Бушуев Денис РЛ2-31 Билет 2 21.01.21.

№1.

$U_{tt} = 4U_{xx}$ $\quad 0<x<3$, $0<t<\infty$ $\quad U(x,0) = 8\sin\pi x$, $U_t(x,0) = 8\pi\sin 4\pi x$, $U(0,t)=0$

$U(3,t)=0$

Пусть $U(x,t) = X(x)\,T(t) \not\equiv 0$

$X T'' = 4TX'' \Rightarrow \frac{T''}{T} = 4\frac{X''}{X} = -\lambda$

разделяющиеся переменные

(I)

$$\begin{cases} X'' + \frac{1}{4}X = 0 \\ X(0)=0 \\ X(3)=0 \end{cases}$$ Решим задачу Штурма - Лиувилля.

1) $\lambda = 0$ $\begin{cases} X''=0 \\ X(0)=0 \\ X(3)=0 \end{cases}$

$X = c_1 x + c_2$

$$\begin{cases} c_2 = 0 \\ 3c_1 = 0 \end{cases}$$

$\begin{cases} c_2 = 0 \\ c_1 = 0 \end{cases} \Rightarrow \lambda=0$ - не С.З., $X=0$ - не С.Ф.

2) $\lambda = -\omega^2 < 0$

$$\begin{cases} X'' - \frac{1}{4}\omega^2 X = 0 \\ X(0)=0 \\ X(3)=0 \end{cases}$$

$X = c_1 e^{\frac{\omega}{2}x} + c_2 e^{-\frac{\omega}{2}x}$

$$\begin{cases} c_1 + c_2 = 0 \\ c_1 e^{\frac{3}{2}\omega} + c_2 e^{-\frac{3}{2}\omega} = 0 \end{cases}$$

$$\begin{pmatrix} 1 & 1 \\ e^{\frac{3}{2}\omega} & e^{-\frac{3}{2}\omega} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \begin{vmatrix} 1 & 1 \\ e^{\frac{3}{2}\omega} & e^{-\frac{3}{2}\omega} \end{vmatrix} = \underbrace{e^{\frac{3}{2}\omega} - e^{-\frac{3}{2}\omega}}_{\text{дисперсионное ур-е.}} = 0 \Rightarrow c_1 = c_2 = 0$$

$\lambda < 0$ - не С.З., $X=0$ - не С.Ф.

3) $\lambda = \omega^2 > 0$

$$\begin{cases} X'' + \frac{\omega^2}{4}X = 0 \\ X(0)=0 \\ X(3)=0 \end{cases}$$

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$X = c_1\cos\frac{\omega}{2}x + c_2\sin\frac{\omega}{2}x$

$$\begin{cases} c_1 = 0 \\ c_2\sin\frac{3\omega}{2} = 0 \end{cases} \Rightarrow c_2 \neq 0 \Rightarrow \sin\frac{3\omega}{2} = 0 \qquad \frac{3\omega}{2} = \pi n,\ n = \overline{1,\infty}$$

$$\omega = \frac{2\pi n}{3},\ n = \overline{1,\infty}$$

$\lambda_n = \left(\frac{2\pi n}{3}\right)^2$ - С.З. $n = \overline{1,\infty}$

$X = \cancel{\frac{\pi n}{3}} \sin\frac{\pi n}{3}x$ - С.Ф. $n = \overline{1,\infty}$

$$\|X_n\|^2 = \int_0^3 \sin^2\frac{\pi n}{3}x\,dx = \frac{1}{2}\left[\int_0^3 dx - \int_0^3 \cos\frac{2\pi n}{3}x\,dx\right] = \frac{3}{2} - \frac{3}{2\pi n}\sin\frac{2\pi n}{3}x\Big|_0^3 = \cancel{\frac{3}{2}} = \frac{3}{2}$$

(II) $\frac{T''}{T} = -\lambda \Rightarrow T'' + \left(\frac{2\pi n}{3}\right)^2 T = 0$

$T_n = c_1\cos\frac{2\pi n}{3}t + c_2\sin\frac{2\pi n}{3}t$

Билет № 2

III $U(x,t) = \sum_{n=1}^{\infty} X(x)\cdot T(t)$

$$U(x,t) = \sum_{n=1}^{\infty} \left(A_n \cos\frac{2\pi n}{3}t + B_n \sin\frac{2\pi n}{3}t\right)\sin\frac{\pi n}{3}x$$

$$U_t(x,t) = \sum_{n=1}^{\infty}\left(-A_n \frac{2\pi n}{3}\sin\frac{2\pi n}{3}t + B_n\cdot\frac{2\pi n}{3}\cos\frac{2\pi n}{3}t\right)\cdot\sin\frac{\pi n}{3}x$$

IV 1 $U(x,0) = 8\sin\pi x$ Учтем условие ортогон. собст фун

$$8\sin\pi x = \sum_{n=1}^{\infty} A_n \sin\frac{\pi n}{3}x \;\Big|\cdot \sin\frac{\pi m}{3}x$$

$$8\int \sin\pi x \sin\frac{\pi m}{3}x\,dx = A_n \int \sin\frac{\pi n}{3}x\cdot\sin\frac{\pi m}{3}x\,dx$$

$$8\cdot\cancel{\frac{3}{2}}\cdot\begin{cases}1, & m=3\\ 0, & m\neq 3\end{cases} = A_n\cdot\cancel{\frac{3}{2}}\begin{cases}1, & m=n\\ 0, & m\neq n\end{cases}$$

$$8 = A_3,\quad A_n = 0 \text{ при } \forall n\neq 3$$

2 $U_t(x,0) = 8\pi\sin 4\pi x$ Учтем условие ортогональности собст фун

$$8\pi\sin 4\pi x = \sum_{n=1}^{\infty} B_n \frac{2\pi n}{3}\cdot\sin\frac{\pi n}{3}x$$

$$8\pi\int \sin 4\pi x\cdot\sin\frac{\pi m}{3}x\,dx = B_n\frac{2\pi n}{3}\int \sin\frac{\pi n}{3}\sin\frac{\pi m}{3}x\,dx$$

$$8\pi\cdot\cancel{\frac{3}{2}}\cdot\begin{cases}1, & m=12\\ 0, & m\neq 12\end{cases} = B_n\frac{2\pi m}{3}\cdot\cancel{\frac{3}{2}}\cdot\begin{cases}1, & n=m\\ 0, & n\neq m\end{cases}$$

$$B_{12} = 1 \qquad B_n = 0 \quad \forall n\neq 12$$

V $U(x,t) = A_3 \cos 2\pi t \sin\pi x + B_{12}\cdot\sin 8\pi t\cdot\sin 4\pi x$

$$U(x,t) = 8\cos 2\pi t\cdot\sin\pi x + \sin 8\pi t\cdot\sin 4\pi x$$

№2.

$\Delta U = 0 \qquad 0 \le r < 4 \qquad 0 \le \varphi \le 2\pi \qquad U(4,\varphi) = \cos^3\varphi + 4\sin^3\varphi + \sin^2\varphi$

Пусть $U = \Phi(\varphi)\cdot R(r) \not\equiv 0$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 U}{\partial \varphi^2} = 0$$

$$\frac{\Phi}{r}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) + \frac{R}{r^2}\frac{\partial \Phi}{\partial \varphi^2} = 0$$

$$\frac{\Phi}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right) + \frac{R}{r^2}\frac{d^2\Phi}{d\varphi^2} = 0 \quad \Big| \cdot \frac{r^2}{R\Phi}$$

$$\frac{r}{R}\frac{d}{dr}\left(r\frac{dR}{dr}\right) + \frac{\Phi''}{\Phi} = -\lambda$$

(I) $\begin{cases} \Phi'' + \lambda\Phi = 0 \\ \Phi(0) = \Phi(2\pi) \\ \Phi'(0) = \Phi'(2\pi) \end{cases}$

(1) $\lambda = 0 \quad \Phi'' = 0 \qquad \Phi = C_1\varphi + C_2, \quad \Phi' = C_1 \qquad \begin{cases} C_2 = 2\pi C_1 + C_2 \\ C_1 = C_1 \end{cases} \Rightarrow \begin{matrix} C_1 = 0 \\ C_2 \ne 0 \end{matrix}$

$\Phi = C$ - С.Ф.

$\lambda = 0$ - С.З.

(2) $\lambda = -\omega^2 < 0$

$\begin{cases} \Phi'' - \omega^2\Phi = 0 \\ \Phi(0) = \Phi(2\pi) \\ \Phi'(0) = \Phi'(2\pi) \end{cases}$

$\Phi = C_1e^{\omega\varphi} + C_2e^{-\omega\varphi}$

$\Phi' = C_1\omega e^{\omega\varphi} - C_2\omega e^{-\omega\varphi}$

$\begin{cases} C_1 + C_2 = C_1e^{2\pi\omega} + C_2e^{-2\pi\omega} \\ C_1\omega - C_2\omega = C_1\omega e^{2\pi\omega} - C_2\omega e^{-2\pi\omega} \quad |:\omega \end{cases}$

$$\begin{pmatrix} 1-e^{2\pi\omega} & 1-e^{-2\pi\omega} \\ 1-e^{2\pi\omega} & -(1-e^{-2\pi\omega}) \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\begin{vmatrix} 1-e^{2\pi\omega} & 1-e^{-2\pi\omega} \\ 1-e^{2\pi\omega} & -(1-e^{-2\pi\omega}) \end{vmatrix} = -2(1-e^{2\pi\omega})(1-e^{-2\pi\omega}) = 0 \Rightarrow C_1 = C_2 = 0 \Rightarrow \lambda < 0$ - не С.З.

$\Phi = 0$ - не С.Ф.

③ $\lambda = \omega^2 > 0$

$\Phi'' + \omega^2 \Phi = 0$

$\Phi = c_1 \cos \omega\varphi + c_2 \sin \omega\varphi$

$\Phi' = -c_1\omega \sin\omega\varphi + c_2 \omega\cos\omega\varphi$

$$\begin{cases} c_1 = c_1\cos 2\pi\omega + c_2 \sin 2\pi\omega \\ c_2\omega = -c_1\omega\sin 2\pi\omega + c_2\omega\cos 2\pi\omega \end{cases}$$

$$\begin{vmatrix} 1-\cos 2\pi\omega & -\sin 2\pi\omega \\ \sin 2\pi\omega & 1-\cos 2\pi\omega \end{vmatrix} = 0$$

$(1-\cos 2\pi\omega)^2 + \sin^2 2\pi\omega = 0$

$1 - 2\cos 2\pi\omega + \cos^2 2\pi\omega + \sin^2 2\pi\omega = 0$

$1 - 2\cos 2\pi\omega + 1 = 0$

$\cos 2\pi\omega = 1$

$2\pi\omega = 2\pi n,\ n = \overline{1,\infty}$

$\omega = n;\ \lambda_n = n^2$ — С.З.

$$\begin{cases} c_1 = c_1 \\ c_2 = c_2 \end{cases}$$

$\Phi = \begin{cases} \cos n\varphi \\ \sin n\varphi \end{cases} \quad n = \overline{1,\infty}$

$\|\Phi_n\|^2 = \pi$

$\Phi_0 = C,\ \lambda_0 = 0;\ \ \|\Phi_0\|^2 = 2\pi$

Ⅱ $\frac{r}{R}\frac{d}{dr}\left(r\frac{dR}{dr}\right) = \lambda \ |\cdot R$

$r\frac{d}{dr}\left(r\frac{dR}{dr}\right) - n^2R = 0$

$r^2R'' + rR' - n^2R = 0$ - уравнение Эйлера.

Пусть $r = e^t$ $\quad R(r) \to y(t)$

$e^t\frac{d}{de^t}\left(e^t\frac{dR}{de^t}\right) - n^2R = 0$

$\frac{d}{dt}\left(\frac{dR}{dt}\right) - n^2R = 0$

$y'' - n^2y = 0$

1) $n \neq 0$

$y = c_1e^{nt} + c_2e^{-nt}$

$\lambda_n = n^2,\ n = \overline{0,\infty}$

2) $n = 0$

$y'' = 0 \quad y = c_1t + c_2$

$n \neq 0 \quad R(r) = c_1r^n + c_2\frac{1}{r^n}$

$n = 0 \quad R(r) = c_1\ln r + c_2$

нас интересуют решения при $r \to 0$.
$\ln r$ при $r \to 0$ расходится.

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$$\begin{cases} R(r) = C_2 \\ R(r) = C_1 r^n \end{cases}$$

$R(r) = r^n, \quad n = \overline{0,\infty}$

III) $u = R(r)\Phi(\varphi)$

$$\cancel{u = \sum_{n=0}^{\infty} r^n \{A_n \cos\varphi n + B_n \sin\varphi n\}} \quad u = A_0 + \sum_{n=1}^{\infty} r^n \{A_n \cos n\varphi + B_n \sin n\varphi\}$$

$$u(4,\varphi) = \cos^3\varphi + 4\sin^3\varphi + \sin^2\varphi$$

$$u(4,\varphi) = \frac{\cos\varphi}{2} + \frac{\cos\varphi\cos 2\varphi}{2} + 2\sin\varphi - 2\sin\varphi\cos 2\varphi + \frac{1}{2} - \frac{\cos 2\varphi}{2} = \frac{\cos\varphi}{2} + \frac{\cos\varphi}{4} + \frac{\cos 3\varphi}{4} + 2\sin\varphi + \sin\varphi - \sin 3\varphi + \frac{1}{2} - \frac{\cos 2\varphi}{2} = \frac{3\cos\varphi}{4} + \frac{\cos 3\varphi}{4} + 3\sin\varphi - \sin 3\varphi - \frac{\cos 2\varphi}{2} + \frac{1}{2}$$

$$A_0 = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{3\cos\varphi}{4} + \frac{\cos 3\varphi}{4} + 3\sin\varphi - \sin 3\varphi - \frac{\cos 2\varphi}{2} + \frac{1}{2}\right]d\varphi = \frac{1}{2\pi}\left[\frac{3\sin\varphi}{4} + \frac{\sin 3\varphi}{12} - 3\cos\varphi + \frac{\cos 3\varphi}{3} + \frac{\sin 2\varphi}{4} + \frac{\varphi}{2}\right]\Big|_0^{2\pi} = \cancel{\frac{1}{2\pi}[0+0+3\pi-(6-3)+\tfrac{1}{3}]} = \frac{1}{2\pi}\left[0 + 0 - 3 + \frac{1}{3} + 0 + \pi - 0 - 0 + 3 - \frac{1}{3} + 0 + 0\right] = \frac{1}{2}$$

$$A_n = \frac{1}{\pi}\int_0^{2\pi}\left[\frac{3\cos\varphi}{4} + \frac{\cos 3\varphi}{4} + 3\sin\varphi - \sin 3\varphi - \frac{\cos 2\varphi}{2} + \frac{1}{2}\right]\cdot\cos n\varphi\, d\varphi$$

Расс-им 6 интегралов:

$$I_1 = \frac{3}{4\pi}\int_0^{2\pi}\cos\varphi\cos n\varphi\, d\varphi = \frac{3}{8\pi}\left[\frac{\sin(1-n)\varphi}{1-n} + \cancelto{0}{\frac{\sin(1+n)\varphi}{1+n}}\right]\Big|_0^{2\pi} = \frac{3}{8\pi}\cdot\frac{2\pi\sin(1-n)2\pi}{(1-n)2\pi} = \frac{3}{4}\begin{cases}1, & n=1\\ 0, & n\neq 1\end{cases}$$

$$I_2 = \frac{1}{4\pi}\int_0^{2\pi}\cos 3\varphi\cos n\varphi\, d\varphi = \frac{1}{8\pi}\left[\frac{\sin(3-n)\varphi}{3-n} + \frac{\sin(3+n)\varphi}{3+n}\right]\Big|_0^{2\pi} = \frac{1}{8\pi}\cdot\frac{2\pi\sin(3-n)2\pi}{(3-n)2\pi} = \frac{1}{4}\begin{cases}1, & n=3\\ 0, & n\neq 3\end{cases}$$

$$I_3 = \frac{3}{\pi}\int_0^{2\pi}\sin\varphi\cos n\varphi\, d\varphi = -\frac{3}{2\pi}\left[\frac{\cos(1-n)\varphi}{1-n} + \frac{\cos(1+n)\varphi}{1+n}\right]_0^{2\pi} = -\frac{3}{2\pi}\left[\frac{\cos(1-n)2\pi}{1-n} + \frac{\cos(1+n)2\pi}{1+n} - \frac{1}{1-n} - \frac{1}{1+n}\right] = 0$$

$$I_4 = -\frac{1}{\pi}\int_0^{2\pi}\sin 3\varphi\cos n\varphi\, d\varphi = \frac{1}{2\pi}\left[\frac{\cos(3-n)\varphi}{3-n} + \frac{\cos(3+n)\varphi}{3+n}\right]_0^{2\pi} = 0, \quad \forall n$$

$$I_5 = \frac{1}{2\pi}\int_0^{2\pi}\cos 2\varphi\cos n\varphi\, d\varphi = \frac{1}{4\pi}\left[\frac{\sin(2-n)\varphi}{2-n} + \frac{\sin(2+n)\varphi}{2+n}\right]\Big|_0^{2\pi} = \frac{1}{4\pi}\cdot\frac{2\pi\sin(2-n)2\pi}{(2-n)2\pi} = \frac{1}{2}\begin{cases}1, & n=2\\ 0, & n\neq 2\end{cases}$$

$$I_6 = \frac{1}{2\pi}\int_0^{2\pi}\cos n\varphi\, d\varphi = \frac{1}{2\pi n}\sin n\varphi\Big|_0^{2\pi} = 0$$

$$B_n = \frac{1}{\pi}\int_0^{2\pi}\left[\frac{3\cos\varphi}{4} + \frac{\cos 3\varphi}{4} + 3\sin\varphi - \sin 3\varphi - \frac{\cos 2\varphi}{2} + \frac{1}{2}\right]\cdot\sin n\varphi\, d\varphi$$

Расс-им 6 интегралов:

$$I_7 = \frac{3}{4\pi}\int_0^{2\pi}\sin n\varphi\cos\varphi\, d\varphi = -\frac{3}{8\pi}\left[\frac{\cos(n-1)\varphi}{n-1} + \frac{\cos(n+1)\varphi}{n+1}\right]\Big|_0^{2\pi} = 0 \quad \forall n$$

$$I_8 = \frac{1}{4\pi}\int_0^{2\pi}\sin n\varphi\cos 3\varphi\, d\varphi = -\frac{1}{8\pi}\left[\frac{\cos(n-3)\varphi}{n-3} + \frac{\cos(n+3)\varphi}{n+3}\right]\Big|_0^{2\pi} = 0 \quad \forall n$$

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$$I_9 = \frac{3}{\pi}\int_0^{2\pi} \sin n\varphi \sin\varphi \, d\varphi = \frac{3}{2\pi}\left[\frac{\sin(n-1)\varphi}{n-1} - \frac{\sin(n+1)\varphi}{n+1}\right]\Bigg|_0^{2\pi} = \frac{3}{2\pi}\cdot\frac{2\pi \sin(n-1)2\pi}{(n-1)2\pi} = 3\begin{cases}1, & n=1\\ 0, & n\neq 1\end{cases}$$

$$I_{10} = -\frac{1}{\pi}\int_0^{2\pi} \sin n\varphi \sin 3\varphi \, d\varphi = -\frac{1}{2\pi}\left[\frac{\sin(n-3)\varphi}{n-3} - \frac{\sin(n+3)\varphi}{n+3}\right]\Bigg|_0^{2\pi} = -\frac{1}{2\pi}\cdot\frac{2\pi \sin(n-3)2\pi}{(n-3)2\pi} = -1\begin{cases}1, & n=3\\ 0, & n\neq 3\end{cases}$$

$$I_{11} = -\frac{1}{2\pi}\int_0^{2\pi} \sin n\varphi \cos 2\varphi \, d\varphi = \frac{1}{4\pi}\left[\frac{\cos(n-2)\varphi}{n-2} + \frac{\cos(n+2)\varphi}{n+2}\right]\Bigg|_0^{2\pi} = 0 \quad \forall n$$

$$I_{12} = \frac{1}{2\pi}\int_0^{2\pi} \sin n\varphi \, d\varphi = -\frac{1}{2\pi n}\cos n\varphi\Big|_0^{2\pi} = 0 \quad \forall n$$

$A_1 = \frac{3}{4}$ $\quad A_3 = \frac{1}{4}$ $\quad A_2 = \frac{1}{2}$

$B_1 = 3$ $\quad B_3 = -1$

$$U = \frac{1}{2} + r\cdot\left(\frac{3}{4}\cdot\cos\varphi + 3\sin\varphi\right) + r^2\cdot\frac{1}{2}\cos 2\varphi + r^3\left(\frac{1}{4}\cos 3\varphi - \sin 3\varphi\right)$$

№3.

а) ~~$\Delta<0 \quad a_{12}^2 - a_{11}a$~~

$$a_{11}U_{xx} + 2a_{12}U_{xy} + a_{22}U_{yy} + F(x,y,U,U_x,U_y) = 0$$

$\Delta < 0 \qquad a_{12}^2 - a_{11}a_{22} < 0$

λ_1, λ_2 – будут различны и комплексно сопряженны

$\Downarrow$

$\xi(x,y) = c_1 \qquad \xi^*(x,y) = c_2$

$\Downarrow$

$\xi(x,y) = c_1$

$\alpha = \operatorname{Re} c_1 \qquad \beta = \operatorname{Im} c_1$

действительная часть $\quad$ мнимая часть

$U_{\alpha\alpha} + U_{\beta\beta} = \bar{F}$ – канонический вид эллиптического типа.

№ $\cancel{2}$3 (б)

$$U_{xx}+4U_{xy}+U_{yy}+U_x\overset{+U_y}{\vee}-x^2y=0$$

$a_{11}=1$ $\cancel{\ }a_{12}=a_{21}=2$ $a_{22}=1$ Гиперболический ~~Параболический~~ тип

$$\Delta=a_{12}^2-a_{11}\cdot a_{22}=4-1=3>0$$

$$\frac{dy}{dx}=\frac{a_{12}+\sqrt{\Delta}}{a_{11}}\Rightarrow dy=(2+\sqrt{3})dx\Rightarrow y=2x+\sqrt{3}x+C_1\Rightarrow C_1=y-2x-\sqrt{3}x$$

$$\frac{dy}{dx}=\frac{a_{12}-\sqrt{\Delta}}{a_{11}}\Rightarrow dy=(2-\sqrt{3})dx\Rightarrow y=2x-\sqrt{3}x+C_2\Rightarrow C_2=y-2x+\sqrt{3}x$$

Введем ξ и η: $\xi_x=-2-\sqrt{3}$ $\xi_y=1$ $\eta_x=\sqrt{3}-2$ $\eta_y=1$

$$U_x=U_\xi\cdot\frac{\partial\xi}{\partial x}+U_\eta\cdot\frac{\partial\eta}{\partial x}=\cancel{U_\eta-U_\xi}\,(-2-\sqrt{3})U_\xi+(\sqrt{3}-2)U_\eta$$

$$U_y=U_\xi\cdot\frac{\partial\xi}{\partial y}+U_\eta\frac{\partial\eta}{\partial y}=U_\eta+U_\xi$$

$$U_{xx}=\cancel{\frac{\partial(U_x)}{\partial x}}=(2+\sqrt{3})^2U_{\xi\xi}+\cancel{2}U_{\xi\eta}+(\sqrt{3}-2)^2U_{\eta\eta}$$

$$U_{yy}=U_{\eta\eta}+2U_{\xi\eta}+U_{\xi\xi}$$

$$U_{xy}=-2U_{\xi\xi}-\sqrt{3}U_{\xi\xi}+\cancel{\sqrt{3}U_{\xi\eta}}-2U_{\xi\eta}\mp2U_{\xi\eta}-\cancel{\sqrt{3}U_{\xi\eta}}+\sqrt{3}U_{\eta\eta}-2U_{\eta\eta}$$

$$=(-2-\sqrt{3})U_{\xi\xi}-4U_{\xi\eta}+(\sqrt{3}-2)U_{\eta\eta}$$

$$7\cancel{U_{\xi\xi}}+4\cancel{\sqrt{3}}U_{\xi\xi}+\cancel{U_{\xi\eta}}+\cancel{7U_{\eta\eta}}-\cancel{4\sqrt{3}U_{\eta\eta}}-\cancel{8U_{\xi\xi}}-\cancel{4\sqrt{3}U_{\xi\xi}}-1\overset{3}{\cancel{6}}U_{\xi\eta}+\cancel{4\sqrt{3}U_{\eta\eta}}$$

$$-\cancel{8U_{\eta\eta}}+\cancel{U_{\eta\eta}}+\cancel{2U_{\xi\eta}}+\cancel{U_{\xi\xi}}-2U_\xi-\sqrt{3}U_\xi\overset{+}{-\cancel{4}}U_\xi+\cancel{4}U_\eta+x^2y=0$$

$$U_{\xi\eta}=\frac{2}{13}U_\xi+\frac{\sqrt{3}}{13}U_\xi-\frac{1}{13}U_\xi-\frac{1}{13}U_\eta+x^2y=0$$

$\xi=y-2x-\sqrt{3}x$

$\eta=y-2x+\sqrt{3}x$