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ТЕТРАДЬ

для ....

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учени .... класса ....

.... школы ....

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Д/З 1 : 6/10

12

клетка

D32

№1 $\int\limits_{(-1;-3)}^{(3;1)} \frac{dx-dy}{(x-y-1)^2} = \int\limits_{(-1;-3)}^{(3;1)} \frac{dx}{(x-3-1)^2} - \frac{dy}{(x-y-1)^2}$

$\int\limits_{-1}^{3} \frac{(1-1)}{(x-(x-2)-1)^2} dx = 0$

Ответ: 0

№3 $\overline{F}(x;y;z) = \{ \overset{P}{x^2}; \overset{Q}{x^2 y}; \overset{R}{(x-2)z} \}$

а) векторные линии

ур-е векторных линий

$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

$\frac{dx}{x^2} = \frac{dy}{x^2 y} = \frac{dz}{xz-2z}$

$\frac{dx}{x^2} = \frac{dy}{x^2 y} \Big| \cdot x^2$

$dx = \frac{dy}{y}$

$\int dx = \int \frac{dy}{y}$

$x = \ln|y| + \ln|C_1|$

$\underline{x = \ln|yC_1|}$

$\frac{dx}{x^2} = \frac{dz}{(x-2)z} \Big| \cdot x-2$

$\int \frac{(x-2)dx}{x^2} = \int \frac{dz}{z}$

$\ln|x| + \frac{z}{x} = \ln|z| + \ln|C_2|$

$\ln|x| + \frac{z}{x} = \ln|z \cdot C_2|$

Ответ: $\begin{cases} x = \ln|y \cdot C_1| \\ \ln|x| + \frac{z}{x} = \ln|z \cdot C_2| \end{cases}$

в) $\operatorname{div} F$ - ? $\operatorname{div} F|_{M_0}$ $M_0(\frac{1}{2}; 0; 3)$

$\operatorname{div} F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$

$\frac{\partial P}{\partial x} = 2x \qquad \frac{\partial Q}{\partial y} = x^2 \qquad \frac{\partial R}{\partial z} = (x-2)$

$\operatorname{div} \vec{F} = 2x + x^2 + x - 2 = x^2 + 3x - 2$

$\operatorname{div} \vec{F}|_{M_0} = \frac{1}{4} + 3 \cdot \frac{1}{2} - 2 = -\frac{1}{4}$

g) $\operatorname{rot} F$ - ? $F\{x^2; x^2 y; (x-2)z\}$

$\operatorname{rot} F = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \vec{i}\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) - \vec{j}\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) +$

$+ \vec{k}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) = \vec{i}(0-0) - \vec{j}(z-0) + \vec{k}(2xy - 0) =$

$= -\vec{j} z + \vec{k} 2xy$

Ответ: $\operatorname{rot} \vec{F}\{0; -z; 2xy\}$

№4 $f(z) = \frac{z}{(z^2-1)^3}$ по степеням $z+1$

$\cancel{\frac{z+1-1}{(z+1)(z+1)-2}}$

$z^2 + 1 + 2z - 2z - z$

$2z^2 + \cancel{z} + 4z - 2z - \cancel{z}$

$f(-1) = 1 - 1 = 0$

$f'(-1) = 2z = -2$

$f''(-1) = 2$

$f(z) = 0(z+1) + (z+1)^2 - 2 + \frac{2}{2}(z+1)^2$

$\frac{z+1-1}{((z+1)^2 - 2(z+1))^3}$

$\frac{z+1}{((z+1)^2 - 2(z+1))^3} - \frac{1}{((z+1)^2 - 2(z+1))^3}$

$(x+1)^{-3} = 1 + \frac{-3}{1!}x + \ldots + \frac{-3(-3-1)(-3-n+1)}{n!}x^n$

$\frac{-3(-4)}{2!} \quad \frac{-3(-4)(-5)}{3!} \quad \sum_{n=0}^{\infty}(-1)^n \frac{x^n}{2n!}(n+2)!$

$(x+1)^{-3} = \sum_{n=0}^{\infty}(-1)^n \frac{x^n (n+2)!}{2n!} \quad |x| < 1$

$\frac{1}{((z+1)^2 - 2(z+1))^3} = \frac{1}{\left(-2(z+1)\left(1 + \frac{(z+1)}{2}\right)\right)^3} =$

$= \frac{1}{\left(-2(z+1)\left(\sum_{n=0}^{\infty}\left(\cancel{\ldots}\frac{(n+2)!}{2n!}\left(\frac{z+1}{-2}\right)^n\right)\right)\right)^3} = \frac{1}{\left(-2(z+1)\sum_{n=0}^{\infty}\frac{(n+2)!(z+1)^n}{2n! \cdot 2^n}\right)^3}$

$$= \frac{1}{-\sum_{n=0}^{\infty} \frac{(n+1)^3(n+2)^3(z+1)^{3n+3}}{2^{3n}}} \qquad \left|\frac{z+1}{-2}\right| < 1 \qquad |z+1| < 2$$

$$\frac{1}{((z+1)^2 \cdot 2(z+1))^3} = \frac{1}{\left((z+1)^2\left(1-\frac{2}{(z+1)}\right)\right)^3} =$$

$$= \frac{1}{\left((z+1)^2 \sum_{n=0}^{\infty} (-1)^n \frac{(n+1)(n+2)}{2}\left(\frac{(-2)^n}{(z+1)^n}\right)\right)^3} =$$

$$= \frac{1}{\left((z+1)^2 \sum_{n=0}^{\infty} (-1)^n \frac{(n+1)(n+2)}{2}\left(\frac{(-2)^n}{(z+1)^n}\right)\right)^3} =$$

$$= \frac{1}{\left(\sum_{n=0}^{\infty} \frac{(n+1)(n+2)2^{n-1}}{(z+1)^{n-2}}\right)^3} = \frac{1}{\sum_{n=0}^{\infty} \frac{(n+1)^3(n+2)^3 8^{n-1}}{(z+1)^{3n-6}}}$$

$$\left|\frac{-2}{z+1}\right| < 1 ;\ 2 < |z+1|$$

$|z+1| < 2$

$$f(z) = \frac{1}{-\sum_{n=0}^{\infty} \frac{(n+1)^3(n+2)^3(z+1)^{3n+2}}{8^n}} + \frac{1}{\sum_{n=0}^{\infty} \frac{(n+1)^3(n+2)^3(z+1)^{3n+3}}{8^n}}$$

$|z+1| > 2$

$$f(z) = \frac{1}{\sum_{n=0}^{\infty} \frac{(n+1)^3(n+2)^3 8^{n-1}}{(z+1)^{3n-6}}} - \frac{1}{\sum_{n=0}^{\infty} \frac{(n+1)^3(n+2)^3 8^{n-1}}{(z+1)^{3n-6}}}$$

№5 $f(z) = \frac{1}{(1-z)^2} \sin\frac{1}{z}$

$z = 1 \qquad z = 0$

$$\lim_{z \to 1} \frac{\sin\left(\frac{1}{z}\right)}{(1-z)^2} = \lim_{z \to 1} \frac{\sin 1}{0} = \infty \text{ – полюс}$$

$$\frac{\sin\frac{1}{z}}{(1-z)^2} \xrightarrow[\substack{z \to 1 \\ z - 1 = t \\ t \to 0}]{} \frac{\sin\left(\frac{1}{t+1}\right)}{t^2} \sim \frac{1}{t^2}$$

полюс 2 пор. нл

$$\lim_{z \to 0} \frac{\sin\frac{1}{z}}{(1-z)^2} = \lim_{z \to 0} \sin\frac{1}{z} \longrightarrow \lim_{t \to \infty} \sin t = \cancel{2}$$

$t \to \infty \quad t = \frac{1}{z} \quad z \to 0$

предела не существует, т.к. sin – огр. функция

⇓

существует особая точка

$$\operatorname{res} f(1) = \lim_{z \to 1} \frac{d}{dz}\left(\frac{\sin\frac{1}{z}}{(1-z)^2}(z-1)^2\right) =$$

$$= \lim_{z \to 1} \frac{-\cos\left(\frac{1}{z}\right)}{z^2} = \frac{\cos 1}{1} = -\cos(1)$$

$$\sin\frac{1}{z} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{z^{2n+1}(2n+1)!}$$

$$\frac{1}{(1-z)^2} = \sum_{n=0}^{\infty} (-1)^n (n+1)! (-z)^n$$

$$f(z) = \sum_{n=0}^{\infty} \frac{(-1)^n (n+1)! (-1)^n}{(2n+1)!\ z^{n+1}}$$

$n=0: \quad \dfrac{1\cdot 1^{1}}{1!\, z^{+1}} = \dfrac{1}{z}$

$\Downarrow$

$\operatorname{res} f(0) = 1$

3) $z_3 = \infty$

$$\lim_{z\to\infty} \frac{\sin\frac{1}{z}}{(1-z)^2} = \lim_{z\to\infty} \frac{1}{z(z-1)^2} = \lim \frac{1}{z\cdot z^2} =$$

$\lim_{z\to\infty} \frac{1}{z^3} = 0$ нуль кратности 3

устранимая особ. точка $\operatorname{res}_{z\to\infty} f(z) = 0$

№6 $\oint \dfrac{(z^2+1)}{z^2(z+2)^2}$ $\quad |z|=1$, $|z|<1$

1) $z=0$ полюс 2-ого пор. $\quad$ 2) $z=-2$ не подх. усл.

$$\operatorname{res}_{z\to 0} f(z) = \lim_{z\to 0} \frac{d}{dz}\left(\frac{z^2+1}{z^2} \cdot \frac{z^2}{(z+2)^2}\right) =$$

$$= \lim_{z\to 0} \frac{4z-2}{(z+2)^3} = \frac{0-2}{2^3} = -\frac{1}{2^2} = -\frac{1}{4}$$

$$\oint_C f(z) = 2\pi i \cdot \left(-\frac{1}{4}\right) = -\frac{\pi i}{2}$$