Experiments in RADAR design (v2)

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1 Introduction

A radar system uses radio-frequency electromagnetic signal reflected from a target to determine information about it. In order to build up a discernible echo, most radar systems emit pulses continuously and the repetition rate of these pulses is determined by the role of the system. The pulse repetition frequency (PRF), denoted by f, is the number of pulses of a repeating signal in a specific time unit, normally measured in pulses per second (Hz). Its inverse $T = 1/f$ is called pulse repetition interval.

We should measure the time delay t between sending the signal and receiving the reflected echo from the target. Due to the periodic nature of transmission, we cannot know for sure which transmission caused the reflection. Therefore we can say that $t \equiv t_0 \pmod{T}$ for some $t_0 \in [0, T)$. [1] We call t_0 *apparent delay*. Therefore we are only able to measure t_0.

We want to calculate the distance r of the target object. We know that

$$r = \frac{ct}{2},$$

where c is the speed of light. Again, since we don't know the exact value of t, but only t_0, we have

$$r \equiv r_0 \pmod{R}$$

where

$$r_0 = \frac{ct_0}{2}, \quad R = \frac{cT}{2}$$

are the *apparent range* and *maximum range*.

Due to the nature of RADAR systems, range measurement only works if $f < 4$ kHz. On the other hand, sending out the pulse signal from the RADAR system takes time, let us assume the pulse width is τ. While the system is transmitting, it cannot receive, so $T > \tau$. For a rule of thumb, we will leave at least 75% of the pulse repetition interval for receiving the echo, therefore $T > 4\tau$. In summary

$$\frac{1}{4\tau} < f < 4\text{kHz}.$$

[1] Here we abuse the notation, since t, t_0 and T are not necessarily integers. What this means that $t = t_0 + kT$ for an integer k.

In summary, our system is only capable of range and velocity determination, when $t \in [0, T)$. In this case, $r \in [0, R)$.

1.1 Range extension with multiple systems

We can use multiple RADAR system to extend the range or velocity detection capabilities. Let us assume that we deploy multiple system with different PRFs. The i-th system PRF is $f^{(i)}$ and the measured time delay of the echo is $t_0^{(i)}$. We calculate the apparent range $r_0^{(i)}$ (based on $t_0^{(i)}$) and the maximum range $R^{(i)}$ (based on $f^{(i)}$) for each transmitter. We can assume that both $r_0^{(i)}$ and $R^{(i)}$ are integers and round to closest integer, if they aren't. We end up with a system of linear congruence relations

$$r \equiv r_0^{(i)} \pmod{R^{(i)}}.$$

2 Assignment description

Your tasks are

- Given a maximum range R and a pulse width τ, design a RADAR system, i.e. select multiple PRFs, which are capable to determine distances based the apparent delays.
- Automate the design process, i.e. create a function for it in Sage.
- Give an example of a badly designed system, in which we cannot determine distance.
- Give an example of a maximum range, which does not yield feasible RADAR system.
- Create a function in Sage, which calculates the distance based on the apparent delays and PRFs.
- Create an *academic report* of your solution. The report shall include the following sections:
  1. Introduction, where you state the problem and its setting.
  2. Methods, where you describe your solution in words. This can include the Sage code as well, if you are working in Notebook form.
  3. Results, where you create example using real numbers. This is the place to test your solution as well.

  Your submitted Sage code must be executable.

2.1 Hints and remarks

- If the hypothesis of Chinese Remainder Theorem holds, then there is always a solution. That solution is unique in a certain range.
- The fundamental theorem of arithmetic can be useful for determining the sub-ranges of the given maximum range.
- There are requirements of PRFs for distance. If you cannot fulfil them for a given maximum range, you can always select larger maximum which works.
- You can use any builtin Sage function to solve the system of equations. Your exercise is to design the system, not to re-create builtin algorithms, for e.g. `crt()`.
- You are free to select the units for the problem. Duration is usually measured in seconds (s), range is measured in kilometers (km) in RADAR systems.