Задача 6

Дано:

$u(t) = u_0 e^{-\alpha t}$

$\|S\| - ?$

E

Решение:

$$E = \|S\|^2 = \int_{-\infty}^{+\infty} S^2(t)\,dt \Rightarrow E = u_0^2 \int_{-\infty}^{+\infty} \frac{1}{e^{2\alpha|t|}}\,dt = \frac{u_0^2}{\alpha}$$

$$I = \int_{-\infty}^{0} \frac{1}{e^{-2\alpha t}}\,dt + \int_{0}^{\infty} \frac{1}{e^{2\alpha t}}\,dt = \frac{e^{2\alpha t}}{2\alpha}\Big|_{-\infty}^{0} = \frac{1}{2\alpha e^{-2t\alpha}}\Big|_{0}^{+\infty} =$$

$$= \frac{1}{2\alpha} + \frac{1}{2\alpha} = \frac{1}{\alpha}$$

Ответ: $E = u_0^2/\alpha$

$\|S\| = u_0 \cdot \sqrt{1/\alpha}$

Задача 7