Филимонов Степан РЛ6-31

Домашнее задание № 1

Вариант № 12

№1

$$4y^3U_{xx} - yU_{yy} + 2y^3U_x + (1+y^2)U_y = 0$$

$a_{11} = 4y^3 \quad a_{12} = 0 \quad a_{22} = -y$

$\Delta = a_{12}^2 - a_{11}\cdot a_{22} = 4y^4 \Rightarrow \begin{cases} 1)\ y \neq 0 & \text{Гиперболический тип } x\in(-\infty;+\infty) \\ 2)\ y = 0 & \text{Параболический тип } x\in(-\infty;+\infty \end{cases}$

1) $y \neq 0$

$$\frac{dy}{dx} = \frac{a_{12}+\sqrt{a_{12}^2 - a_{11}\cdot a_{22}}}{a_{11}} \quad \Big| \quad \frac{dy}{dx} = \frac{a_{12}-\sqrt{a_{12}^2 - a_{11}\cdot a_{22}}}{a_{11}}$$

$$\frac{dy}{dx} = \frac{1}{2y} \quad \Big| \quad \frac{dy}{dx} = -\frac{1}{2y}$$

$$2ydy = dx \quad \Big| \quad 2ydy = -dx$$

$$y^2 = x + C_1 \quad \Big| \quad y^2 = -x + C_2$$

$$C_1 = y^2 - x \quad \Big| \quad C_2 = y^2 + x$$

Введем ξ и η:

$$\xi_x = -1 \quad \xi_y = 2y \quad \Big| \quad \eta_x = 1 \quad \eta_y = 2y$$

$$U_x = U_\xi\cdot\frac{\partial\xi}{\partial x} + U_\eta\cdot\frac{\partial\eta}{\partial x} = U_\eta - U_\xi$$

$$U_y = U_\xi\cdot\frac{\partial\xi}{\partial y} + U_\eta\cdot\frac{\partial\eta}{\partial y} = 2y[U_\xi + U_\eta]$$

$$U_{xx} = \frac{\partial}{\partial x}(U_\eta - U_\xi) = U_{\eta\eta} - 2U_{\eta\xi} + U_{\xi\xi}$$

$$U_{yy} = \frac{\partial}{\partial y}(2y[U_\xi + U_\eta]) = 4y^2[U_{\xi\xi} + 2U_{\eta\xi} + U_{\eta\eta}]$$

Подставим в ур-е:

$$\cancel{4y^3U_{\xi\xi}} - 8y^3U_{\eta\xi} + \cancel{4y^3U_{\eta\eta}} - \cancel{4y^3U_{\xi\xi}} - 8y^3U_{\eta\xi} - \cancel{4y^3U_{\eta\eta}} - \cancel{2y^3U_\xi} +$$

$$+ 2y^3U_\eta + 2yU_\xi + 2yU_\eta + \cancel{2y^3U_\xi} + 2y^3U_\eta = 0$$

$$-16y^3U_{\xi\eta} + 2yU_\xi + (4y^3 + 2y)U_\eta = 0 \quad |:2y$$

$$8y^2U_{\xi\eta} = U_\xi + (2y^2+1)U_\eta \qquad \begin{cases} \xi = y^2 - x \\ \eta = y^2 + x \end{cases}$$

$$U_{\xi\eta} = \frac{U_\xi}{8y^2} + \left(\frac{1}{4} + \frac{1}{8y^2}\right)U_\eta \qquad 2y^2 = \xi + \eta$$

Канонический вид:

$$U_{\xi\eta} = \frac{U_\xi}{4(\xi+\eta)} + \frac{1}{4}\left(1 + \frac{1}{\xi+\eta}\right)U_\eta$$

2) $y = 0$

$U_y = 0$

$U = 0$

$U = C_3$

№2

Дано: $L = -\frac{d^2}{dx^2} + 4I \qquad U(0) = U'\left(\frac{3\pi}{2}\right) = 0$

Решение:

$Ly = \lambda y$

$-\frac{d^2y}{dx^2} + 4y = \lambda y$

$y'' - 4y = -\lambda y$

$y'' + (\lambda - 4)y = 0$

$$\begin{cases} y'' + \cancel{\lambda}\overset{(\lambda-4)}{}y = 0 \\ y(0) + y'\left(\frac{3\pi}{2}\right) = 0 \end{cases}$$

Исследование:

1) $\overset{-4}{\cancel{\lambda}} = 0$

$$\begin{cases} y'' = 0 \\ y(0) = y'\left(\frac{3\pi}{2}\right) = 0 \end{cases} \qquad y = C_1x + C_2 \qquad \begin{cases} 0 = C_1 \\ 0 = C_2\cdot\frac{3\pi}{2} \end{cases} \Rightarrow \begin{cases} C_1 = 0 \\ C_2 = 0 \end{cases} \Rightarrow \begin{array}{l} y = 0 - \text{не. собст. фун} \\ \lambda = 0 - \text{не собст. знач.} \end{array}$$

② $\lambda - 4 = -\omega^2 < 0$

$$y = C_1 e^{\omega x} + C_2 e^{-\omega x}$$

$$\begin{cases} y'' - \omega^2 y = 0 \\ y(0) = y'\left(\frac{3\pi}{2}\right) = 0 \end{cases}$$

$$\begin{cases} 0 = C_1 - C_2 \\ 0 = \omega C_1 e^{\omega \frac{3\pi}{2}} - C_2 e^{-\omega\frac{3\pi}{2}} \end{cases} \quad y' = C_1 e^{\omega x} - C_2 e^{-\omega x}$$

$$\begin{pmatrix} 1 & -1 \\ e^{\omega\frac{3\pi}{2}} & -e^{-\omega\frac{3\pi}{2}} \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$ где диспер ур-ие?

$y = 0$ — не собст. функ.

$\lambda = \omega^2 + 4$ — не собст. знач.

③ $\lambda - 4 = \omega^2 > 0$

$$\begin{cases} y'' + \omega^2 y = 0 \\ y(0) = y'\left(\frac{3\pi}{2}\right) = 0 \end{cases}$$

$$y = C_1 \sin\omega x + C_2 \cos\omega x$$

$$C_2 = 0 \Leftarrow y(0)$$

$$y = C_1 \sin\omega x \Rightarrow y' = C_1 \omega \cdot \cos\omega x \Rightarrow$$

$$\Rightarrow \omega\frac{3\pi}{2} = \pi k - \frac{\pi}{2},\ k = \overline{1,\infty}$$

$\lambda = \left(\frac{2n-1}{3}\right)^2 + 4$, $n = \overline{1,\infty}$ — собственное значение

$y = \sin\left(\left(\frac{2n-1}{3} + 4\right)x\right)$ ← не верно!, $n = \overline{1,\infty}$ — собств. функций

$$\|y_k\|^2 = \int_0^{3\pi/2} \sin^2\left(\left(\frac{2n-1}{3}+4\right)x\right)dx = \frac{1}{2}\int_0^{3\pi/2}\left(1 - \cos\left(2\cdot\left(\frac{2n-1}{3}+x\right)x\right)\right)dx =$$

$$= \frac{x}{2}\Big|_0^{3\pi/2} - \left(\frac{3}{2n-1} + \frac{1}{4}\right)\cdot x \cdot \sin\left(2\left(\frac{2n-1}{3}+4\right)x\right)\Big|_0^{3\pi/2} = \frac{3\pi}{4} - 0 = \frac{3\pi}{4}$$

$y_k = \frac{2\pi}{\sqrt{3}}\sin\left(\left(\frac{2n-1}{3}+4\right)x\right)$ ← не верно и далее

N3

① $f_1 = \sin 3x$ ② $f_2 = x^2 + 1$ ③ $f_3 = \cos x$

① $$\sin 3x = \sum_{n=1}^{\infty} A_n \cdot \sin\left(\left(\frac{2n-1}{3}+4\right)x\right) \Big| \cdot \sin\left(\left(\frac{2m-1}{3}+4\right)x\right)$$

$$\int_0^{3\pi/2} \sin 3x \sin\left(\left(\frac{2m-1}{3}+4\right)x\right)dx = \sum_{n=1}^{\infty} A_n \cdot \int_0^{3\pi/2}\sin\left(\left(\frac{2n-1}{3}+4\right)x\right)\sin\left(\left(\frac{2m-1}{3}+4\right)x\right)dx$$

$$\int_0^{3\pi/2} \sin 3x \sin\left(\left(\frac{2m-1}{3}+4\right)x\right)dx = \sum_{n=1}^{\infty} A_n \begin{cases} \frac{3\pi}{4}, & n = m \\ 0, & n \neq m \end{cases}$$

$$A_n = \frac{4}{3\pi}\int_0^{3\pi/2} \sin 3x \sin\left(\left(\frac{2m-1}{3}+4\right)x\right)dx = \frac{2}{3\pi}\int_0^{3\pi/2}\left(\cos\left(\frac{2m-3}{3}x\right) - \cos\left(\frac{2m+20}{3}x\right)\right)dx =$$

$$= \frac{2}{3\pi}\left(\frac{3}{2m-3}\cdot\frac{3\pi}{2}\cdot\sin\left(\frac{2m-3}{3}\cdot\frac{3\pi}{2}\right) - 0\right) = (-1)^m\cdot\frac{3}{2m-3},\ m = \overline{1,\infty}$$

? $$\sin 3x = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{3}{2m-3}\cdot\sin\left(\left(\frac{2n-1}{3}+4\right)x\right)$$

② $$x^2 - 1 = \sum_{n=1}^{\infty} A_n \cdot \sin\left(\left(\frac{2n-1}{3}+4\right)x\right) \Big| \cdot \sin\left(\left(\frac{2m-1}{3}+4\right)x\right)$$

$$\int_0^{3\pi/2}(x^2-1)\cdot\sin\left(\left(\frac{2m-1}{3}+4\right)x\right)dx = \sum_{n=1}^{\infty} A_n \begin{cases} 3\pi/4, & n = m \\ 0, & n \neq m \end{cases}$$

$$A_n = \frac{4}{3\pi}\int_0^{3\pi/2}(x^2-1)\sin\left(\left(\frac{2m-1}{3}+4\right)\cdot x\right)dx = \frac{4}{3\pi}\left(\int_0^{3\pi/2} x^2\sin\left(\frac{2m+11}{3}x\right)dx - \int_0^{3\pi/2}\sin\left(\frac{2m+11}{3}\right)dx\right) =$$

$$= \frac{4}{3\pi}\left[\frac{3x^2}{2m+11}\cdot\cos\left(\frac{2m+11}{3}\cdot x\right) + \frac{2x\cdot 9}{(2m+11)^2}\cdot\sin\left(\frac{2m+11}{3}x\right) + \frac{2\cdot 27}{(2m+11)^3}\cdot\cos\left(\frac{2m+11}{3}x\right) - \right.$$

$$\left. - \frac{3}{2m+11}\cdot\cos\left(\frac{2m+11}{3}\cdot x\right)\right]_0^{3\pi/2} = \frac{4}{3\pi}\left(\frac{2\cdot 3\pi\cdot 3^2}{2(2m+11)^2}\cdot(-1)^m - \frac{2\cdot 3^2}{(2m+11)^3} + \frac{1}{(2m+11)}\right) =$$

$$= (-1)^m\cdot\frac{36}{(2m+11)^2} - \frac{72}{\pi(2m+11)^3} + \frac{4}{\pi(2m+11)},\ m = \overline{1,\infty}$$

$n = m$

$$x^2 - 1 = \sum_{n=1}^{\infty} \cancel{A_n} \cdot \sin\left(\left(\frac{2n-1}{3}+4\right)x\right)$$

③ $$\cos x = \sum_{n=1}^{\infty} A_n \sin\left(\left(\frac{2n-1}{3}+4\right)x\right) \Big| \cdot \sin\left(\left(\frac{2m-1}{3}+4\right)x\right)$$

$$\int_0^{3\pi/2}\cos x\cdot\sin\left(\left(\frac{2m-1}{3}+4\right)x\right)dx = \sum_{n=1}^{\infty} A_n \begin{cases} 3\pi/4, & n = m \\ 0, & n \neq m \end{cases}$$

$$A_n = \frac{4}{3\pi}\int_0^{3\pi/2}\cos x\cdot\sin\left(\left(\frac{2m-1}{3}+4\right)x\right) = \frac{2}{3\pi}\int_0^{3\pi/2}\left(\sin\left(\frac{2(m+7)}{3}x\right) - \sin\left(\frac{2(m+4)}{3}x\right)\right)dx =$$

$$= \frac{2}{3\pi}\left[-\frac{3}{2m+14}\cos\left(\frac{2(m+7)}{3}\cdot x\right) - \frac{3}{2m+8}\cos\left(\frac{2(m+4)}{3}x\right)\right]_0^{3\pi/2} =$$

$$= \frac{2}{3\pi}\left(\cancel{(\ldots)}\cdot\frac{3}{2m+14} + \cancel{(\ldots)}\cdot\frac{3}{2m+8} + \frac{3}{2m+14} + \frac{3}{2m+8}\right) = \quad m = 2, 4, 6, \ldots$$

$$= \frac{2}{\pi(m+7)} + \frac{2}{\pi(m+4)} \quad \begin{cases} m = 2, 4, 6, 8, 10, \ldots \\ 0, \ m = 1, 3, 5, 7 \ldots \end{cases}$$

$$\cos x = \sum_{n=2}^{\infty}\left(\frac{2}{\pi(m+7)} + \frac{2}{\pi(m+4)}\right)\cdot\sin\left(\left(\frac{2n\pm1}{3}+4\right)x\right)$$, при n, с шагом 2.

$= \frac{2}{3\pi}\cancel{\left(\frac{3 \cdot 1}{2m-3}\cdot 1 + \frac{3 \cdot 1}{2m+20} \ldots \cos(m \ldots)\right)}$