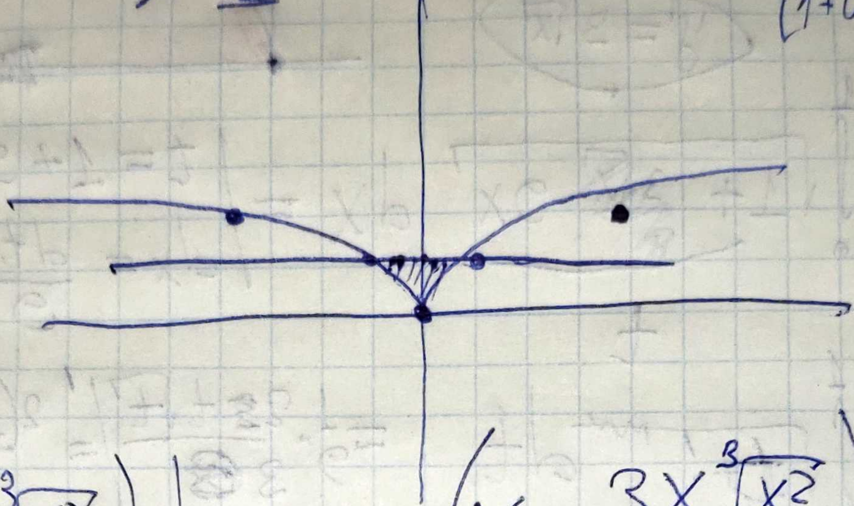
ИиДУ РК1 Билет №7 (20)

$(1+\cos\varphi)(1+2\cos\varphi+\cos^2\varphi)$

№1

$y=\sqrt[3]{x^2}$



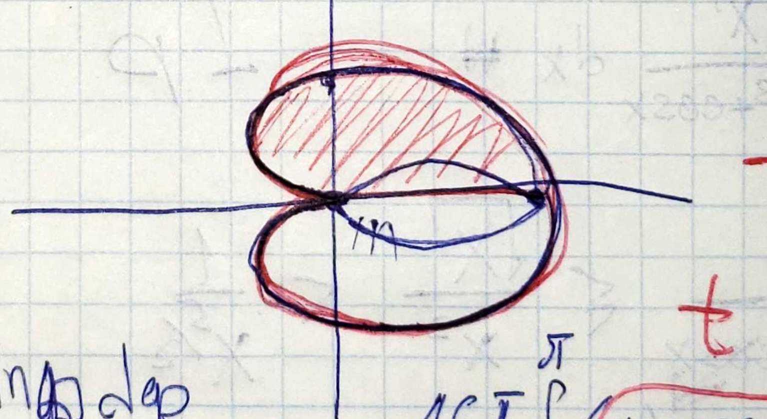
$$S=\int_{-1}^{1}\left(1-\sqrt[3]{x^2}\right)dx=\left(x-\frac{3x\sqrt[3]{x^2}}{5}\right)\Bigg|_{-1}^{1}=$$

$$=1-\frac{3}{5}+1-\frac{3}{5}=\frac{4}{5}$$

№2

$\rho=2\cos^2\frac{\varphi}{2}$

$V=\frac{2}{3}\pi\int_{\alpha}^{\beta}r^3\sin\varphi\,d\varphi$



$-\int t^3\,dt$

$t \quad -\frac{(1+\cos\varphi)^4}{4}\Big|_0^{\pi}$

$$V=\frac{2\pi}{3}\int_0^{\pi}8\cos^6\frac{\varphi}{2}\sin\varphi\,d\varphi=\frac{16\pi}{3}\int_0^{\pi}\left(\frac{1+\cos\varphi}{2}\right)^3 d\cos\varphi$$

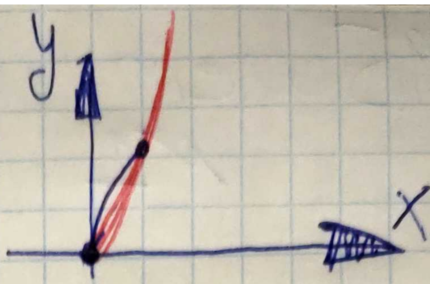
$$=\frac{16\pi}{3}\int_0^{\pi}1+2\cos\varphi+\cos^2\varphi+\cos\varphi+2\cos^2\varphi+\cos^3\varphi\,d\cos\varphi$$

$$=\cos\varphi+\cos^2\varphi+\frac{3\cos^3\varphi}{2}+\cos^3\varphi+\frac{\cos^4\varphi}{4}\Big|_0^{\pi}$$

$-1+\frac{3}{2}-1+\frac{1}{4}-1$

Отв: $\frac{8\pi}{3}$

№3



$y=2x\sqrt{x}$

$2x^{\frac{3}{2}}$

$y'=3\sqrt{x}$

$$L=\int_0^1 \sqrt{\underbrace{1+9x}_{t}}\,dx=\left|\begin{matrix} t=1+9x \\ dx=\frac{dt}{9}\end{matrix}\right|=$$

$$=\frac{1}{9}\int_0^1 \sqrt{t}\,dt=\frac{1}{9}\cdot\frac{2t\sqrt{t}}{3}\Bigg|_0^1=\frac{2(1+9x)\sqrt{1+9x}}{27}\Bigg|_0^1=$$

$$=\boxed{\frac{20\sqrt{10}-2}{27}}$$

№4

$$\int_1^{+\infty}\frac{\sqrt{x}}{x^3+\cos x}\,dx$$

I р

$$\frac{\sqrt{x}}{x^3+\cos x}\le\frac{\sqrt{x}}{x^3}<\frac{1}{x^{5/2}}$$

$\cos x \to 1$

$q=5/2$ $q>1$

сход

№5

$$\int_0^1\frac{\sqrt{x}}{\sin^2 x}\,dx$$

II род

$q=\frac{3}{2}$ $q>1$

$$\frac{\sqrt{x}}{|\sin^2 x|}\le\frac{\sqrt{x}}{x^2}<\frac{1}{x^{3/2}}$$

$|\sin^2 x| \sim x$

несход