Филимонов Степан РЛ6-31

Вариант 12

Домашнее задание №2

Задача 3.2.1

Дано:

$y(t) = a e^{-\frac{R_0}{2L}t}$

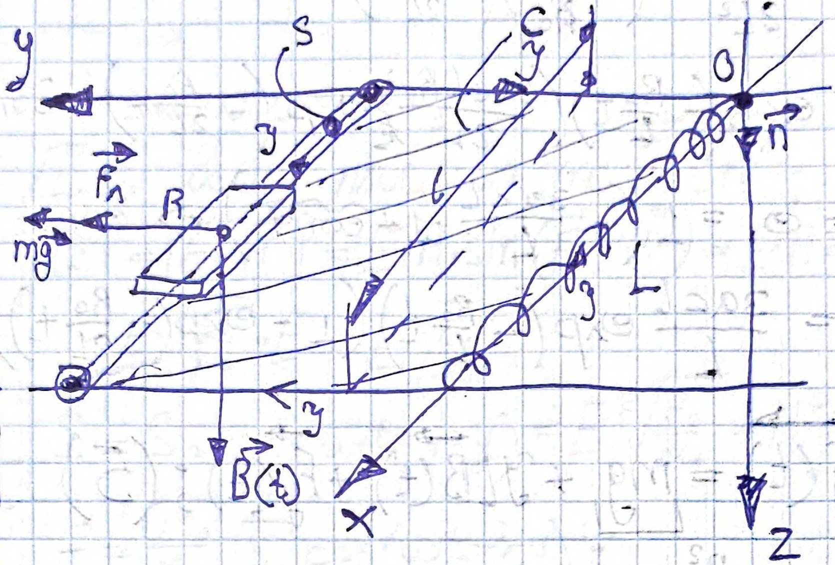
$B_z = c\cdot e^{-\frac{R_0}{L}t}$

$m, n_0, R_0, L, l, a, c,$

$S, \rho_{уд}, e, \mathcal{J}(0) = 0\,A$

$\mathcal{J}(t) - ?, \mathcal{J}_{max} - ?, \mathcal{E}(t) - ?$

$F(t) - ?, F_{Lx}(t) - ?, F_{Ly}(t) - ?$



Решение:

1) Выберем $\vec{n} \uparrow\uparrow \vec{B}$, тогда $\Phi_B > 0$

$$\Phi = \oint\!\!\oint_S \vec{B}\, d\vec{S} = \oint\!\!\oint_S \vec{B}\vec{n}\, dS = \oint_C \vec{B}\vec{n}\, dS = \vec{B}|\vec{n}|\, l y \Big|_{\substack{|\vec{n}|=1\\ B=B(t)\\ y=y(t)}} = B(t)\cdot y(t)\cdot l$$

$$\Phi(t) = acl\, e^{-\frac{3R_0}{2L}t}; \quad (1)$$

$$\mathcal{E}_i(t) = -\frac{d\Phi}{dt} = acl\left(-\frac{3R_0}{2L}\right)e^{-\frac{3R_0}{2L}t} = \frac{3aclR_0}{2L}e^{-\frac{3R_0}{2L}t} \quad (2)$$

$$\mathcal{J}(t) = \frac{\mathcal{E}_i(t) + \mathcal{E}_{si}(t)}{R_0} = \frac{\mathcal{E}_i(t) - L\frac{d\mathcal{J}}{dt}}{R_0}$$

$$L\frac{d\mathcal{J}}{dt} = \mathcal{E}_i - \mathcal{J}R_0 \Rightarrow \frac{d\mathcal{J}}{dt} = \frac{\mathcal{E}_i}{L} - \mathcal{J}\frac{R_0}{L}; \quad (3)$$

Решим ДУ(3) методом Лагранжа:

решим однородное ДУ: $\frac{d\mathcal{J}}{dt} = -\mathcal{J}\frac{R_0}{L} \Rightarrow \frac{d\mathcal{J}}{\mathcal{J}} = -\frac{R_0}{L}dt \Rightarrow \ln\mathcal{J} = -\frac{R_0}{L}t + const$

$const = const(t)$: $\mathcal{J} = \exp\left(-\frac{R_0}{L}t\right)\cdot\underbrace{\exp(const(t))}_{A}$; A

$$\mathcal{J}' = -\frac{R_0}{L}\cdot\exp\left(-\frac{R_0}{L}t\right)\cdot e^A + \exp\left(-\frac{R_0}{L}t\right)\cdot A'\cdot e^A;$$

$$\exp\left(-\frac{R_0}{L}t + A\right)\cdot\left(A' - \frac{R_0}{L}\right) = \frac{\mathcal{E}_i}{L} - \exp\left(-\frac{R_0}{L}t + A\right)\cdot\frac{R_0}{L};$$

$$\exp\left(-\frac{R_0}{L}\cdot t\right)\cdot\exp(A)\cdot A' = \frac{\mathcal{E}_i}{L}$$

$$d\,const \cdot e^{A} = \frac{1}{l} \cdot \exp\left(\frac{R_0}{L} \cdot t\right) \cdot \frac{3acl R_0}{2L} \left(-\frac{3R_0}{2L} t\right) dt =$$

$$= \frac{3acl R_0}{2L^2} \exp\left(-\frac{R_0}{2L} t\right) dt$$

$$e^{A} = \frac{3acl R_0}{2L^2} \left(-\frac{2L}{R_0}\right)\left(-\frac{R_0}{2L} t\right) + \widetilde{const}$$

$$\mathcal{I}(t) = \exp\left(-\frac{R_0}{L} t\right)\left(\frac{3acl R_0}{2L^2} \exp\left(-\frac{R_0}{2L} t\right) + \widetilde{const}\right)$$

$$\mathcal{I}(0) = 0 = 1\left(-\frac{3acl}{L} \cdot 1 + \widetilde{const}\right) \Rightarrow \widetilde{const} = \frac{3acl}{L}$$

$$\mathcal{I}(t) = \frac{3acl}{L} \exp\left(-\frac{R_0}{L} t\right)\left(1 - \exp\left(-\frac{R_0}{2L} t\right)\right); (4)$$

2) $m\vec{\ddot{y}}(t) = \underbrace{m\vec{g}} + \mathcal{I} l \vec{B}(t) + \vec{F_y}(t); (5)$

$$\ddot{y}(t) = \frac{a R_0^2}{4L^2} \cdot \exp\left(-\frac{R_0}{2L} t\right);$$

$$m\ddot{y}(t) = \underbrace{mg} + \mathcal{I} l B(t) + F_y(t); (5.1)$$

$$F_y(t) = m\ddot{y}(t) - \underbrace{mg} - \mathcal{I} l B(t);$$

$$F_y(t) = \frac{ma R_0^2}{4L^2} \exp\left(-\frac{R_0}{2L} t\right) - \underbrace{mg} - \frac{3ac^2 l^2}{L} \exp^2\left(-\frac{R_0}{L} t\right)\left(1 - \exp\left(-\frac{R_0}{2L} t\right)\right) =$$

$$= \frac{ma R_0^2}{4L^2} \exp\left(-\frac{R_0}{2L} t\right) - \underbrace{mg} - \frac{3ac^2 l^2}{L} \exp\left(-\frac{2R_0}{L} t\right)\left(1 - \exp\left(-\frac{R_0}{2L} t\right)\right); (6)$$

3) $E = j\rho_{уд} = \frac{\mathcal{I}(t) \cdot \rho_{уд}}{S}; (7)$

4) $\vec{j} = |e| \cdot n_0 \langle \vec{v} \rangle;$

$$\langle v \rangle = \frac{j}{|e| n_0} = \frac{\mathcal{I}(t)}{|e| \cdot n_0 \cdot S}; \quad \dot{y}(t) = -\frac{a R_0}{2L} \exp\left(-\frac{R_0}{2L} t\right)$$

$$F_L(t) = |e| [\vec{v} \times \vec{B}] = \left| \begin{matrix} v = \langle \vec{v} \rangle + \dot{y}(t) \\ \text{- полная скор.} \\ \text{электронов} \end{matrix} \right| = |e| [(\langle \vec{v} \rangle + \dot{y}(t)) \times \vec{B}] =$$

$$= |e| \cdot [\langle \vec{v} \rangle \times \vec{B}] + |e| [\dot{y}(t) \times \vec{B}]; \text{ причем } [\langle \vec{v} \rangle \times \vec{B}] \perp [\dot{y}(t) \times \vec{B}]$$

$$F_L(t) = |e| \cdot \sqrt{([\langle \vec{v} \rangle \times \vec{B}])^2 + ([\dot{y}(t) \times \vec{B}])^2} \quad (8)$$

$$F_L(t)_\Sigma = S h_0 |e| \sqrt{([\langle \vec{v} \rangle \times \vec{B}])^2 + ([\dot{y}(t) \times \vec{B}])^2} . \quad (9)$$

Задача 4

Дано:

$\varepsilon = 1$

$\mu = 1$

$\vec{S} = S_0 \cos^2(\omega t - ky)$

$k;\ S_0;$

$E = ?,\ \vec{H} = ?;\ \langle S \rangle;$

$\langle \vec{S} \rangle = ?,\ \langle |j_{em}| \rangle = ?,$

$w = ?\ \ k_{ed} = ?$

Решение:

Т.к. волна плоская, то $\vec{E} \perp \vec{H}$;

$$|\vec{S}| = |\vec{E} \times \vec{H}| = E \cdot H \cdot \sin(\vec{E} \wedge \vec{H}) = E \cdot H = $$

$$= E \cdot \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot E = E^2 \sqrt{\frac{\varepsilon_0}{\mu_0}}$$

$$S = S_0 \cos^2(\omega t - ky) = E^2 \sqrt{\frac{\varepsilon_0}{\mu_0}} = H^2 \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$E = \sqrt[4]{\frac{\mu_0}{\varepsilon_0}} \sqrt{S_0} \cos(\omega t - ky);$$

$$H = \sqrt[4]{\frac{\varepsilon_0}{\mu_0}} \sqrt{S_0} \cos(\omega t - ky)$$

$$E_0 = \sqrt[4]{\frac{\mu_0}{\varepsilon_0}} \cdot \sqrt{S_0};\quad H_0 = \sqrt[4]{\frac{\varepsilon_0}{\mu_0}} \sqrt{S_0}$$

$$w = \frac{\vec{E} \times \vec{H}}{c^2} \cdot c = \frac{\vec{S}}{c^2} \cdot c = \frac{\vec{S}}{c} = \frac{S_0}{c} \cos^2(\omega t - ky),$$ c – скор света в вакуме

$$\langle \vec{S} \rangle = \langle S_0 \cos^2(\omega t - ky) \rangle \vec{l}_y = S_0 \langle \cos^2(\omega t - ky) \rangle \vec{l}_y =$$

$$= S_0 \cdot \frac{1}{2} \vec{l}_y = \frac{S_0}{2} \vec{l}_y = \frac{S_0}{2}$$

$$\langle |\vec{j}_{em}| \rangle = \left\langle \left| \frac{\partial \vec{D}}{\partial t} \right| \right\rangle = \left\langle \varepsilon \varepsilon_0 \frac{\partial E}{\partial t} \right\rangle \Big|_{\varepsilon = 1} = \varepsilon_0 \left\langle \frac{\partial E}{\partial t} \right\rangle = \varepsilon_0 E_0 \left\langle \frac{\partial}{\partial t} (\cos(\omega t - k\right.$$

$$\langle \cos(\omega t - kx) \rangle = \frac{1}{T} \int_0^T \cos(\omega t - kx) dt = \frac{1}{T} [\sin(\omega T - kx) - \sin(-kx)] = \frac{1}{T} \cdot 0 = 0$$

$$|k_{ed}| = \frac{|\vec{S}|}{c^2} = \frac{S_0 \cos^2(\omega t - ky)}{c^2}$$