The following separable problem will be considered:

$$\min_{x_1,x_2,x_3} \qquad f_1(x_1,x_3)+f_2(x_2,x_3)$$

or equivalently:

$$\begin{aligned}\min_{x_1,\xi_1,x_2,\xi_2} \qquad & f_1(x_1,\xi_1)+f_2(x_2,\xi_2)\\ s.t. \qquad & \xi_1=\xi_2,\end{aligned}$$

where $x_1 \in \mathbb{R}^n$, $x_2 \in \mathbb{R}^m$ and $x_3, \xi_1, \xi_2 \in \mathbb{R}$, for some n and m. Then the dual function is

$$\begin{aligned}q(\lambda) &= \inf_{x_1,\xi_1,x_2,\xi_2}[f_1(x_1,\xi_1)+f_2(x_2,\xi_2)-\lambda^\top(\xi_1-\xi_2)]\\ &= \inf_{x_1,\xi_1}[f_1(x_1,\xi_1)-\lambda^\top(\xi_1)]+\inf_{x_2,\xi_2}[f_2(x_2,\xi_2)+\lambda^\top(\xi_2)]\\ &= q_1(\lambda)+q_2(\lambda)\end{aligned}$$

We seek a λ that maximises this dual function, which can be found using the subgradient method. Here the negative subgradient of the dual function at a given point λ_k is given as $g_k = \xi_1 - \xi_2$. The subgradient method involves iteratively

- finding $(x_1,\xi_1,x_2,\xi_2) \in \arg\min_{x_1,\xi_1,x_2,\xi_2}[f_1(x_1,\xi_1)+f_2(x_2,\xi_2)-\lambda^\top(\xi_1-\xi_2)]$, and
- updating $\lambda_{k+1} = \lambda_k - \alpha_k(\xi_1-\xi_2)$.

The method of dual decomposition is simply making use of the fact that the dual function can be decomposed into $q_1(\lambda)$ and $q_2(\lambda)$ as shown above, and so finding

$$(x_1,\xi_1,x_2,\xi_2) \in \arg\min_{x_1,\xi_1,x_2,\xi_2}[f_1(x_1,\xi_1)+f_2(x_2,\xi_2)-\lambda^\top(\xi_1-\xi_2)]$$

can instead be done by two individual agents finding the minimisers

$(x_1,\xi_1) \in \arg\min_{x_1,\xi_1}[f_1(x_1,\xi_1)-\lambda^\top(\xi_1)]$, and

$(x_2,\xi_2) \in \arg\min_{x_2,\xi_2}[f_2(x_2,\xi_2)+\lambda^\top(\xi_2)]$

separately.