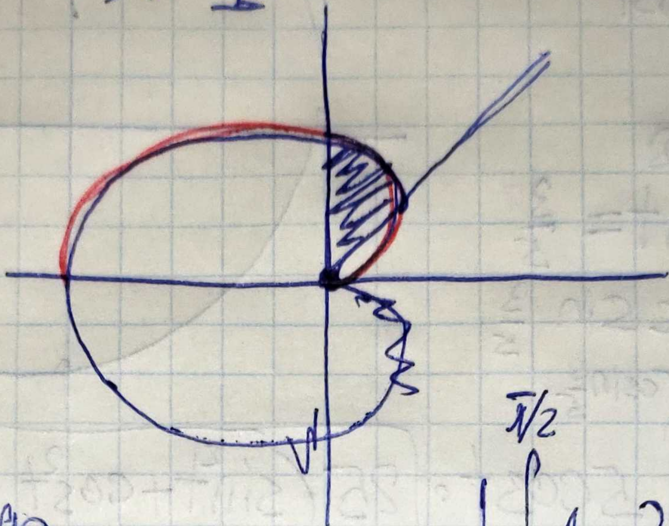
ИиДУ РК1 Билет № 11 (24)

№ 1

$\rho = 2\sin^2\frac{\varphi}{2}$



$S = \frac{1}{2}\int_{\pi/4}^{\pi/2} 4\sin^4\frac{\varphi}{2}\, d\varphi =$

$= 2\int_{\pi/4}^{\pi/2} \frac{(1-\cos\varphi)^2}{4}\, d\varphi = \frac{1}{2}\int_{\pi/4}^{\pi/2} 1 - 2\cos\varphi + \cos^2\varphi\, d\varphi$

$\cos^2\varphi = \frac{1+\cos 2\varphi}{2}$

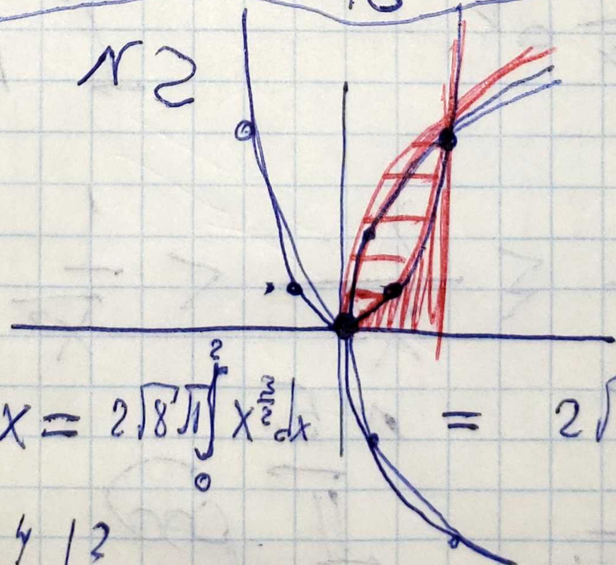
$= \left(\frac{\varphi}{2} - \sin\varphi + \frac{\varphi}{4} + \frac{\sin 2\varphi}{8}\right)\Big|_{\pi/4}^{\pi/2} = \frac{\pi}{4} - 1 + \frac{\pi}{8} + 0 - \frac{\pi}{8} + \frac{\sqrt{2}}{2} - \frac{\pi}{16} - \frac{1}{8}$

$= \frac{3\pi}{16} - \frac{9}{8} + \frac{\sqrt{2}}{2} = \frac{3\pi - 18 + 8\sqrt{2}}{16}$

№ 2

$y = x^2$

$y = \pm\sqrt{8x}$



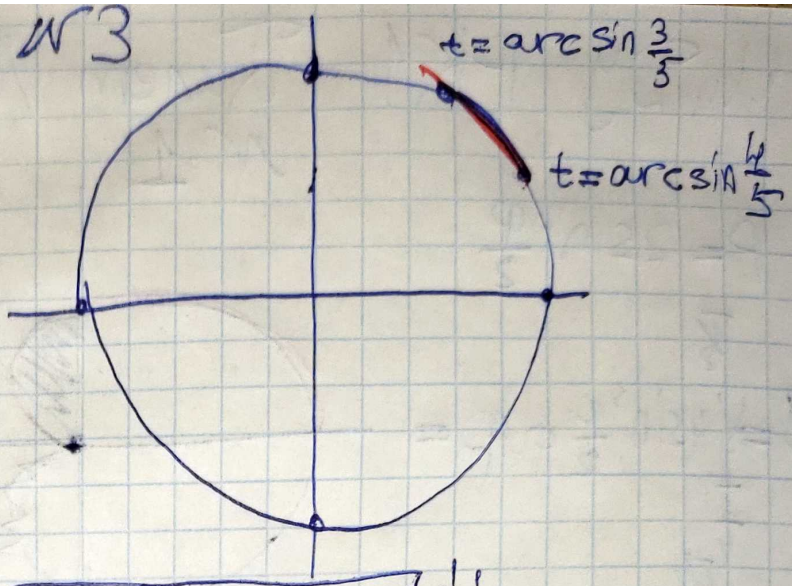
$V_{y_1} = 2\pi\int_0^2 x \cdot \sqrt{8x}\, dx = 2\sqrt{8}\pi\int_0^2 x^{\frac{3}{2}} dx = 2\sqrt{8}\pi \cdot \frac{2 \cdot x^2 \cdot \sqrt{x}}{5}\Big|_0^2 = \frac{64\pi}{5}$

$V_{y_2} = 2\pi\int_0^2 x^3 dx = \frac{2\pi x^4}{4}\Big|_0^2 \quad 8\pi$

$V_y = V_{y_1} - V_{y_2} = \frac{24\pi}{5}$

№3

$$\begin{cases} x = 5\sin t \\ y = 5\cos t \end{cases}$$



$\sin t = \frac{3}{5}$

$t = \arcsin\frac{3}{5}$

$\arcsin\frac{3}{5}$

$$S = 2\pi \int_{\arcsin\frac{3}{5}}^{\arcsin\frac{4}{5}} 5\cos t \cdot \sqrt{25\underbrace{(\sin^2 t + \cos^2 t)}_{1}}\, dt =$$

$$= 50\pi \int_{\arcsin\frac{3}{5}}^{\arcsin\frac{4}{5}} \cos t\, dt = 50\pi \sin t \Big|_{\arcsin\frac{3}{5}}^{\arcsin\frac{4}{5}} = 10\pi$$

№4

$$\int_1^{+\infty} \frac{x+1}{x^3+\sin x}\, dx$$

I рое

$q = 2 \quad q > 1$

сходе

$$\frac{x+1}{|x^3+\sin x|} \sim \frac{x}{x^3} < \frac{1}{x^2}$$

№5

$$\int_2^3 \frac{x-2}{x^3-3x^2+4}$$

II рое

$q = 2 \quad q > 1$

$$\frac{x-2}{x^3-3x^2+4} \sim \frac{1}{x^2-x+2} < \frac{1}{x^2}$$

не сходе