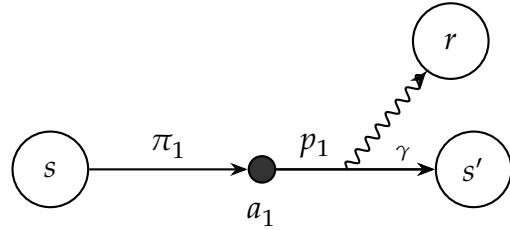


Bellman Equation as Dot Products



$$\begin{aligned}
 g &= \vec{\gamma} \cdot \vec{r} \\
 \hline
 \vec{o} &= \vec{r} + \gamma \vec{v}' \\
 q &= \vec{p} \cdot \vec{o} & = E [g | s, a] \\
 v &= \vec{\pi} \cdot \vec{q} & = E [g | s]
 \end{aligned}$$

$$\begin{aligned}
 \vec{\gamma} &= \begin{bmatrix} \gamma^0 \\ \gamma^1 \\ \vdots \end{bmatrix} & \vec{p} &= \begin{bmatrix} p_1 \\ p_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} p(s'_1 | a, s) \\ p(s'_2 | a, s) \\ \vdots \end{bmatrix} \\
 \vec{r} &= \begin{bmatrix} r_1 \\ r_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} r(s, a, s'_1) \\ r(s, a, s'_2) \\ \vdots \end{bmatrix} & \vec{q} &= \begin{bmatrix} q_1 \\ q_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} \vec{p}_1 \cdot \vec{o}_1 \\ \vec{p}_2 \cdot \vec{o}_2 \\ \vdots \end{bmatrix} \\
 \vec{v} &= \begin{bmatrix} v_0 \\ v_1 \\ \vdots \end{bmatrix} = \begin{bmatrix} v(s_0) \\ v(s_1) \\ \vdots \end{bmatrix} = \begin{bmatrix} \vec{\pi}_1 \cdot \vec{q}_1 \\ \vec{\pi}_2 \cdot \vec{q}_2 \\ \vdots \end{bmatrix} & \vec{\pi} &= \begin{bmatrix} \pi_1 \\ \pi_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} p(a_1 | s) \\ p(a_2 | s) \\ \vdots \end{bmatrix} \\
 \vec{v}' &= \begin{bmatrix} v'_0 \\ v'_1 \\ \vdots \end{bmatrix} = \begin{bmatrix} v(s'_0) \\ v(s'_1) \\ \vdots \end{bmatrix} \\
 \vec{o} &= \begin{bmatrix} o_1 \\ o_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} r_1 + \gamma v'_1 \\ r_2 + \gamma v'_2 \\ \vdots \end{bmatrix}
 \end{aligned}$$

r = reward (rewarded **after** action a is taken)
 γ = discounting factor
 $g = g_0 = r_1 + \gamma r_2 + \gamma^2 r_3 + \dots$ = return (reward now plus discounted future reward)
 $o = r + \gamma v'$ = outcome ($v = E[g|s] = E[o|s]$ ← Crucial Bellman insight!)
 $s = s_0$ = current state
 $s' = s_1$ = next state
 $v = v(s) = v(s_0)$
 $v' = v(s') = v(s_1)$
 a = action
 p = transition probability, dynamics
 v = state value function
 q = state-action value function
 π = policy

