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CSC 380

Homework 7 Part 2

Due November 18, 2020

Part II – Simulated Annealing for TSP

Part A

For this section, we wish to perform experiments to estimate how many guesses, on average, simulated annealing requires for a graph of a given number of vertices. From this information we will try to estimate the Big-O efficiency of the simulated annealing algorithm, as applied to the traveling salesman problem. Due to the rather lengthy running times encountered during the course of these experiments, the number of vertices examined was kept to under 20. To estimate the Big-O of the simulated annealing algorithm, the number of vertices were graphed (on the x-axis) against the average number of guesses (on the y-axis), as shown in Figure 1. Also graphed with the simulated annealing curve is the graph of the function 1.8^n, where n represents the number of vertices. The two curves seem similar and seem to have a similar pattern. It is hypothesized, then, that the Big-O for simulated annealing, as applied to the traveling salesman problem, is $O(1.8^n)$.

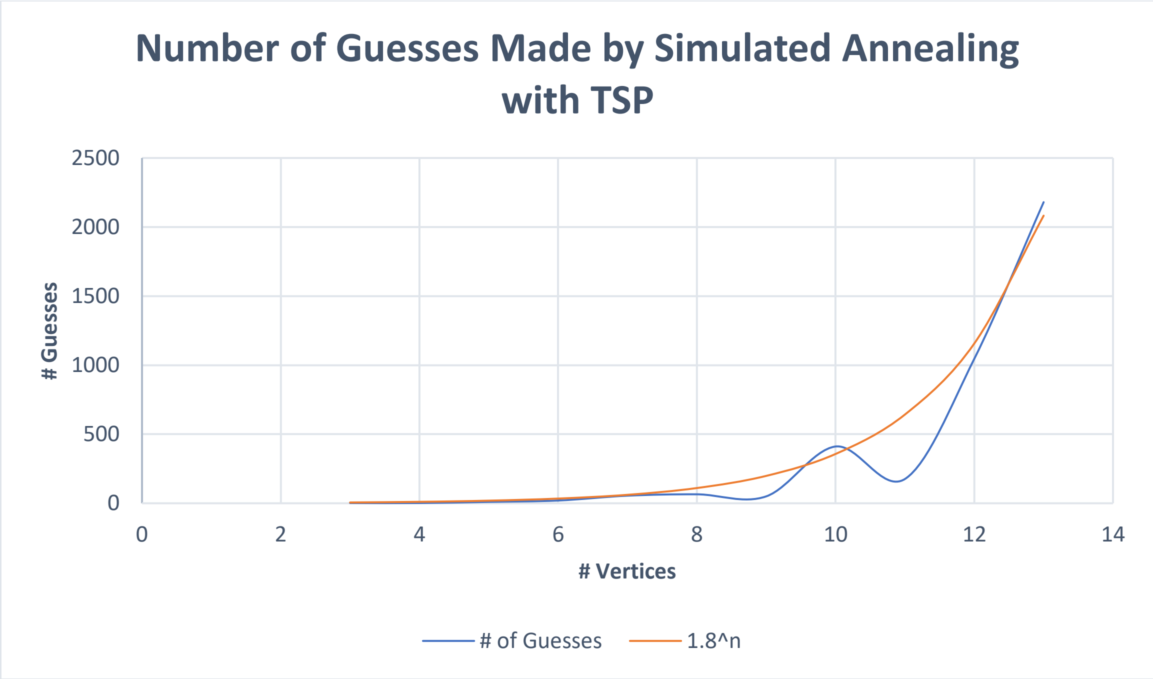

Figure 1. Graph of number of vertices versus number of guesses made by simulated annealing implementation in trying to solve the traveling salesman problem for a graph of size n.

Based on this estimation of the Big-O, with the function 1.8^n, we will try to predict the number of guesses made by simulated annealing for 15, 17, and 19 vertices (which as can be seen from Figure 1 is an extrapolation). These predictions and the actual number of guesses made are shown in Table 1 below. Percent error shown in Table 1 was calculated using the following formula:

$$\% \, Error = \frac{experimental\ value - theoretical\ value}{theoretical\ value} \times 100\%$$

The predicted number of guesses was treated as the theoretical value, while the actual number of guesses was considered as the experimental value.

Value of n	*Predicted Number of Guesses*	*Actual Number of Guesses*	*Percent Error*
15	6747	16840	149.59%
17	21860	60899	-7.55%
19	70824	76046	7.37%

Table 1. Results of extrapolation of the function estimation for the Big-O of the simulated annealing algorithm, compared with actual results.

The rather high percent error for n = 15 could be because of a particularly difficult graph that happened to be randomly generated during this set of experiments. The function estimation of 1.8^n seemed to work well for n = 17 and n = 19. From these results, it seems likely that a good estimation of the Big-O for simulated annealing, as applied to the traveling salesman problem, could be $O(1.8^n)$.

Part B: Suboptimality of Simulated Annealing For TSP with Different Cooling Rates

In this section, different cooling rates will be tested for the simulated annealing algorithm. The experiments in the previous section were run with starting temperature of 75. The two cooling rates to be tested in this section are T = 25, T = 50, T = 75, T = 100, and T = 250. We wish to see if the optimality of simulated annealing changes depending on the starting temperature. To determine optimality, we will be calculating percent error based on the percent error formula shown in Part A, with experimental value being defined as the actual path cost and theoretical value being defined as the optimal path cost. To determine the optimal solution to compare against, this was calculated by multiple 10 by the number of vertices in the graph. For example, for a graph with 4 vertices, the "optimal solution" is considered to be 10 * 4 = 40. The results of these experiments with different cooling values are shown in Tables 2 through 6 below.

# Vertices	Cost	Opt Cost	Percent Error
3	30	30	0
4	40	40	0
5	50	50	0
6	51	60	-15
7	70	70	0
8	80	80	0
9	90	90	0
10	100	100	0
11	110	110	0
12	120	120	0
13	129	130	-0.76923
14	140	140	0
15	147	150	-2
16	147	160	-8.125
17	167	170	-1.76471

Table 2. Results of experiments on simulated annealing with T = 25.

# Vertices	Cost	Opt Cost	Percent Error
3	30	30	0
4	40	40	0
5	50	50	0
6	60	60	0
7	70	70	0
8	76	80	-5
9	87	90	-3.33333
10	96	100	-4
11	110	110	0
12	120	120	0
13	130	130	0
14	134	140	-4.28571
15	150	150	0
16	153	160	-4.375
17	162	170	-4.70588

Table 3. Results of experiments on simulated annealing with T = 50.

# Vertices	Cost	Opt Cost	Percent Error
3	30	30	0
4	40	40	0
5	50	50	0
6	60	60	0
7	68	70	-2.85714
8	80	80	0
9	86	90	-4.44444
10	100	100	0
11	104	110	-5.45455
12	115	120	-4.16667
13	128	130	-1.53846
14	140	140	0
15	150	150	0
16	151	160	-5.625
17	159	170	-6.47059

Table 4. Results of experiments on simulated annealing with T = 75.

# Vertices	Cost	Opt Cost	Percent Error
3	30	30	0
4	40	40	0
5	50	50	0
6	60	60	0
7	70	70	0
8	69	80	-13.75
9	90	90	0
10	82	100	-18
11	104	110	-5.45455
12	120	120	0
13	130	130	0
14	140	140	0
15	144	150	-4
16	152	160	-5
17	164	170	-3.52941

Table 5. Results of experiments on simulated annealing with T = 100.

# Vertices	Cost	Opt Cost	Percent Error
3	30	30	0
4	40	40	0
5	50	50	0
6	58	60	-3.33333
7	70	70	0
8	80	80	0
9	90	90	0
10	89	100	-11
11	108	110	-1.81818
12	120	120	0
13	130	130	0
14	140	140	0
15	150	150	0
16	160	160	0
17	169	170	-0.58824

Table 6. Results of experiments on simulated annealing with T = 250.

All of the starting temperature values tested generally had good performance, with most experiments having a percent error of 0. However, the smallest initial temperature value of T = 25 did have a few large percent errors for a few observations. Other than this suboptimality for the lowest initial temperature value, the other starting T values generally seemed to perform equally well, at least for the vertex values tested. Graphs with a larger number of vertices may indicate different levels of performance for different starting temperatures.