ME 473

Project 2:
Continuous Time Low Pass Filter

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Executive Summary

In this project, I implemented a continuous time low pass filter (Butterworth filter) whose inputs and outputs are connected to a voltage follower. After deriving the transfer function on paper, I derived the transfer function to be in the form of a second order system with the natural frequency or cutoff frequency of 9.99 Hz. This result is confirmed by the fact the fact that when a sinusoidal input of larger than 10 Hz is inputted to the filter, the output voltage is quickly attenuated.

To generate the output voltage, I connected the analog MyDAQ’s output 0 into the non-inverting lead of the op-amp. I measured the signal generated by connecting the output of the op-amp to analog input 1. The filtered voltage is measured by connecting MyDAQ’s analog input 0 to the output of the output voltage follower op-amp. I powered the op-amp by connecting the source voltages into the +15V and -15V source in MyDAQ.

I measured both the output and the input voltage by measuring the output voltage of their respective voltage followers. The reason for this is because not only the output voltage of the voltage follower is the same as the voltage of interest, but also because the output voltage has low output impedance so that loading effect will be minimized (due to the voltage divider rule).

Below the cutoff frequency, the gain of the filter is relatively flat with gain value of 1. At the natural frequency or the cutoff frequency, the gain of the filter is at approximately -3 dB. Then, the gain shows a rolloff rate of 40dB/decade. When the measured gain values are compared with the bode plot based on the transfer function, the measured gains lie approximately on the theoretical bode plot (maximum error percentage of 2.5%).

In addition, the waveform plots of the unfiltered and filtered voltages are phase shifted. This is due to the phase difference associated with each of the input frequency.

In summary, I learned how to implement low pass filter, which quickly attenuates input signal for input frequencies larger than the natural frequency.

Derivation

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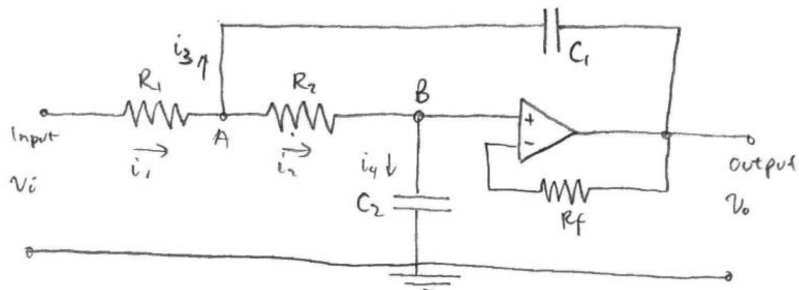
Project 2

Given:



- $R_1 = 18{,}000\ \Omega$, $R_2 = 30{,}000\ \Omega$, $C_1 = 10^{-6}$ F, $C_2 = 0.47 \times 10^{-6}$ F

Analysis

measured values:

$R_1 = 17{,}990\ \Omega$ $R_2 = 30{,}000\ \Omega$ $C_1 = 0.999 \times 10^{-6}$ F $C_2 = 0.4699 \times 10^{-6}$ F

- Derive the transfer function, and find ω_n

Assume: $i^- = i^+ = 0$, $v^+ = v^-$

$v^- = v_0 = v_B$, since $i^- = 0$

$i_1 = i_2 + i_3$ (node equation)

$$\frac{v_i - v_A}{R_1} = \frac{v_A - \overset{v_o}{\cancel{v_B}}}{R_2} + \frac{v_A - v_o}{1/sC_1} \quad ①$$

$i_2 = i_4$ (node equation, $i^+ = 0$)

$$\frac{v_A - \overset{v_o}{\cancel{v_B}}}{R_2} = \frac{\overset{v_o}{\cancel{v_B}}}{1/sC_2} \Rightarrow v_A = R_2 v_0 \left(\frac{1}{R_2} + sC_2\right) \quad ②$$

sub ② to ①

$$\frac{v_i}{R_1} = v_A\left(\frac{1}{R_2} + \frac{1}{R_1} + sC_1\right) + v_0\left(\frac{-1}{R_2} - sC_1\right)$$

$$\frac{v_i}{R_1} = R_2 v_0 \left(\frac{1}{R_2} + sC_2\right)\left[\frac{1}{R_2} + \frac{1}{R_1} + sC_1\right] + v_0\left[-\frac{1}{R_2} - sC_1\right]$$

$$\frac{V_0}{V_i} = \frac{1}{R_1\left[(1 + sC_2R_2)\left(\frac{1}{R_2} + \frac{1}{R_1} + sC_1\right) - \frac{1}{R_2} - sC_1\right]}$$

$$= \frac{1}{R_1\left[\frac{1}{R_2} + \frac{1}{R_1} + sC_1 + sC_2 + \frac{sC_2R_2}{R_1} + s^2C_2C_1R_2 - \frac{1}{R_2} - sC_1\right]}$$

$$\frac{V_o}{V_i} = \frac{1}{1 + sC_2R_1 + sC_2R_2 + s^2C_2C_1R_1R_2}$$

$$= \frac{1/C_1C_2R_1R_2}{s^2 + \frac{R_1C_2 + R_2C_2}{C_1C_2R_1R_2}s + \frac{1}{C_1C_2R_1R_2}}$$

$$\boxed{\frac{V_o}{V_i} = \frac{1/C_1C_2R_1R_2}{s^2 + \frac{R_1+R_2}{C_1R_1R_2}s + \frac{1}{C_1C_2R_1R_2}}}$$

$$\omega_n = \sqrt{\frac{1}{C_1C_2R_1R_2}}$$

plugging in the values,

$$\boxed{\omega_n = 62.8258 \text{ rad/s} = 9.999 \text{ Hz}}$$

Theoretical Gain

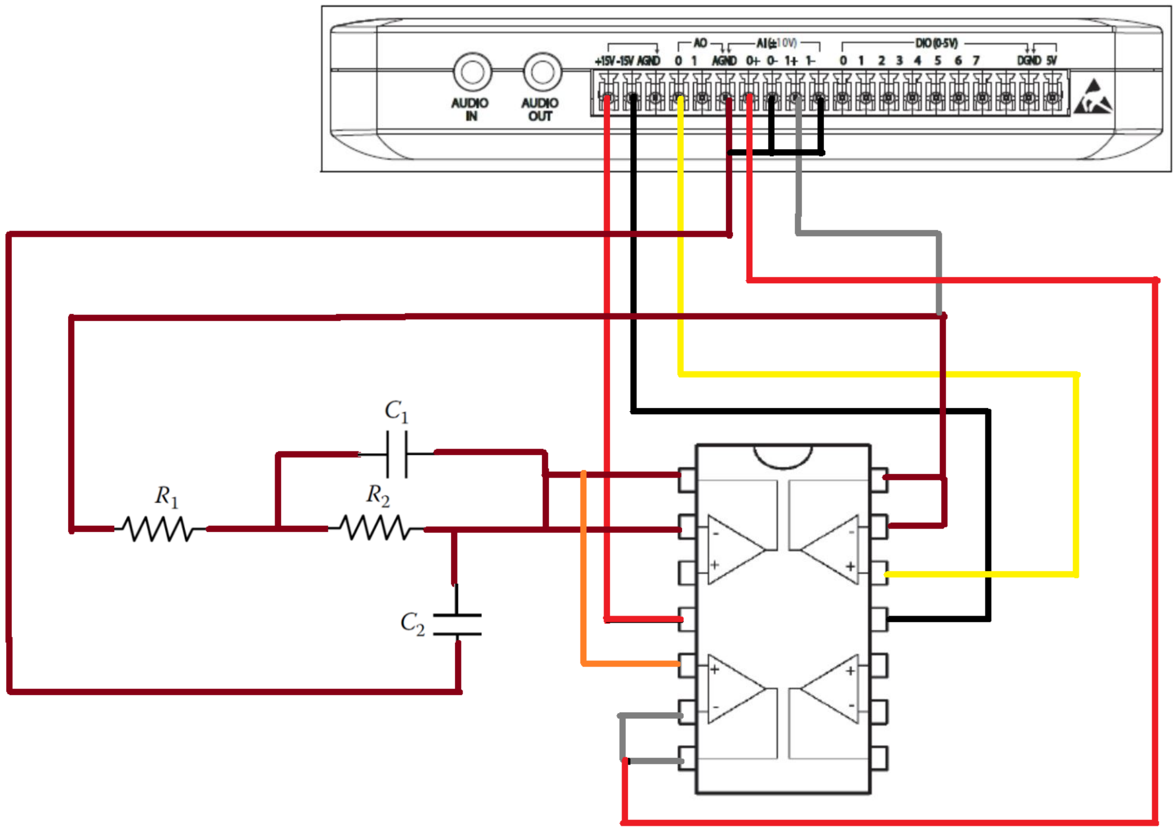
$$\frac{V_o}{V_i}(j\omega) = \frac{1/C_1C_2R_1R_2}{-\omega^2 + \frac{R_1+R_2}{C_1R_1R_2}\omega j + \frac{1}{C_1C_2R_1R_2}}$$

$$\left|\frac{V_o}{V_i}(j\omega)\right| = \frac{1/C_1C_2R_1R_2}{\sqrt{\left(\frac{1}{C_1C_2R_1R_2} - \omega^2\right)^2 + \left(\frac{R_1+R_2}{C_1R_1R_2}\omega\right)^2}}$$

for 10V amplitude sinosoidal signal

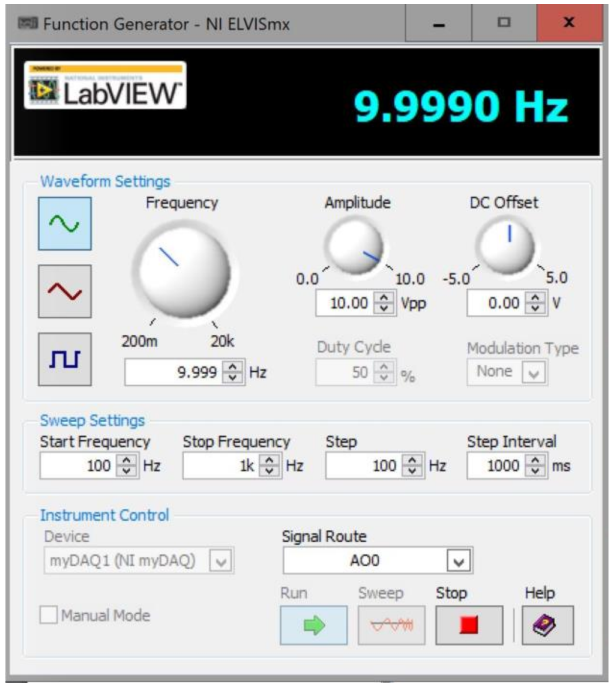
input frequency (Hz)	Theoretical Amplitude	unfiltered V_{p-p}	filtered V_{p-p}	Amplitude gain
$10^{-1}\omega_n$	0.9999	9.533	9.181	0.9631
$10^{-1/2}\omega_n$	0.9947	9.995	9.930	0.9935
ω_n	0.7058	9.995	6.979	0.6982
$10^{1/4}\omega_n$	0.3012	9.995	2.966	0.2967
$10^{1/2}\omega_n$	0.0995	9.995	0.983	0.0983
$10^{3/4}\omega_n$	0.0316	9.995	0.315	0.0315
$10\omega_n$	0.0100	9.995	9.102	0.0102

Wiring Diagram

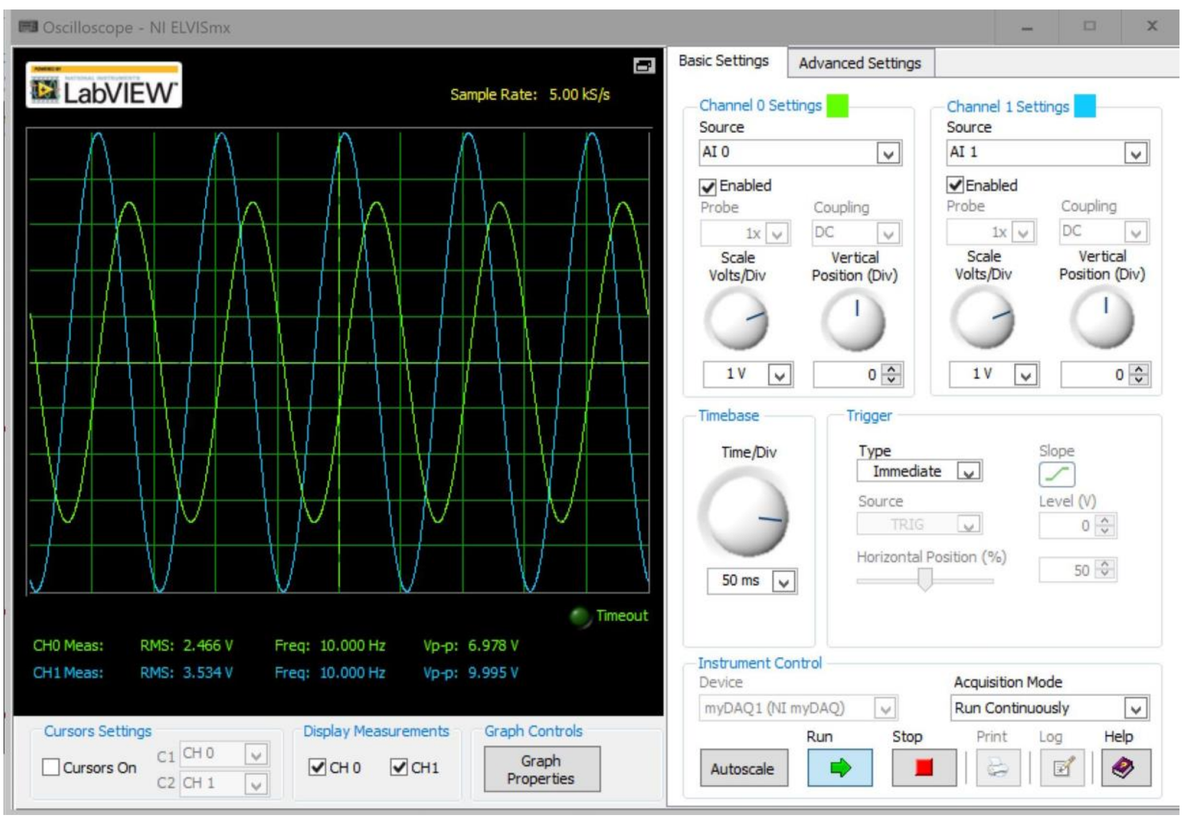


Wiring Diagram

Unfiltered Signal and Filtered Signal

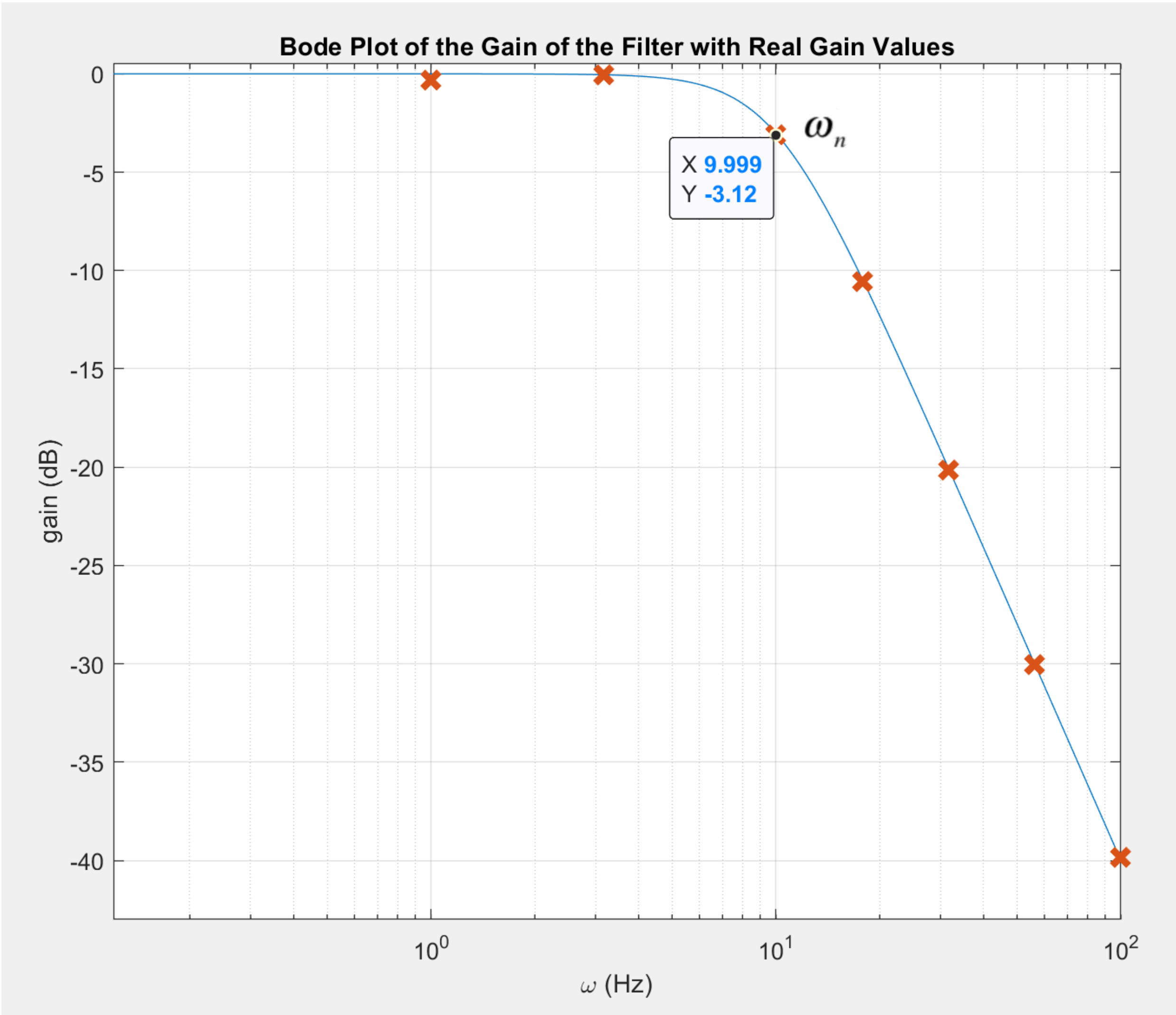


Virtual Signal Generator



Virtual Oscilloscope

Bode Plot with Measured Gain Superimposed

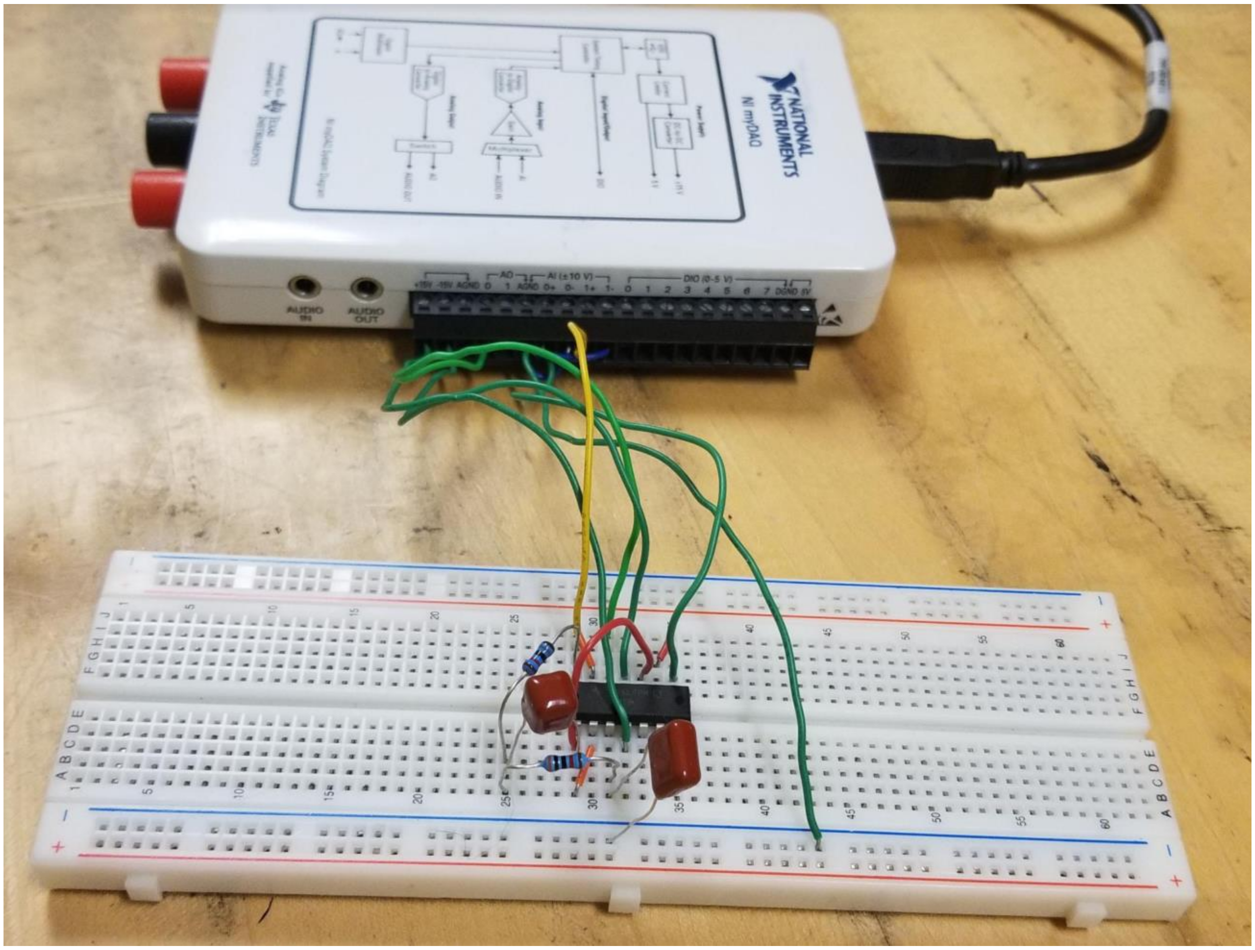


Bode Plot

MATLAB Code

```
%% ME 473 Project 2
clear all; close all; clc;

R1 = 17.99e+03;
R2 = 30e+03;
C1 = 0.999e-06;
C2 = 0.4699e-06;

wn = sqrt(1/(C1*C2*R1*R2)) %/(2*pi)

p = [1 (R1+R2)/(C1*R1*R2) 1/(C1*C2*R1*R2)];
r = roots(p);

w = [10^(-1)*wn 10^(-0.5)*wn wn 10^(1/4)*wn 10^(1/2)*wn 10^(3/4)*wn 10*wn];
f = w./(2*pi)
%G = (1/(C1*C2*R1*R2))./(s.^2 + s.*(R1+R2)/(C1*R1*R2) + 1/(C1*C2*R1*R2));

gain = (1/(C1*C2*R1*R2))./sqrt((1./(C1*C2*R1*R2) -w.^2).^2 +
((R1+R2)/(C1*R1*R2).*w).^2)

num = 1/(C1*C2*R1*R2);
denom = [1 (R1+R2)/(C1*R1*R2) 1/(C1*C2*R1*R2)];
sys = tf(num,denom);
bode(sys)

vi = [9.533 9.995 9.995 9.995 9.995 9.995 9.995];
vo = [9.181 9.930 6.979 2.966 0.983 0.315 0.102];
realgain = vo./vi

error = (gain - realgain)./gain * 100

%%
close all;
w = linspace(0,120,1000)*2*pi;
gain = (1/(C1*C2*R1*R2))./sqrt((1./(C1*C2*R1*R2) -w.^2).^2 +
((R1+R2)/(C1*R1*R2).*w).^2);
semilogx(w/(2*pi), 20*log10(gain))
hold on
w = [10^(-1)*wn 10^(-0.5)*wn wn 10^(1/4)*wn 10^(1/2)*wn 10^(3/4)*wn 10*wn];
gain = (1/(C1*C2*R1*R2))./sqrt((1./(C1*C2*R1*R2) -w.^2).^2 +
((R1+R2)/(C1*R1*R2).*w).^2);
semilogx(w/(2*pi), 20*log10(realgain),"x","MarkerSize",10,"LineWidth",3)
xlabel("\omega (Hz)");
ylabel("gain (dB)");
ylim([-43 0.5]);
xlim([0 100]);
grid on
title("Bode Plot of the Gain of the Filter with Real Gain Values")
```

Appendix



Picture of the Filter