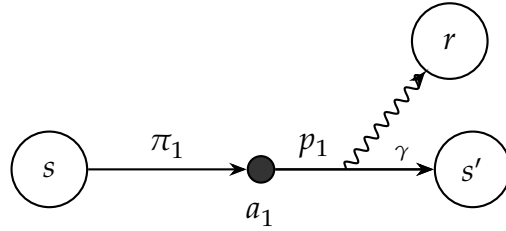


# Bellman Equation as Dot Products



$$g = \vec{\gamma} \cdot \vec{r}$$

$$\vec{o} = \vec{r} + \gamma \vec{v}'$$

$$q = \vec{p} \cdot \vec{o} \quad = E [ g \mid s, a ]$$

$$v = \vec{\pi} \cdot \vec{q} \quad = E [ g \mid s ]$$

$$\vec{\gamma} = \begin{bmatrix} \gamma^0 \\ \gamma^1 \\ \vdots \end{bmatrix}$$

$$\vec{p} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} p(s'_1 \mid a, s) \\ p(s'_2 \mid a, s) \\ \vdots \end{bmatrix}$$

$$\vec{r} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} r(s, a, s'_1) \\ r(s, a, s'_2) \\ \vdots \end{bmatrix}$$

$$\vec{q} = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} \vec{p}_1 \cdot \vec{o}_1 \\ \vec{p}_2 \cdot \vec{o}_2 \\ \vdots \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} v_0 \\ v_1 \\ \vdots \end{bmatrix} = \begin{bmatrix} v(s_0) \\ v(s_1) \\ \vdots \end{bmatrix} = \begin{bmatrix} \vec{\pi}_1 \cdot \vec{q}_1 \\ \vec{\pi}_2 \cdot \vec{q}_2 \\ \vdots \end{bmatrix}$$

$$\vec{\pi} = \begin{bmatrix} \pi_1 \\ \pi_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} p(a_1 \mid s) \\ p(a_2 \mid s) \\ \vdots \end{bmatrix}$$

$$\vec{v}' = \begin{bmatrix} v'_0 \\ v'_1 \\ \vdots \end{bmatrix} = \begin{bmatrix} v(s'_0) \\ v(s'_1) \\ \vdots \end{bmatrix}$$

$$\vec{o} = \begin{bmatrix} o_1 \\ o_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} r_1 + \gamma v'_1 \\ r_2 + \gamma v'_2 \\ \vdots \end{bmatrix}$$

$r$  = reward (rewarded **after** action  $a$  is taken)  
 $\gamma$  = discounting factor  
 $g = g_0 = r_1 + \gamma r_2 + \gamma^2 r_3 + \dots = \text{return}$  (reward now plus discounted future reward)  
 $o = r + \gamma v' = \text{outcome}$  (  $v = E[g|s] = E[o|s] \leftarrow \text{Crucial Bellman insight!}$  )  
 $s = s_0 = \text{current state}$   
 $s' = s_1 = \text{next state}$   
 $v = v(s) = v(s_0)$   
 $v' = v(s') = v(s_1)$   
 $a = \text{action}$   
 $p = \text{transition probability, dynamics}$   
 $v = \text{state value function}$   
 $q = \text{state-action value function}$   
 $\pi = \text{policy}$

