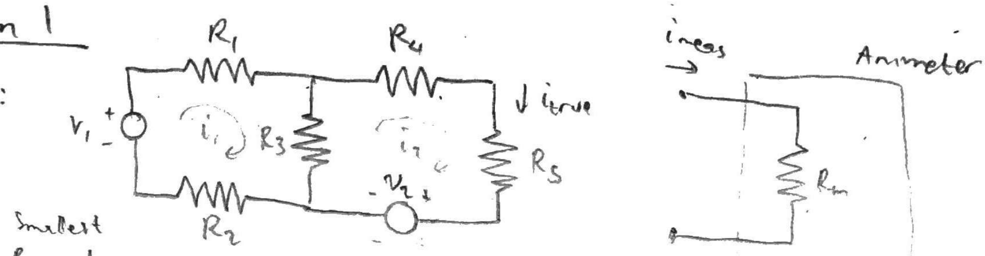
Problem 1

Given:



Find: R_m to guarantee that i_{meas} is within 1%

$0.99\, i_{true} \leq i_{meas} \leq 1.01\, i_{meas} \Rightarrow 0.99 \leq \left|\frac{i_{meas}}{i_{true}}\right| \leq 1.01$

loop 1 $\quad V_1 = i_1 R_1 + (i_1 - i_2) R_3 + i_1 R_2$

loop 2 $\quad 0 = i_2 R_4 + i_2 R_5 + V_2 + R_3 (i_2 - i_1)$

1) $\quad V_1 = i_1 (R_1 + R_3 + R_2) - i_2 R_3$

$$i_1 = \frac{V_1 + i_2 R_3}{R_1 + R_2 + R_3}$$

2)
$$0 = i_2 R_4 + i_2 R_5 + V_2 + R_3 \left(i_2 - \frac{V_1 + i_2 R_3}{R_1 + R_2 + R_3} \right)$$

$$0 = i_2 \left(R_4 + R_5 - \frac{R_3^2}{R_1 + R_2 + R_3} \right) + V_2 - \frac{R_3 V_1}{R_1 + R_2 + R_3}$$

$$i_2 = \frac{-V_2 + \frac{R_3 V_1}{R_1 + R_2 + R_3}}{\left(R_4 + R_5 - \frac{R_3^2}{R_1 + R_2 + R_3} \right)}$$

with R_m,

loop 2 $\quad 0 = i_2 R_4 + i_2 R_5 + i_2 R_m + V_2 + R_3 (i_2 - i_1)$

$$i_2 = \frac{-V_2 + \frac{R_3 V_1}{R_1 + R_2 + R_3}}{R_4 + R_5 + R_m - \frac{R_3^2}{R_1 + R_2 + R_3}}$$

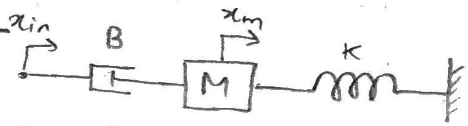
$$\frac{i_{2\,meas}}{i_{2\,true}} = \frac{R_4 + R_5 - \frac{R_3^2}{R_1 + R_2 + R_3}}{R_4 + R_5 + R_m - \frac{R_3^2}{R_1 + R_2 + R_3}} \leq 0.99$$

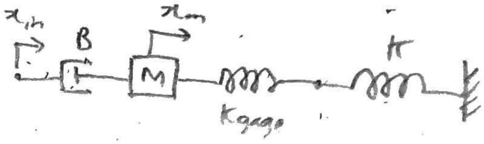
$$0.01 \left(R_4 + R_5 - \frac{R_3^2}{R_1 + R_2 + R_3} \right) \leq 0.99\, R_m$$

$$\boxed{R_m \geq \frac{1}{99} \left(R_4 + R_5 - \frac{R_3^2}{R_1 + R_2 + R_3} \right)}$$

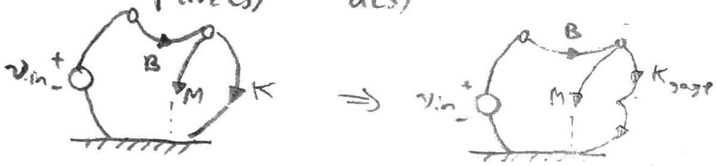
Problem 2

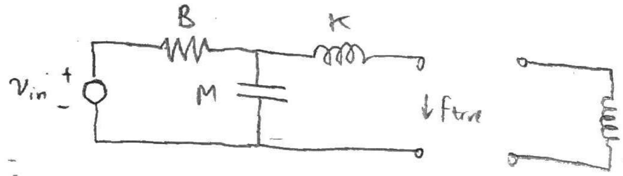
Given:



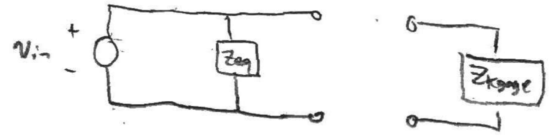


Find: $\dfrac{f_{gage}(s)}{f_{true}(s)} = \dfrac{n(s)}{d(s)}$





$$Z_{eq} = (Z_B^{-1} + Z_M^{-1})^{-1} + Z_K$$
$$= (B + Ms)^{-1} + \frac{s}{K}$$
$$= \frac{1}{B+Ms} + \frac{s}{K} \qquad \frac{K + Bs + Ms^2}{K(B+Ms)}$$



$$Z_{gage} = \frac{s}{K_{gage}}$$

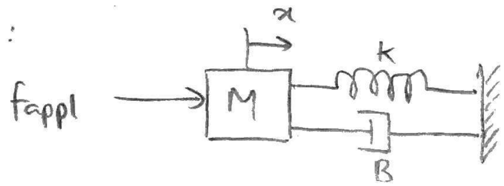
$$\frac{f_{gage}(s)}{f_{true}(s)} = \frac{Y_{gage}}{Y_{gage} + Y_{eq}} = \frac{1}{1 + \frac{Y_{eq}}{Y_{gage}}}$$

$$= \frac{1}{1 + \frac{\frac{K(B+Ms)}{K+Bs+Ms^2}}{\frac{K_{gage}}{s}}} = \frac{K_{gage}(K+Bs+Ms^2)}{K_{gage}(K+Bs+Ms^2) + Ks(B+Ms)}$$

$$\boxed{\begin{aligned} n(s) &= K_{gage}(K + Bs + Ms^2) \\ d(s) &= K_{gage}(K + Bs + Ms^2) + Ks(B + Ms) \end{aligned}}$$

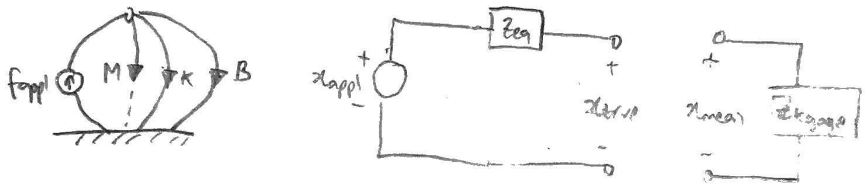
Problem 3

Given:



$$\frac{x_{gage}}{x_{true}} = \frac{n(s)}{d(s)}$$

a) Find : $n(s)$ & $d(s)$



$$Z_{eq} = \left(Z_M^{-1} + Z_K^{-1} + Z_B^{-1}\right)^{-1}$$

$$= \left(\frac{1}{Ms} + \frac{s}{K} + B\right)^{-1} = \left[\frac{K + Ms^2 + BMKs}{MKBs}\right]^{-1}$$

$$Z_{kgage} = \frac{K_{gage}}{s}$$

$$\frac{x_{gage}}{x_{true}} = \frac{1}{1 + \frac{Z_{gage}}{Z_{eq}}} = \frac{1}{1 + \frac{\frac{K_{gage}}{s}}{\frac{MKBs}{K+Ms^2+BMKs}}}$$

$$= \frac{MKBs^2}{MKBs^2 + K_{gage}(K + Ms^2 + BMKs)}$$

$$n(s) = MKBs^2$$

$$d(s) = MKBs^2 + K_{gage}(K + Ms^2 + BMKs)$$

b) Find : Steady-state loading relationship

$$\left.\frac{x_{gage}}{x_{true}}\right|_{steady} = \lim_{s \to 0} \frac{\cancel{MKBs^2}}{\cancel{MKBs^2} + K_{gage}(K + \cancel{Ms^2} + \cancel{BMKs})}$$

$$\boxed{\left.\frac{x_{gage}}{x_{true}}\right|_{steady} = \frac{1}{K K_{gage}}}$$

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c) with $f_{appl} = \sin(\omega t)$,

$x_{true} = A_{true}\sin(\omega t + \theta_{true})$, $x_{gage} = A_{gage}\sin(\omega t + \theta_{gage})$

For $M = 1$ kg, $K = 9$ N/m, $B = 3/5$ N/ms, $k_{gage} = k/10$, $\omega = 3$ rad/s

Find A_{gage}/A_{true}

$$f_{appl} = \operatorname{Im}\{e^{\omega j t}\} \quad , s = \omega j$$

$$\frac{|x_{gage}|}{|x_{true}|} = \frac{A_{gage}}{A_{true}}$$

$$\frac{x_{gage}}{x_{true}} = \frac{-MKB\omega^2}{-MKB\omega^2 + k_{gage}(K - M\omega^2 + BMK\omega j)}$$

$$= \frac{-MKB\omega^2}{-MKB\omega^2 + k_{gage}K - M\omega^2 k_{gage} + BMKk_{gage}\omega j}$$

$$\frac{|x_{gage}|}{|x_{true}|} = \frac{MKB\omega^2}{\sqrt{(k_{gage}K - M\omega^2 k_{gage} - MKB\omega^2)^2 + (BMKk_{gage}\omega)^2}}$$

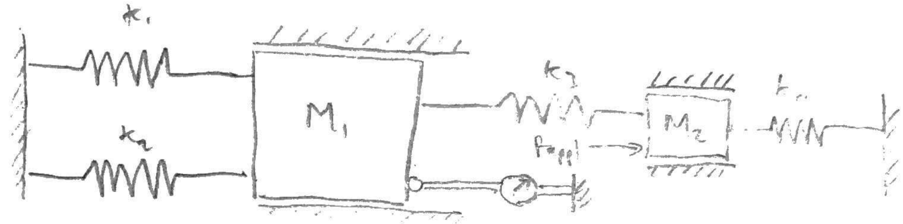
$$\frac{|x_{gage}|}{|x_{true}|} = \frac{1 \times 9 \times \frac{3}{5} \times 3^2}{\sqrt{(\frac{1}{10}(9^2) - 1 \times 3^2 \times \frac{1}{10} \times 9 - 1 \times 9 \times \frac{3}{5} \times 3^2)^2 + (\frac{3}{5} \times 1 \times \frac{1}{10} \times 9^2 \times 3)^2}}$$

$$\frac{|x_{gage}|}{|x_{true}|} = \frac{48.6}{\sqrt{(-48.6)^2 + (14.58)^2}}$$

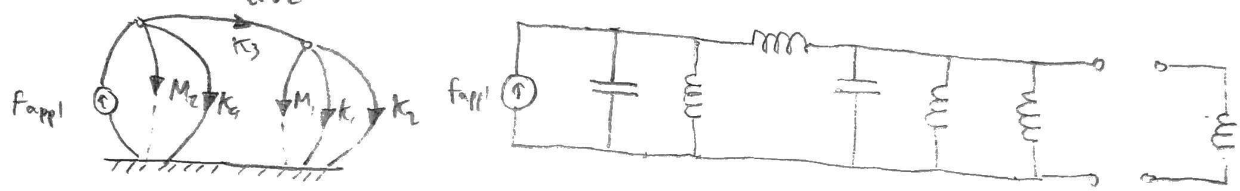
$$\boxed{\frac{A_{gage}}{A_{true}} = 0.9578}$$

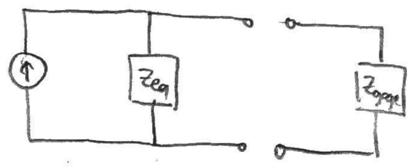
Problem 4

Given:



Find: $\dfrac{x_{gage}}{x_{true}}$





$$Z_{eq} = \left(Z_{M_2}^{-1} + Z_{k_4}^{-1} + \left(Z_{k_3} + \left(Z_{M_1}^{-1} + Z_{k_1}^{-1} + Z_{k_2}^{-1}\right)^{-1}\right)^{-1}\right)^{-1}$$

$$Z_{gage} = \frac{s}{k_{gage}}$$

$$Z_{eq} = \left(M_2 s + \frac{k_4}{s} + \left(\frac{s}{k_3} + \left(M_1 s + \frac{k_1}{s} + \frac{k_2}{s}\right)^{-1}\right)^{-1}\right)^{-1}$$

$$= \left(\frac{M_2 s^2 + k_4}{s} + \left(\frac{s}{k_3} + \frac{s}{M_1 s^2 + k_1 + k_2}\right)^{-1}\right)^{-1}$$

$$= \left[\frac{M_2 s^2 + k_4}{s} + \frac{k_3 (M_1 s^2 + k_1 + k_2)}{s(M_1 s^2 + k_1 + k_2) + k_3 s}\right]^{-1}$$

$$= \left[\frac{(M_2 s^2 + k_4)\left[(M_1 s^2 + k_1 + k_2) + k_3 s\right] + k_3 (M_1 s^2 + k_1 + k_2)}{s^2 (M_1 s^2 + k_1 + k_2) + k_3 s}\right]^{-1}$$

$$\frac{x_{gage}}{x_{true}} = \frac{1}{1 + \frac{Z_{gage}}{Z_{eq}}} = \frac{1}{1 + \dfrac{\frac{s}{k_{gage}}}{\dfrac{s^2(M_1 s^2 + k_1 + k_2) + k_3 s}{(M_2 s^2 + k_4)\left[(M_1 s^2 + k_1 + k_2) + k_3 s\right] + k_3 (M_1 s^2 + k_1 + k_2)}}}$$

Static $s = 0$

$$\frac{x_{gage}}{x_{true}} = \frac{1}{1 + \dfrac{\frac{1}{k_{gage}}}{\dfrac{s(\cancel{M_1 s^2} + k_1 + k_2) + k_3}{(\cancel{M_2 s^2} + k_4)\left[(\cancel{M_1 s^2} + k_1 + k_2) + \cancel{k_3 s}\right] + k_3(\cancel{M_1 s^2} + k_1 + k_2)}}} = \frac{1}{1 + \dfrac{\frac{1}{k_{gage}}}{\frac{k_3}{(k_4 + k_3)(k_1 + k_2)}}}$$

$$\boxed{\frac{x_{gage}}{x_{true}} = \frac{k_3 k_{gage}}{k_3 k_{gage} + (k_4 + k_3)(k_1 + k_2)}}$$