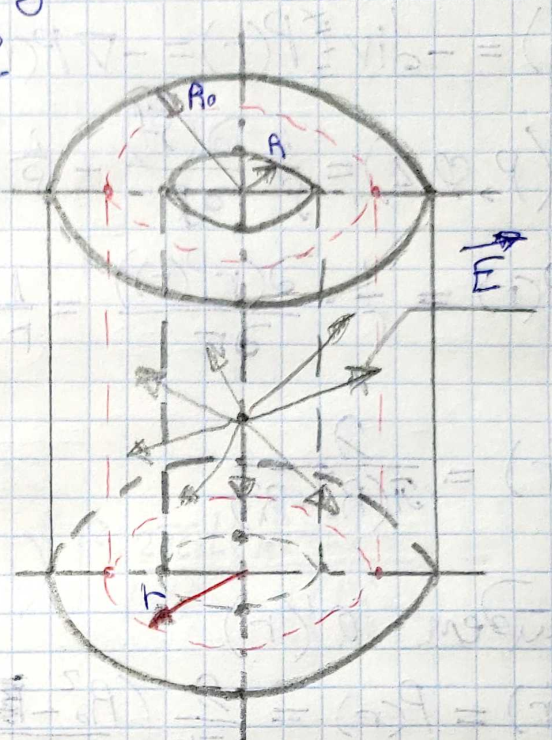
Филимонов Степан РЛ6-31

Вариант 12

Домашнее задание 1

Задача 1.2



Дано:

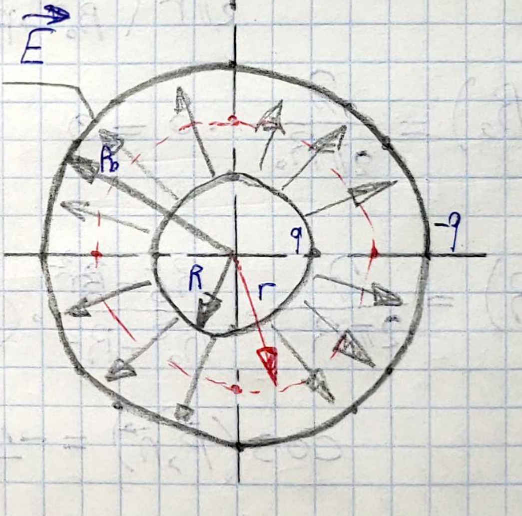
$\varepsilon(r)=\frac{R_0^2+R}{R^2+r^2}$, $\frac{R_0}{R}=\frac{3}{2}$

Найти:

$\sigma'(R)$, $\sigma'(R_0)$, $\rho'(r)$, E_{max}, $\frac{C}{l}$

Решение:

Поверхность — ~~соосный~~ соосный цилиндр радиуса ~~[...]~~ $R_0 .. R$. $r\in[R_0;R]$ В каждой точке $\vec{E}\perp dS$ в силу цилиндрической симметрии, тогда $dS=d(2\pi rh)=2\pi h dS$, где h - высота цилиндра ($h\to\infty$).



1) Определим C/l:

$$\oint_S \vec{D}d\vec{S}=q=\lambda l$$

$$2\pi r l D=q=\lambda l \Rightarrow D(r)=\frac{q}{2\pi r l}=\frac{\lambda}{2\pi r}$$

$$D=\varepsilon\varepsilon_0 E \Rightarrow E(r)=\frac{q}{2\pi\varepsilon(r)\varepsilon_0 r l}=\frac{\lambda}{2\pi\varepsilon(r)\varepsilon_0 r}$$

$$U=\int_R^{R_0}E(r)dr=\int_R^{R_0}\frac{\lambda dr}{2\pi\varepsilon(r)\varepsilon_0 r}=\frac{\lambda}{2\pi\varepsilon_0}\int_R^{R_0}\frac{\frac{R^2}{r}+r}{R_0^2+R^2}dr=\frac{\lambda}{2\pi\varepsilon_0(R_0^2+R^2)}\left[R^2\ln\frac{R_0}{R}+\frac{1}{2}(R_0^2-R^2)\right]$$

$$U=\frac{\lambda}{2\pi\varepsilon_0\cdot\left(\frac{R_0^2}{R^2}+1\right)}\cdot\left[\ln\frac{R_0}{R}+\frac{1}{2}\left(\frac{R_0^2}{R^2}-1\right)\right]$$

$$\frac{C}{l}=\frac{q}{Ul}=\frac{2\pi\varepsilon_0\left(\frac{R_0^2}{R^2}+1\right)}{\ln\frac{R_0}{R}+\frac{1}{2}\left(\frac{R_0^2}{R^2}-1\right)}\approx 1{,}75\cdot 10^{-10}\left[\frac{Ф}{м}\right]$$

2) Определим $\rho'(r)$:

$$P(r) = \varkappa(r)\varepsilon_0 E(r) = \frac{(\varepsilon(r)-1)\varepsilon_0 q}{2\pi\varepsilon(r)\varepsilon_0 r L} = \frac{\lambda}{2\pi r}\left(1-\frac{1}{\varepsilon(r)}\right) = \frac{\lambda}{2\pi r}\left(\frac{R_0^2-r^2}{R_0^2+R^2}\right)$$

$$\rho'(r) = -\mathrm{div}\,P(r) = -\nabla P(r)$$

$$\nabla f(\rho,\varphi,z) = \frac{1}{\rho}\frac{\partial(\rho f)}{\partial \rho} + \frac{1}{\rho}\frac{\partial f}{\partial \varphi} + \frac{\partial f}{\partial z}\ ;\ \frac{\partial f}{\partial \varphi} \equiv 0,\ \frac{\partial f}{\partial z} \equiv 0$$

$$\nabla P(r) = \frac{1}{r}\cdot\frac{\partial(rP(r))}{\partial r} = \frac{1}{r}\cdot\frac{\partial}{\partial r}\left(\frac{\lambda}{2\pi}\left(\frac{R_0^2-r^2}{R_0^2+R^2}\right)\right) = \frac{1}{r}\cdot\frac{\lambda(-2r)}{2\pi(R_0^2+R^2)} =$$

$$= -\frac{\lambda}{\pi(R_0^2+R^2)}$$

$$\rho'(r) = \frac{\lambda}{\pi(R_0^2+R^2)}$$

3) Найдем $\sigma'(r)$

$$\sigma'(r) = P(r) = \frac{\lambda}{2\pi r}\left(\frac{R_0^2-r^2}{R_0^2+R^2}\right)\cos(\widehat{\vec{P},\vec{n}})$$

$$\sigma'(R_0) = \frac{\lambda}{2\pi R_0}\cdot 0 = 0 \Rightarrow \cos(\widehat{\vec{P},\vec{n}}) = 1$$

$$\sigma'(R) = \frac{\lambda}{2\pi R}\cdot\frac{R_0^2-r^2}{R_0^2+R^2}\cdot(-1) = -\frac{\lambda}{2\pi R}\cdot\frac{\frac{R_0^2}{R^2}-1}{\frac{R_0^2}{R^2}+1} \approx 0{,}06\frac{\lambda}{R}$$

$$\cos(\widehat{\vec{P},\vec{n}}) = -1$$

4) Найдем E_{max}

Введем $f(r) = \varepsilon(r)\cdot r$, $E(r) = \frac{const}{f(r)}$

$f_{min} \Rightarrow E_{max}$

$$f'(r) = const\,\frac{R^2+r^2-2r^2}{(R^2+r^2)^2} = 0 \Rightarrow r = R,\ r>0$$

$$f(R) = \frac{R_0^2+R^2}{R_0^2-R^2}\cdot R = 1{,}625R$$

5) Проведём проверку

а) $q' = \int_V \rho'(r)dV + \sigma'(R)\cdot S(R) = 0$

$$\int_V \rho'(r)dV = \int_R^{R_0} \frac{\lambda}{\pi(R_0^2+R^2)}\, d(\pi r^2 l) = \frac{q}{R_0^2+R^2}(R_0^2-R^2)$$

$$\sigma'(R)\cdot S(R) = \left(-\frac{\lambda}{2\pi R}\cdot\frac{R_0^2-R^2}{R_0^2+R^2}\right)\cdot(2\pi R l) = -\frac{q}{R_0^2+R^2}(R_0^2-R^2)$$

$$\int_V \rho'(r)dV = -\sigma'(R)\cdot S(R)$$

б) $\frac{CU^2}{2} = \int_V w\,dV = \int_V \frac{\vec E\vec D}{2}dV$

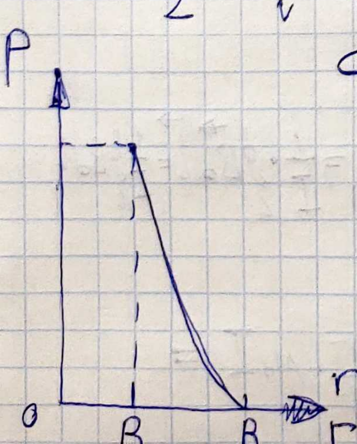
$$\frac{CU^2}{2} = \frac{\lambda^2}{8\pi^2\varepsilon_0^2\left(\frac{R_0^2}{R^2}+1\right)^2}\left[\ln\frac{R_0}{R}+\frac12\left(\frac{R_0^2}{R^2}-1\right)\right]^2\cdot\frac{2\pi\varepsilon_0 l\left(\frac{R_0^2}{R^2}+1\right)}{\ln\left(\frac{R_0}{R}\right)+\frac12\left(\frac{R_0^2}{R^2}-1\right)} =$$

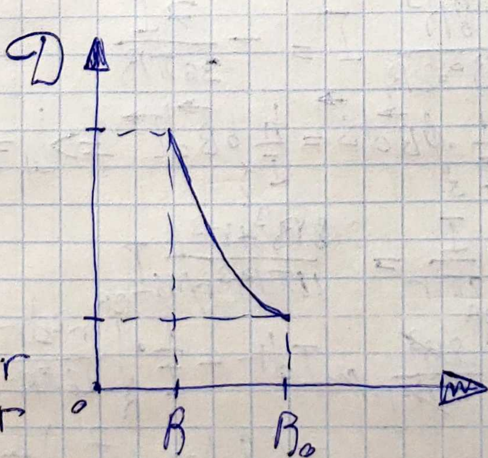
$$= \frac{q^2\left(\ln R_0/R+\frac12\left(\frac{R_0^2}{R^2}-1\right)\right)}{4\pi\varepsilon_0 l\left(\frac{R_0^2}{R^2}+1\right)}$$

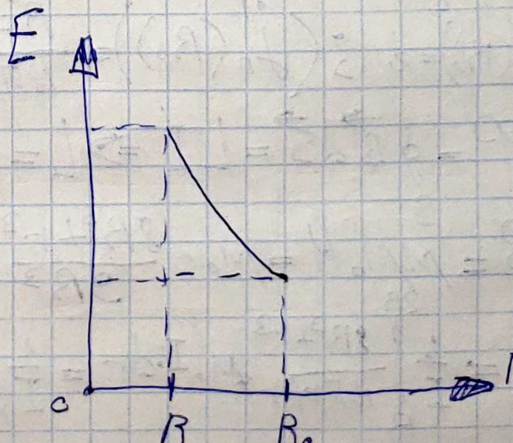
$$\int_V\frac{\vec E\vec D}{2}dV = \int_R^{R_0}\frac{\frac{\lambda^2}{4\pi^2\varepsilon_0\varepsilon(r)r^2}\cdot d(\pi r^2 l)}{2} = \int_R^{R_0}\frac{\frac{\lambda^2 l}{4\pi\varepsilon_0\cdot\varepsilon(r)r^2}dr^2}{2} = \frac{\frac{\lambda^2 l}{4\pi\varepsilon_0}\cdot\frac{1}{R_0^2+R^2}}{2}\int_R^{R_0}\left(\frac{R^2}{r^2}+1\right)dr^2$$

$$= \frac{q^2}{4\pi\varepsilon_0 l}\cdot\frac{1}{R_0^2+R^2}\left(\frac{R^2\ln\frac{R_0^2}{R^2}+(R_0^2-R^2)}{2}\right) = \frac{q^2\left(\ln\frac{R_0}{R}+\frac12(R_0^2/R^2-1)\right)}{4\pi\varepsilon_0 l\left(\frac{R_0^2}{R^2}+1\right)}$$

$$\frac{CU^2}{2} = \int_V\frac{\vec E\vec D}{2}dV$$



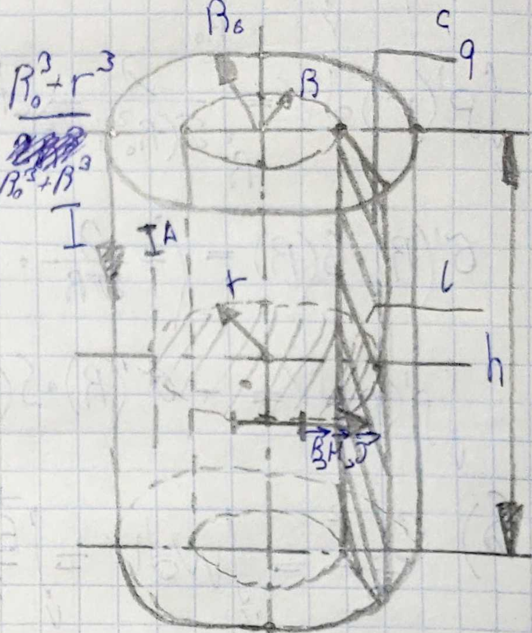




Задача 2.2

Дано:

коаксиальный кабель $\frac{R_0}{R} = \frac{2}{1}$; I; $\mu(r) = \frac{R_0^3 + r^3}{R_0^3 + R^3}$



Найти:

$j'_n(r) = ?$; $j'_{об}(r) = ?$; $L_h = ?$

Решение: $\mu(r) = \frac{8R^3 + r^3}{9R^3}$

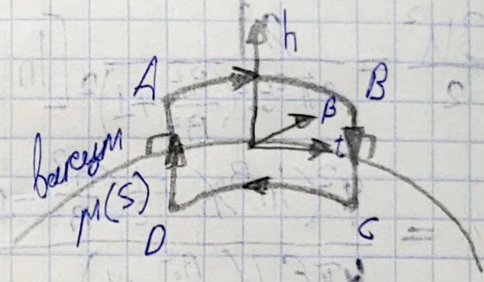
1) $\oint\limits_l \vec{H} d\vec{l} = H \cdot 2\pi r = I \Leftrightarrow H(r) = \frac{I}{2\pi r}$

$J(r) = (\mu(r) - 1) H(r) = \frac{r^3 - R^3}{18\pi R^3 r} I$

$\oint\limits_l \vec{J}(r) d\vec{l} = I' = d(J(r) 2\pi r) = dI'$

$dI' = 2\pi d(rJ) = j'_{об} \cdot 2\pi r dr \Rightarrow$

$\Rightarrow j'_{об} = \frac{1}{r} \cdot \frac{d(rJ)}{dr} = \frac{I}{6\pi R^3} \cdot r$



$j'_{об}(R) = \frac{I}{6\pi R^3}$; $j'_{об}(R_0) = \frac{I}{3\pi R^2}$

2) $\oint\limits_{ABCD} \vec{J} d\vec{l} = I' = \underbrace{\int\limits_{AB} \vec{J} d\vec{l}}_{0} + \underbrace{\int\limits_{BC} \vec{J} d\vec{l}}_{0} + \int\limits_{CD} \vec{J} d\vec{l} + \underbrace{\int\limits_{DA} \vec{J} d\vec{l}}_{0} \Rightarrow \oint\limits_{ABCD} \vec{J} d\vec{l} = \int\limits_{CD} \vec{J} d\vec{l} =$

$= \int\limits_{CD} J dl = \vec{\tau}(\underbrace{J_{2t}}_{0} - J_{1t}) l$; $dI'_n = (\vec{j}'_n, \vec{v}) dl = j_n \to dl$

$I'_n = \oint\limits_l i'_n dl \Rightarrow \underbrace{J_{2t}}_{0} - J_{1t} = (\vec{j}'_n)_v = \frac{R^3 - r^3}{18\pi R^3 r} I$

$(j'_n(R)) = 0$; $(j'_n(R_0)) = \frac{R^3 - 8R^3}{18\pi R^3 \cdot 2R} I = -\frac{7I}{36\pi R}$

3) $\Phi = \oint\limits_S \vec{B} d\vec{S} = LI \Rightarrow L = \frac{1}{I} \cdot \oint\limits_S \vec{B} d\vec{S} = \frac{h}{I} \oint\limits_l \vec{B} d\vec{l} \Rightarrow L_h = \frac{1}{I} \cdot \oint\limits_l \vec{B} d\vec{l} = \frac{1}{I} \oint\limits_l B dl$

$B = \mu \mu_0 H = \mu_0 \cdot \frac{8R^3 + r^3}{9R^3} \cdot \frac{I}{2\pi r} = \frac{8R^3 + r^3}{18\pi R^3 r} \mu_0 I$

$L_h = \frac{1}{I} \int\limits_R^{2R} \frac{8R^3 + r^3}{18\pi R^3 r} \mu_0 I dr = \frac{1}{I} \int\limits_R^{2R} \frac{8R^3}{18\pi R^3 r} \mu_0 I dr + \frac{1}{I} \cdot \int\limits_R^{2R} \frac{r^2 dr}{18\pi R^3} \cdot \mu_0 I =$

$= \frac{\frac{8}{9} \mu_0 I \cdot \ln 2 \frac{1}{I}}{2} + \frac{\frac{\mu_0 I \cdot 7}{27 I}}{2\pi} = \frac{\mu_0 I \left(\frac{8}{9} \ln 2 + \frac{7}{27}\right) \frac{1}{I}}{2\pi}$