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A survey on pickup and delivery problems

Part II: Transportation between pickup and delivery locations

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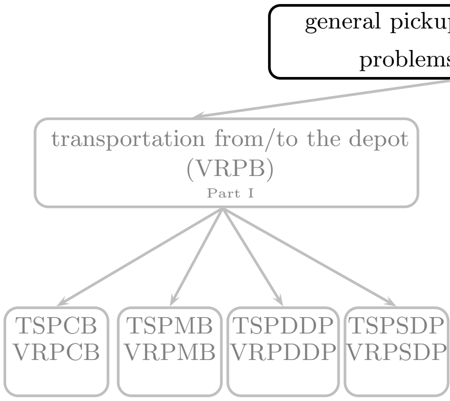
Abstract This paper is the second part of a comprehensive survey on pickup and delivery models. Basically, two problem classes can be distinguished. The first part dealt with the transportation of goods from the depot to linehaul customers and from backhaul customers to the depot. In this class four subtypes were considered, namely the Vehicle Routing Problem with Clustered Backhauls (VRPCB - all linehauls before backhauls), the Vehicle Routing Problem with Mixed linehauls and Backhauls (VRPMB - any sequence of linehauls and backhauls permitted), the Vehicle Routing Problem with Divisible Delivery and Pickup (VRPDDP - customers demanding delivery and pickup service can be visited twice), and the Vehicle Routing Problem with Simultaneous Delivery and Pickup (VRPSDP - customers demanding both services have to be visited exactly once). The second part now considers all those problems where goods are transported between pickup and delivery locations, denoted as Vehicle Routing Problems with Pickups and Deliveries (VRPPD). These are the Pickup and Delivery VRP (PDVRP - unpaired pickup and delivery points), the classical Pickup and Delivery Problem (PDP - paired pickup and delivery points), and the Dial-A-Ride Problem (DARP - paired pickup and delivery points and user inconvenience taken into consideration). A single as well as a multi vehicle mathematical problem formulation for all three VRPPD types is given, and the respective exact, heuristic, and metaheuristic solution methods are discussed.

1 Basic definitions

The aim of this paper is to present a classification scheme as well as a comprehensive survey on pickup and delivery problems and their variants. Part one of this survey presented all problem types belonging to the class

Fig. 1 Pickup and delivery problems. The numbers indicated refer to the sections covering the respective problems.



of Vehicle Routing Problems with Backhauls (VRPB). Furthermore, the motivation for this survey as well as its limitations were given. Part two will now cover all problem types where goods are transported between pickup and delivery locations, referred to as Vehicle Routing Problems with Pickups and Deliveries (VRPPD).

The two pickup and delivery problem classes as well as their subclasses are depicted in Figure 1. The gray part was subject to discussion in part one of this survey. The numbers indicated in the boxes refer to the sections covering the respective problems. The first two indicators refer to the modeling part while the last refer to the sections on solution methods.

1.1 VRPB subclass definitions

The first category can be subdivided into four subclasses. In the first subclass the cluster of delivery customers has to be visited before the first pickup customer can be served. Delivery customers are also denoted as linehaul customers, pickup customers as backhaul customers, respectively. This subclass will be referred to as VRP with all linehauls before backhauls or VRP with Clustered Backhauls (VRPCB). The second subclass does not consider this restriction. Mixed visiting sequences are explicitly allowed, denoted as VRP with Mixed linehauls and Backhauls (VRPMB).

The third subclass covers situations where every customer is associated with a linehaul as well as a backhaul quantity. However, customers may be visited twice, first for delivery and second for pickup service. We denote this problem type as VRP with Divisible Delivery and Pickup. In subclass four as in subclass three every customer can demand both services. However, in contrast to the VRPDDP it is imposed that every customer can only be visited exactly once, denoted as VRP with Simultaneous Delivery and

Pickup (VRPSDP). For a more detailed description with references as well as for the different denotations used in the literature we refer to the first part of this survey.

1.2 VRPPD subclass definitions

The class we denote VRPPD refers to problems where goods are transported from pickup to delivery points. It can be further divided into two subclasses. The first subclass refers to situations where pickup and delivery points are unpaired. An identical good is considered. Each unit picked up can be used to fulfill the demand of any delivery customer. In the literature mostly the single vehicle case is tackled, denoted as Capacitated Traveling Salesman Problem with Pickups and Deliveries in (Anily and Bramel, 1999), One-Commodity Pickup and Delivery Traveling Salesman Problem (1-PDTSP) in (Hernández-Pérez and Salazar-González, 2003), and Traveling Salesman Problem with Pickup and Delivery in (Hernández-Pérez and Salazar-González, 2004a). Since also a multi vehicle application has been reported in the literature, see (Dror et al., 1998), we will denote this problem class as Pickup and Delivery VRP (PDVRP) and Pickup and Delivery TSP (PDTSP), in the multi and in the single vehicle case, respectively.

The second VRPPD subclass comprises the classical Pickup and Delivery Problem (PDP) and the Dial-A-Ride Problem (DARP). Both types consider transportation requests, each associated with an origin and a destination, resulting in paired pickup and delivery points. The PDP deals with the transportation of goods while the DARP deals with passenger transportation. This difference is usually expressed in terms of additional constraints or objectives that explicitly refer to user (in)convenience. The single vehicle case of the PDP has also been referred to as Pickup-Delivery Traveling Salesman Problem by (Kalantari et al., 1985) and the multi vehicle case as Pickup and Delivery Vehicle Routing Problem in (Malca and Semet, 2004) and Vehicle Routing Problem with Pickup and Delivery in (Derigs and Döhmer, 2006). However, a majority of the work published refers to this problem class as Pickup and Delivery Problem (PDP), compare, e.g. (Dumas et al., 1991) or (van der Bruggen et al., 1993). We will follow this wording. Dial-a-ride problems are also mostly referred to as such. However, some authors, such as Toth and Vigo (1996), denote the same problem as the Handicapped persons Transportation Problem. The dynamic case is also referred to as Demand Responsive Transport, compare, e.g. (Mageean and Nelson, 2003). We denote the single vehicle case of the PDP as SPDP, the single vehicle case of the DARP as SDARP.

The remainder of this paper is organized as follows. First, in order to clearly define the different VRPPD types, a consistent mathematical problem formulation for the single and for the multi vehicle case is given. Then, solution methods are discussed in subsections devoted to the corresponding problems. In each of these subsections the solution methods presented will be divided into exact, heuristic and metaheuristic approaches. Also related work will be mentioned.

Section 4 gives an overview of existing benchmark instances for the different problem classes as well as some information regarding best known solutions. Concluding remarks and possible directions for future research are provided at the very end of this survey.

2 Mathematical problem formulation

In the following section a consistent mathematical problem formulation will be presented . First, the notation used throughout the survey is given. After that two basic problem formulations are introduced, one for the single, and one for the multi vehicle case, that are adjusted to the unpaired pickup and delivery problem, the classical pickup and delivery problem, and the dial-a-ride problem, respectively.

Note that the mathematical models are only given for definition purposes regardless the strength of their LP relaxations or whether they were used in the context of additive bounding, branch and cut algorithms or Lagrange relaxations. We refer the interested reader to the work summarized in the sections on exact solution procedures for additional information w.r.t. these aspects.

2.1 Notation

n ...	number of pickup vertices
$\tilde{n}$ ...	number of delivery vertices in case of paired pickups and deliveries $n = \tilde{n}$
P ...	set of backhauls or pickup vertices, $P = \{1, \dots, n\}$
D ...	set of linehauls or delivery vertices, $D = \{1, \dots, \tilde{n}\}$
q_i ...	demand/supply at vertex i; pickup vertices are associated with a positive value, delivery vertices with a negative value; at the start depot 0 and the end depot $n + \tilde{n} + 1$ the supply is $q_0 = q_{n+\tilde{n}+1} = 0$
e_i ...	earliest time to begin service at vertex i
l_i ...	latest time to begin service at vertex i
d_i ...	service duration at vertex i
c_{ij}^k...	cost to traverse arc or edge (i, j) with vehicle k
t_{ij}^k...	travel time from vertex i to vertex j with vehicle k
K ...	set of vehicles
C^k...	capacity of vehicle k
T^k...	maximum route duration of vehicle/route k

Note that this notation is valid for the symmetric as well as for the asymmetric case. In the symmetric case $t_{ij}^k = t_{ji}^k$ and $c_{ij}^k = c_{ji}^k$, arc (i, j) and arc (j,i) could thus be modeled by one edge. Consequently, fewer variables would be needed to formulate the symmetric case. However, since we focus on problem definition and not on computational efficiency we refrain from presenting these variants here. VRPPD are modeled on complete graphs $G = (V, A)$ where V is the set of all vertices $V = \{0, n+\tilde{n}+1\} \cup P \cup D$, and A the set of all arcs.

During the optimization process some or all of the following decision variables are determined, depending on the problem considered.

$$x_{ij}^k \;\ldots\; = \begin{cases} 1, & \text{if arc } (i,j) \text{ is traversed by vehicle } k \\ 0, & \text{else} \end{cases}$$

Q_i^k ... load of vehicle k when arriving at vertex i

B_i^k ... beginning of service of vehicle k at vertex i

Note that vehicle dependent start as well as end vertices can easily be introduced into the model. However, for the sake of simplicity we will not consider this extension in our formulation.

In the single vehicle problem formulation the superscript k can be omitted, resulting in the parameter notations c_{ij}, C , T and the decision variables x_{ij}, Q_i, B_i.

2.2 Single vehicle pickup and delivery problem formulations

The single vehicle model for the different VRPPD is based on an open TSP formulation.

$$\min \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij} \tag{1}$$

subject to:

$$\sum_{i \in V} x_{ij} = 1 \qquad \forall j \in V \setminus \{0\} \tag{2}$$

$$\sum_{j \in V} x_{ij} = 1 \qquad \forall i \in V \setminus \{n+\tilde{n}+1\} \tag{3}$$

$$x_{i0} = 0 \qquad \forall i \in V \tag{4}$$

$$x_{n+\tilde{n}+1,j} = 0 \qquad \forall j \in V \tag{5}$$

$$\sum_{i \in S} \sum_{j \notin S} x_{ij} \geq 1 \qquad \forall S \subseteq V \setminus \{n+\tilde{n}+1\}, S \neq \emptyset \tag{6}$$

$$x_{ij} \in \{0,1\} \qquad \forall i \in V, j \in V \tag{7}$$

The objective function (1) minimizes total routing cost. Constraint sets (2) and (3) ensure that each vertex is visited exactly once. No arcs enter the origin depot 0 and no arcs leave the destination depot $n+\tilde{n}+1$ because of

(4) and (5), respectively. Constraints (6) present one of several possibilities to ensure route-connectivity. For other options we refer to part one of this article.

2.2.1 PDTSP Here it is assumed that every unit picked up can be used to satisfy every delivery customer's demand. Similar to the TSPMB described in part one of this survey, vehicle loading constraints have to be amended to formulation (1) – (7),

$$Q_j \geq (Q_i + q_i)x_{ij} \qquad \forall i \in V, j \in V, \tag{8a}$$
$$\max\{0, q_i\} \leq Q_i \leq \min\{C, C + q_i\} \qquad \forall i \in V. \tag{8b}$$

The only difference to the TSPMB consists in the initial load of the vehicle which is free here (see Hernández-Pérez and Salazar-González, 2004a).

2.2.2 SPDP The SPDP considers situations where pickup and delivery vertices are paired, i.e. $n = \tilde{n}$. In the literature it is common to refer to such a vertex pair as a *request*, indexed by $i = 1, ..., n$ with i being the origin or pickup point and $n + i$ the corresponding destination or delivery point. To ensure that every destination is only visited after its origin, in addition to (1) – (5), (7), and (8), precedence constraints are needed which are usually modeled via time variables,

$$B_i \leq B_{n+i} \qquad \forall i \in P, \tag{9}$$
$$B_j \geq x_{ij}(B_i + d_i + t_{ij}) \qquad \forall i \in V, j \in V \tag{10}$$

Constraint set (9) states that every origin is to be visited before its destination and (10) ensures that time variables are consistent with travel and service times. Note that (10) also guarantee that short cycles are avoided and therefore constraints (6) are not needed, given that $t_{ij} + d_i > 0$ for all i, j.

2.2.3 SDARP This pickup and delivery problem class deals with the transportation of people. Problems of this kind arise, e.g., in connection with the transportation of handicapped or elderly persons. Another possible application could be, however, the transportation of perishable goods, that also require maximum ride time constraints. In addition to the basic model, given in (1) – (5) and (7) – (10), user inconvenience must be considered which can be handled by another term in the objective function or by additional constraints. Here we choose to formulate a constraint set concerning maximum user ride time,

$$B_{n+i} - (B_i + d_i) \leq L_i \qquad \forall i \in P. \tag{11}$$

2.2.4 Time window constraints Constraints referring to the compliance with time windows,

$$e_i \leq B_i \leq l_i \qquad \forall i \in V, \tag{12}$$

can be added to all of the above problems and almost always are in case of SDARP.

2.3 Multi vehicle pickup and delivery problem formulations

The basic model for multi vehicle pickup and delivery problems is an adapted three index VRP formulation of the one proposed in (Cordeau et al., 2002, p. 158f.) for the VRPTW.

$$\min \sum_{k \in K} \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij}^k \tag{13}$$

subject to:

$$\sum_{k \in K} \sum_{j \in V} x_{ij}^k = 1 \qquad \forall i \in V \setminus \{0, n + \tilde{n} + 1\} \tag{14}$$

$$\sum_{j \in V} x_{0j}^k = 1 \qquad \forall k \in K \tag{15}$$

$$\sum_{i \in V} x_{i,n+\tilde{n}+1}^k = 1 \qquad \forall k \in K \tag{16}$$

$$\sum_{i \in V} x_{ij}^k - \sum_{i \in V} x_{ji}^k = 0 \qquad \forall j \in V \setminus \{0, n + \tilde{n} + 1\}, k \in K \tag{17}$$

$$B_j^k \geq x_{ij}^k (B_i^k + d_i + t_{ij}^k) \qquad \forall i \in V, j \in V, k \in K \tag{18}$$

$$Q_j^k \geq (Q_i^k + q_i) x_{ij}^k \qquad \forall i \in V, j \in V, k \in K \tag{19}$$

$$\max\{0, q_i\} \leq Q_i^k \leq \min\left\{C^k, C^k + q_i\right\} \qquad \forall i \in V, k \in K \tag{20}$$

$$x_{ij}^k \in \{0, 1\} \qquad \forall i \in V, j \in V, k \in K \tag{21}$$

Total routing cost are minimized by the objective function, given in (13). Constraints (14) state that every vertex has to be served exactly once. Constraint sets (15) and (16) guarantee that every vehicle starts at the depot and returns to the depot at the end of its route. Note that this does not mean that every vehicle has to be used. A vehicle may only use arc $(0, n + \tilde{n} + 1)$, i.e. it does not leave the depot. Flow conservation is ensured by (17). Time variables are introduced in constraint set (18) to ensure that no subtours occur and to facilitate the introduction of time related constraints later on. Constraint sets (19) and (20) guarantee that a vehicle's capacity is not violated throughout its tour. It should be noted that a model formulation in the above form requires the introduction of additional decision variables, Q_i^k, corresponding to the total load of vehicle k at vertex i, that is not needed in the basic VRP but essential for its extension to a pickup and delivery problem. However, they are sometimes used in VRP formulations to ensure route connectivity (accomplished by (18) in the above formulation).

2.3.1 VRPPD The special characteristic of this pickup and delivery problem class refers to the fact that every unit picked up can be used to satisfy every customer's demand. While the above formulation (13) – (21) was one of the possible VRP formulations, if all q_i have the same sign, it automatically becomes the appropriate formulation of the VRPPD when both pickups and deliveries can occur and no modifications are needed.

2.3.2 PDP Here every pickup point is associated with a delivery point and therefore $n = \tilde{n}$. In addition to (13) – (21), two more sets of constraints are needed. First, both origin and destination of a request must be served by the same vehicle:

$$\sum_{j \in V} x_{ij}^k - \sum_{j \in V} x_{n+i,j}^k = 0 \qquad \forall i \in P, k \in K \tag{22}$$

Furthermore, delivery can only occur after pickup, i.e.

$$B_i^k \leq B_{n+i}^k \qquad \forall i \in P, k \in K. \tag{23}$$

2.3.3 DARP To extend the multi vehicle PDP to the multi vehicle DARP, again, as in the single vehicle case, constraints related to the minimization of user inconvenience need to be added. As in the single vehicle case we will restrict this requirement to adding maximum user ride time constraints to formulation (13) – (23):

$$B_{n+i}^k - (B_i^k + d_i) \leq L_i \qquad \forall i \in P \tag{24}$$

2.3.4 Additional constraints Two more sets of constraints can be added to all of the above problem classes. These correspond to time window and maximum route duration restrictions,

$$e_i \leq B_i^k \leq l_i \qquad \forall i \in V, k \in K, \tag{25}$$

$$B_{2n+1}^k - B_0^k \leq T^k \qquad \forall k \in K. \tag{26}$$

The above formulations are based on the DARP formulation presented in (Cordeau, 2006). Non linear constraints, given in (8a) and (10) in the single vehicle case, and (18) and (19) in the multi vehicle case, can easily be reformulated as a linear program by means of the usual big M formulation (cf. Cordeau, 2006).

3 Solution methods for VRPPD

In the following section an overview of the different solution methods for the PDVRP, the PDP and the DARP are presented. Solution methods are classified according to exact, heuristic and metaheuristic approaches. A description of the benchmark instances used is given in Section 4. For references to general information on the solution concepts applied in the field of vehicle routing we refer to the first part of this survey. A survey on different solution methods can be found in (Berbeglia et al., 2007). If it was not possible to describe all the work published in detail because of maximum paper length restrictions an overview of the proposed methods is given in tabular form and only contributions we considered especially important, for recency or originality reasons, are described in further detail. These are marked by an asterisk in the different tables. Solution methods are ordered chronologically within the tables according to single and multi vehicle approaches. Further details on solution algorithms are given in the same order.

3.1 Unpaired pickups and deliveries

The PDVRP, i.e. the problem class where every good can be picked up and transported anywhere, did not receive as much attention in the literature as the other problem classes. Moreover, most of the literature is restricted to the PDTSP. Therefore, with the exception of (Dror et al., 1998), all the solution methods presented are only applicable to the one vehicle case. To the authors' knowledge no metaheuristic approach for the PDTSP has been proposed until today.

3.1.1 Exact methods The only exact method proposed for the problem at hand was introduced in (Hernández-Pérez and Salazar-González, 2003, 2004a). It is a branch and cut algorithm using a cutting plane approach. To speed up the algorithm construction and improvement heuristics are used to generate the initial solution. The construction heuristic applied is an adaptation of the nearest insertion algorithm which is improved by 2-opt and 3-opt exchanges (Lin, 1965). The test instances solved are adaptations of the ones used in (Mosheiov, 1994) and (Gendreau et al., 1999), containing up to 75 customers.

3.1.2 Heuristics A special case of the PDTSP is considered in (Chalasani and Motwani, 1999). In this case the number of goods to be picked up is equal to the goods to be delivered and the demand (supply) at every delivery (pickup) location is equal to one. This problem is an extension of the swapping problem where the vehicle's capacity is also set to one. Chalasani and Motwani propose an approximation algorithm with a worst case bound of 9.5. They use Christofides' heuristic to construct two traveling salesman tours, one containing only pickup locations, and one containing only delivery locations. These two tours are then combined by means of decomposition and matching. In (Anily and Bramel, 1999) a polynomial time iterated tour matching algorithm for the same problem is proposed.

Two heuristic methods can be found in (Hernández-Pérez and Salazar-González, 2004b). The first algorithm is of the construction-improvement type, using a greedy construction procedure that is improved by 2-opt and 3-opt exchanges. The second heuristic is based on incomplete optimization. The branch and cut procedure, described in (Hernández-Pérez and Salazar-González, 2004a), with restrictions on the search space is applied. Random instances with up to 500 customers were solved.

An approximation algorithm on a tree graph was proposed by (Lim et al., 2005). It is based on a recurrent construction process and has a worst case bound of 2.

The PDTSP on a tree and on a line is also subject to investigation in (Wang et al., 2006). They propose an $O(n^2/min\,\{Q,n\})$ algorithm for the line case. The unit capacity as well as the uncapacitated version can be solved in linear time. On a tree an $O(n)$ algorithm is devised for the case of unit capacity and an $O(n^2)$ algorithm for the uncapacitated case.

Finally, Dror et al. (1998) propose a heuristic algorithm for the application of the multi vehicle PDVRP to the redistribution of self-service cars that is related to Dijkstra's algorithm (Dijkstra, 1959). Also other solution approaches are briefly discussed.

3.2 The static pickup and delivery problem

Solution methods for the classical pickup and delivery problem (PDP), where every transportation request is associated with a pickup and a delivery point, are presented in this section. Lokin (1978) was the first to discuss the incorporation of precedence constraints into the traditional TSP which are needed to formulate the PDP. The first attempt to generalize the PDP in unified notation was proposed in (Savelsbergh and Sol, 1995), covering all possible versions of the PDP including the dial-a-ride problem. They also provide a brief survey on existing solution methods until 1995. Mitrović-Minić (1998) presents a survey on the PDP with Time Windows (PDPTW). An early survey on vehicle routing problems already including the PDP was given in (Desrochers et al., 1988). Cordeau et al. (2004) provide a survey on demand responsive transport covering PDP and DARP. Further surveys on solution methods can be found in (Assad, 1988, Desaulniers et al., 2002, Desrochers et al., 1988). A survey on recent solution methods focusing on exact approaches is given in (Cordeau et al., 2007). In the following paragraphs the various solution techniques proposed for the static PDP are summarized according to exact, heuristic and metaheuristic approaches.

3.2.1 Exact methods A number of exact solution procedures have been proposed for the static PDP. In Table 1 the different solution techniques

Table 1 Exact methods for the static PDP

Reference	Type	Obj.	Con.	Algorithm	Bench./Size
The single vehicle case					
* Ruland and Rodin (1997)	-	min. RC	-	branch and cut algorithm, greedy route construction	up to 15 req.
* Cordeau et al. (2006)	-	min. RC	TW, LIFO	branch and bound algorithms	up to 51 req.
The multi vehicle case					
* Kalantari et al. (1985)	-	min. RC	-	extension of branch and bound algorithm of Little et al. (1963)	up to 15 req.
Desrosiers and Dumas (1988)	-	min. RC	TW	study of constr. shortest path problem, column generation, heur.	-
* Dumas et al. (1991)	HF, MD	min. RC	TW	exact algorithm using column generation, constr. shortest path subproblems	up to 19 req.
Sigurd et al. (2004)	MD	min. RC	TW	pricing, branch and bound	up to 205 req.
Lu and Dessouky (2004)	-	min. RC	soft TW	branch and bound, valid inequalities	up to 25 req.
* Ropke and Cordeau (2006)	-	min. RC	TW	branch and cut and price algorithm	RCL07, partly LL01
* Ropke et al. (2007)	-	min. RC	TW	2 branch and cut algorithms	RCL07

Bench. = Benchmark, Con. = Constraints, constr. = constrained, heur. = heuristic(s), HF = Heterogeneous Fleet, LIFO = Last-In-First-Out, MD = Multi depot, Obj. = Objective(s), RC = Routing Cost, req. = requests, TW = Time Windows; The respective benchmark instances are described in Section 4. Entries marked by an asterisk (*) are described in further detail in the text.

proposed are briefly described. The literature reference, objective(s) as well as additional constraints considered, and either the benchmark instances used or the size of the largest instance solved to optimality, in terms of number of requests, are given. Entries marked by an asterisk are described in further detail below.

Ruland and Rodin (1997) propose a branch and cut algorithm to solve the single vehicle PDP. It includes cutting plane generation in the bounding phase as well as a greedy route construction procedure to generate the first upper bound.

A branch and bound algorithm for the single vehicle PDPTW with Last-In-First-Out (LIFO) loading, i.e. goods have to be delivered in the reverse order they were picked up, is presented in (Cordeau et al., 2006). They propose three mathematical problem formulations and various valid inequalities based on existing inequalities for the basic PDP as well as new problem specific ones.

The first exact solution method applicable to both the single as well as the multi vehicle case dates back to (Kalantari et al., 1985). The branch and bound algorithm proposed is an extension of the one developed by (Little et al., 1963).

A column generation approach was proposed in (Dumas et al., 1991) to tackle the multi vehicle PDP considering heterogeneous vehicles, time windows as well as multiple depots. The constrained shortest path problems are solved by means of a forward dynamic programming algorithm.

Ropke and Cordeau (2006) propose a branch and cut and price algorithm using a set partitioning formulation of the PDPTW. The elementary constrained shortest path problem with time windows, capacity and pickup and delivery, being the natural pricing problem of the PDPTW, is solved by means of a label setting shortest path algorithm. Valid inequalities are added during the search in a branch and cut fashion. The relationship between the set partitioning formulation and valid inequalities is also discussed.

A branch and cut algorithm departing from two different 2-index PDPTW formulations is studied in (Ropke et al., 2007). Formulation one makes use of time variables. In formulation two time related constraints are modelled by means of infeasible path inequalities. The latter formulation proves more efficient when used as a basis for the branch and cut algorithm. New valid inequalities to strengthen the proposed formulations are discussed.

3.2.2 Heuristics Heuristics for the static PDP have first been proposed in the 1980s. Table 2 gives an overview of the various heuristic solution techniques developed since then. Information concerning the type of problem considered, objective(s) used, additional constraints, the type of algorithm proposed as well as either the benchmark instances solved or the largest instance considered for testing purposes is given.

Sexton and Choi (1986) used Bender's decomposition procedure to solve the static SPDP. To save CPU time the initial solutions were constructed using a space-time heuristic. A route improvement phase was also implemented. As soft time windows were considered, the objective function takes into account the total operating time as well as time window violation penalties.

A construction-improvement heuristic algorithm for the static single vehicle PDP is discussed in van der Bruggen et al. (1993). First a feasible initial solution is constructed. Then this solution is improved by exchange procedures maintaining feasibility at all times.

Renaud et al. (2000) also propose a construction-improvement algorithm for the same problem using a double insertion construction heuristic improved by deletion and re-insertion (4-opt* (Renaud et al., 1996)).

Renaud et al. (2002) present seven different perturbation heuristics to generate near optimal solutions for the static SPDP. In all seven implementations, first, an initial solution is computed which is improved by a 4-opt** heuristic, an adaptation of the 4-opt* heuristic proposed in (Renaud et al., 1996). Then, a perturbation scheme is applied (instance, algorithmic or solution perturbation) followed by a post-optimization phase. The last two

Table 2 Heuristics for the static PDP

Reference	Type	Obj.	Con.	Algorithm	Bench./Size
The single vehicle case					
* Sexton and Choi (1986)	-	min. RC, min. TWV	soft TW	heur. algorithm using Bender's decomposition	up to 17 req.
* van der Bruggen et al. (1993)	-	min. RC	TW	variable depth-search based algorithm	up to 50 req.
* Renaud et al. (2000)	-	min. RC	-	2-phase algorithm. (1) double insertion (2) deletion and re-insertion (4-opt*)	RBO00
* Renaud et al. (2002)	-	min. RC	-	7 perturbation heur. (instance, algorithmic, solution perturbation)	RBO00
The multi vehicle case					
Shang and Cuff (1996)	transf.	min. UC	soft TW	concurrent insertion, mini-clustering algorithm	SC96
Lim et al. (2002)	-	min. NV, min. RC, min. RD	TW	squeaky wheel optimization[1]	LL01
* Xu et al. (2003)	HF, HG	min. RC	mTW, RD, LIFO	column generation based heur.	up to 500 req.
Mitrović-Minić and Laporte (2006)	trans-ship-ment	min. RC	TW	cheapest insertion, local search improvement	up to 100 req.
* Lu and Dessouky (2006)	-	min. RC	TW	construction heur. based on distance increase, TW slack reduction, visual attractiveness	LL01
Thangiah and Awan (2006)	transf., split	min. UC	TW	(Shang and Cuff, 1996) plus improvement heur.	SC96

Bench. = Benchmark, Con. = Constraints, heur. = heuristic(s), HF = Heterogeneous Fleet, HG = Heterogeneous goods, LIFO = Last-In-First-Out, MD = Multi Depot, NV = Number of Vehicles, Obj. = Objective(s), req. = requests, RC = Routing Cost, RD = Route Duration, split = split deliveries, (m)TW = (multiple) Time Windows, TWV = TW Violation, transf. = transfers between vehicles, UC = Unsatisfied Customers; The respective benchmark instances are described in Section 4. Entries marked by an asterisk (*) are described in further detail in the text.

[1] i.e. an iterative procedure based on an insertion algorithm that prioritizes requests that caused difficulties in previous iterations.

steps are repeated until a stopping criterion is met. The best results are obtained with the solution perturbation scheme.

Xu et al. (2003) propose a column generation based heuristic algorithm for the multi vehicle case. They consider several additional constraints, such as multiple time windows at pickup and delivery locations, loading restrictions, compatibility of goods and vehicles as well as driver working hours. By column generation the master problem can be solved using a commercial LP solver. However, the resulting subproblems have to be solved heuristically by means of two heuristics called merge and two-phase, i.e. merging trips and greedy deletion and insertion of requests.

Lu and Dessouky (2006) propose a construction heuristic. The algorithm proposed does not only incorporate distance increase into the evaluation criterion but also time window slack reduction as well as visual attractiveness (referred to as crossing length percentage).

3.2.3 Metaheuristics Table 3 gives an overview of the different metaheuristic solution methods proposed for the static PDP. The same information as in the previous tables is provided.

A variable neighborhood search for an extension of the SPDP is presented in (Carrabs et al., 2006). The problem tackled includes a further constraint regarding rear loading, i.e. items can only be delivered in a LIFO matter. Eight different construction techniques are used to generate an initial solution. The neighborhoods are defined by couple exchange, block exchange, relocate-block, 2-opt-L and multi-relocate operators.

Li and Lim (2001) develop a tabu embedded simulated annealing approach to solve the static multi vehicle PDP. Pickup and delivery pair swap neighborhoods are defined. These are based on a shift, an exchange and a rearrange operator. The first two serve as the neighborhoods searched by the metaheuristic, the third is used for postoptimization purposes.

Pankratz (2005b) proposes a grouping genetic algorithm for the PDP. The grouping genetic algorithm differs from traditional genetic algorithms in that a group-oriented genetic encoding is used. The encoding used by Pankratz (2005b) corresponds to the cluster of requests forming a route. The routing aspect not comprised in the encoding is added while decoding the chromosome.

Ropke and Pisinger (2006a) present an adaptive large neighborhood search algorithm for multi vehicle PDPTW. Multiple depots as well as the existence of service times can be handled by the approach at hand. In (Pisinger and Ropke, 2007, Ropke and Pisinger, 2006b) the proposed method is used to solve VRP and VRPB instances by transforming them into rich PDPTW. We refer to part one of this survey for further details on this solution procedure.

A two-stage hybrid algorithm for the static PDPTW has recently been presented in (Bent and van Hentenryck, 2006). The first phase uses simulated annealing to decrease the number of vehicles needed. The second phase consists of a large neighborhood search algorithm in order to reduce total travel cost.

3.2.4 Summary Summarizing, the largest static PDP problem instance solved to optimality with state-of-the-art solution methods, a branch and cut and price algorithm proposed in (Sigurd et al., 2004), comprises 205 requests. However, the size of the largest instance solved is not always a good indicator since tightly constrained problems are more easy to solve than less tightly constrained ones. The benchmark data set most often used to

Table 3 Metaheuristics for the static PDP

Reference	Type	Obj.	Con.	Algorithm	Bench./Size
The single vehicle case					
Landrieu et al. (2001)	S	min. RC	TW	(probabilistic) tabu search	up to 40 req.
* Carrabs et al. (2006)	-	min. RC	LIFO	variable neighborhood search	up to 375 req.
The multi vehicle case					
Nanry and Barnes (2000)	S	min. RC, min. TWV, min. LV	TW	reactive tabu search, insertion and swap neighborhoods	NB00
Jung and Haghani (2000)	-	min. RC, min. TWV	TW	genetic algorithm	up to 30 req.
Lau and Liang (2001, 2002)	-	min. RC	TW	tabu search, adapted (Nanry and Barnes, 2000) neighborhoods	adapted (Solomon, 1987), NB00
* Li and Lim (2001)	S	min. NV, min. RC, min. RD, min. WT	TW	tabu embedded simulated annealing	LL01, NB00
Schönberger et al. (2003)	reject req.	max. profit	TW	hybrid genetic algorithm	adapted NB00
Caricato et al. (2003)	MD	min. mRD	track contention	parallel tabu search	up to 50 req.
Ambrosini et al. (2004)	-	min. RC	LIFO	greedy randomized adaptive search (GRASP)	up to 100 req.
Creput et al. (2004)	-	min. NV, min. RC	TW	genetic algorithm combined with local search	LL01
* Pankratz (2005b)	-	min. NV, min. RC	TW	grouping genetic algorithm	NB00, LL01
Derigs and Döhmer (2006)	-	min. NV, min. RC	TW	indirect search	LL01
* Ropke and Pisinger (2006a)	MD, S	min. RC	TW	adaptive large neighborhood search	LL01+
* Bent and van Hentenryck (2006)	-	min. NV, min. RC	TW	hybrid algorithm. (1) simulated annealing (2) large neighborhood search	LL01+

Bench. = Benchmark, Con. = Constraints, LIFO = Last-In-First-Out, LV = Load Violation, MD = Multi Depot, NV = Number of Vehicles, Obj. = Objective(s), RC = Routing Cost, (m)RD = (maximum) Route Duration, S = Service Time, TW = Time Windows, TWV = TW Violation, WT = Waiting Time; The respective benchmark instances are described in Section 4. Entries marked by an asterisk (*) are described in further detail in the text.

assess the performance of heuristic as well as metaheuristic solution methods for the static PDP with TW is the one proposed in (Li and Lim, 2001) (LL01, LL01+, see Table 6). Recent new best results have been presented in (Ropke and Pisinger, 2006a) and (Bent and van Hentenryck, 2006), two metaheuristic solution procedures. The metaheuristic of (Li and Lim, 2001) still holds the best results for a part of the smaller instances. However, also exact methods advanced. The more tightly constrained part of the LL01 data set has recently been solved by a state-of-the-art branch and cut and price algorithm (Ropke and Cordeau, 2006).

3.2.5 Related work Pickup and delivery problems do not only arise in the context of vehicle routing but also in ocean borne transportation. Christiansen and Nygreen (1998a) propose a combined PDPTW and multi inventory model arising in the context of ship routing. It is solved by means of a branch and bound embedded iterated solution procedure based on Dantzig-Wolfe decomposition. Similar solution approaches are presented in (Christiansen and Nygreen, 1998b, Christiansen, 1999). A column generation approach is presented in (Christiansen and Nygreen, 2005), assuming uncertainties in sailing times and considering inventory constraints to be soft. An optimal solution method for the traditional PDPTW in the context of ship routing is reported in (Christiansen and Fagerholt, 2002). Brønmo et al. (2007) propose a multi-start local search heuristic. A relaxed version of the multi-ship PDPTW, considering soft time windows, is tackled in (Fagerholt, 2001). Fagerholt and Christiansen (2000a) also study a combined ship scheduling and allocation problem. Optimal solutions are computed for several real life cases. The subproblem, a TSPTW with allocation and precedence constraints, of the combined problem is studied by Fagerholt and Christiansen (2000b). For an extensive survey on ship routing problems we refer to (Christiansen et al., 2004).

An interesting extension of the PDPTW was proposed by Recker (1995), namely the household activity pattern problem. It involves ridesharing as well as vehicle-switching options. Its objective refers to the minimization of household travel disutility. Recker solved the problem defined by means of a genetic algorithm.

Research dedicated to polyhedral analysis is presented in (Dumitrescu, 2005) w.r.t. the SPDP. New valid inequalities are discussed. Combinatorially simple pickup and delivery paths, i.e. paths consisting of several patterns defined by visiting at most two requests, are studied in (Lübbecke, 2004).

Gambardella and Dorigo (2000) discuss another problem related to the PDP, namely the sequential ordering problem. Its objective is to determine a minimum weight Hamiltonian path in a directed graph, with weights on arcs and vertices, respecting precedence constraints between vertices. In contrast to the PDPTW one vertex can have multiple predecessors. Gambardella and Dorigo (2000) propose an ant colony optimization based approach to solve this problem. Other solution methods involve, e.g. those of (Escudero, 1988, Ascheuer et al., 1993).

3.3 The dynamic pickup and delivery problem

The dynamic PDP has not received as much attention as its static counterpart. However, a number of heuristic and metaheuristic solution procedures have been proposed. Surveys on dynamic routing can be found in, e.g. (Ghiani et al., 2003, Psaraftis, 1988). So far exact procedures have not been used to solve the dynamic PDP. In the following the different heuristic and metaheuristic methods proposed for the dynamic single as well as the multi vehicle PDP will be discussed.

3.3.1 Heuristics A stochastic and dynamic SPDP is discussed in (Swihart and Papstavrou, 1999). The objective minimized consists of the expected time the requests remain in the system. They test three routing policies, a sectoring, a nearest neighbor and a stacker crane policy. The stacker crane policy refers to grouping arriving demands into contiguous sets of equal size and serving them according to the first-come-first-serve rule. Lower bounds under light and heavy traffic conditions are computed.

The first heuristic procedure determined for the dynamic multi vehicle PDP was proposed in (Savelsbergh and Sol, 1998). Their solution methodology called DRIVE (Dynamic Routing of Independent VEhicles) incorporates a branch and price algorithm based on a set partitioning problem formulation that generates approximate solutions via incomplete optimization. The problem tackled consists of a ten days real life simulation. Up to 354 active requests are considered at the various re-optimization runs.

An insertion based heuristic procedure for the dynamic multi vehicle PDP is proposed in (Popken, 2006). The heuristic is combined with different types of order circuity control in order to increase the efficient utilization of the vehicle's capacity. Results for test instances with up to 2500 initial orders are reported.

Fabri and Recht (2006) present an adaptation of the heuristic algorithm initially designed for the dynamic DARP in (Caramia et al., 2002). They explicitly allow for waiting times. To enhance the procedure an additional local search phase is introduced. This phase is initiated whenever a new request has been inserted and ends when the next request comes in. Fabri and Recht (2006) report solutions to instances with up to 1000 requests, arriving at an average rate of one request per minute.

Thangiah and Awan (2006) test their algorithm (see Table 2) also in a real-time setting considering up to 159 requests.

3.3.2 Metaheuristics A population based metaheuristic approach, namely a hybrid genetic algorithm, for the dynamic single vehicle PDP with TW is proposed in (Jih and Hsu, 1999). It is called hybrid, as the genetic algorithm is combined with a dynamic programming algorithm. The data sets used for testing purposes consist of up to 50 requests.

Early research on dynamic multi vehicle PDP is conducted in (Shen et al., 1995) and (Potvin et al., 1995). Both articles focus on neural networks with learning capabilities to support vehicle dispatchers in real-time. In (Potvin et al., 1995) the neural network based learning techniques are compared to a linear programming based method. Both articles use real life data sets with 200 and 140 requests in (Potvin et al., 1995, Shen et al., 1995), respectively, to assess the performance of their algorithms.

The first neighborhood based metaheuristic solution method for the multi vehicle case is proposed in (Gendreau et al., 1998). They propose a tabu search heuristic for the dynamic PDPTW that uses an ejection chain neighborhood (Glover, 1996), i.e. one request is removed from its current route and inserted into another route forcing the ejection of another request of this route to a third route and so on. A lateness criterion is incorporated into the objective function. To speed up the optimization procedure a parallel implementation is conducted. The resulting program is tested on simulations over 7.5 hours with 20 vehicles and 23 requests per hour, and over four hours with ten vehicles and 33 requests per hour.

Another tabu search algorithm for the same type of problem is proposed in (Malca and Semet, 2004). The neighborhood used is of the request to vehicle assignment type. In order to speed up the search an elimination matrix that memorizes the compatibility of two requests is used. Thus, only promising moves are considered. The proposed procedure is tested on some adapted instances of (Li and Lim, 2001).

A two-phase solution procedure using a tabu search algorithm for the dynamic multi vehicle PDPTW is presented in (Mitrović-Minić and Laporte, 2004). In the first phase an initial solution is constructed via cheapest insertion. Then, a tabu search algorithm is run to improve the initial solution. In the second phase different waiting strategies are used to schedule the requests. The waiting strategies tried are referred to as drive first, wait first, dynamic waiting and advanced dynamic waiting. They differ regarding the vehicle's location when waiting occurs. When applying the drive first strategy, the vehicle leaves every vertex as early as possible. If it arrives too early at the subsequent stop it waits there until service is possible. When applying the wait first strategy, the vehicle leaves every vertex as late as possible w.r.t. time windows of subsequent vertices. Dynamic waiting refers to a strategy where customers are clustered according to time windows. The vehicle waits as long as possible before moving on to the first customer of the next cluster. Advanced dynamic waiting refers to a strategy where waiting time before visiting the first cluster depends on the latest possible time to begin service at the last cluster, without violating time windows at intermediate clusters. Mitrović-Minić and Laporte (2004) report solutions to problem instances with a total of up to 1000 requests all occurring during the service period. The results indicate that the advanced dynamic waiting strategy is the most efficient.

The advanced dynamic waiting strategy is also used in (Mitrović-Minić et al., 2004). They propose a double horizon based heuristic. The routing part is solved by means of a construction heuristic improved by tabu search. Scheduling is conducted according to advanced dynamic waiting. Routes are segmented. The first segment corresponds to the short term horizon, the remainder of the route to the long term horizon. Also the case of several depots is considered. Again instances with a total of up to 1000 requests are solved.

Gutenschwager et al. (2004) compare a steepest descent, a reactive tabu search, and a simulated annealing algorithm to solve the dynamic PDP on an electric monorail system by means of simulation. The best results are obtained with tabu search.

In (Pankratz, 2005a) a grouping genetic algorithm for the static PDP is embedded in a rolling horizon framework in order to solve the dynamic PDP with time windows. It is tested on data sets with different degrees of dynamism.

3.3.3 Summary To summarize, over the last decades in the field of dynamic PDP a number of solution procedures have been developed. However, the proposed algorithms cannot be directly compared since so far no standardized simulation environment to assess the performance of heuristic and metaheuristic solution algorithms has been used by more than one group of authors. Benchmark instances are available, e.g. those used in (Mitrović-Minić and Laporte, 2004, Mitrović-Minić et al., 2004), see Section 4.

3.4 The static dial-a-ride problem

The dial-a-ride problem class has received considerable attention in the literature. The first publications in this area date back to the late 1960s and early 1970s (cf. Rebibo, 1974, Wilson and Weissberg, 1967, Wilson et al., 1971, Wilson and Colvin, 1977). Surveys on solution methods can be found in (Cordeau and Laporte, 2003a, 2007, Cordeau et al., 2004, Gendreau and Potvin, 1998). Different local search based heuristic methods for the SDARP are compared in (Kubo and Kasugai, 1990).

3.4.1 Exact methods An early exact dynamic programming algorithm for the single vehicle DARP is proposed in (Psaraftis, 1980). Service quality is taken care of by means of maximal position shift constraints w.r.t. a first-come-first-serve policy. The largest instance solved comprises nine requests. In Psaraftis (1983b) a modified version of the above algorithm is presented. Instead of backward recursion forward recursion is used and also time windows are considered.

A forward dynamic programming algorithm for the static SDARP is introduced in (Desrosiers et al., 1986). Possible states are reduced by eliminating those that are incompatible w.r.t. vehicle capacity, precedence and time window restrictions. User inconvenience w.r.t. ride times is incorporated into time window construction, resulting in tight time windows on both origin and destination of the transportation request. The largest instance solved contains 40 transportation requests.

Kikuchi (1984) develops a balanced LP transportation problem for the multi vehicle case minimizing empty vehicle travel as well as idle times and thus fleet size. In a preprocessing step the service area is divided into zones and the time horizon into several time periods. Every request is classified according to an origin and a destination zone as well as a departure and an arrival time period. An example with four zones is presented.

Cordeau (2006) proposes a branch and cut algorithm for the static DARP. The algorithm is based on a 3-index mixed-integer problem formulation and uses new valid inequalities as well as previously developed ones for the PDP and the VRP. The largest instance that could be solved to optimality comprises 32 requests. Two branch and cut algorithms are presented in (Ropke et al., 2007). Instead of the 3-index formulation, two more efficient 2-index problem formulations and additional valid inequalities are used. The largest instance solved to optimality consists of 96 requests.

3.4.2 Heuristics A large number of heuristic algorithms have been proposed over the last decades for the static DARP. Table 4 gives an overview of the various solution methods proposed in chronological order divided into single and multi vehicle approaches.

A heuristic routing and scheduling algorithm for the SDARP using Bender's decomposition is proposed in (Sexton and Bodin, 1985a,b). The scheduling problem can be solved optimally while the routing problem is solved by means of a heuristic algorithm.

One of the first heuristic solution procedures for static multi vehicle DARP is developed in (Cullen et al., 1981). They propose an interactive heuristic algorithm that follows the cluster first route second approach. It is based on a set partitioning formulation solved by means of column generation. The location-allocation subproblem is only solved approximately. However, user related constraints or objectives are not explicitly considered. The same applies to the work of (Healy and Moll, 1995).

Jaw et al. (1986) propose a sequential insertion procedure for the static multi vehicle DARP. Customers are ordered by increasing earliest time for pickup and inserted w.r.t. the cheapest feasible insertion criterion using the notion of active vehicle periods.

An optimization based mini-clustering algorithm is presented in (Ioachim et al., 1995). It uses column generation to obtain mini-clusters and an enhanced initialization procedure to decrease processing times. As in (Desrosiers et al., 1988) also the case of multiple depots is considered.

A multi-objective approach is followed in (Madsen et al., 1995). They propose an insertion based algorithm called REBUS. The objectives considered are the total driving time, the number of vehicles, the total waiting time, the deviation from promised service times as well as cost.

A classical cluster first route second algorithm is discussed in (Borndörfer et al., 1997). The clustering as well as the routing problem are modeled as set partitioning problems. The clustering problem can be solved optimally while the routing subproblems are solved approximately by a branch and bound algorithm. Customer satisfaction is taken care of in terms of punctual

Table 4 Heuristics for the static DARP

Reference	Type	Obj.	Constr.	Algorithm	Bench./Size
The single vehicle case					
Psaraftis (1983a)	-	min. RC	-	MST heur., local interchanges	up to 50 req.
Psaraftis (1983c)	-	min. RC	-	adapted 2-opt and 3-opt	up to 30 req.
* Sexton and Bodin (1985a,b)	-	min. CI	DDT	routing and scheduling algorithm based on Bender's decomposition	up to 20 req.
Healy and Moll (1995)	-	min. RC	-	2-opt improvement, optimizing/sacrificing phases	up to 100 req.
The multi vehicle case					
Stein (1978a,b)	transfers	min. RC	TW	cluster first route second	-
* Cullen et al. (1981)	-	min. RC	-	cluster first route second, column generation	up to 50 req.
Roy et al. (1985b,a)	HF	min. RC, min. CI	TW	parallel insertion	up to 578 req.
Bodin and Sexton (1986)	-	min. CI	DDT	cluster first route second	up to 85 req.
* Jaw et al. (1986)	-	min. RC, min. CI	TW, RT	sequential feasible insertion algorithm	up to 2617 req.
Alfa (1986)	HF	min. RC	TW, RT	adapted heur. of (Jaw et al., 1986)	up to 49 req.
Psaraftis (1986)	-	min. RC, min. CI	TW, RT	comparison of Jaw's heur. and grouping-clustering-routing heur.	-
Desrosiers et al. (1988), Dumas et al. (1989)	MD	min. RC	TW	mini-clustering algorithm, column generation	up to 200 req.
Kikuchi and Rhee (1989)	-	max. NCS	TW	sequential insertion	up to 200 req.
Desrosiers et al. (1991)	HF	min. RC	TW, RD	improved mini-clustering algorithm of (Desrosiers et al., 1988)	up to 2411 req.
Potvin and Rousseau (1992)	-	min. RC	TW, RT	constraint directed search (beam search)	up to 90 req.
* Ioachim et al. (1995)	HF, MD, S	min. NV, min. RC	TW	mini-clustering, column generation	up to 2545 req.

continued on next page

Table 4 Heuristics for the static DARP (cont.)

Reference	Type	Obj.	Constr.	Algorithm	Bench./Size
* Madsen et al. (1995)	HF, S	min. RC, NV, TWT, DPS	TW, RD, RT	REBUS. insertion based algorithm	up to 300 req.
Toth and Vigo (1996)	HF	min. RC	TW, RT	parallel insertion, improved by trip insertion, exchange, double insertion, moves	TV96a
* Borndörfer et al. (1997)	HF, MD, S	min. RC	TW, (RT), RD	cluster first route second, set partitioning, branch and bound	up to 1771 req.
Fu (2002a)	HF, S	min. RC, min. CI	TW, RT	parallel insertion, stochastic travel times	up to 2800 req.
* Diana and Dessouky (2004)	-	min. RC, min CI, idle times	TW, RT	parallel regret insertion heur.	up to 1000 customers
Xiang et al. (2006)	HF, S	min. RC	TW, RT, RD, BR	construction-improvement; clustering by TW, ideas of sweep heur.; local search improvement	up to 2000 req.
Wong and Bell (2006)	S, HF	min. RC, min. CI	TW, RT, RD	parallel insertion, improved by trip insertion	up to 150 req.
Wolfler Calvo and Colorni (2007)	-	max. NCS, max. SL	TW	cluster first route second, assignment heur., vertex reinsertions	up to 180 req.

Bench. = Benchmark, BR = BReak Time between two trips, CI = Customer Inconvenience, Constr. = Constraints, DDT = Desired Delivery Time, DPS = Deviation from Promised Service, HF = Heterogeneous Fleet, heur. = heuristic(s), MD = Multi Depot, NCS = Number of Customers Served, NV = Number of Vehicles, Obj. = Objective(s), PoC = Position of Customer, RC = Routing Cost, req. = requests, TL = Tour Length, TW = Time Windows, TWT = Total Waiting Time, RT = Ride Time, S = Service time, SL = Service Level; The respective benchmark instances are described in Section 4. Entries marked by an asterisk (*) are described in further detail in the text.

service and customer ride times are implicitly considered by means of time windows.

A regret insertion algorithm for the static DARP is proposed in (Diana and Dessouky, 2004). First all requests are ranked according to ascending pickup times allowing some swaps in this order, giving preference to requests that might be difficult to insert later on w.r.t. their spatial location. The first m requests are used as seed customers, with m being the number of vehicles. All the remaining requests are inserted following a regret insertion strategy, as described in (Potvin and Rousseau, 1993). The regret insertion based process is also subject to analysis in a study by Diana (2004) to determine why the performance of this heuristic is superior to that of other insertion rules.

Table 5 Metaheuristics for the static DARP

Reference	Type	Obj.	Con.	Algorithm	Bench./Size
The multi vehicle case					
Colorni et al. (1996)	-	max. NCS, min CI	RD	simulated annealing	up to 100 req.
* Toth and Vigo (1997)	HF, MD	min. RC	TW, RT	parallel insertion algorithm, tabu thresholding	TV96a
Baugh et al. (1998)		min. NV, min. RC, min. CI	TW	simulated annealing	up to 300 req.
Uchimura et al. (1999)	-	min. RC	RT, RD	genetic algorithm	10 req.
* Cordeau and Laporte (2003b)	-	min. RC	TW, RT, TL	tabu search algorithm	CL03, up to 295 req.
Aldaihani and Dessouky (2003)	mix with FRT	min. RC, min. CI	TW	tabu search	up to 155 req.
Ho and Haugland (2004)	proba-bilistic	min. RC	TW, RT	tabu search, hybrid GRASP-tabu search	adapted CL03
Melachrinoudis et al. (2007)	HF, MD	min. RC, min CI	TW	tabu search	up to 8 req.
Bergvinsdottir et al. (2006)	-	min. RC, min. RT, min. TWV, ...	TW, RT, RD	genetic algorithm, space-time nearest neighbor heur.	CL03
Rekiek et al. (2006)	S, MD, HF	min. RC	TW, RT, VA	grouping genetic algorithm	up to 164 req.

Bench. = Benchmark, CI = Customer Inconvenience, Con. = Constraints, DDT = Desired Delivery Time, FRT = Fixed Route Transit, HF = Heterogeneous Fleet, MD = Multi depot, NV = Number of Vehicles, Obj. = Objective(s), RT = Ride Time, RC = Routing Cost, S = Service time, TL = Tour Length, TW = Time Windows, TWV = TW Violation, VA = Vehicle Availability; The respective benchmark instances are described in Section 4. Entries marked by an asterisk (*) are described in further detail in the text.

3.4.3 Metaheuristics Also metaheuristic solution methods have been developed for the static DARP. Table 4 provides an overview of the different algorithms proposed. The same information as in the previous tables is given.

Toth and Vigo (1997) propose a local search based metaheuristic algorithm, i.e. a tabu thresholding algorithm, for the static multi vehicle DARP using the neighborhoods defined in (Toth and Vigo, 1996). The tabu thresholding algorithm uses parallel insertion to obtain an initial solution.

Cordeau and Laporte (2003b) propose a tabu search algorithm. Time windows are considered at either origin or destination depending on the type of request (inbound or outbound). The neighborhood considered in the tabu search is defined by moving one request to another route. The best possible move serves to generate a new incumbent solution. Reverse moves are declared tabu. However, an aspiration criterion is defined, such that tabu moves that provide a better solution, w.r.t. all other solutions already constructed by the same move, can constitute a new incumbent solution.

3.4.4 Summary State-of-the-art exact methods for the static DARP solve some instances with up to 96 request to optimality (Ropke et al., 2007). However, the same limitation applies as stated in Section 3.2.4, i.e. the size of the test instance is not a very meaningful indicator since tightly constrained instances are easier to solve than those with less tight constraints and no standardized data set has been solved by the proposed approaches. In case of heuristic and metaheuristic methods comparison becomes even harder since a large part of the solution procedures developed are motivated by real world problem situations differing w.r.t. problem type (single and multi depot, homogeneous and heterogeneous fleet), constraints considered as well as objective(s) optimized. Moreover, even when the same data sets are used different objectives are considered, compare e.g. (Cordeau and Laporte, 2003a, Bergvinsdottir et al., 2006). Consequently, we can only state that in general heuristic methods run faster whereas metaheuristics demand more computation time but usually outperform basic heuristic procedures w.r.t. solution quality.

3.4.5 Related work Dealing with the transportation of people, especially handicapped or elderly, research has also been dedicated to the comparison of dial-a-ride systems with other modes of transportation, e.g. public bus systems or mixed systems. Early studies of dial-a-ride transportation systems are discussed in (Carlson, 1976, Teixeira and Karash, 1975). Elmberg (1978) already tested a robot dispatcher dial-a-ride system in Sweden. Daganzo (1984) compares fixed route transit systems with checkpoint dial-a-ride and door-to-door dial-a-ride systems. He concludes that most of the time either fixed route systems or door-to-door transportation is the appropriate choice. Belisle et al. (1986) investigate the impact of different operating scenarios on the quality of transportation systems for the handicapped. More recent studies comparing dial-a-ride and traditional bus systems by means of simulation were conducted by (Noda et al., 2003, Noda, 2005). A study by means of simulation w.r.t. the usability of dial-a-ride systems in urban areas was presented in (Shinoda et al., 2003). Mageean and Nelson (2003) study and evaluate telematics based demand responsive transport services in Europe. Palmer et al. (2004) study the impact of management practices as well as advanced technologies in the context of demand responsive transport systems. The impact of information flows, e.g. the percentage of real time requests or the length of the interval between the arrival of a new request and its requested pickup TW, is investigated in (Diana, 2006). In (Diana et al., 2006) the optimal fleet size w.r.t. predetermined service quality is studied.

Research has also been dedicated to possible ways of computation time reduction. Hunsaker and Savelsbergh (2002), e.g., propose a fast feasibility check for the DARP. The proposed procedure can deal with waiting times, ride times as well as time window restrictions. Castelli et al. (2002) discuss three algorithms granting 2-opt-improvement feasibility.

A problem class related to the DARP is the *car pooling* problem. It consists of finding subsets of employees that share a car and of determining the path the driver should follow and possibly also who should be the driver. In contrast to the DARP either origin or destination are the same for all users depending on whether the trip is from home to the office or back. Two variants can be investigated, either one car pool for both ways or differing to-work and from-work problems. Baldacci et al. (2004) propose an exact as well as a heuristic procedure to solve the car pooling problem. A real life application was reported in (Wolfler Calvo et al., 2004). (Maniezzo et al., 2004) propose an ant colony optimization algorithm for the long-term problem.

3.5 Dynamic dial-a-ride problems

Less research has been dedicated to the domain of dynamic DARP. The different solution techniques developed are depicted in the following paragraphs. Predominantly heuristic methods have been used to solve the dynamic version of the DARP.

3.5.1 Exact methods Exact methods have not been explicitly developed for the dynamic DARP. However, in (Psaraftis, 1980) the static version of his algorithm is adapted to the dynamic case. Transportation requests are ordered according to the time of call. Up to ten customer requests can be handled by the dynamic solution method.

3.5.2 Heuristics Early heuristic algorithms to solve the dynamic DARP are discussed in (Daganzo, 1978). Daganzo analyzes three different insertion algorithms. The first algorithm refers to visiting the closest stop next, the second heuristic consists in visiting the closest origin or the closest destination in alternating order and the third algorithm only allows the insertion of delivery locations after a fixed number of passengers have been picked up.

Another early heuristic method applied to the dynamic case with transfer possibilities in the context of bus routing is presented in (Stein, 1978a,b). In a first step the relevant region is clustered and each subregion is assigned to a vehicle. In a second step traveling salesman tours are constructed within each subregion.

The dynamic case of the multi vehicle DARP is also studied in (Dial, 1995). New transportation requests are assigned to clusters according to least cost insertion. Routes are then optimized using dynamic programming. Results for a real life problem instance are reported.

Teodorovic and Radivojevic (2000) propose a two-stage fuzzy logic based heuristic algorithm. One approximate reasoning heuristic decides which vehicle a new request is assigned to and a second heuristic handles the adjustment of this vehicle's route. Results for instances with up to 900 requests are reported.

Another cluster first route second algorithm for the dynamic problem is proposed in (Colorni and Righini, 2001). Considering only the most urgent requests the routing subproblems can be solved to optimality using a branch and bound algorithm.

Caramia et al. (2002) use a dynamic programming algorithm to iteratively solve the single vehicle subproblems to optimality. Results for instances with up to 50 clients per hour are reported.

Horn (2002a) provide a software environment for fleet scheduling and dispatching of demand responsive services. The system can handle advance as well as immediate requests. New incoming requests are inserted into existing routes according to least cost insertion. A steepest descent improvement phase is run periodically. Also automated vehicle dispatching procedures to achieve a good combination of efficient vehicle deployment as well as customer service are included. The system was tested using the modeling framework LITRES-2 (Horn, 2002b) on a real life data set covering a 24 hour time period of taxi operations and 4282 customer requests. Another simulation environment to test solution methods for the dynamic DARP is proposed in (Fu, 2002b).

Coslovich et al. (2005) propose a two-phase insertion heuristic. A simple insertion procedure allows for quick answers with respect to inclusion or rejection of a new customer. The subsequent step consists in improving the initial solution by means of a local search operator, i.e. a 2-opt arc swap. Instances with a total of up to 50 (unexpected and expected requests) were solved.

Xiang et al. (2007) propose a fast heuristic for the dynamic DARP in a stochastic environment. The stochastics involved is due to possible travel time fluctuations, absent customers, vehicle breakdowns, cancellation of requests, traffic jams etc. To solve this complex problem situation the heuristic proposed in (Xiang et al., 2006) is adapted to the dynamic case. For testing purposes several simulations under varying parameter settings are conducted. Up to 610 requests are considered with different proportions of them already known in advance.

3.5.3 Metaheuristics Metaheuristic solution methods, as exact algorithms, have not been explicitly developed for the dynamic DARP since short response times are necessary in dynamic settings. Only one algorithm developed for the static version has been used to solve the dynamic case. The tabu search of (Cordeau and Laporte, 2003b) is adapted to the dynamic DARP by means of parallelization in (Attanasio et al., 2004). Different parallelization strategies are tested on instances with up to 144 requests.

3.5.4 Summary To summarize, only a limited number of solution algorithms have been proposed for the dynamic DARP. Most of them are based on repeated calls of static solution routines. Comparison across the different methods proposed becomes a difficult task since a majority of the work presented is motivated by real life applications. Consequently, each solution methodology was tested on different data sets with varying problem specific characteristics. Whenever a solution method originally developed for the static case was used we refer to Section 3.4 of this survey for further details on its performance.

3.5.5 Related work A problem class referred to as *online DARP* in the literature deals with the real time scheduling of server moves, see (Ascheuer et al., 2000, Feuerstein and Stougie, 2001, Hauptmeier et al., 2000, Krumke et al., 2005). The transportation requests consist of objects not people. However, since the objective minimized is the completion time, the proposed approximation algorithms might also be applicable in the context of passenger transportation. Lipmann et al. (2004) study the influence of restricted information on the online DARP, i.e. the destination of a request is only revealed after the object has been picked up. Its extension to the time window case is tackled in (Yi and Tian, 2005). The static version on a caterpillar graph, i.e. a graph consisting of a path and leaves attached to this path (one leaf per node of the path), is studied in (Coja-Oghlan et al., 2005). A heuristic algorithm for an extended static version was proposed by Hauptmeier et al. (2001), considering a single vehicle with unit capacity traveling between the different origins and destinations. At the origins more than one object waits to be transported. These objects are ordered according to the First-In-First-Out (FIFO) or the LIFO rule.

4 Benchmark Instances for VRPPD

In order to provide the interested reader with information on available benchmark instances used in the literature, we provide the following information in Table 6. Column one gives the literature reference for the respective benchmark instance. Column two through four state the problem type, the size of the different instances in terms of number of requests per instance, and the number of instances provided. Then, a brief description of the data set as well as the abbreviations used in this article are given.

The data set predominantly used to asses the performance of PDPTW is the one proposed by (Li and Lim, 2001). The latest new best results for both the primary and the extended data sets can be found in (Ropke and Pisinger, 2006a) and (Bent and van Hentenryck, 2006).

In contrast to the field of PDP, solution methods developed for the DARP have not been tested on standardized benchmark instances. This might be due to the fact that most methods vary considerably w.r.t. the constraints considered as well as the objectives minimized. However, since data sets for rather standard problems settings do exist now, this might change in the near future.

Table 6 Benchmark Instances for VRPPD

Literature Ref.	Type	Req.	#	Characteristics	Abbr.
Shang and Cuff (1996)	PDPTW, trans-fers	159	1	real world data	SC96
Nanry and Barnes (2000)	PDPTW	13-50	43	based on the VRPTW instances of Solomon (1987), optimal solution schedules by procedure of (Carlton, 1995), customers randomly paired	NB00
Renaud et al. (2000)	SPDP	25-249	108	based on 36 TSPLIB instances (Reinelt, 1991), for each pickup a delivery chosen among the 5 (10) closest or all unselected neighbors.	RBO00
Li and Lim (2001)	PDPTW	56	50	based on those of (Solomon, 1987), customers appearing on the same route in a solution of the VRPTW, using the solution procedure of (Li et al., 2001), were randomly paired;	LL01
		100-500		extended data set	LL01+
Toth and Vigo (1996)	DARP	276-312	5	real life data, Municipality of Bologna	TV96a
Cordeau and Laporte (2003b)	DARP	24-144	20	randomly generated around seed points, half of the requests have a tight TW at the origin, half a tight TW at the destination, 10 instances with narrow, 10 with wider TW.	CL03
Mitrović-Minić and Laporte (2004)	dyn. PDPTW	100-1000	40	ten hours service period, 60 x 60 km^2 area, vehicle move at 60 km/h, requests occur according to a continuous uniform distribution, no requests are known in advance	-
Cordeau (2006)	DARP	16-48	30	randomly generated; 15 instances with $C = 3$, unit user demand and $L = 30$; 15 instances with $C = 6$, varying user demand and $L = 60$.	-
		16-96	42	extended data set	-
Ropke et al. (2007)	PDPTW	30-75	40	randomly generated as described in (Savelsbergh and Sol, 1998).	RCL07

= number of instances, Abbr. = abbreviation used, bh. = backhaul customers, MD = Multi Depot, Req. = approximate number of requests of each instance, SD = Single Depot, TW = Time Window

5 Conclusion

Right now the solution methods presented in this survey are state-of-the-art in the field of VRPPD. In line with the first part of this survey we believe that future research will involve the incorporation of additional real life constraints, the consideration of stochastics as well as knowledge about future events, e.g. that in a certain region some requests are very likely to occur. This information could be exploited by incorporating probability distributions. Moreover, the DARP is, in contrast to many other routing problems, a natural multiobjective problem. This aspect will also be part of future investigations. As mentioned PDP and DARP are in most applied situations inherently dynamic. This is why we have grouped this survey into subsections w.r.t. static and dynamic approaches.

We hope and trust that this survey will lead to future research in the area of vehicle routing involving pickups and deliveries.

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