Problem 1

This is a pedagogical toy problem. It is designed to show how Process and Pipe objects from the multiprocessing module on Python can be used in a separable optimisation problem.

The following is the separable unconstrained optimisation problem to be solved:

$$\min_{x_1,x_2,x_3} \qquad x_1^2 + x_2^2 + 2x_3^2,$$

where $x_1, x_2, x_3 \in \mathbb{R}$. Let x_1 and x_2 both be private variables, i.e., they can only be accessed by Agent 1 and Agent 2 respectively. Let x_3 be a shared or public variable that can be accessed by all agents. The minimiser of this problem is obviously $(x_1^*, x_2^*, x_3^*) = (0, 0, 0)$, but remember, the point of this problem is really to give Python's multiprocessing module a test run on an optimisation problem.

By applying a change of variables, the cost function can be separated and the new formulation of the problem will be

$$\begin{aligned} \min_{x_1,\xi_1,x_2,\xi_2} \qquad & [x_1^2 + \xi_1^2] + [x_2^2 + \xi_2^2] \\ s.t. \qquad & \xi_1 = \xi_2, \end{aligned}$$

where $x_1, \xi_1, x_2, \xi_2 \in \mathbb{R}$. The dual function will then be

$$\begin{aligned} q(\lambda) &= q_1(\lambda) + q_2(\lambda) \\ &= \inf_{x_1,\xi_1} [x_1^2 + \xi_1^2 - \lambda\xi_1] + \inf_{x_2,\xi_2} [x_2^2 + \xi_2^2 + \lambda\xi_2]. \end{aligned}$$

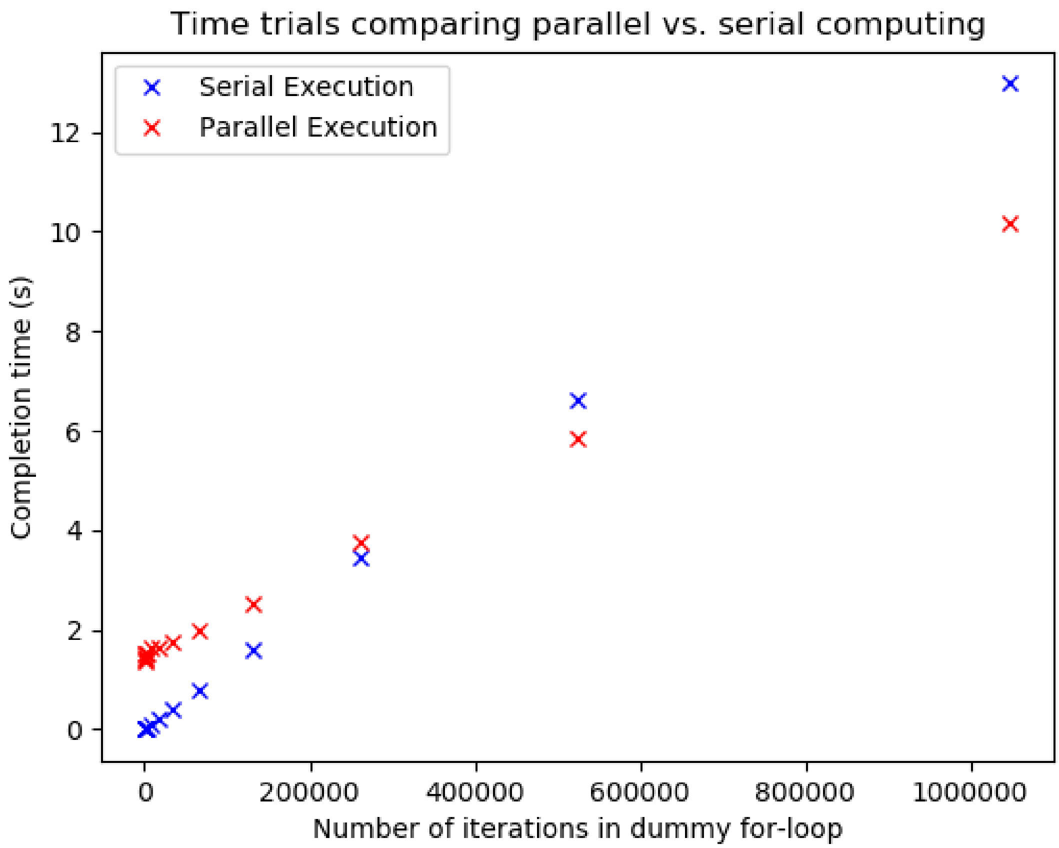
The minimisers of $q_1(\lambda)$ and $q_2(\lambda)$ will be $(x_1^*, \xi_1^*) = (0, \lambda/2)$ and $(x_2^*, \xi_2^*) = (0, -\lambda/2)$ respectively. The algorithm will be as follows:

At step $k = 0$, some master processor chooses an initial $\lambda(0) = 1.0$ and sends $\lambda(0)$ to Agents 1 and 2. Agent 1 calculates $\xi_1^*(0) = \lambda(0)/2$, while (in parallel) Agent 2 calculates $\xi_2^*(0) = -\lambda(0)/2$. Agents 1 and 2 send $\xi_1^*(0)$ and $\xi_2^*(0)$ back to the master processor. At step $k = 1$, the master processor updates $\lambda(1) = \lambda(0) - \alpha(\xi_1^*(0) - \xi_2^*(0))$ and sends $\lambda(1)$ to Agents 1 and 2. Keep looping until $\xi_1^*(k) - \xi_2^*(k)$ is small enough, or we reach some chosen maximum number of iterations.

Computational Overhead

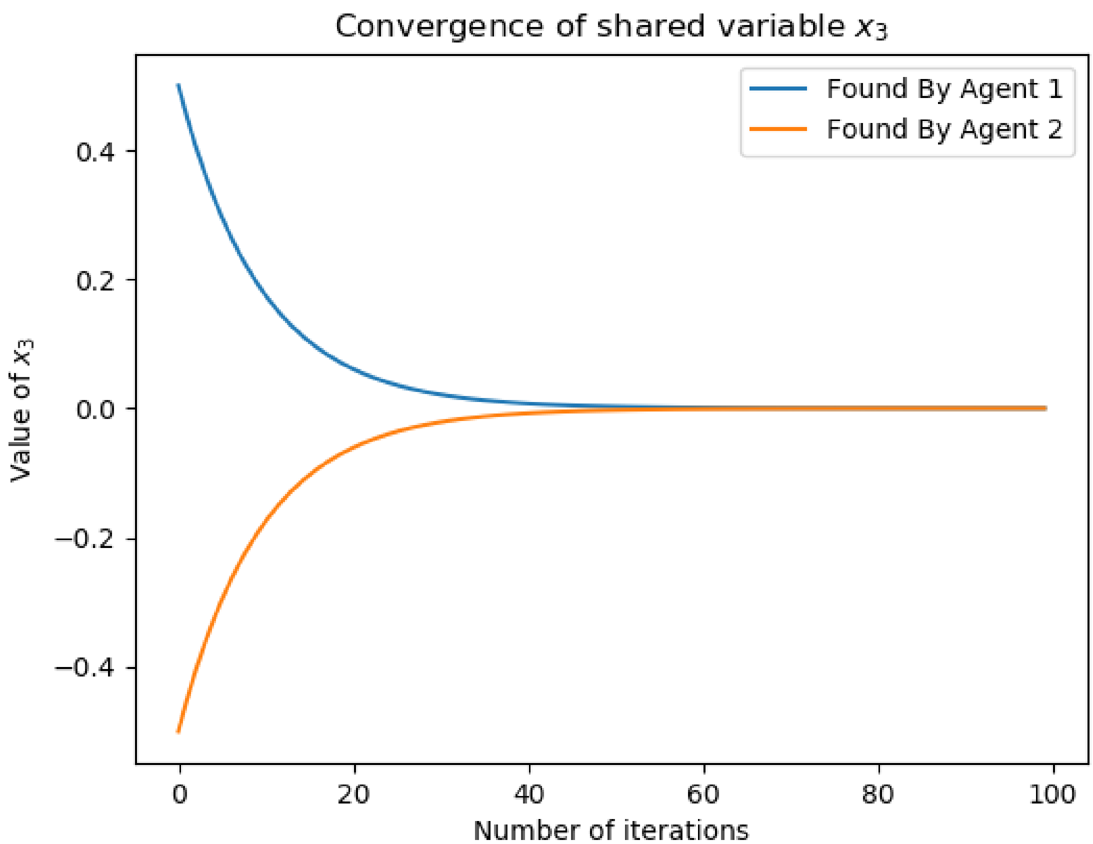
There is some computational overhead to spawn a Process on Python. Since the parallel processes in this problem are computationally cheap compared to this overhead, it is faster to just carry out the processes in series. For the sake of testing, we want to simulate a more "difficult" problem, where the minimisers $\xi_1^*(k)$ and $\xi_2^*(k)$ may be more computationally expensive to compute.

This can be done by introducing a for-loop that in the parallel processes so that on top of calculating $\xi_1^*(k)$ and $\xi_2^*(k)$, the agents must also get through a large "dummy" calculation. When the for-loop is made large enough (of the order 1000000 iterations), we start to see the parallel method overtake the serial method in terms of speed. The key observation in Problem 1 is that is the cost of computation per process must be larger the computational cost of spawning that process.



Convergence of Algorithm

We get convergence to $(x_1^*, x_2^*, x_3^*) = (0, 0, 0)$ as expected. It turns out that convergence occurs regardless of how the variable x_3^2 in the cost function is separated. Given $f = x_1^2 + x_2^2 + 2x_3^2$, convergence occurs for $f = [x_1^2 + 2ax_3^2] + [x_2^2 + 2(1-a)x_3^2]$ for all $a \in [0, 1]$, where the square brackets show how the cost function is separated. This will be observed in more detail, in Problem 2.



How to Run

The above examples

Make sure you are in the correct directory. Then to run the test that generated the above plots, execute the **main.py** file, i.e. use the command

>>>**python main.py**

Function Descriptions

The function **parallel.do_parallel**, description.

Syntax: do_parallel(max_iter,alpha,size_problem=0,verbose=False)

Parameter values:

- max_iter, Required. Number of iterations for the subgradient method.
- alpha, Required. Step size for the subgradient method.
- size_problem, Default 0. Number of iterations for the dummy for-loop.
- verbose, Default False. Print results to screen.

Outputs:

- Output 1. List containing ξ_1^* for all iterations of the subgradient method.
- Output 2. List containing ξ_1^* for all iterations of the subgradient method.
- Output 3. Completion time.