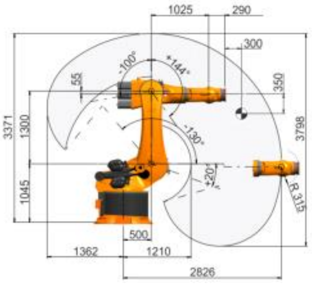
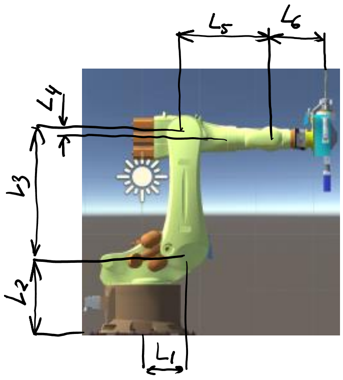
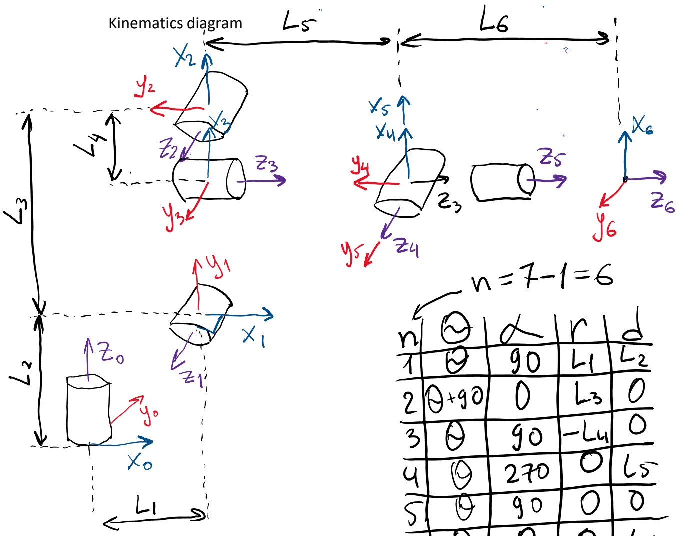
Kuka





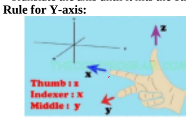
Kinematics diagram



$n = 7 - 1 = 6$

n	Θ	α	r	d
1	θ	90	L_1	L_2
2	θ+90	0	L_3	0
3	θ	90	$-L_4$	0
4	θ	270	0	L_5
5	θ	90	0	0
6	θ	0	0	L_6

D-H Frame Rules
Rule 1: The Z-axis must be the axis of rotation for a revolute joint
Rule 2: The X-axis must be perpendicular to its own Z-axis and to the Z-axis of the previous frame
Rule 3: Each X-axis must intersect the Z-axis of the previous frame
Note if Rule 3 is not satisfied:
- Translate the axis until it hits the other
Rule for Y-axis:



Rotation and Translation Parameters

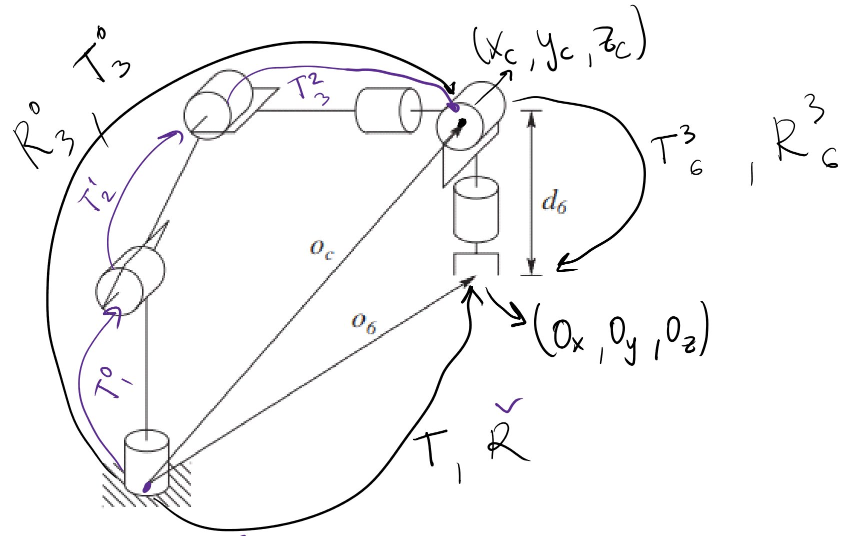
Θ
Rotation around z_{n-1} by Θ, that is required to match x_{n-1} with x_n

α
Rotation around x_n by α, that is required to match z_{n-1} with z_n

d
Distance between origins n-1 and n, along axis z_{n-1}

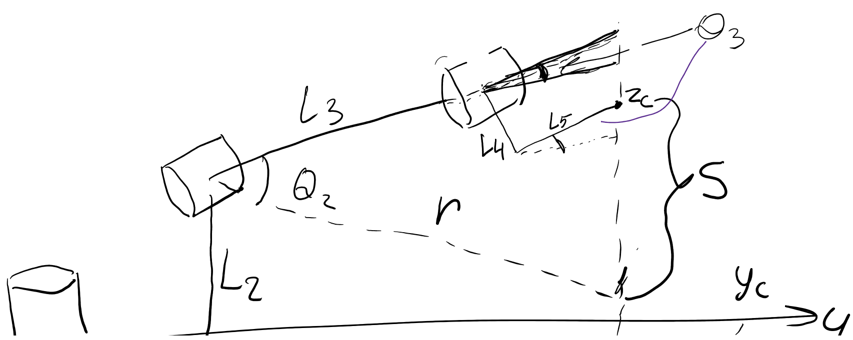
r
Distance between origins n-1 and n, along axis x_n

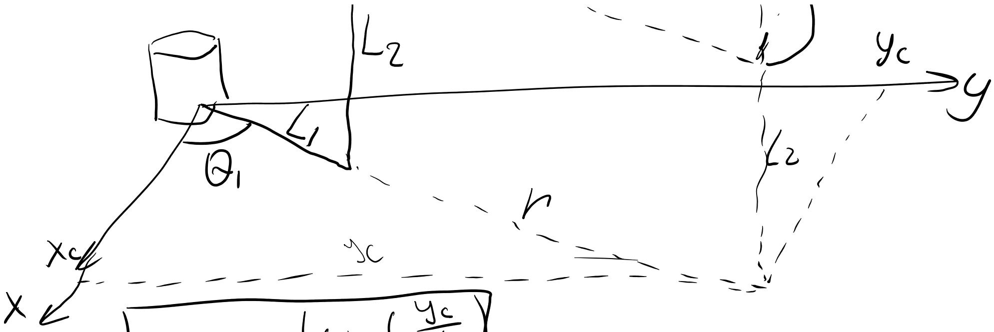
Inverse kinematics calculation



$$T^0_3 = T^0_1 \cdot T^1_2 \cdot T^2_3$$

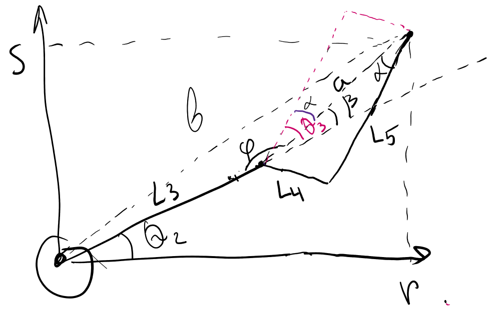
$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} O_x - L_6 R_{13} \\ O_y - L_6 R_{23} \\ O_z - L_6 R_{33} \end{bmatrix}$$





$$\boxed{\Theta_1 = \operatorname{atan}\left(\frac{y_c}{x_c}\right)}$$

I

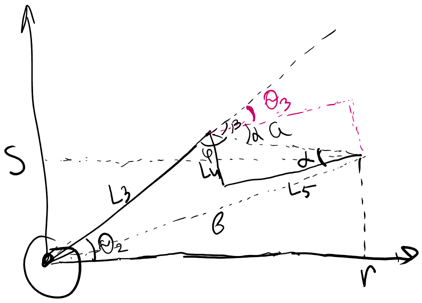


$$a = \sqrt{L_4^2 + L_5^2}$$
$$r = \sqrt{x_c^2 + y_c^2} - L_1$$
$$s = z_c - L_2$$
$$b = \sqrt{r^2 + s^2}$$

$$\cos\varphi = \frac{L_3^2 + a^2 - b^2}{2 L_3 a} := D$$
$$\varphi = \operatorname{acos}(D)$$
$$\beta = 180 - \varphi$$
$$\alpha = \operatorname{atan}\left(\frac{L_4}{L_5}\right)$$

$$\boxed{\Theta_3 = \beta + \alpha + 90}$$

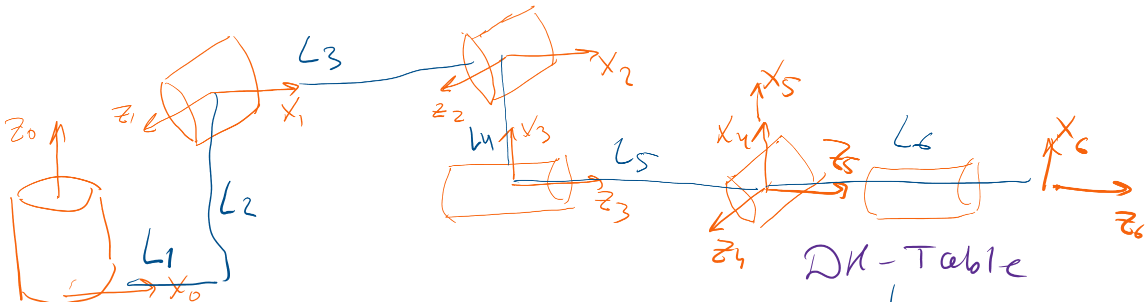
II



$$a = \sqrt{L_4^2 + L_5^2}$$
$$r = \sqrt{x_c^2 + y_c^2} - L_1$$
$$s = z_c - L_2$$
$$b = \sqrt{r^2 + s^2}$$

$$\cos\varphi = \frac{L_3^2 + a^2 - b^2}{2 L_3 a} := D$$
$$\varphi = \operatorname{acos}(D)$$
$$\beta = 180 - \varphi$$
$$\alpha = \operatorname{atan}\left(\frac{L_4}{L_5}\right)$$

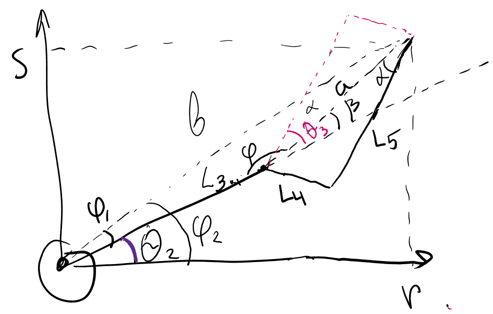
$$\boxed{\Theta_3 = \beta - \alpha + 90}$$



DH-Table

n	Θ	α	r	d
1	Θ	90	L_1	L_2
2	Θ	0	L_3	0
3	$90+\Theta$	90	$-L_4$	0
4	Θ	270	0	L_5
5	Θ	90	0	0
6	Θ	0	0	L_6

I



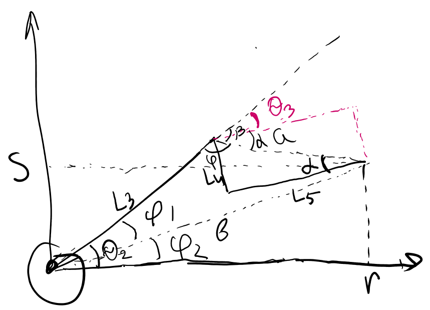
$$\varphi_2 = \operatorname{atan}\left(\frac{s}{r}\right)$$

$$\cos\varphi_1 = \frac{L_3^2 + b^2 - a^2}{2 \cdot L_3 \cdot b} := D1$$

$$\varphi_1 = \operatorname{acos}(D1)$$

$$\boxed{\Theta_2 = \varphi_2 - \varphi_1}$$

II



$$\varphi_2 = \operatorname{atan}\left(\frac{s}{r}\right)$$

$$\cos\varphi_1 = \frac{L_3^2 + b^2 - a^2}{2 \cdot L_3 \cdot b} := D1$$

$$\varphi_1 = \operatorname{acos}(D1)$$

$$\boxed{\Theta_2 = \varphi_2 + \varphi_1}$$

$\Theta_4, \Theta_5, \Theta_6$ – ?

100
010 – identity matrix
001

no rotation //

$$\operatorname{Trans}_{z_{n-1}}(d_n) = \begin{array}{c} \begin{array}{cc} R & T \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \end{array} \begin{array}{l} x \\ y \\ z \\ \end{array}$$

$$\operatorname{Rot}_{z_{n-1}}(\theta_n) = \left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n & 0 & 0 \\ \sin\theta_n & \cos\theta_n & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\operatorname{Trans}_{x_n}(r_n) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & r_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} x \\ y \\ z \\ \end{array}$$

$$\operatorname{Rot}_{x_n}(\alpha_n) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_n & -\sin\alpha_n & 0 \\ 0 & \sin\alpha_n & \cos\alpha_n & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

This gives:

$$ {}^{n-1}T_n = \left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & r_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & r_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} & \boxed{R} & & T \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Rotation and Translation Parameters

Θ
Rotation around z_{n-1} by Θ, that is required to match x_{n-1} with x_n

α
Rotation around x_n by α, that is required to match z_{n-1} with z_n

d
Distance between origins n-1 and n, along axis z_{n-1}

r
Distance between origins n-1 and n, along axis x_n

n	Θ (rot)	α (rot)	r (dist)	d (dist)
1	Θ	90	L1	L2
2	Θ	0	L3	0
3	90+Θ	90	-L4	0
4	Θ	270	0	L5
5	Θ	90	0	0
6	Θ	0	0	L6

T_1 T_2'

$$\mathbf{R}_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$\mathbf{R}_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\mathbf{R}_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R = \boxed{R_3^0} R_6^3 \quad ?$$

$$R_6^3 = (R_3^0)^{-1} R = (R_3^0)^T R = \begin{bmatrix} \cdots & \cdots & (1,3) \\ \cdots & \cdots & (2,3) \\ \cdots & \cdots & \cdots \end{bmatrix}$$

(3,1) (3,2) (3,3)

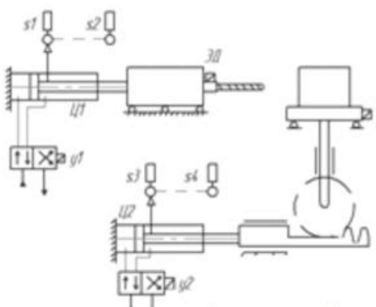
$$\cdot 3 \quad \Big[\, C_4C_5C_6 - S_4S_6 \qquad -C_4C_5S_6 - S_4C_6 \qquad C_4S_5 \,\Big]$$

$$R^3_6 = \begin{bmatrix} c_4 c_5 c_6 - s_4 s_6 & -c_4 c_5 s_6 - s_4 c_6 & c_{\boxed{4}} s_5 \\ s_4 c_5 c_6 + c_4 s_6 & -s_4 c_5 s_6 + c_4 c_6 & s_{\boxed{4}} s_5 \\ -\cancel{s_5}\, c_{\boxed{6}} & \cancel{s_5}\, s_{\boxed{6}} & c_5 \end{bmatrix}$$

$$\theta_5 = a\cos\left(R^3_6(3,3)\right)$$

$$\theta_4 = a\tan\left(\frac{R^3_6(2,3)}{R^3_6(1,3)}\right)$$

$$\theta_6 = a\tan\left(\frac{R^3_6(3,2)}{R^3_6(3,1)}\right)$$



s4 s5