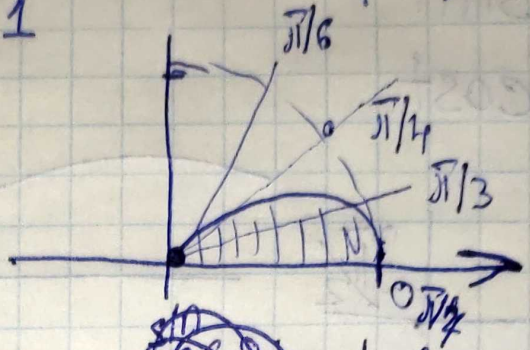
РК1 ИиДУ Билет №5 (18)

№1

$\rho^2 = \cos 2\varphi$

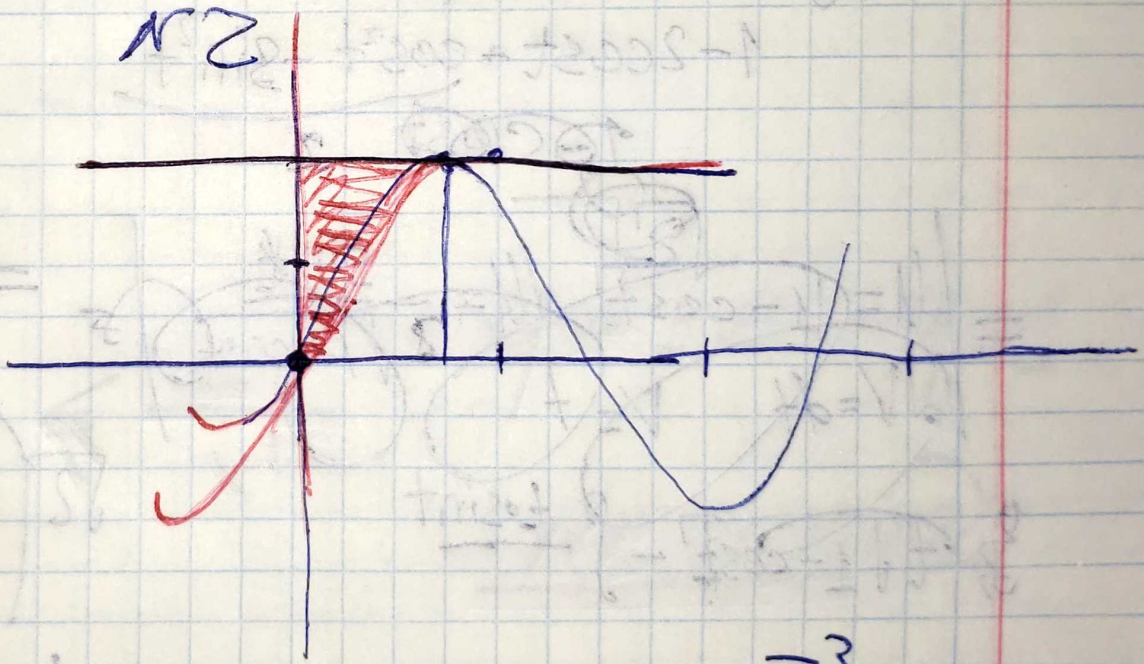


$$S = \frac{1}{2}\int_0^{\pi/4} \cos^2 2\varphi \, d\varphi = \frac{1}{8}\int_0^{\pi/4} 1+\cos t \, dt$$

$$= \left(\frac{\varphi}{4} + \frac{\sin 4\varphi}{8}\right)\Big|_0^{\pi/4} = \boxed{\frac{\pi}{16}}$$

№2

$$\begin{cases} y = \sin x \\ y = 1 \end{cases}$$



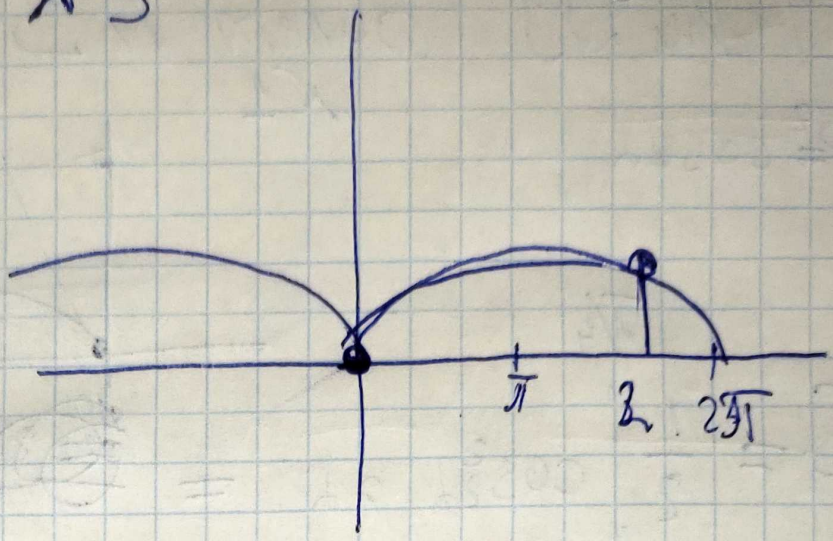
$$V_y = 2\pi\int_0^{\pi/2} x\cdot(1-\sin x)\,dx = \ldots = \frac{\pi^3}{4} - 2\pi$$

Отв > 0

№3

$$\begin{cases} x = t - \sin t \\ y = 1 - \cos t \end{cases}$$

t	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\cdots$	$\frac{3\pi}{2}$
x	0	$\frac{\pi}{6} - \frac{1}{2}$			
y	0	$1 - \frac{\sqrt{3}}{2}$			



$$l = \int_0^{3\pi/2} \sqrt{(1-\cos t)^2 + (\sin t)^2}\, dt = \sqrt{2} \int_0^{3\pi/2} \sqrt{1-\cos t}\, dt =$$

$$1 - 2\cos t + \cos^2 t + \sin^2 t$$

$$1 - \cos t \qquad 1$$

$$= \left| \begin{array}{l} u = \sqrt{1-\cos t} \\ dv = dt \end{array} \right| \begin{array}{l} du = \frac{\sin t}{2\sqrt{1-\cos t}}\, dt \\ v = t \end{array} = \cdots =$$

$$t\sqrt{1-\cos t} - \int \frac{t \sin t}{\sqrt{1-\cos t}}$$

$$\sqrt{2}\left(-2\sqrt{1+\cos(x)} \right) \Big|_0^{3\pi/2}$$

$$= -2\sqrt{2} + 4 = \boxed{4 - 2\sqrt{2}}$$

№4

$$\int_1^{+\infty} \frac{\operatorname{arctg} x}{\sqrt{x^3+5}}\,dx$$

$$\frac{\operatorname{arctg} x \cdot x^{\frac{3}{2}}}{(x^3+5)^{\frac{1}{2}}} \leq \frac{\pi}{2x^{\frac{3}{2}}}$$

$q = \frac{3}{2}$ $q > 1$

сходится

№5

$$\int_0^{\pi/2} \frac{dx}{x^3\sqrt{\sin x}}$$

$$\frac{1}{x^3\sqrt{\sin x}} \sim \frac{1}{x^{3,5}}$$

$x \to 0$

$q = 3,5$ $q > 1$

не сходится