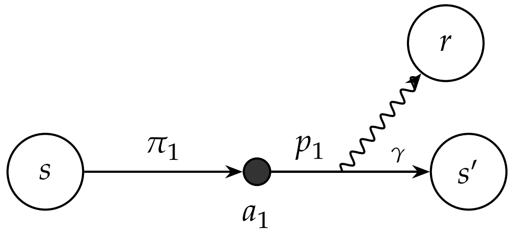
Bellman Equation as Dot Products



$$g = \vec{\gamma} \cdot \vec{r}$$

$$\vec{o} = \vec{r} + \gamma \vec{v'}$$

$$q = \vec{p} \cdot \vec{o} \qquad = E\ [\ g \mid s, a\ ]$$

$$v = \vec{\pi} \cdot \vec{q} \qquad = E\ [\ g \mid s\ ]$$

$$\vec{\gamma} = \begin{bmatrix} \gamma^0 \\ \gamma^1 \\ \vdots \end{bmatrix}$$

$$\vec{r} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} r(s,a,s'_1) \\ r(s,a,s'_2) \\ \vdots \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} v_0 \\ v_1 \\ \vdots \end{bmatrix} = \begin{bmatrix} v(s_0) \\ v(s_1) \\ \vdots \end{bmatrix} = \begin{bmatrix} \vec{\pi}_1 \cdot \vec{q}_1 \\ \vec{\pi}_2 \cdot \vec{q}_2 \\ \vdots \end{bmatrix}$$

$$\vec{v}' = \begin{bmatrix} v'_0 \\ v'_1 \\ \vdots \end{bmatrix} = \begin{bmatrix} v(s'_0) \\ v(s'_1) \\ \vdots \end{bmatrix}$$

$$\vec{o} = \begin{bmatrix} o_1 \\ o_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} r_1 + \gamma v'_1 \\ r_2 + \gamma v'_2 \\ \vdots \end{bmatrix}$$

$$\vec{p} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} p(s'_1 \mid a,s) \\ p(s'_2 \mid a,s) \\ \vdots \end{bmatrix}$$

$$\vec{q} = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} \vec{p}_1 \cdot \vec{o}_1 \\ \vec{p}_2 \cdot \vec{o}_2 \\ \vdots \end{bmatrix}$$

$$\vec{\pi} = \begin{bmatrix} \pi_1 \\ \pi_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} p(a_1 \mid s) \\ p(a_2 \mid s) \\ \vdots \end{bmatrix}$$

r = reward (*rewarded **after** action a is taken*)

γ = discounting factor

$g = g_0 = r_1 + \gamma r_2 + \gamma^2 r_3 + \cdots$ = return (*reward now plus discounted future reward*)

$o = r + \gamma v'$ = outcome ($v = E[g|s] = E[o|s]$ ← *Recursion: Bellman's insight!*)

$s = s_0$ = current state

$s' = s_1$ = next state

$v = v(s) = v(s_0)$

$v' = v(s') = v(s_1)$

a = action

p = transition probability, dynamics

v = state value function

q = state-action value function

π = policy