Билет № 2

III $U(x,t) = \sum_{n=1}^{\infty} X(x) \cdot T(t)$

$$U(x,t) = \sum_{n=1}^{\infty} \left(A_n \cos \frac{2\pi n}{3} t + B_n \sin \frac{2\pi n}{3} t\right) \sin \frac{\pi n}{3} x$$

$$U_t(x,t) = \sum_{n=1}^{\infty} \left(-A_n \frac{2\pi n}{3} \sin \frac{2\pi n}{3} t + B_n \cdot \frac{2\pi n}{3} \cos \frac{2\pi n}{3} t\right) \cdot \sin \frac{\pi n}{3} x$$

IV 1) $U(x,0) = 8 \sin \pi x$ Учтем условие ортогон. собст фун

$$8 \sin \pi x = \sum_{n=1}^{\infty} A_n \sin \frac{\pi n}{3} x \, \Big| \cdot \sin \frac{\pi m}{3} x$$

$$8 \int \sin \pi x \sin \frac{\pi m}{3} x \, dx = A_n \int \sin \frac{\pi n}{3} x \cdot \sin \frac{\pi m}{3} x \, dx$$

$$8 \cdot \cancel{\frac{3}{2}} \cdot \begin{cases} 1, & m = 3 \\ 0, & m \neq 3 \end{cases} = A_n \cdot \cancel{\frac{3}{2}} \begin{cases} 1, & m = n \\ 0, & m \neq n \end{cases}$$

$$8 = A_3, \quad A_n = 0 \text{ при } \forall n \neq 3$$

2) $U_t(x,0) = 8\pi \sin 4\pi x$ Учтем условие ортогональности собст фун

$$8\pi \sin 4\pi x = \sum_{n=1}^{\infty} B_n \frac{2\pi n}{3} \cdot \sin \frac{\pi n}{3} x$$

$$8\pi \int \sin 4\pi x \cdot \sin \frac{\pi m}{3} x \, dx = B_n \frac{2\pi n}{3} \int \sin \frac{\pi n}{3} \sin \frac{\pi m}{3} x \, dx$$

$$8\pi \cdot \cancel{\frac{3}{2}} \cdot \begin{cases} 1, & m = 12 \\ 0, & m \neq 12 \end{cases} = B_n \frac{2\pi n}{3} \cdot \cancel{\frac{3}{2}} \cdot \begin{cases} 1, & n = m \\ 0, & n \neq m \end{cases}$$

$$B_{12} = 1 \qquad B_n = 0 \quad \forall n \neq 12$$

V) $$U(x,t) = A_3 \cos 2\pi t \sin \pi x + B_{12} \cdot \sin 8\pi t \cdot \sin 4\pi x$$

$$U(x,t) = 8\cos 2\pi t \cdot \sin \pi x + \sin 8\pi t \cdot \sin 4\pi x$$

№ 3 (б)

$U_{xx} + 4U_{xy} + U_{yy} + U_x + U_y - x^2 y = 0$

$a_{11} = 1 \quad a_{12} = a_{21} = 2 \quad a_{22} = 1$ Гиперболический ~~Параболический~~ тип

$\Delta = a_{12}^2 - a_{11} \cdot a_{22} = 4 - 1 = 3 > 0$

$\frac{dy}{dx} = \frac{a_{12} + \sqrt{\Delta}}{a_{11}} \Rightarrow dy = (2+\sqrt{3})dx \Rightarrow y = 2x + \sqrt{3}x + C_1 \Rightarrow C_1 = y - 2x - \sqrt{3}x$

$\frac{dy}{dx} = \frac{a_{12} - \sqrt{\Delta}}{a_{11}} \Rightarrow dy = (2-\sqrt{3})dx \Rightarrow y = 2x - \sqrt{3}x + C_2 \Rightarrow C_2 = y - 2x + \sqrt{3}x$

Введем ξ и η: $\xi_x = -2-\sqrt{3} \quad \xi_y = 1 \quad \eta_x = \sqrt{3}-2 \quad \eta_y = 1$

$U_x = U_\xi \cdot \frac{\partial \xi}{\partial x} + U_\eta \cdot \frac{\partial \eta}{\partial x} = (-2-\sqrt{3})U_\xi + (\sqrt{3}-2)U_\eta$

$U_y = U_\xi \cdot \frac{\partial \xi}{\partial y} + U_\eta \frac{\partial \eta}{\partial y} = U_\eta + U_\xi$

$U_{xx} = \cancel{\frac{\partial}{\partial x}(U_x)} = (2+\sqrt{3})^2 U_{\xi\xi} + [illegible] U_{\xi\eta} + (\sqrt{3}-2)^2 U_{\eta\eta}$

$U_{yy} = U_{\eta\eta} + 2U_{\xi\eta} + U_{\xi\xi}$

$U_{xy} = -2U_{\xi\xi} - \sqrt{3}U_{\xi\xi} + \cancel{\sqrt{3}U_{\xi\eta}} - 2U_{\xi\eta} - 2U_{\xi\eta} - \cancel{\sqrt{3}U_{\xi\eta}} + \sqrt{3}U_{\eta\eta} - 2U_{\eta\eta}$

$= (-2-\sqrt{3})U_{\xi\xi} - 4U_{\xi\eta} + (\sqrt{3}-2)U_{\eta\eta}$

$7\cancel{U_{\xi\xi}} + 4\cancel{\sqrt{3}}U_{\xi\xi} + \cancel{U_{\xi\eta}} + \cancel{7U_{\eta\eta}} - \cancel{4\sqrt{3}U_{\eta\eta}} - \cancel{8U_{\xi\xi}} - \cancel{4\sqrt{3}U_{\xi\xi}} - 13U_{\xi\eta} + \cancel{4\sqrt{3}U_{\eta\eta}}$

$- \cancel{8U_{\eta\eta}} + \cancel{U_{\eta\eta}} + \cancel{2U_{\xi\eta}} + \cancel{U_{\xi\xi}} - 2U_\xi - \sqrt{3}U_\xi - U_\xi + U_\eta + x^2 y = 0$

$U_{\xi\eta} = \frac{2}{13}U_\xi + \frac{\sqrt{3}}{13}U_\xi - \frac{1}{13}U_\xi - \frac{1}{13}U_\eta + x^2 y = 0$

$\xi = y - 2x - \sqrt{3}x$

$\eta = y - 2x + \sqrt{3}x$