Филимонов Степан РЛ6-41

Вариант 12

Задача 1 (*5 баллов*). Задан случайный процесс $x(t) = Ut^2 + Vt^3$, где U и V – некоррелированные случайные величины, $M[U] = 0$, $M[V] = 0$, $D[U] = 2$, $D[V] = 2$.

Найдите:

1) $M[x(t)]$; $K_x(t_1, t_2)$; $D[x(t)]$,

2) математическое ожидание, корреляционную функцию случайного процесса $y_1 = \dfrac{dx(t)}{dt}$,

3) математическое ожидание, корреляционную функцию и дисперсию случайного процесса $y_2 = x(t) + \dfrac{dx(t)}{dt}$,

4) математическое ожидание, корреляционную функцию и дисперсию случайного процесса $y_3 = \int_0^t x(s)ds$,

5) $R_{x\dot{x}}$ и $R_{\dot{x}x}$.

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Решение:

1) $M[X(t)] = M[Ut^2 + Vt^3] = t^2 M[U] + t^3 M[V] = 0$

$K_x(t,t') = M[\overset{\circ}{X}(t)\overset{\circ}{X}(t')]$

$\overset{\circ}{X}(t) = X(t) - m_x(t) = Ut^2 + Vt^3 - 0 = Ut^2 + Vt^3$

$K_x(t,t') = M[(Ut^2+Vt^3)\cdot(2\cdot Ut\cdot t' + 3Vt^2t')] =$

$= M[2U^2t^3t' + \underbrace{3VUt^4t' + 2V\cdot Ut^4 t'}_{5VUt^4t' \Rightarrow K_{uv} = M[U]\cdot M[V] = 0} + 3V^2t^5t'] =$

$= 2t^3t'D[U] + 5\cdot t^4\cdot t'\cdot K_{uv} + 3t^5t'\cdot D[V] = 4t^3t' + 6t^5t'$

$D[X(t)] = D[Ut^2+Vt^3] = t^2 D[U] + t^3 D[V] = 4t^4 + 6t^6$

2) $y_1 = \dfrac{dX(t)}{dt} = 2Ut + 3Vt^2$

$m_{y_1}(t) = \dfrac{dm_x(t)}{dt} = \dfrac{d}{dt}(0) = 0$

$K_{y_1}(t,t') = \dfrac{\partial^2}{\partial t \partial t'}(K_x(t,t')) = 4\dfrac{\partial}{\partial t}(t^3t') + 6\dfrac{\partial}{\partial t}(t^5t') =$

$= 12t^2t't'' + 30t^4t't''$

3) $y_2 = X(t) + \frac{dX(t)}{dt} = X(t) + y_1(t)$

$m_{y_2}(t) = m_x(t) + m_{y_1}(t) = 0 + 0 = 0$

$K_{y_2}(t,t') = K_x(t,t') + K_{y_1}(t,t') + R_{xy_1}(t,t') + R_{y_1x}(t,t')$

$R_{xy_1}(t,t') = M[\mathring{X}(t)\cdot\mathring{y}_1(t')] =$

$\mathring{y}_1(t) = y_1(t) - m_{y_1}(t) = 2Ut + 3Vt^2 - 0 = 2Utt' + 3Vt^2t'$

$= M[(Ut^2+Vt^3)(2Ut' + 3Vt'^2)] = \quad t^2t'^2$

$= M[2U^2t^2t' + 3VUt^2t'^2 + 2VUt^3t' + 3V^2 t^3t'^2] = 4t^3t' + 6\ldots$

$R_{y_1x}(t,t') = M[\mathring{y}_1(t)\cdot\mathring{X}(t')] = M[(2Ut + 3Vt^2)(Ut'^2 + Vt'^3)] =$

$= M[2U^2tt'^2 + 2UVt\cdot t'^3 + 3VUt^2t'^2 + 3V^2t^2t'^3] =$

$= 4tt'^2 + 6t^2t'^3$

$\boxed{=}\ 4t^3t' + 6t^5t' + 12t^2t't'' + 30t^4t't'' + 4t^3t' + 6t^3t'^2 +$

$+ 4tt'^2 + 6t^2t'^3$

4) $y_3 = \int_0^t x(s)\,ds$

$m_{y_3}(t) = \int_0^t m_x(s)\,ds = \int_0^t 0\,ds = 0$

$K_{y_3}(t,t') = \int_0^t ds \int_0^{t'} ds'\, K_x(s,s') =$

$= \int_0^t ds \int_0^{t'} ds' \cdot (4s^3s' + 6s^5s') = \int_0^t ds \left((4s^3+6s^5)\left[\frac{s'^2}{2}\right]_0^{t'}\right) =$

$(4s^3 + 6s^5)s'$

$= \frac{t'^2}{2}\left[s^4 + s^6\right]_0^t = \frac{t'^2}{2}(t^4+t^6)$

$D_{y_3}(t) = K_{y_3}(t,t) = \frac{t^2}{2}\cdot(t^4+t^6)$

Задача 2 (*5 баллов*). Найти $K_y(\tau)$, если известна $R_x(\tau) = 1e^{-13|\tau|}\left(\cos 5\tau + \frac{13}{5}\sin 5|\tau|\right)$ и $4\frac{dx}{dt} = 2\frac{dy}{dt} + 5y(t)$

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1) $S_x^*(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} R_x(\tau)e^{-i\omega\tau}\,d\tau = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-13|\tau|}\left(\cos 5\tau + \frac{13}{5}\sin 5|\tau|\right)e^{-i\omega\tau}\,d\tau$

$= \left| \begin{matrix} |\tau| = \tau, \tau > 0 \\ |\tau| = -\tau, \tau < 0 \end{matrix} \right| = \frac{1}{2\pi}\left[\int_0^{+\infty} e^{-13\tau}\left(\cos 5\tau + \frac{13}{5}\sin 5\tau\right)e^{-i\omega\tau}\,d\tau + \int_{-\infty}^{0} e^{13\tau}\left(\cos 5\tau + \frac{13}{5}\sin\right.\right.$

$\left.\cdot e^{-i\omega\tau}\,d\tau\right] = \frac{1}{2\pi}\left[\int_0^{+\infty} e^{-(13+i\omega)\tau}\left(\cos 5\tau + \frac{13}{5}\sin 5\tau\right)d\tau + \right.$ I

$\left.+\int_{-\infty}^{0} e^{(13-i\omega)\tau}\left(\cos 5\tau + \frac{13}{5}\sin 5\tau\right)d\tau\right] = \frac{1}{2\pi}\left[\int_0^{+\infty} e^{-(13+i\omega)\tau}\cdot\cos 5\tau\,d\tau + \right.$ II

$\left. + \int_0^{+\infty} e^{-(13+i\omega)\tau}\cdot\frac{13}{5}\sin 5\tau\,d\tau + \int_{-\infty}^{0} e^{(13-i\omega)\tau}\cos 5\tau\,d\tau + \frac{13}{5}\int_{-\infty}^{0} e^{(13-\omega i)}\sin 5\tau\,d\tau\right] =$ III IV

I $= \cancel{\cos 5x \cdot \frac{e^{-(13+i\omega)\tau}}{13+i\omega} - 5\int \frac{e^{-(13+i\omega)\tau}}{13+i\omega}\sin 5\tau\,d\tau} = 0$

II $= 0$

III $= \dfrac{(13-i\omega)}{(13-i\omega)^2+25}$ $\cancel{\frac{5}{(13-i\omega)^2+25}}$

IV $= -\dfrac{5}{(13-i\omega)^2+25}$

$= \dfrac{13-i\omega-5}{(13-i\omega)^2+25}\cdot\dfrac{1}{2\pi}$

2) $S_Y^*(\omega) = S_x^*(\omega)\cdot|\Phi(i\omega)|^2$

Если $Y(t) = \frac{d}{dt}X(t)$, то $\Phi(i\omega) = i\omega \Rightarrow$

$\Rightarrow 4i\omega = 2i\omega + 5 \Rightarrow \Phi(i\omega) = \dfrac{4i\omega}{2i\omega+5} \Rightarrow$

$\Rightarrow |\Phi(i\omega)|^2 = \dfrac{16\omega^2}{25+4\omega^2} \Rightarrow$

$$S_\gamma^*(\omega) = \frac{1}{8\pi} \cdot \frac{8-i\omega}{(13-i\omega)^2-25} \cdot \frac{8/6\,\omega^2}{25+4\omega^2}$$

$$169 - 26i\omega - \omega^2 - 25$$

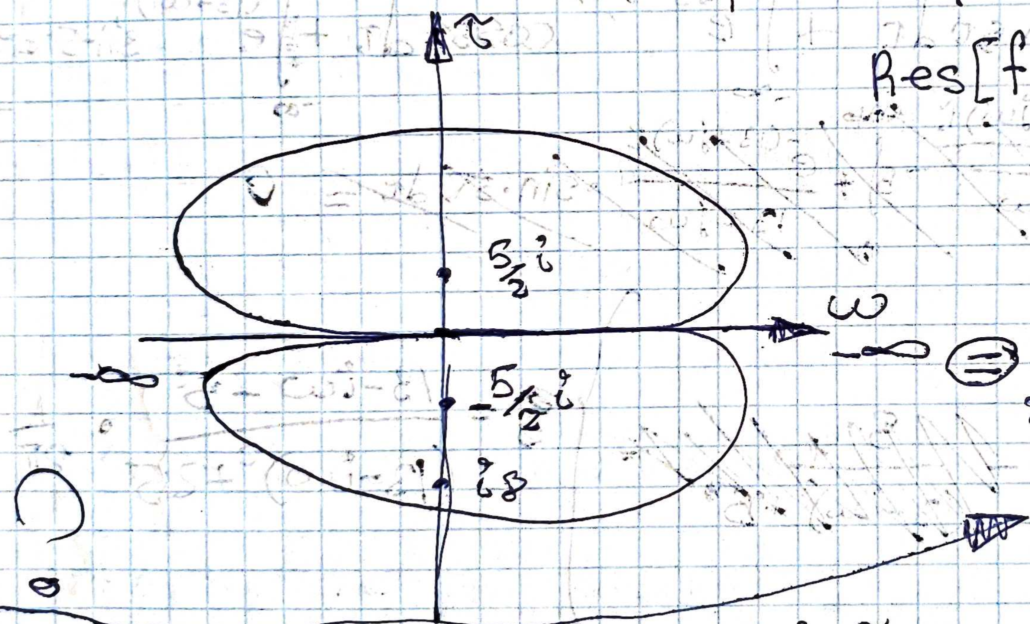
$$144 - 26i\omega - \omega^2$$

$$R(\tau) = \int_{-\infty}^{+\infty} S^*(\omega) \cdot e^{i\omega\tau}\, d\omega$$

$$\Downarrow$$

$$R_\gamma(\tau) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{(8-i\omega)(8\omega^2)}{\underbrace{\cancel{(144-26i\omega-\omega^2)}}_{(13-i\omega)^2-25}(25+4\omega^2)} e^{i\omega\tau}\, d\omega =$$

$$= \left[\begin{array}{l} \omega = -i8 \\ \omega = \pm i\frac{5}{2} \end{array}\right] = \frac{1}{\pi} \cdot \frac{1}{2\pi i} \begin{cases} \tau > 0, \ \mathrm{Res}\left[f(z), i\frac{5}{2}\right] + \\ \tau < 0, \ -\left(\mathrm{Res}\left[f(z), -i8\right] + \mathrm{Res}\left[f(z), -\frac{5}{2}i\right]\right) \end{cases} =$$



$$= \frac{1}{2\pi^2 i} \begin{cases} \tau > 0, \ -b_1 \\ \tau < 0, \ -a - b_2 \end{cases}$$

$$\mathrm{Res}[f(z), -i8] = \lim_{\omega \to i8} \frac{d}{d\omega}\left(\frac{e^{i\omega\tau} \cdot (8-i\omega) \cdot (8\omega^2)\,\cancel{((13-i\omega)^2-25)}}{\cancel{((13-i\omega)^2-25)}((13+i\omega)^2-25)(25+4\omega^2)}\right)$$

$$= \_\_ = a$$

$$\mathrm{Res}\left[f(z), \frac{5}{2}i\right] = \lim_{z \to \frac{5}{2}i} \frac{e^{i\omega\tau} \cdot (8-i\omega)(8\omega^2)\,\cancel{(5-4\frac{5}{2}i)}}{((13-i\omega)^2-25)\cancel{(5-4\frac{5}{2}i)}(5+4\frac{5}{2}i)} \ldots = b_1$$

$$\mathrm{Res}\left[f(z), -\frac{5}{2}i\right] = \cancel{\lim_{z\to-\frac{5}{2}i} \frac{e^{i\omega\tau}(8-i\omega)(8\omega^2)\cdot(5+(-4\cdot\frac{5}{2}i))}{((13-i\omega)^2-25)}} = \_\_ = b_2$$