivation

When working on an data sonification, I wanted to algorihtmically assign input data to certain output notes. The input dataset was a discrete set of values and the expected output was another discrete set of values. Each individual element of a dataset has a frequency of occurrence within the set. After figuring out the frequency of occurrences of elements for each respective set, the goal was as follows:

> Map elements of the input data set to the output data set in such a way that as much of the input data set as possible is proportionally mapped into the output data set, and the output data set's frequencies of occurrences are as close to the actual representation as possible.

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Literature Review

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Modeling Approach

4.1 Background Model

4.2 Model Formulation

The linear programming representation I developed offhand was as follows.

4.2.1 Definitions and Notation

Sets

The sets of phonemes, $\mathcal{P}$, and scale degrees, $\mathcal{S}$ are used in defining this linear program.

Symbol	Composition	Name	Description
$\mathcal{P}$	$\{p_1, p_2, \ldots, p_k\}$	Phonemes	the set of k alphabet-phonemes extracted from the input text
$\mathcal{S}$	$\{s_1, s_2, \ldots, s_l\}$	Scale Degrees	the set of all scale degrees from a given musical database

Decision Variables

The variables, $X(p, s)$ and $Y(p, s)$ help us determine how much of each phoneme is assigned to which scale degree. We will determine their values through the optimization.

Symbol	Description
$X(p, s)$	amount of phoneme p assigned to scale degree s
$Y(p, s)$	binary indicator variable indicating if phoneme p was assigned to scale degree s

Parameters

Our parameters include A, W, $C(p)$, and $D(s)$. A and W are constants. $\bar{C}$ and $\bar{D}$ are fixed vectors composed of $C(p)$ and $D(s)$. As the sum over all probabilities in a space must be one, $\sum_p C(p) = 1$ and $\sum_s D(s) = 1$.

Symbol	Description
A	number of scale degrees one phoneme p can be assigned to
W	large weight
$C(p)$	probability of phoneme p occurring in the text
$D(s)$	probability of scale degree s occurring in the musical corpus

4.2.2 Objective

Our goal is to maximize the amount of allocated phonemes, or minimize the amount of unallocated phonemes.

$$\min \qquad -\sum_p \sum_s X(p,s). \tag{4.1}$$

4.2.3 Constraints

The amount assigned to a scale degree s cannot exceed the probability of that scale degree occurring, $D(s)$.

$$\sum_p X(p,s) \quad \le \quad D(s), \quad \forall s \in \mathcal{S}. \tag{4.2}$$

Similarly, the amount of phoneme p assigned to a set of scale degrees cannot exceed the probability of that phoneme occurring, $C(p)$.

$$\sum_s X(p,s) \quad \le \quad C(p), \quad \forall p \in \mathcal{P}. \tag{4.3}$$

All phonemes p such that $C(p) \ge 0$ need to be assigned to some scale degree... but this will happen because of the objective? Check - July 5 in notebook, but probably unnecessary constraint

Phoneme p must be assigned to A buckets. *This constraint is an equality... hm... bad.*

$$\sum_s Y(p,s) \quad = \quad A, \quad \forall p \in \mathcal{P}. \tag{4.4}$$

The amount of phoneme p assigned to A buckets is restricted.

$$X(p,s) \quad \le \quad W \times Y(p,s), \quad \forall p \in \mathcal{P}, s \in \mathcal{S}. \tag{4.5}$$

And, we need nonnegativity constraints.

$$X(p,s) \quad \ge \quad 0, \quad \forall p \in \mathcal{P}, s \in \mathcal{S}, \tag{4.6}$$
$$Y(p,s) \quad \ge \quad 0, \quad \forall p \in \mathcal{P}, s \in \mathcal{S}. \tag{4.7}$$

4.2.4 Linear Program

Therefor, our linear program is

$$\begin{array}{llcrr}
\min & -\sum_p \sum_s X(p,s) & & & \\
\text{s.t.} & \sum_p X(p,s) & \le & D(s), & \forall s \in \mathcal{S}, \\
 & \sum_s X(p,s) & \le & C(p), & \forall p \in \mathcal{P}, \\
 & \sum_s Y(p,s) & = & A, & \forall p \in \mathcal{P}, \\
 & X(p,s) & \le & W \times Y(p,s), & \forall p \in \mathcal{P}, s \in \mathcal{S}, \\
 & X(p,s) & \ge & 0, & \forall p \in \mathcal{P}, s \in \mathcal{S}, \\
 & Y(p,s) & \ge & 0, & \forall p \in \mathcal{P}, s \in \mathcal{S}.
\end{array}$$

4.3 Simplifications

4.3.1 Process Overview

As the LP is difficult to implement, consider using the following heuristic.

Given

- Take $\bar{x} =$ **phonemeProportions** and $\bar{y} =$ **scaleDegProportions**. They should contain the proportions of each phoneme and each scale degree given the data set.

Process

1. Create a new data frame,**assignments**

5
Testing

6
Output

7
Future Work and Conclusions