Khrisna Kamarga | ME 473 HW3 | [illegible] page [illegible]

Problem 1

Assumptions: $v^+ = v^-$ & $i^- = i^+ = 0$

a) When an Op-Amp is powered and no signal is supplied, why is the output found to oscillate between $\pm 12V$?

Answer:

① The ideal output voltage formula is $V_0 = K(V_A - V_B)$

② However, in reality the average DC voltage at the inputs affect output. $\rightarrow$ $V_0 = A(V_A - V_B - V_{of})$

③ V_{of} (offset voltage) is highly influenced by noise & bias current

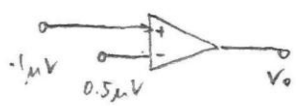
④ Noise would cause the offset voltage to slightly fluctuate between 0 V. (positive or negative small value).

⑤ Since the gain of an op-amp is very high, slight deviation of V_{of} from 0 will cause the output voltage to fluctuate between the positive and negative value of the saturation voltage.

b) An op-amp has an open-loop gain of 5×10^5, $V_{sat} = \pm 10V$

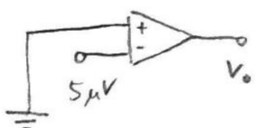
$V^+ = -1\mu V$, $V^- = +0.5\mu V$

Find: V_0



$$V_0 = K(V_A - V_B)$$
$$= 5\times10^5(-1\times10^{-6} - 0.5\times10^{-6})$$
$$\boxed{V_0 = -0.75\ V}$$

If $V^- = 5\mu V$ and V^+ is grounded, what is the output?



$$V_0 = K(V_A - V_B)$$
$$= 5\times10^5(0 - 5\times10^{-6})$$
$$\boxed{V_0 = -2.5\ V}$$

Problem 2

Define & provide typical values

a) Offset current: the difference in the two bias currents

$\boxed{\text{typical: } 10^{-6} - 10^{-14}\ A}$ (bias current: average dc current through one input lead)

b) Offset voltage: The voltage difference that is needed at the two input terminals (for imperfect op-amps) to make the output voltage zero.

$\boxed{\text{typical: } \pm 1mV \text{ at } 25°C}$

c) Unequal gains: The inverting gain not equal to the non-inverting gain

typical: 10^5 V/V

d) Slew rate: Maximum possible rate of change of output voltage, without significantly distorting the output.

$\boxed{\text{typical: } 160\ V/\mu s}$

High slew rate is desirable

the reason is because of it's high ~~o~~ impedance compared to the input impedance that the variance will become negligible

Problem 3

What are passive filters?

Passive filters are filters (a device that passes signals in a certain range of frequency) that does not require any power source

Advantage & disadvantages? + (advantage) - (disadvantage)

+ Does not need a power supply
+ There is no saturation non-linearities
- Variable / uncontrolled impedance characteristics
- Can't multitask, one dedicated function
- Sensitive to noise

What is the purpose of a voltage follower?

Voltage follower can amplify the current of the input signal to provide higher resolution.

Problem 4

Give one application for each filter

a) Low pass : eliminating high frequency noise from a signal
b) High pass : used as a part of audio crossover to direct high frequencies to a tweeter
c) Band pass : used in wireless transmitters and receivers (radio)
d) Notch filter : used to suppress a certain frequency in audio applications

Resonance peak is possible since multiple active single pole filters will form second order filters.

Problem 5

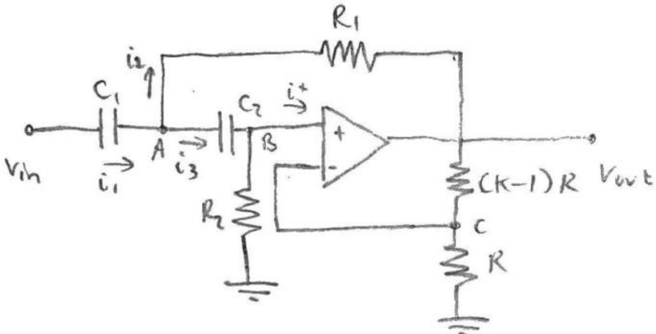
Butterworth filter has a maximally flat magnitude (high bandwidth). Explain

Flatness refers to the constant gain area in the bode plot of the filter. Flatness is desired so that the output signal is simply proportional to the input signal. Butterworth filters a large flat region for this purpose with a relatively sharp knee with high rolloff rate after the cutoff frequency

Another desired character of a filter is "hard" knee which quickly rejects frequencies higher than the cutoff frequency

Problem 6

Given:



a) Obtain the transfer function and the order.

Assume : $i^+ = i^- = 0$, $V^+ = V^-$

$i_1 = i_2 + i_3$ (node equation)

$$\frac{V_{in} - V_A}{1/sC_1} = \frac{V_A - V_o}{R_1} + \frac{V_A - V_B}{1/sC_2} \quad ①$$

$$V_B = V_C = \frac{R}{(K-1)R + R} V_o = \frac{V_o}{K} \quad ②$$

$T = \frac{V_o}{V_i}$ ④

$i_3 = i_{BG}$ (node equation)

$$\frac{V_A - V_B}{1/sC_2} = \frac{V_B}{R_2}, \quad V_A(sC_2) = V_B\left(\frac{1}{R_2} + sC_2\right) \quad ③$$

$$V_A = V_B\left(\frac{1}{R_2C_2s} + 1\right)$$

② → ③

$$V_A = \frac{V_o}{K}\left[\frac{1}{R_2} + sC_2\right]$$

②, ③ → ①

$$V_{in}(sC_1) = V_A\left(\frac{1}{R_1} + sC_1 + sC_2\right) + V_o\left(\frac{-1}{R_1}\right) + V_B(-sC_2)$$

$$V_{in}(sC_1) = \frac{V_o}{K}\left[\frac{1}{R_2C_2s} + 1\right]\left(\frac{1}{R_1} + sC_1 + sC_2\right) - \frac{V_o}{R_1} + \frac{V_o}{K}(-sC_2)$$

$$V_{in}(sC_1) = V_o\left[\frac{1}{K}\left(\left[\frac{1}{R_2C_2s} + 1\right]\left[\frac{1}{R_1} + sC_1 + sC_2\right] - \frac{K}{R_1} - sC_2\right)\right]$$

$$\frac{V_o}{V_{in}} = \frac{sC_1}{\frac{1}{K}\left(\left[\frac{1}{R_1R_2C_2s} + \frac{(C_1+C_2)}{R_2C_2} + \frac{1}{R_1} + s(C_1+C_2) - \frac{K}{R_1} - sC_2\right)}$$

$$\boxed{\frac{V_o}{V_{in}} = \frac{Ks^2C_1}{\frac{1}{R_1\tau_2} + \left(\frac{C_1+C_2}{\tau_2} + \frac{1-K}{R_1}\right)s + s^2C_1}}$$

$$\tau_2 = R_2C_2$$

let's not look at this, let's solve it with matlab → next page

put eq ① ② ③ ④ to MATLAB solver

$$\frac{V_o}{V_i} = \frac{C_1 C_2 R_1 R_2 k s^2}{C_1 C_2 R_1 R_2 s^2 + (C_1 R_1 + C_2 R_1 + C_2 R_2 - C_2 R_2 k)s + 1}$$

$\tau_1 = C_1 R_1$, $\tau_2 = C_2 R_2$, $\tau_3 = R_1 C_2$

$$\frac{V_o}{V_i} = \frac{\tau_1 \tau_2 k s^2}{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_3 + \tau_2 - \tau_2 k)s + 1}$$ 2nd order

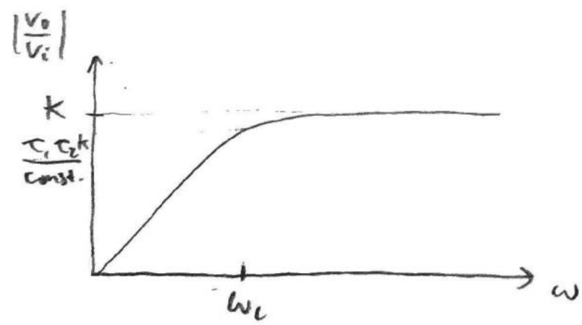
b) Sketch $\left|\frac{V_o}{V_i}\right|$, what type of filter?

$$\left|\frac{V_o}{V_i}\right|(\omega j) = \frac{\tau_1 \tau_2 k \omega^2}{\sqrt{(1-\tau_1\tau_2\omega^2)^2 + [(\tau_1+\tau_3+\tau_2-\tau_2 k)\omega]^2}}$$

$\omega \to 0$, $\left|\frac{V_o}{V_i}\right| = 0$

$\omega \to \frac{1}{\sqrt{\tau_1\tau_2}}$, $\left|\frac{V_o}{V_i}\right| = \frac{\tau_1\tau_2 k}{\text{constant}}$

$\omega \to \infty$, $\left|\frac{V_o}{V_i}\right| = \frac{\cancel{\tau_1\tau_2} k \cancel{\omega^2}}{\cancel{\tau_1\tau_2}\cancel{\omega^2}} = k$



Second order high pass filter

c) Rolloff rate

$$\left|\frac{V_o}{V_i}\right| = \frac{\tau_1\tau_2 k \omega^2}{\sqrt{(1-\tau_1\tau_2\omega^2)^2 + \text{nn}\,\omega^2}} = \frac{\tau_1\tau_2 k \omega^2}{\text{nn}\ \cancel{\omega}}$$ proportional

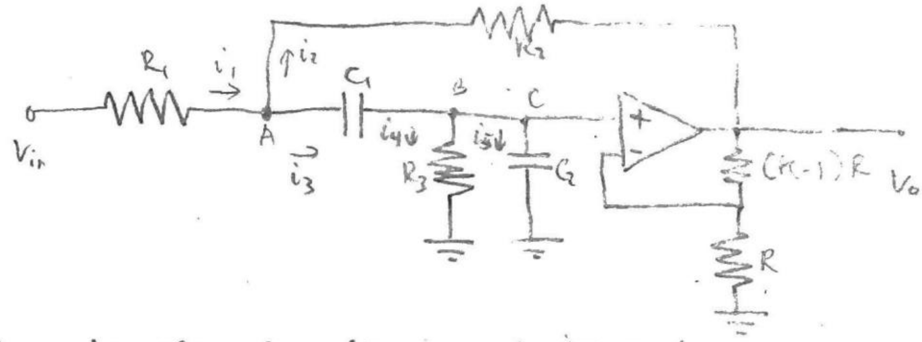
$\to 0$

$\boxed{\text{20 dB/decade roll-up}}$

cutoff ω $\to$ $\boxed{\omega_c = \frac{1}{\sqrt{\tau_1\tau_2}}}$

Problem 7

Given :



a) Obtain the transfer function and its order

Assume : $i^- = i^+ = 0$, $v^+ = v^-$

$i_1 = i_2 + i_3$

$$\frac{V_i - V_A}{R_1} = \frac{V_A - V_0}{R_2} + \frac{V_A - V_B}{1/sC_1} \quad ①$$

$i_3 = i_4 + i_5$

$$\frac{V_A - V_B}{1/sC_1} = \frac{V_B}{R_3} + \frac{V_C}{1/sC_2} \quad ②$$

$$V_C = V^- = \frac{R}{(K-1)R + R} \times V_0 = \frac{V_0}{K} \quad , \quad ③$$

$V_B = V_C \quad ④$

$$V_A (sC_1) = V_B \left(\frac{1}{R_3} + sC_1 + \frac{1}{R_3}\right) = \frac{V_0}{K}\left(\frac{1}{R_3} + sC_1 + \frac{1}{R_3}\right)$$

Put ① ② ③ ④ in MATLAB

$$\frac{V_0}{V_i} = \frac{C_1 R_2 R_3 K s}{R_1 + R_2 + (C_1 R_1 R_2 + C_1 R_1 R_3 + C_1 R_2 R_3 + C_2 C_1 R_3 + C_2 R_2 R_3 K)s + C_1 C_2 R_1 R_2 R_3 s^2}$$

$$\boxed{\frac{V_0}{V_i} = \frac{C_1 R_2 R_3 K s}{R_1 + R_2 + (C_1(R_1R_2 + R_1R_3 + R_2R_3) + C_2(C_1R_3 + R_2R_3K))s + C_1C_2R_1R_2R_3 s^2}}$$

second order filter

$$\frac{ks}{a + bs + cs^2}$$

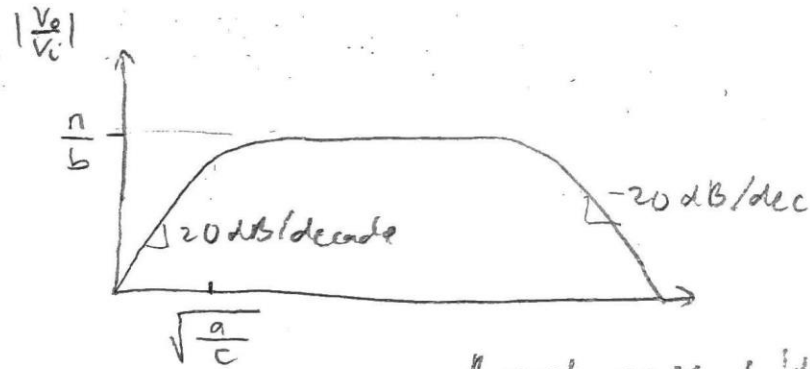
b) Sketch the magnitude and, frequency. What is the filter type?

$$\left|\frac{V_o}{V_i}\right|(\omega j) = \frac{n\omega}{\sqrt{(a - c\omega^2)^2 + b^2\omega^2}}$$

$$\omega \to 0, \quad \left|\frac{V_o}{V_i}\right| = 0$$

$$\omega \to \sqrt{\frac{a}{c}} \quad \left|\frac{V_o}{V_i}\right| = \frac{n\cancel{\omega}}{b\cancel{\omega}} = \frac{n}{b} = \frac{C_1 R_2 R_3}{C_1(R_1R_2 + R_1R_3 + R_2R_3) + C_2(C_1R_3 + R_2R_3 +)}$$

$$\omega \to \infty \quad \left|\frac{V_o}{V_i}\right| = \frac{n\cancel{\omega}}{c\omega^{\cancel{2}}} = \frac{n}{c\omega} = 0$$



Band-pass filter

c) Estimate ω_c & roll off slope

roll off slope $= -20\,dB/dec$

since $\omega \gg \sqrt{\frac{a}{c}} = \frac{n}{c\omega} \leftarrow -20\,dB/dec$

$$\boxed{\omega_c = \sqrt{\frac{a}{c}} = \sqrt{\frac{R_1 + R_2}{C_1 C_2 R_1 R_2 R_3}}}$$

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%% ME 473 HW3 Prob 6

clear all; close all; clc;

syms k s Vo Vi VA VB R1 R2 C1 C2 T;

Eqs = [(VA - Vo)/R1 + (VA - VB)*s*C2 - (Vi - VA)*s*C1;
        VB - Vo/k;
        VA*s*C2 - VB*(1/R2 + s*C2);
        T - Vo/Vi];
Sol = solve(Eqs, VA, VB, T, Vo, Vi);
ans = simplify(Sol.T, 5) pretty(ans)
  %% num = [4 0
0]; den = [3 1 1];
sys = tf(num,den);
bode(sys)

%% ME 473 HW3 Prob 7

clear all; close all; clc;

syms k s Vo Vi VA VB VC R1 R2 R3 C1 C2 T;

Eqs = [(VA-Vo)/R2 + (VA-VB)*s*C1 - (Vi-VA)/R1;
        -(VA-VB)*s*C1 + VB/R3 + VC*s*C2;
        VC - Vo/k;
        T - Vo/Vi;
        VB - VC];
Sol = solve(Eqs, VA, VB, VC, T, Vo, Vi); ans
= simplify(Sol.T, 6)
  %% num = [2 0];
den = [1 100 1];
sys = tf(num,den);
bode(sys);
```