ИиДУ РК1 Билет №9 (22)

№1

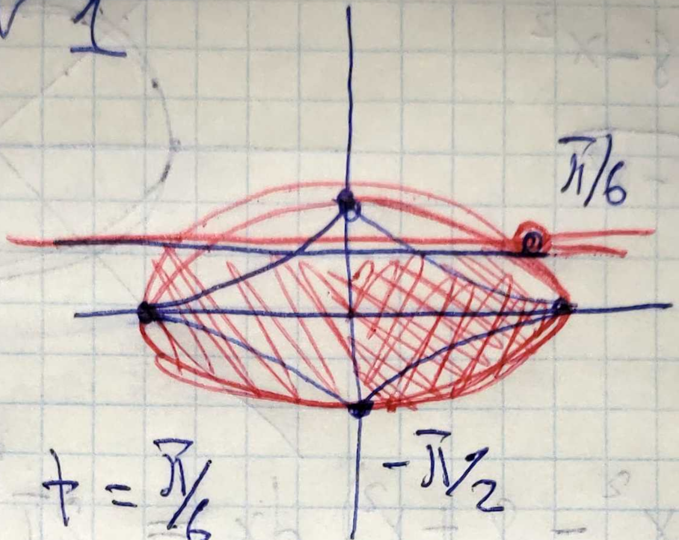
$$\begin{cases} x = 4\cos t \\ y = 2\sin t \end{cases}$$

t	0	$\pi/2$
x	4	0
y	0	2

$y = 1$

$2\sin t = 1$

$\sin t = \frac{1}{2} \quad t = \pi/6$

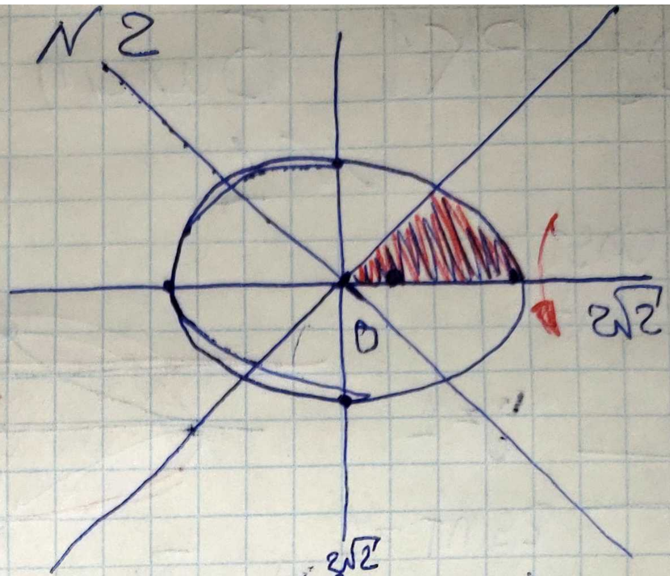


$$\frac{S}{2} = \int_{\pi/6}^{-\pi/2} -8\sin^2 t\,dt = -8\int_{\pi/6}^{-\pi/2} \frac{1-\cos 2t}{2}\,dt = \left(-4t + 2\sin 2t\right)\Big|_{\pi/6}^{-\pi/2}$$

$$= 2\pi + 0 + \frac{2\pi}{3} - \sqrt{3} = \frac{8\pi}{3} - \sqrt{3} > 0$$

$$S = 2\cdot\frac{S}{2} = \frac{16\pi}{3} - 2\sqrt{3}$$

№2



$y^2 = x^2$

$y^2 = 8 - x^2$

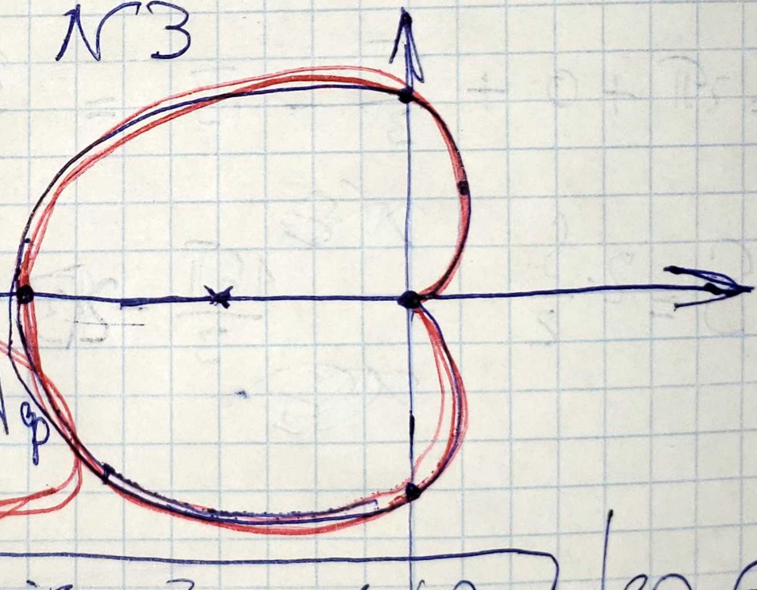
$y = \pm\sqrt{8 - x^2}$

$$V = \pi\int_0^{2\sqrt{2}} (x^2 - 8 + x^2)\, dx = 2\pi \int_0^{2\sqrt{2}} (x^2 - 4)\, dx =$$

$$= \left(\frac{2\pi x^3}{3} - 8\pi x\right)\Big|_0^{2\sqrt{2}} = \frac{32\pi\sqrt{2}}{3} - 1$$

№3

$\rho = 4(1 - \cos\varphi)$



$$l = \int \sqrt{r^2 + (r')^2}\, d\varphi$$

$$l = \int_0^{2\pi} \sqrt{(4 - 4\cos\varphi)^2 + 16\sin^2\varphi}\, d\varphi$$

$$= 4\int_0^{2\pi} \sqrt{\underset{1}{\cancel{16}} - \underset{2}{\cancel{32}}\cos\varphi + \underbrace{\cancel{16\cos^2\varphi + 16\sin^2\varphi}}_{1}}\; d\varphi =$$

$$\sqrt{\underbrace{2 - 2\cos\varphi}_{t}} \neq$$

$$\textcircled{=} 4\sqrt{2}\int_0^{2\pi} \boxed{\sqrt{1-\cos\varphi}}\, d\varphi = -8\sqrt{2}\sqrt{1+\cos x}\Big|_0^{2\pi}$$?

$$\int \sqrt{1-\cos x}\, d\varphi = \int \frac{\sqrt{1-\cos^2\varphi}}{\sqrt{1+\cos\varphi}}\, d\varphi = \int \frac{\sin\varphi}{\sqrt{1+\cos\varphi}}\, d\varphi =$$

$$= \left| \begin{array}{l} t = 1+\cos t \\ dx = -\frac{dt}{\sin x} \end{array} \right| = \int \frac{1}{\sqrt{t}}\, dt = \ldots = -2\sqrt{1+\cos\varphi} + C$$

N4

$$\int_1^{+\infty} \frac{\operatorname{arctg} x^3}{x^2+x}$$

I-род

$q = 2 \quad q > 1$

$$\frac{\pi}{2(x^2+2x)} \leq \frac{\pi}{2x^2}$$

сход

N5

$$\int_0^1 \frac{x\sqrt{x}}{\ln(1+x^2)}\, dx =$$

I-р

$q = \frac{1}{2} \quad q < 1$

$$\frac{\sqrt[3]{x}}{\ln(1+x^2)} \leq \frac{x^{\frac{3}{2}}}{x^2} \leq \frac{1}{\sqrt{x}}$$

$\sim x^2$

сход