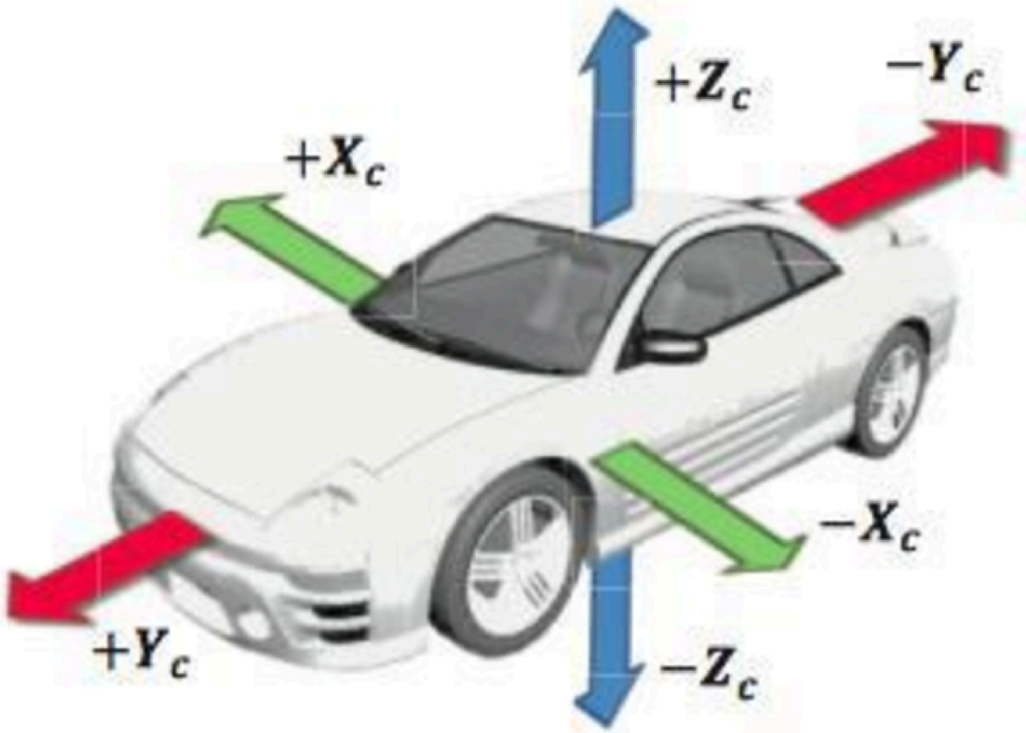
German University in Cairo (GUC)
Faculty of Engineering & Materials Science (EMS)
Mechatronics Department (MCTR)
Numerical Analysis ENME 602

Vehicle Trajectory Reconstruction and Analysis

Members:
Khalid Waleed 58-2224 *T38*
Rimaz Mohamed 58-15938 *T38*

The purpose of this project is to reconstruct and analyze the trajectory of a vehicle based on time-series accelerometer data along the X and Y axes. The dataset consists of acceleration measurements over time, and the goal is to estimate the vehicle's position and examine its movement across a two-dimensional plane.



In this project, we tackle three key tasks:

1. **Numerical Double Integration**: We apply numerical integration techniques to compute the velocity and position of the vehicle in the X and Y directions. This task involves using acceleration data to reconstruct the vehicle's trajectory over time, which will be visualized in position-time graphs.

2. **Polynomial Trajectory Approximation**: Once the position data is obtained, we use polynomial interpolation to model the trajectory in terms of time. By fitting polynomials of varying degrees to the data, we aim to determine the most suitable degree based on the accuracy of the fit.

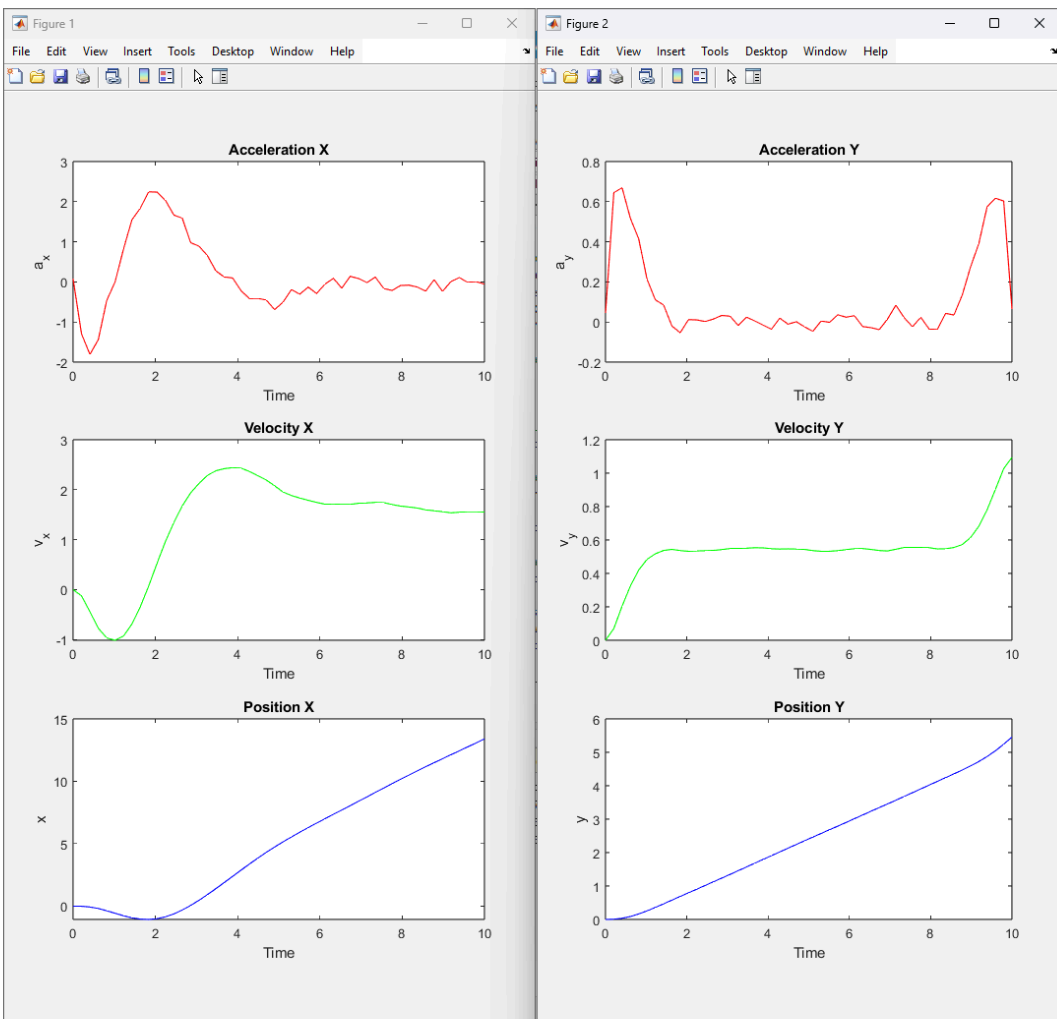
3. **Position Query via Root-Finding**: In the final task, we use several root-finding algorithms (such as Bisection Method, Newton Method, Secant Method, and False Position Method) to estimate the time at which the vehicle reaches a specific target position, calculated as 10.5 meters from the starting point.

<u>Task 1:</u>

The **composite trapezoidal rule** was selected for integrating acceleration data to compute velocity and position because:

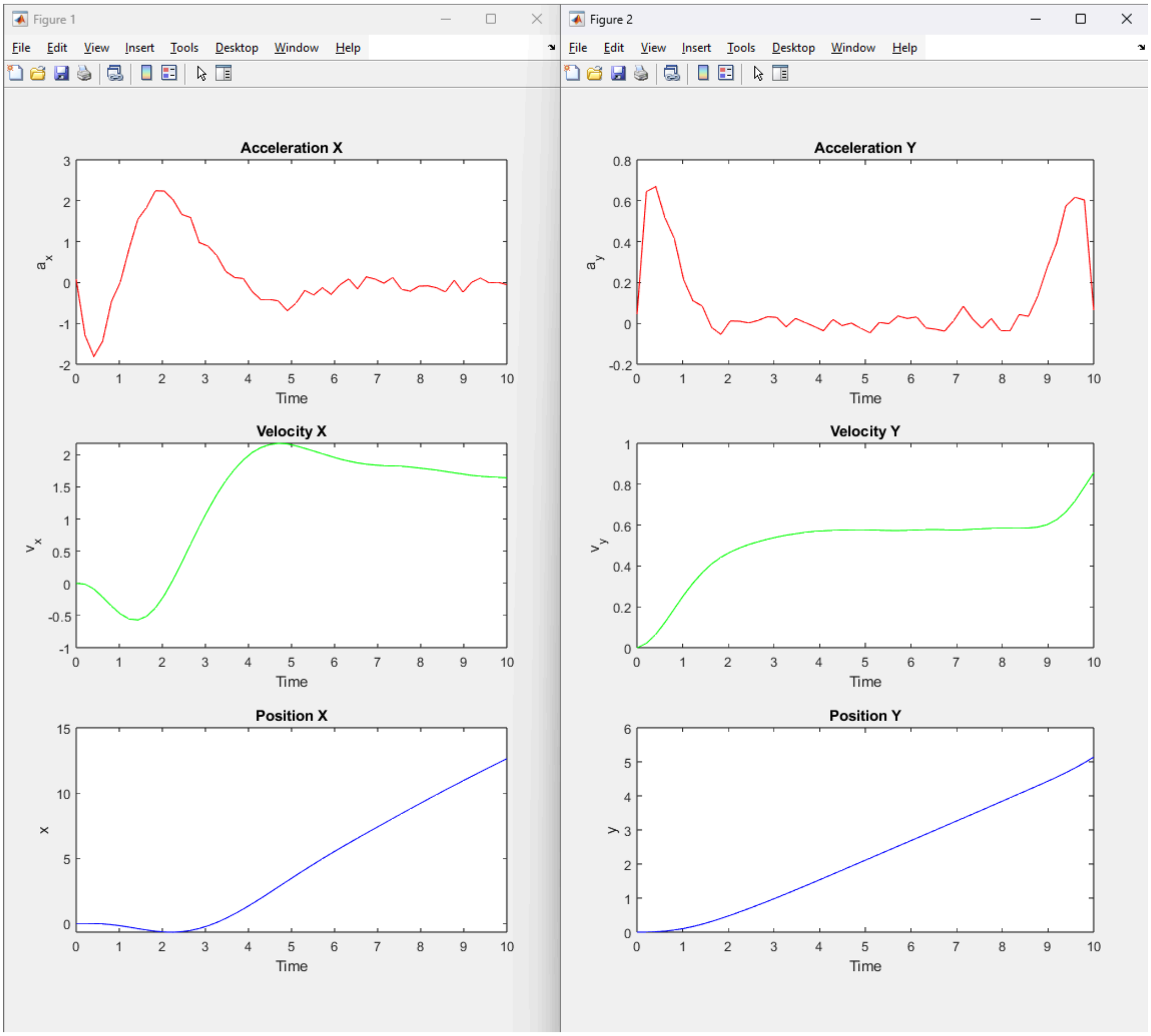
- It is **simple to implement** and computationally efficient.
- It works well for **uniformly spaced time-series data**, which matches the format of the accelerometer dataset.

Our initial results looked like this:



These were the initial results. However, we can clearly see that there are some vibrations caused by spikes in data. This could be due to sensor imperfections, road vibrations, or sudden jerks and slight movements.

By adding a low pass filter, we obtained the following results:



As you can see, we obtain a smoother curve which indicates more realistic accelerometer data.

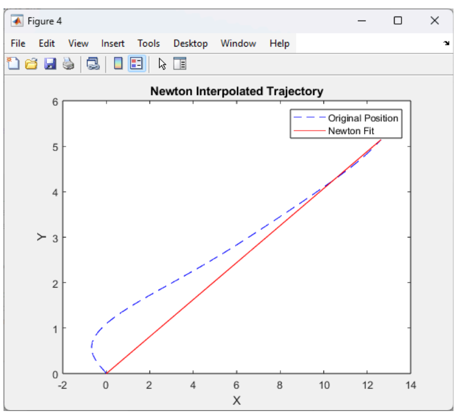
Task 2:

Newton's Divided Difference method was used to fit polynomials because:

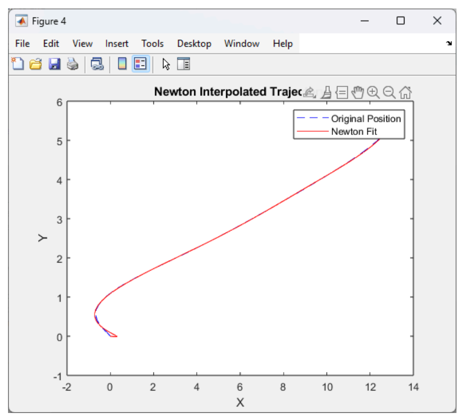
- It builds the polynomial **incrementally**, making it easier to compare polynomials of different degrees.
- The method allows for **efficient reuse of previous computations**, which is useful when adjusting polynomial degree for best fit.

Our dataset consists of 50 data points, and it isn't efficient to use all 50. This concluded in us using the following numbers:

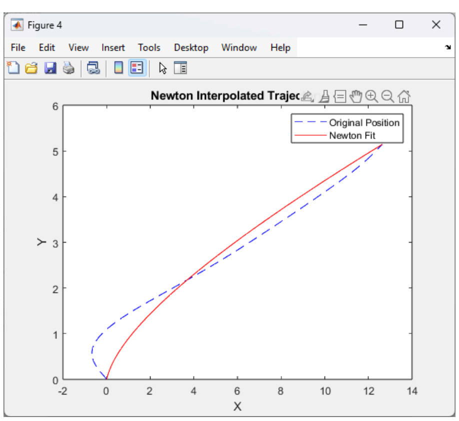
For n=2:



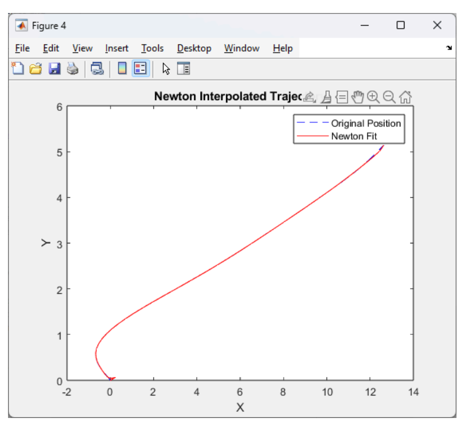
For n=8:



For n=3:



For n=15:



As we can observe, an n number of points gives us an n-1 degree polynomial. Moreover, as we increase the number of points we can better fit the curve.

Task 3:

In task 3, we used a couple of methods to find the time taken to reach 10.5m by taking:

$$x^2 + y^2 - 10.5 = 0$$

We obtained the following results:

```
Number of iterations:
Bisection: 43
Newton: 5
Secant: 6
False Position: 8
Time to reach 10.5m:
Bisection: 8.2693 s
Newton: 8.2693 s
Secant: 8.2693 s
False Position: 8.2693 s
```

As we can see, they all converge to the same value for a tolerance of 1e-12. However, the Newton method is one iteration faster than the secant method, the secant method is two iterations faster than the false position method, and the false position method is much faster than the bisection method. However, if the derivative is 0, we can't proceed with Newton's method. Moreover, the secant method doesn't doesn't bracket the root. This concludes that False Position is the best compromise between speed and convergence.

Conclusion:

We reconstructed the vehicle's motion using trapezoidal integration, fitted its trajectory with Newton's divided difference, and estimated target time using root-finding methods. Each technique balanced accuracy and efficiency, showing the practical value of numerical methods in motion analysis.