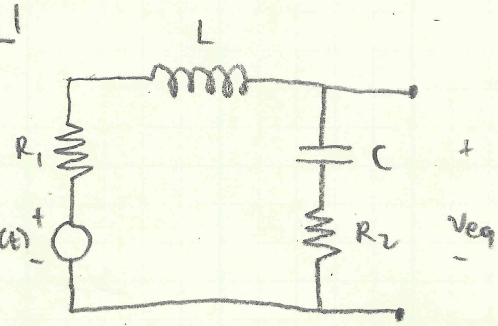
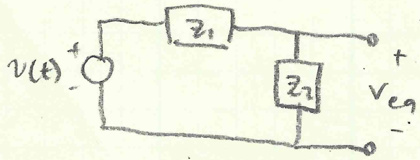
Problem 1

Given:



Find: V_{eq} and Z_{eq}



$$Z_1 = Z_L + Z_{R_1} = sL + R_1$$

$$Z_2 = Z_C + Z_{R_2} = \frac{1}{sC} + R_2 = \frac{sR_2C + 1}{sC}$$

$$V_{eq} = \frac{Z_2}{Z_1 + Z_2} v(t)$$

$$= \frac{\frac{sR_2C+1}{sC}}{sL + R_1 + \frac{sR_2C+1}{sC}} v(t)$$

$$= \frac{sR_2C+1}{s^2LC + sR_1C + sR_2C + 1} v(t) = \frac{sR_2C+1}{s^2LC + sC(R_1+R_2) + 1} v(t)$$

$$\boxed{V_{eq} = \frac{sR_2C+1}{s^2LC + sC(R_1+R_2)+1} v(t)}$$

$$Z_{eq} = Z_1 + Z_2$$

$$= sL + R_1 + \frac{sR_2C+1}{sC}$$

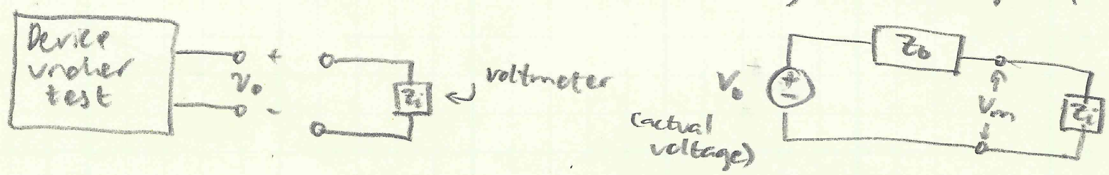
$$= \frac{s^2LC + sR_1C + sR_2C + 1}{sC}$$

$$\boxed{Z_{eq} = \frac{s^2LC + sC(R_1+R_2)+1}{sC}}$$

Problem 2

- Why should a voltmeter have high resistance?

Ans: Below is a diagram of measuring the voltage of a device.



The transfer function of the measured voltage to the actual voltage is $G(s) = \frac{V_m}{V_o} = \frac{1}{1+\frac{Z_o}{Z_i}}$. Since we want V_m to be as close to V_o as possible, $G(s)$ should be close to 1.

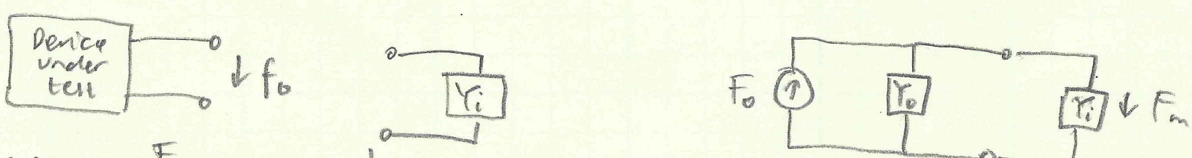
when $Z_i >> Z_o$, $G(s) = \frac{1}{1+\underbrace{\frac{Z_o}{Z_i}}_{\to 0}} = 1$.

$Z_i = R_i$

Therefore, the voltmeter needs higher resistance to increase its impedance.

- Why should the resistance of an ammeter should be high?

Below is the diagram of measuring the current of a system.



$$G(s) = \frac{F_m}{F_o} = \frac{1}{1+\frac{Y_o}{Y_i}}$$

We want $G(s) = 1$ to accurately measure current.

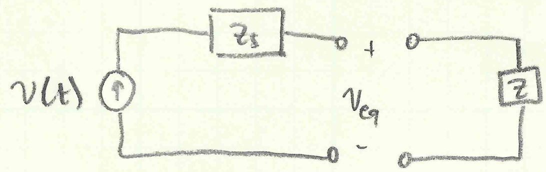
when $Y_i >> Y_o$, $G(s) = \frac{1}{1+\underbrace{\frac{Y_o}{Y_i}}_{\to 0}} = 1$

$Y_i = \frac{1}{R_i}$

Therefore, the ammeter needs to have really low resistance to ensure that its admittance is as low as possible

Problem 3

Given:



Show that the power dissipated is maximum when $R = R_s$

$$p = v\,i$$

$$p_{dissipated} = v_{eq} \times i$$

$$v_{eq} = \frac{Z}{Z_s + Z} = \frac{R}{R_s + R} v \quad , \quad i = \frac{v}{R_s + R}$$

$$p_{dissipated} = \frac{R}{R_s + R} v \times \frac{v}{R_s + R} = \frac{R}{(R_s + R)^2} v^2$$

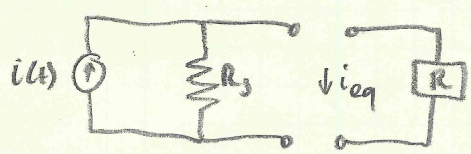
maximize $p_{dissipated}$ while charging R

$$\frac{dp(R)}{dR} = -\frac{R - R_s}{(R - R_s)^3} v^2 = 0 \leftarrow \text{maximum value}$$

$$\therefore \boxed{R = R_s} \text{ for maximum value}$$

Problem 4

Given:



Show that $p_{dissipated}$ in the resistive element is max when $R = R_s$

$$p = v\,i \quad , \quad p_{dissipated} = v_{eq} \times i_{eq}$$

$$i_{eq} = \frac{Y}{Y_s + Y} i = \frac{\frac{1}{R}}{\frac{1}{R_s} + \frac{1}{R}} i = \frac{\frac{1}{R}}{\frac{R + R_s}{R_s R}} i = \frac{R_s}{R + R_s} i \quad , \quad v = i\left(\frac{1}{R} + \frac{1}{R_s}\right)^{-1}$$

$$p_{dissipated} = \frac{R_s}{R + R_s} i \times i \times \frac{R R_s}{R + R_s} = \frac{R R_s^2}{(R + R_s)^2} i^2$$

maximize $p_{dissipated}$ while charging R

$$\frac{dp(R)}{dR} = -\frac{R_s^2 (R - R_s)}{(R + R_s)^3} = 0 \quad , \quad \boxed{\therefore R = R_s} \text{ for maximum value}$$