Задача 1

Решить смешанную задачу для неоднородного уравнения теплопроводности с нулевыми граничными условиями $u(0,t) = u(\pi,t) = 0$ *(1 балл)*

1. $u_t = \frac{1}{4}u_{xx} + 5\cos 2t \sin 2x, \qquad u(x,0) = \sin 4x, \quad 0 < t < \infty, \quad 0 < x < \pi$

2. $u_t = \frac{1}{9}u_{xx} + 5\sin 2t \sin 3x, \qquad u(x,0) = 2\sin 9x, \quad 0 < t < \infty, \quad 0 < x < \pi$

3. $u_t = \frac{1}{16}u_{xx} + 10\cos 3t \sin 4x, \qquad u(x,0) = 3\sin 16x, \quad 0 < t < \infty, \quad 0 < x < \pi$

4. $u_t = \frac{1}{25}u_{xx} + 10\sin 3t \sin 5x, \qquad u(x,0) = 4\sin 10x, \quad 0 < t < \infty, \quad 0 < x < \pi$

5. $u_t = \frac{1}{36}u_{xx} + 17\sin 4t \sin 2x, \qquad u(x,0) = 5\sin 18x, \quad 0 < t < \infty, \quad 0 < x < \pi$

6. $u_t = \frac{1}{4}u_{xx} + 17\sin 4t \sin 2x, \qquad u(x,0) = 6\sin 8x, \quad 0 < t < \infty, \quad 0 < x < \pi$

7. $u_t = \frac{1}{9}u_{xx} + 26\cos 5t \sin 3x, \qquad u(x,0) = 7\sin 6x, \quad 0 < t < \infty, \quad 0 < x < \pi$

8. $u_t = \frac{1}{16}u_{xx} + 26\sin 5t \sin 4x, \qquad u(x,0) = 8\sin 12x, \quad 0 < t < \infty, \quad 0 < x < \pi$

9. $u_t = \frac{1}{25}u_{xx} + 37\cos 6t \sin 5x, \qquad u(x,0) = 9\sin 20x, \quad 0 < t < \infty, \quad 0 < x < \pi$

10. $u_t = \frac{1}{36}u_{xx} + 37\sin 6t \sin 6x, \qquad u(x,0) = 10\sin 12x, \quad 0 < t < \infty, \quad 0 < x < \pi$

11. $u_t = \frac{1}{4}u_{xx} + 26\cos 5t \sin 2x, \qquad u(x,0) = 11\sin 6x, \quad 0 < t < \infty, \quad 0 < x < \pi$

12. $u_t = \frac{1}{9}u_{xx} + 26\sin 5t \sin 3x, \qquad u(x,0) = 12\sin 12x, \quad 0 < t < \infty, \quad 0 < x < \pi$

13. $u_t = \frac{1}{16}u_{xx} + 17\cos 4t \sin 4x, \qquad u(x,0) = 13\sin 8x, \quad 0 < t < \infty, \quad 0 < x < \pi$

14. $u_t = \frac{1}{25}u_{xx} + 17\sin 4t \sin 5x, \qquad u(x,0) = 14\sin 15x, \quad 0 < t < \infty, \quad 0 < x < \pi$

15. $u_t = \frac{1}{36}u_{xx} + 10\cos 3t \sin 6x, \qquad u(x,0) = 15\sin 18x, \quad 0 < t < \infty, \quad 0 < x < \pi$

16. $u_t = \frac{1}{4}u_{xx} + 10\sin 3t \sin 2x, \qquad u(x,0) = 16\sin 4x, \quad 0 < t < \infty, \quad 0 < x < \pi$

17. $u_t = \frac{1}{9}u_{xx} + 5\cos 2t \sin 3x, \qquad u(x,0) = 17\sin 9x, \quad 0 < t < \infty, \quad 0 < x < \pi$

18. $u_t = \frac{1}{16}u_{xx} + 5\sin 2t \sin 4x, \qquad u(x,0) = 18\sin 16x, \quad 0 < t < \infty, \quad 0 < x < \pi$

19. $u_t = \frac{1}{25}u_{xx} + 10\cos 3t \sin 5x, \qquad u(x,0) = 19\sin 10x, \quad 0 < t < \infty, \quad 0 < x < \pi$

20. $u_t = \frac{1}{36}u_{xx} + 10\sin 3t \sin 6x, \qquad u(x,0) = 20\sin 18x, \quad 0 < t < \infty, \quad 0 < x < \pi$

21. $u_t = \frac{1}{4}u_{xx} + 17\cos 4t \sin 2x, \qquad u(x,0) = 21\sin 8x, \quad 0 < t < \infty, \quad 0 < x < \pi$

22. $u_t = \frac{1}{9}u_{xx} + 17\sin 4t \sin 3x, \qquad u(x,0) = 22\sin 6x, \quad 0 < t < \infty, \quad 0 < x < \pi$

23. $u_t = \frac{1}{16}u_{xx} + 26\cos 5t \sin 4x, \qquad u(x,0) = 23\sin 12x, \quad 0 < t < \infty, \quad 0 < x < \pi$

24. $u_t = \frac{1}{25}u_{xx} + 26\cos 5t \sin 5x, \qquad u(x,0) = 24\sin 20x, \quad 0 < t < \infty, \quad 0 < x < \pi$

25. $u_t = \frac{1}{36}u_{xx} + 37\cos 6t \sin 6x, \qquad u(x,0) = 25\sin 12x, \quad 0 < t < \infty, \quad 0 < x < \pi$

26. $u_t = \frac{1}{4}u_{xx} + 37\sin 6t \sin 2x, \qquad u(x,0) = 26\sin 12x, \quad 0 < t < \infty, \quad 0 < x < \pi$

27. $u_t = \frac{1}{16}u_{xx} + 26\sin 5t \sin 3x, \qquad u(x,0) = 27\sin 12x, \quad 0 < t < \infty, \quad 0 < x < \pi$

28. $u_t = \frac{1}{9}u_{xx} + 26\cos 5t \sin 4x, \qquad u(x,0) = 28\sin 8x, \quad 0 < t < \infty, \quad 0 < x < \pi$

29. $u_t = \frac{1}{25}u_{xx} + 17\sin 4t \sin 6x, \qquad u(x,0) = 29\sin 18x, \quad 0 < t < \infty, \quad 0 < x < \pi$

30. $u_t = \frac{1}{36}u_{xx} + 17\cos 4t \sin 5x, \qquad u(x,0) = 30\sin 20x, \quad 0 < t < \infty, \quad 0 < x < \pi$

Задача 2

Решить смешанную задачу для однородного волнового уравнения с неоднородными начальными и граничными условиями (*1 балл*)

1. $u_{tt} = 9u_{xx}$, $\quad 0 < t < \infty$, $\quad 0 < x < 2$

   $u(0,t) = -8$, $\quad u(2,t) = 2$, $\qquad u(x,0) = \sin 6\pi x - 8 + 5x$, $\quad u_t(x,0) = 0$.

2. $u_{tt} = 9u_{xx}$, $\quad 0 < t < \infty$, $\quad 0 < x < 1$

   $u(0,t) = 7$, $\quad u(1,t) = 2$, $\qquad u(x,0) = 7 - 5x$, $\quad u_t(x,0) = 12\pi \sin 4\pi x$.

3. $u_{tt} = 9u_{xx}$, $\quad 0 < t < \infty$, $\quad 0 < x < 3$

   $u(0,t) = -6$, $\quad u(3,t) = 6$, $\qquad u(x,0) = 3\sin 3\pi x - 6 + 4x$, $\quad u_t(x,0) = 0$.

4. $u_{tt} = 9u_{xx}$, $\quad 0 < t < \infty$, $\quad 0 < x < 2$

   $u(0,t) = 5t$, $\quad u(2,t) = -3t$, $\qquad u(x,0) = 0$, $\quad u_t(x,0) = 3\pi \sin 4\pi x + 5 - 4x$.

5. $u_{tt} = 16u_{xx}$, $\quad 0 < t < \infty$, $\quad 0 < x < 1$

   $u(0,t) = -4$, $\quad u(1,t) = -1$, $\qquad u(x,0) = 5\sin 2\pi x - 4 + 3x$, $\quad u_t(x,0) = 0$.

6. $u_{tt} = 16u_{xx}$, $\quad 0 < t < \infty$, $\quad 0 < x < 2$

   $u(0,t) = 3$, $\quad u(2,t) = 7$, $\qquad u(x,0) = 3 + 2x$, $\quad u_t(x,0) = 12\pi \sin 3\pi x$.

7. $u_{tt} = 16u_{xx}$, $\quad 0 < t < \infty$, $\quad 0 < x < 3$

   $u(0,t) = -2$, $\quad u(3,t) = 1$, $\qquad u(x,0) = 7\sin 4\pi x - 2 + x$, $\quad u_t(x,0) = 0$.

8. $u_{tt} = 16u_{xx}$, $\quad 0 < t < \infty$, $\quad 0 < x < 2$

   $u(0,t) = t$, $\quad u(2,t) = -t$, $\qquad u(x,0) = 0$, $\quad u_t(x,0) = 28\pi \sin 7\pi x + 1 - x$.

9. $u_{tt} = 25u_{xx}$, $\quad 0 < t < \infty$, $\quad 0 < x < 1$

   $u(0,t) = -1$, $\quad u(1,t) = -3$, $\qquad u(x,0) = 9\sin 3\pi x - 1 - 2x$, $\quad u_t(x,0) = 0$.

10. $u_{tt} = 25u_{xx}$, $\quad 0 < t < \infty$, $\quad 0 < x < 2$

    $u(0,t) = 3$, $\quad u(2,t) = -5$, $\qquad u(x,0) = 3 - 4x$, $\quad u_t(x,0) = 20\pi \sin 4\pi x$.

11. $u_{tt} = 25u_{xx}$, $\quad 0 < t < \infty$, $\quad 0 < x < 1$

    $u(0,t) = -5$, $\quad u(1,t) = 1$, $\qquad u(x,0) = 11\sin 3\pi x - 5 + 6x$, $\quad u_t(x,0) = 0$.

12. $u_{tt} = 25u_{xx}$, $\quad 0 < t < \infty$, $\quad 0 < x < 2$

    $u(0,t) = 7t$, $\quad u(2,t) = -3t$, $\qquad u(x,0) = 0$, $\quad u_t(x,0) = 10\pi \sin 2\pi x + 7 - 5x$.

13. $u_{tt} = 36u_{xx}$, $\quad 0 < t < \infty$, $\quad 0 < x < 3$

    $u(0,t) = -9$, $\quad u(3,t) = 3$, $\qquad u(x,0) = 13\sin 3\pi x - 9 + 4x$, $\quad u_t(x,0) = 0$.

14. $u_{tt} = 36u_{xx}$, $\quad 0 < t < \infty$, $\quad 0 < x < 2$

    $u(0,t) = 8$, $\quad u(2,t) = 2$, $\qquad u(x,0) = 8 - 3x$, $\quad u_t(x,0) = 18\pi \sin 3\pi x$.

15. $u_{tt} = 36u_{xx}$, $\quad 0 < t < \infty$, $\quad 0 < x < 3$

    $u(0,t) = -6$, $\quad u(3,t) = 0$, $\qquad u(x,0) = 15\sin 3\pi x - 6 + 2x$, $\quad u_t(x,0) = 0$.

16. $u_{tt} = 36u_{xx}$, $\quad 0 < t < \infty$, $\quad 0 < x < 4$

    $u(0,t) = 4t$, $\quad u(4,t) = 8t$, $\qquad u(x,0) = 0$, $\quad u_t(x,0) = 24\pi \sin 4\pi x + 4 + x$.

17. $u_{tt} = 49u_{xx}, \qquad 0 < t < \infty, \quad 0 < x < 3$

$u(0,t) = -2, \quad u(3,t) = -5, \qquad u(x,0) = 17\sin 3\pi x - 2 - x, \quad u_t(x,0) = 0.$

18. $u_{tt} = 49u_{xx}, \qquad 0 < t < \infty, \quad 0 < x < 2$

$u(0,t) = 3, \quad u(2,t) = -1, \qquad u(x,0) = 3 - 2x, \quad u_t(x,0) = 35\pi\sin 5\pi x.$

19. $u_{tt} = 49u_{xx}, \qquad 0 < t < \infty, \quad 0 < x < 1$

$u(0,t) = -1, \quad u(1,t) = -4, \qquad u(x,0) = 19\sin 7\pi x - 1 - 3x, \quad u_t(x,0) = 0.$

20. $u_{tt} = 49u_{xx}, \qquad 0 < t < \infty, \quad 0 < x < 2$

$u(0,t) = 2t, \quad u(2,t) = -6t, \qquad u(x,0) = 0, \quad u_t(x,0) = 28\pi\sin 4\pi x + 2 - 4x.$

21. $u_{tt} = 64u_{xx}, \qquad 0 < t < \infty, \quad 0 < x < 1$

$u(0,t) = -4, \quad u(1,t) = -9, \qquad u(x,0) = 21\sin 3\pi x - 4 - 5x, \quad u_t(x,0) = 0.$

22. $u_{tt} = 64u_{xx}, \qquad 0 < t < \infty, \quad 0 < x < 3$

$u(0,t) = 2, \quad u(3,t) = -7, \qquad u(x,0) = 2 - 3x, \quad u_t(x,0) = 24\pi\sin 3\pi x.$

23. $u_{tt} = 64u_{xx}, \qquad 0 < t < \infty, \quad 0 < x < 1$

$u(0,t) = -3, \quad u(1,t) = 1, \qquad u(x,0) = 23\sin 2\pi x - 3 + 4x, \quad u_t(x,0) = 0.$

24. $u_{tt} = 64u_{xx}, \qquad 0 < t < \infty, \quad 0 < x < 2$

$u(0,t) = 4t, \quad u(2,t) = -6t, \qquad u(x,0) = 0, \quad u_t(x,0) = 32\pi\sin 4\pi x + 4 - 5x.$

25. $u_{tt} = 81u_{xx}, \qquad 0 < t < \infty, \quad 0 < x < 3$

$u(0,t) = -5, \quad u(3,t) = 1, \qquad u(x,0) = 25\sin 3\pi x - 5 + 2x, \quad u_t(x,0) = 0.$

26. $u_{tt} = 81u_{xx}, \qquad 0 < t < \infty, \quad 0 < x < 4$

$u(0,t) = 6, \quad u(4,t) = -2, \qquad u(x,0) = 6 - 2x, \quad u_t(x,0) = 27\pi\sin 3\pi x.$

27. $u_{tt} = 81u_{xx}, \qquad 0 < t < \infty, \quad 0 < x < 3$

$u(0,t) = -7, \quad u(3,t) = 2, \qquad u(x,0) = 27\sin 2\pi x - 7 + 3x, \quad u_t(x,0) = 0.$

28. $u_{tt} = 81u_{xx}, \qquad 0 < t < \infty, \quad 0 < x < 4$

$u(0,t) = 8t, \quad u(4,t) = -4t, \qquad u(x,0) = 0, \quad u_t(x,0) = 27\pi\sin 3\pi x + 8 - 3x.$

29. $u_{tt} = 4u_{xx}, \qquad 0 < t < \infty, \quad 0 < x < 2$

$u(0,t) = -9, \quad u(2,t) = 1, \qquad u(x,0) = 29\sin 4\pi x - 9 + 5x, \quad u_t(x,0) = 0.$

30. $u_{tt} = 4u_{xx}, \qquad 0 < t < \infty, \quad 0 < x < 3$

$u(0,t) = 9, \quad u(3,t) = -3, \qquad u(x,0) = 9 - 4x, \quad u_t(x,0) = 10\pi\sin 5\pi x.$

Задача 3

Решить краевую задачу для уравнения Лапласа в круге. (*1 балл*)

1. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 2, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = 2\cos^3\varphi - \sin^3\varphi + \sin\varphi. \end{cases}$

2. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 1, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=1} = 3\cos^3\varphi + \sin^3\varphi - \sin\varphi. \end{cases}$

3. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 4, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=4} = \cos^3\varphi + 4\sin^3\varphi + \sin^2\varphi. \end{cases}$

4. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 3, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=3} = 4\sin^3\varphi - \sin^2\varphi + \sin\varphi. \end{cases}$

5. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 2, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=2} = 2\cos^3\varphi + \sin\varphi. \end{cases}$

6. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 1, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=1} = \cos^3\varphi - \cos^2\varphi + \sin\varphi. \end{cases}$

7. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 3, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=3} = 3\sin^3\varphi - \cos^3\varphi + \sin\varphi. \end{cases}$

8. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 4, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=4} = 2\cos^3\varphi + 4\sin^3\varphi - \sin^2\varphi. \end{cases}$

9. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 1, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=1} = -4\cos^3\varphi - \sin^3\varphi + 2\sin\varphi. \end{cases}$

10. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 2, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = 4\cos^3\varphi - 2\sin^3\varphi - 3\cos\varphi + 2\sin\varphi. \end{cases}$

11. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 3, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=3} = \sin^3\varphi - \cos^3\varphi + 3\sin\varphi. \end{cases}$

12. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 6, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=6} = 2\sin^3\varphi - 3\cos^3\varphi + \cos\varphi. \end{cases}$

13. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 4, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=4} = 2\cos^3\varphi + 2\sin^3\varphi + 2\sin\varphi. \end{cases}$

14. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 1, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=1} = -\sin^3\varphi - 2\sin^2\varphi + 3\sin\varphi. \end{cases}$

15. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 2, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = 4\cos^3\varphi - 2\sin^3\varphi + \cos^2\varphi. \end{cases}$

16. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 3, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=3} = 2\cos^3\varphi - 4\sin^3\varphi + \cos\varphi. \end{cases}$

17. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 4, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=4} = 2\cos^2\varphi - \sin^3\varphi + \cos\varphi. \end{cases}$

18. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 5, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=5} = 2\sin^3\varphi - \sin\varphi + 2\cos\varphi. \end{cases}$

19. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 2, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = 2\cos^3\varphi + 4\sin^3\varphi + 2\sin\varphi. \end{cases}$

20. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 1, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=1} = 6\sin^3\varphi - \cos^3\varphi + 4\sin\varphi. \end{cases}$

21. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 3, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=3} = \cos^3\varphi + \sin^3\varphi + \sin^2\varphi. \end{cases}$

22. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 4, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=4} = 4\sin^3\varphi - \sin\varphi - \cos\varphi. \end{cases}$

23. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 5, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=5} = -\cos^3\varphi - \sin^3\varphi + \cos\varphi. \end{cases}$

24. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 2, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=2} = 4\cos^3\varphi + 2\sin^3\varphi + \sin\varphi - 3\cos\varphi. \end{cases}$

25. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 1, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=1} = 2\cos^3\varphi - \sin^3\varphi + 3\sin\varphi. \end{cases}$

26. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 1, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=1} = \sin^3\varphi + 2\sin^2\varphi - 3\sin\varphi - \cos\varphi. \end{cases}$

27. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 2, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = \cos^3\varphi + \sin^3\varphi + 3\sin^2\varphi. \end{cases}$

28. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 5, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=5} = 4\sin^3\varphi + \sin^2\varphi - \sin\varphi. \end{cases}$

29. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 3, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=3} = 4\cos^3\varphi + 2\sin^3\varphi + 2\cos\varphi. \end{cases}$

30. $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 1, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=1} = 4\cos^3\varphi - 4\sin^3\varphi + \cos^2\varphi. \end{cases}$

Задача 4

Решить краевую задачу для уравнения Гельмгольца в круге. (*1 балл*)

1. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 2, \ \ 0 \leqslant \varphi < 2\pi; \\ u_r'\big|_{r=2} = 2\cos^3\varphi - 3\sin\varphi. \end{cases}$

2. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 1, \ \ 0 \leqslant \varphi < 2\pi; \\ u_r'\big|_{r=1} = \cos^3\varphi + \sin\varphi. \end{cases}$

3. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 2, \ \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = 3\cos^3\varphi - \sin\varphi. \end{cases}$

4. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 3, \ \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=3} = \cos^2\varphi - 3\sin\varphi. \end{cases}$

5. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 2, \ \ 0 \leqslant \varphi < 2\pi; \\ u_r'\big|_{r=2} = \cos^3\varphi - 2\sin^3\varphi. \end{cases}$

6. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 2, \ \ 0 \leqslant \varphi < 2\pi; \\ u_r'\big|_{r=2} = \sin^3\varphi + 5\cos\varphi. \end{cases}$

7. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 1, \ \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=1} = \cos^2\varphi - 3\sin\varphi. \end{cases}$

8. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 2, \ \ 0 \leqslant \varphi < 2\pi; \\ u_r'\big|_{r=2} = \cos^3\varphi + \sin\varphi. \end{cases}$

9. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 3, \ \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=3} = \cos^3\varphi + \sin^2\varphi - \cos\varphi. \end{cases}$

10. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 1, \ \ 0 \leqslant \varphi < 2\pi; \\ u_r'\big|_{r=1} = \sin^3\varphi + 3\cos\varphi. \end{cases}$

11. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 1, \ \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=1} = \cos^2\varphi - 2\sin\varphi. \end{cases}$

12. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 2, \ \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = \cos^2\varphi + 5\sin\varphi. \end{cases}$

13. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 2, \ \ 0 \leqslant \varphi < 2\pi; \\ u_r'\big|_{r=2} = \cos^3\varphi - \sin\varphi. \end{cases}$

14. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 1, \ \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=1} = \cos^2\varphi - 6\sin^3\varphi. \end{cases}$

15. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 2, \ \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = \cos^3\varphi - \sin^2\varphi. \end{cases}$

16. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 3, \ \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=3} = \cos^2\varphi - 3\sin\varphi. \end{cases}$

17. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 2, \ \ 0 \leqslant \varphi < 2\pi; \\ u_r'\big|_{r=2} = 2\cos^3\varphi - \sin\varphi. \end{cases}$

18. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 2, \ \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = \cos^2\varphi + 5\sin\varphi. \end{cases}$

19. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 2, \ \ 0 \leqslant \varphi < 2\pi; \\ u_r'\big|_{r=2} = 3\cos^3\varphi + 5\cos\varphi. \end{cases}$

20. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 2, \ \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = \cos^2\varphi + 4\sin\varphi. \end{cases}$

21. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 1, \ \ 0 \leqslant \varphi < 2\pi; \\ u_r'\big|_{r=1} = \cos^3\varphi + \sin^3\varphi. \end{cases}$

22. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 2, \ \ 0 \leqslant \varphi < 2\pi; \\ u_r'\big|_{r=2} = \cos^3\varphi + 12\sin^3\varphi. \end{cases}$

23. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 3, \ \ 0 \leqslant \varphi < 2\pi; \\ u_r'\big|_{r=3} = \cos^3\varphi - \sin^3\varphi. \end{cases}$

24. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 2, \ \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = \cos^2\varphi - 3\sin^2\varphi. \end{cases}$

25. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 2, \ \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = \cos^2\varphi + 5\sin\varphi. \end{cases}$

26. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 2, \ \ 0 \leqslant \varphi < 2\pi; \\ u_r'\big|_{r=2} = \cos^3\varphi - 2\sin\varphi. \end{cases}$

27. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 1, \ \ 0 \leqslant \varphi < 2\pi; \\ u_r'\big|_{r=1} = \cos^2\varphi + 2\sin\varphi. \end{cases}$

28. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 3, \ \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=3} = \sin^2\varphi - 3\sin\varphi. \end{cases}$

29. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 2, \ \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = \cos^3\varphi + 2\sin\varphi. \end{cases}$

30. $\begin{cases} \Delta u + u = 0, \ \ 0 \leqslant r < 4, \ \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=4} = \sin^2\varphi + 6\sin^3\varphi. \end{cases}$