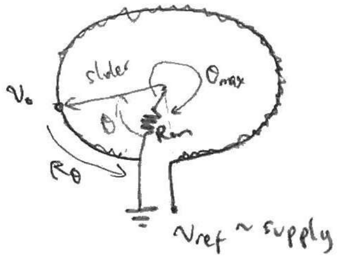
Problem 1

- Derive the expression for electrical-loading nonlinearity error in a rotary potentiometer in terms of θ, θ_{max}, R_θ, R_M
- Plot the percentage error as a function of the fractional displacement for $R_m/R_{max} = 0.1, 1.0$ and 10.0.
- Determine the angular displacement of a rotary potentiometer at which the loading nonlinearity error is the largest.



$$\frac{V_{ref} - V_o}{R_{max} - R_\theta} = \frac{V_o}{R_\theta} + \frac{V_o}{R_m}, \quad R_\theta = \frac{\theta}{\theta_{max}} R_{max}$$

$$\frac{V_{ref} - V_o}{R_{max} - \frac{\theta}{\theta_{max}} R_{max}} = \frac{V_o}{\frac{\theta}{\theta_{max}} R_{max}} + \frac{V_o}{R_m}$$

$$\Rightarrow \frac{V_o}{V_{ref}} = \frac{\theta/\theta_{max} \cdot R_m/R_{max}}{\frac{R_m}{R_{max}} + \frac{\theta}{\theta_{max}} - \left(\frac{\theta}{\theta_{max}}\right)^2}$$

non-linearity error

$$\varepsilon = \frac{V_o/V_{ref} - \theta/\theta_{max}}{\theta/\theta_{max}} \cdot 100\%$$

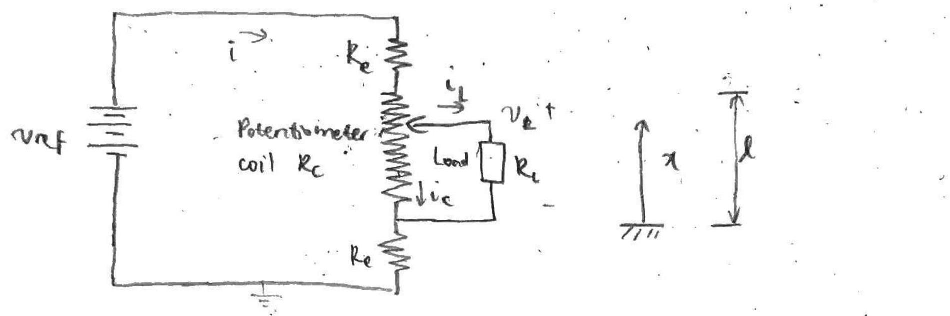
$$\varepsilon = \frac{\dfrac{\theta/\theta_{max} \cdot R_m/R_{max}}{\frac{R_m}{R_{max}} + \frac{\theta}{\theta_{max}} - \left(\frac{\theta}{\theta_{max}}\right)^2} - \dfrac{\theta}{\theta_{max}}}{\theta/\theta_{max}} \times 100\%$$

$$\boxed{\varepsilon = \left(\frac{R_m/R_{max}}{\frac{R_m}{R_{max}} + \frac{\theta}{\theta_{max}} - \left(\frac{\theta}{\theta_{max}}\right)^2} - 1 \right) \times 100\%}$$

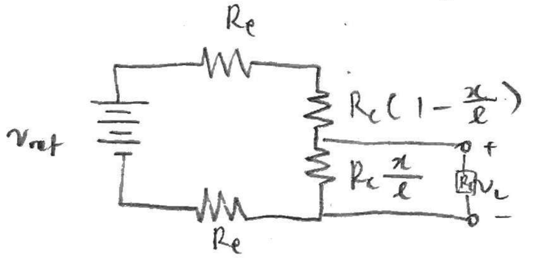
- See appendix for the plots

- From the plot, we can tell that the lowest point for every R_m/R_{max} value is at $\frac{\theta}{\theta_{max}} = 0.5$

Problem 2



- Derive the corresponding displacement output voltage relation.
- Normalize the relationship with respect to the maximum displacement and the maximum output voltage.
- Comment on the effect of the end resistors on the sensor output (or sensitivity) and variations in the supply voltage.



$$v_L = \frac{\left(\frac{1}{R_c\frac{x}{L}}+\frac{1}{R_L}\right)^{-1}}{2R_e + R_c(1-\frac{x}{L}) + \left(\frac{1}{R_c\frac{x}{L}}+\frac{1}{R_L}\right)^{-1}} v_{ref}$$

$$= \frac{\frac{R_c R_L \frac{x}{L}}{R_c\frac{x}{L}+R_L}}{2R_e + R_c(1-\frac{x}{L}) + \frac{R_c R_L \frac{x}{L}}{R_c\frac{x}{L}+R_L}} v_{ref}$$

$$\boxed{\frac{v_L}{v_{ref}} = \frac{R_c R_L \frac{x}{L}}{\left[2R_e + R_c(1-\frac{x}{L})\right](R_c\frac{x}{L}+R_L) + R_c R_L\frac{x}{L}}}$$

$=$

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Problem 3

- Derive an expression for the sensitivity of a rotary potentiometer as a function of displacement.
- Plot when $R_L/R_c = 0.1, 1.0,$ and 10.0
- Where does the maximum sensitivity occur?
- Verify with analytical expression

$$S = \frac{\partial y_0}{\partial x} = \frac{\partial}{\partial x}\left[\frac{\theta/\theta_{max} \; R_m/R_{max}}{\frac{R_m}{R_{max}} + \frac{\theta}{\theta_{max}} - \left(\frac{\theta}{\theta_{max}}\right)^2}\right] \qquad y = \frac{v_0}{v_{ref}} \qquad x = \frac{\theta}{\theta_{max}}$$

$$\boxed{S = \frac{\frac{R_m}{R_{max}}\left(\left(\frac{\theta}{\theta_{max}}\right)^2 + \frac{R_m}{R_{max}}\right)}{\left(\left(\frac{\theta}{\theta_{max}}\right)^2 - \frac{\theta}{\theta_{max}} - \frac{R_m}{R_{max}}\right)^2}}$$

- See MATLAB Plot

- Maximum sensitivity

max at $\theta/\theta_{max} = 1$

$$S_{max} = \frac{\frac{R_m}{R_{max}}\left(1 + \frac{R_m}{R_{max}}\right)}{\left(1 - 1 - \frac{R_m}{R_{max}}\right)^2} \Rightarrow S_{max} = \frac{\left(1 + \frac{R_m}{R_{max}}\right)}{\frac{R_m}{R_{max}}}$$

$$\boxed{S_{max} = \frac{R_{max}}{R_m} + 1}$$

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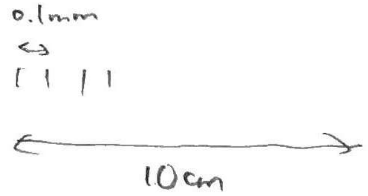
Problem 4

The range of a coil-type pot is 10cm
If the wire diameter = 0.1 mm, find the resolution.

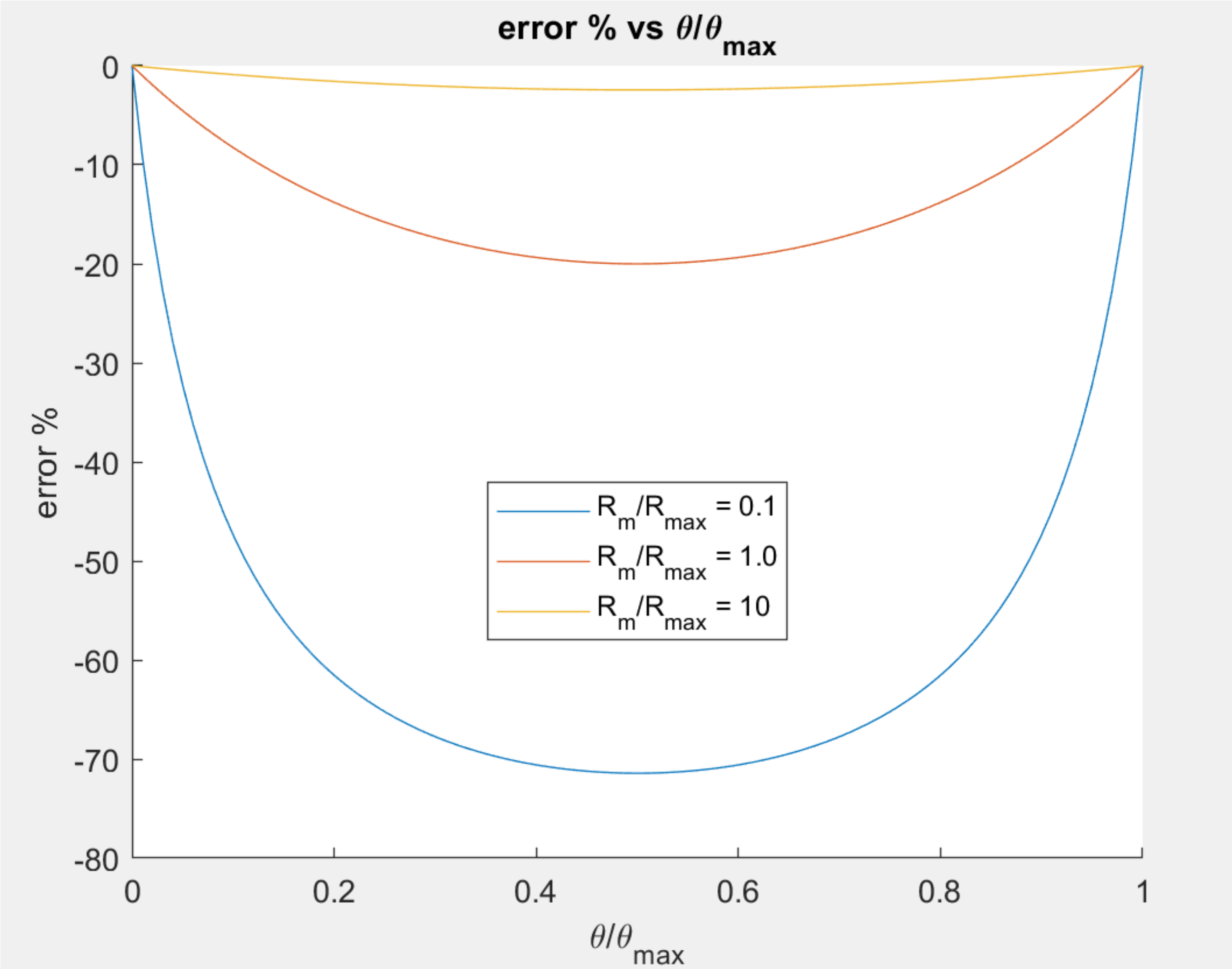
$$r = \frac{100}{N} \%$$



$$N = \frac{10 \times 10^{-2}}{0.1 \times 10^{-3}} = 1000$$

$$r = \frac{100}{1000} \% , \quad \boxed{r = 0.1\%}$$

Problem 1

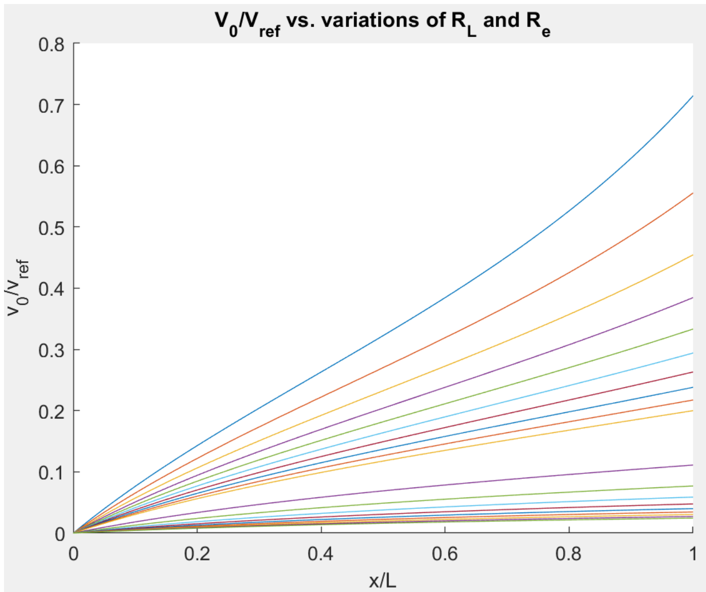


We ideally would like R_m/R_{max} to be as large as possible to avoid high error percentage.

However, we observe in problem 3 that making R_m/R_{max} causes higher nonlinearity.

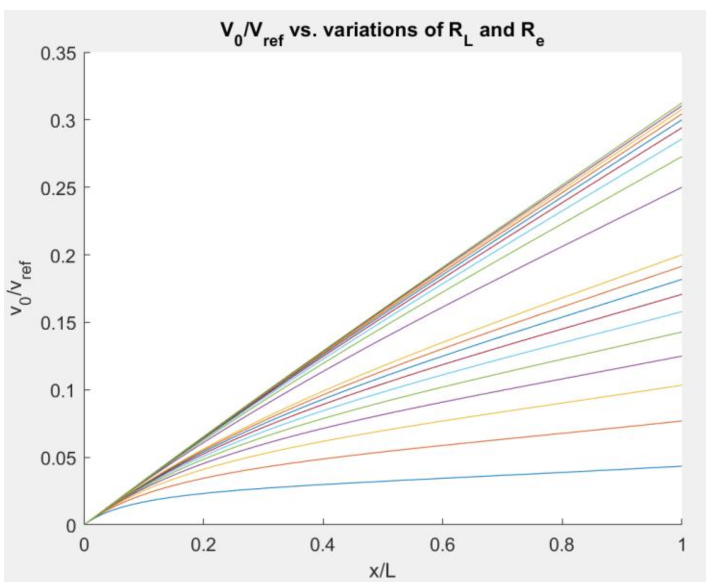
Problem 2

The figure below describes the variation of the normalized output voltage while changing the end resistances. We can see that the larger the end resistance, the more linear the variation of the output voltage will be. However, we are sacrificing sensitivity with increasing end resistance because the slope of the graph decreases with increasing end resistance.



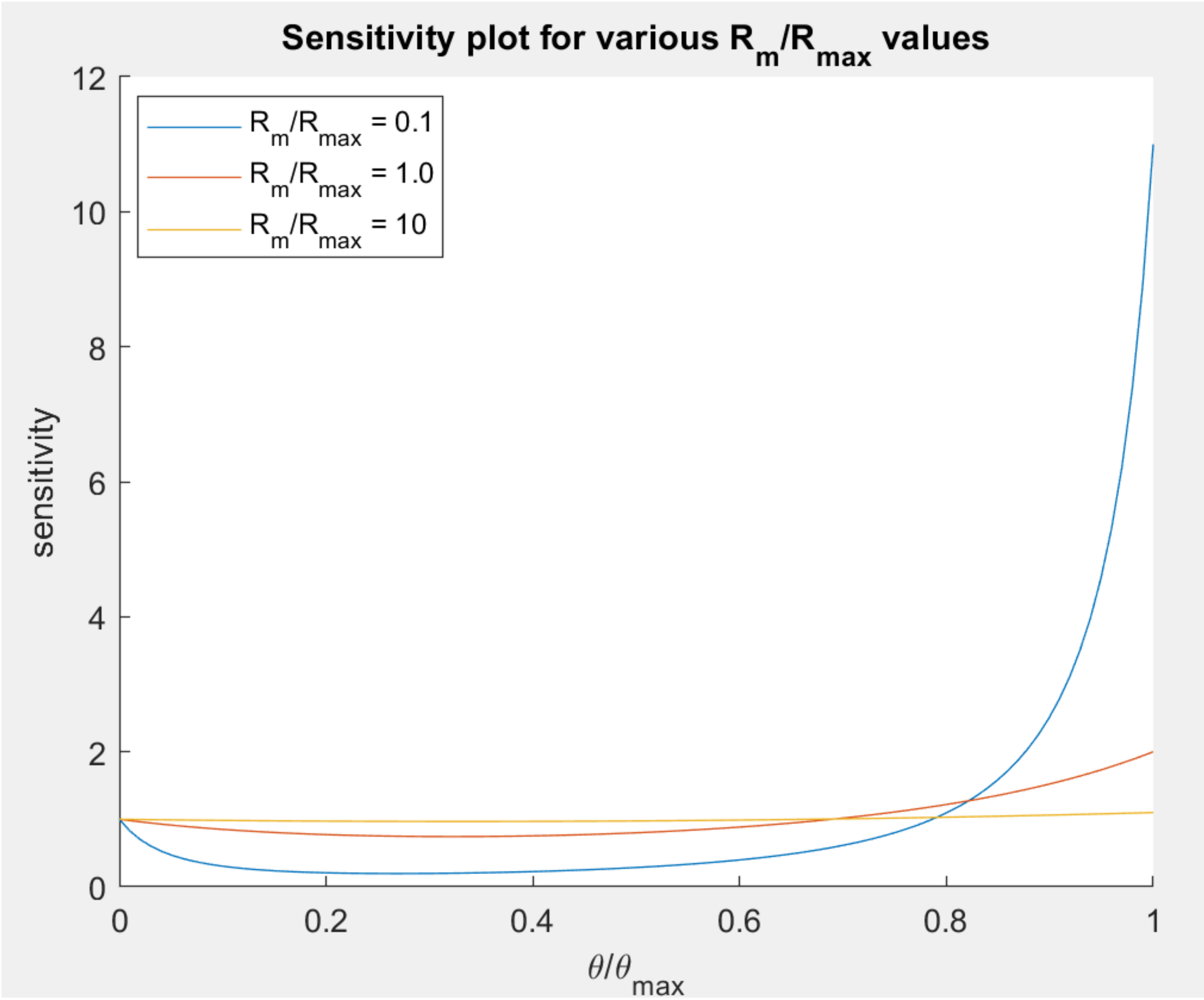
Effect of variation of R_e on V_0/V_{ref}

The figure below describes the normalized output voltage while varying the load resistance. As learned previously in class, increasing the load resistance will cause the normalized output voltage to be more linear since the test point will be closer to an open circuit. When the load resistance go lower, the measurement will both be less sensitive and less linear.



Effect of variation of R_L on V_0/V_{ref}

Problem 3



Upon observing the graph, we learn that the sensitivity of the rotary potentiometer is the highest at

$$\theta / \theta_{max} = 1$$

Solving the derivative of the sensitivity and finding its roots to find out the maximum point will not work since there is no maximum point observed in the graph, only minimum points.

MATLAB Code

```
%ME 473 - HW5
%Khrisna Kamarga

%% Problem 1

clear all; close all; clc;

RmRmax = [0.1, 1, 10];
thetaThetaMax = linspace(0, 1);

hold on
for i = RmRmax
    error = (i./(i + thetaThetaMax - thetaThetaMax.^2) - 1) * 100;
    plot(thetaThetaMax, error);
end
title("error % vs \theta/\theta_{max}");
legend("R_m/R_{max} = 0.1", "R_m/R_{max} = 1.0", "R_m/R_{max} = 10", "Location",
"best");
xlabel("\theta/\theta_{max}");
ylabel("error %");

%% Problem 3
clc;

hold on
for i = RmRmax
    sensitivity = (i*(thetaThetaMax.^2 + i))./(thetaThetaMax.^2 - thetaThetaMax -
i).^2;
    plot(thetaThetaMax, sensitivity);
end
title("Sensitivity plot for various R_m/R_{max} values");
legend("R_m/R_{max} = 0.1", "R_m/R_{max} = 1.0", "R_m/R_{max} = 10", "Location",
"best");
xlabel("\theta/\theta_{max}");
ylabel("sensitivity");

%% Problem 2
clear all; clc; close all;

Rc = 1;
a = linspace(0,1);
ratio = linspace(0.1,0.9,9);
ratio = [ratio linspace(1,10,10)];


hold on
for i = 1
    Rl = i*Rc;
    for j = ratio
        Re = j*Rc;
        for k = a
            y = Rc*Rl*a./((2*Re+Rc*(1-a)).*(Rc*a+Rl)+Rc*Rl*a);
            plot(a,y);
        end
    end
end
title("V_0/V_{ref} vs. variations of R_L and R_e");
xlabel("x/L");
ylabel("v_0/v_{ref}");
```