ME 473

Project 3:
Ball on the Track

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Abstract

In the Ball on the Track Project, the objective of the experiment is to perform system identification by calculating the R/r ratio from the measured data, as well as to apply a Butterworth filter to the measurement to filter out noise in the measurement.

We started the experiment by developing an analytical model of the Ball on the Track that helped us recover the equation relating time and the displacement of the ball. After calibrating voltage reading to displacement of the ball, and taking measurements of the variation of displacement through time of the rolling metal ball, we can confirm the analytical model. The measured data is curve fitted so that we can recover the quadratic relationship between the displacement and time of the metal ball. The coefficient in the quadratic relationship is compared with the analytical model which yields R/r value of 1.8.

After acquiring the R/r and the angle of the track, we can confirm our system by plotting the measured and theoretical value of the displacement. With the R/r value of 1.8, the measured and theoretical plots closely match.

The next step of experiment is to apply a Butterworth filter in the VI we previously used. Applying the filter will result in smoothening the noisy measured data if the right cutoff frequency is implemented (around 5 Hz). If the cutoff frequency is too low, the resulting plot will be too low compared to the measured data, and therefore is inaccurate. If the cutoff frequency is too high, the resulting filtered plot will have almost no difference from the measured plot, which would render the filter obsolete. Lastly, higher order filters result in sharper knees and therefore can smooth out the measurement better, with the trade off of a lag in the measurement.

In conclusion the experiment has successfully demonstrated system identification through calibration and noise filtering sensor measurements. The topic in this lab is relevant to real world applications such as providing feedback measurement for a ball position controller system in which we need the velocity data of the ball to control the position of the ball in the track.

This experiment can be improved by incorporating other types of measurement device such as an optical encoder.

The appendix contains the MATLAB code that I implemented to work on this lab.

Introduction

In this experiment, we are interested to learn how to confirm our data acquisition technique by performing system identification, and to apply a filter which would refine the feedback data provided by the DAQ.

The equipment used in this project is the Ball on Track system that is available at the MEB 115. The specific track that is used in this experiment is **Track #2**. Other than the ball on track system, we also used NI myDAQ to perform our data acquisition, which follows with the implementation of VI for the parts of this project.

Before starting the experiment, system identification is performed. We started by performing analytical model using free body diagrams while implementing newton's second law on both rotational and translational model. In theory, the displacement of the ball should be quadratic in relation to the time when the ball is released.

Next, calibration is peformed. We learned that ideal potentiometer will have a linear relationship between the voltage measured across it and the displacement of the object of intererst. Therefore, we took measurements in various known ball positions, so that we were able to recover the linear relationship between the ball's position and the voltage reading acquired my NI myDAQ.

After calibration, we can continue performing system identification. We recorded measurements of the ball rolling down in a time period. The data is then recorded and using linear regression, we recovered the coefficient to the quadratic time term which helps us decipher R/r ratio (which is the actual radius and the distance between the centroid to the contact point ratio). If we observe consistent R/r and if the expression that would count the theoretical displacement matches with the measurement data, we have successfully performed system identification.

In the last part, we are interested in designing a control system that controls the ball's position. However, if we are utilizing only the measured data, the feedback response would not be reliable since the measured data is polluted with noise; this is especially bad if we are relying on the velocity feedback to control our system. Therefore, we ran the measured data through a low pass second order Butterworth filter in order to filter out the unwated noise.

We can tune our filter depending on the sampling rate, cutoff frequency, and the order of the filter. Theoretically, the higher the sampling rate, the higher variation of the cutoff frequency we can apply since there are more bandwidth that the signal covers up to its Nyquist frequency. Low cutoff frequency will cause the filter to eliminate most of the useful data, and will produce an output that does not resemble the original data. Extremly high cutoff frequency will cause the filter to include every data points, which will eliminate the function of the filter. Thus, we need to find a perfect cutoff frequency that filters out the right amount of noise. Lastly, higher filter order will smoothen the data more, with the tradeoff of having a phase lag.

Procedure

Part 1

1. Develop an analytical model of the displacement of the rolling ball over time.

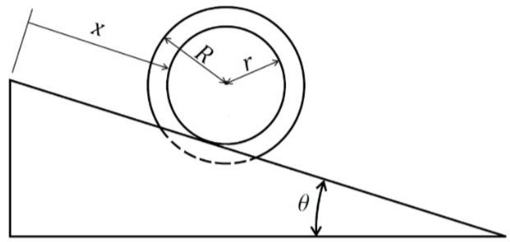


Figure 1. Ball on the Track Diagram

2. Calibrate the potentiometer on the track
   a. Connect myDAQ to the track by following the wiring diagram shown below.

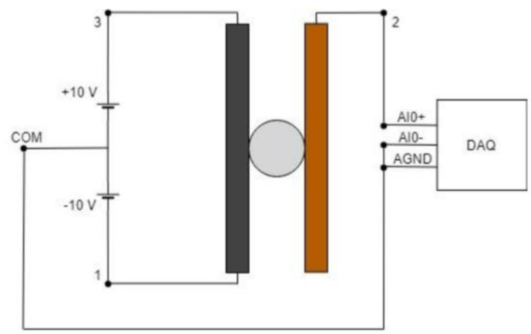


Figure 2. Wiring Diagram

   b. Set the function generator to have -25V negative terminal to -10V and the +25V positive terminal to +10V. Make sure that the function generator is ON.
   c. Place the ball in a known position on the track, then measure the voltage value at the specific location.
   d. Be sure to document the location versus voltage everytime a measurement is taken.
   e. From the data collected, plot a graph, and apply a linear regression to find the approximate gradient and offset to obtain the relationship between the voltage and the actual position.
3. From the metal ball position equation derived previously, perform a system identification by obtaining the value of R/r based on the analytical model.
   a. Measure the angle of the track by using the digital level shown below. Make sure to zero the digital level on the flat table.

Figure 3. Digital Level

b. Place the digital level on top of the track and record the track's angle.
c. Connect myDAQ to the track.
d. Place the metal ball on top of the track and let the ball roll down while taking voltage measurements with the myDAQ (use the VI provided to take measurements).
e. Export the voltage vs. time measurement to an excel document and process the data in the MATLAB script (in the appendix).
f. The MATLAB script plots the distance vs. time squared, then it takes a linear regression of the plot to find out the coefficient of interest that will help us find R/r.
g. Solve the analytical model, and recover the value for R/r.
h. Repeat step a. until h. but with another track angle.

<u>Part 2</u>

In this part, it is sufficient to utilize the data recovered from the previous part containing the displacement vs. time values of both of the track's angle.

1. Create a VI that takes in the displacement vs. time data.
2. Split the displacement vs. time data into three bundles and plot them all together.
    a. The first group should contain the actual displacement vs. time values
    b. The second group should contain the filtered displacement vs. time values. Set the filter order, sampling frequency, and the cutoff frequency with a numerical control in the front panel.
    c. The third group should contain the theoretical displacement vs. time. Calculate the theoretical displacement by applying the relationship between the displacement vs. time using the analytical model.
3. Contain everything in a while loop so that when the VI is running, changes in the filter settings can be clearly shown.
    a. Vary the cutoff frequency between 0.1 Hz and 100 Hz.
    b. Vary the filter order (while setting the final order to be 2).

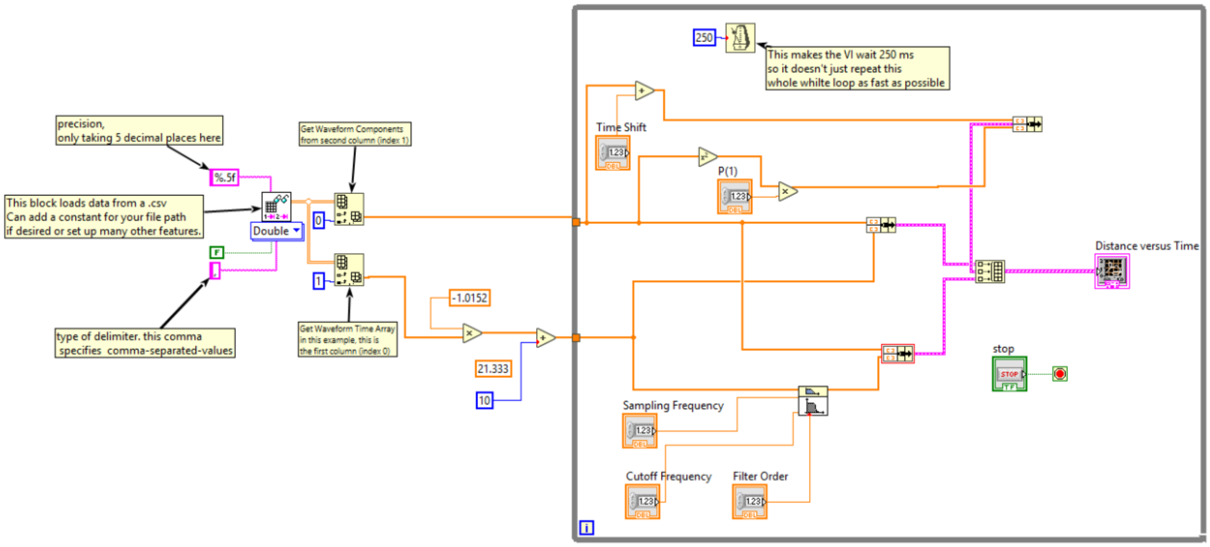


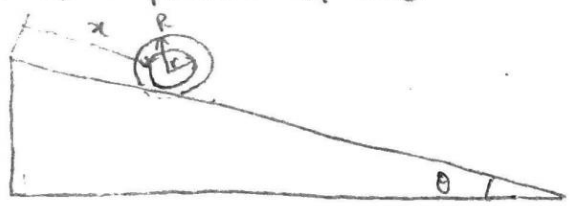
Figure 4. LabView VI for Part 2

Results

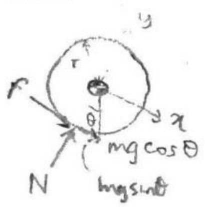
Analytical Model

Below is the result of the analysis of the Ball on the Track system. As a result, I recovered the expression for the displacement of the ball through time. The resulting equation shows that the displacement is directly proportional to the time squared.

· Develop an analytical model to calculate the position of the ball as a function of time.



FBD-



$$\Sigma M = I\alpha$$

$$-mg\sin\theta \times r = I\alpha$$

$$I_0 = \frac{2}{5} mR^2, \quad I = \frac{2}{5} mR^2 + mr^2$$

$$-mg\sin\theta \times r = \left(\frac{2}{5} mR^2 + mr^2\right)\alpha$$

$$-mg\sin\theta \times \frac{r}{R^2} = \left[\frac{2}{5} m + m\left(\frac{r}{R}\right)^2\right]\alpha$$

$$\alpha = \frac{-mg\sin\theta \frac{r}{R^2}}{\frac{2}{5} m + m\left(\frac{r}{R}\right)^2}$$

$$\ddot{x}(t) = -\alpha r$$

$$\dot{x}(t) = -\alpha r t + \cancel{\dot{x}_0}$$

$$x(t) = -\frac{\alpha r}{2} t^2 + \cancel{x_0}$$

$$x(t) = \frac{1}{2} \frac{g\sin\theta \frac{r}{R^2}}{\frac{2}{5} + \left(\frac{r}{R}\right)^2} r t^2$$

$$\boxed{x(t) = \frac{r^2 g\sin\theta}{\frac{2}{5}R^2 + r^2} \times \frac{t^2}{2}}$$

$$x(t) = \frac{\left(\frac{r}{R}\right)^2 g\sin\theta}{\frac{2}{5} + \left(\frac{r}{R}\right)^2} \times \frac{t^2}{2}$$

compare with polyfit

Calibration Data

Below is the plot of voltage measured versus the ball's position. Linear regression is applied to the resulting plot to obtain the linear relationship between the voltage measured and the ball's position. This resulting relationship (highlighted inside the red rectangle) will be used to track the ball's position.

Voltage (V)	Distance (in)
9.8	11
9	12
8.1	13
7.2	14
6.3	15
5.3	16
4.4	17
3.5	18
2.5	19
1.5	20
0.5	21
-0.5	22
-1.5	23
-2.6	24
-3.6	25
-4.7	26
-5.8	27
-6.9	28

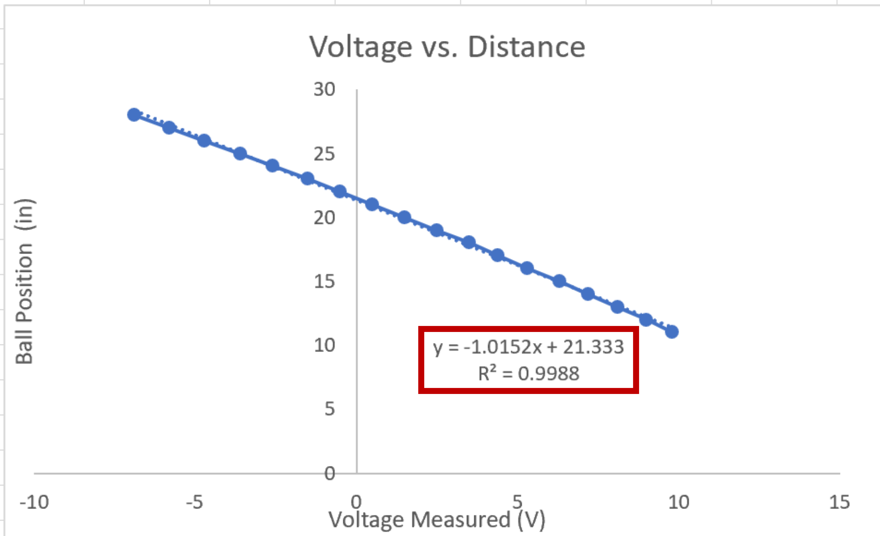


Figure 5. Calibration Data

Rolldown Data

Below is the plot of the position of the ball versus time. The rolldown data is acquired with two different track angle.

7.34 Degrees Track

Firstly, below is the rolldown data acquired with the track angle of 7.34 degrees (the result has been concatenated because I excluded the data acquired when the ball is either stationary or reached the end of the track).

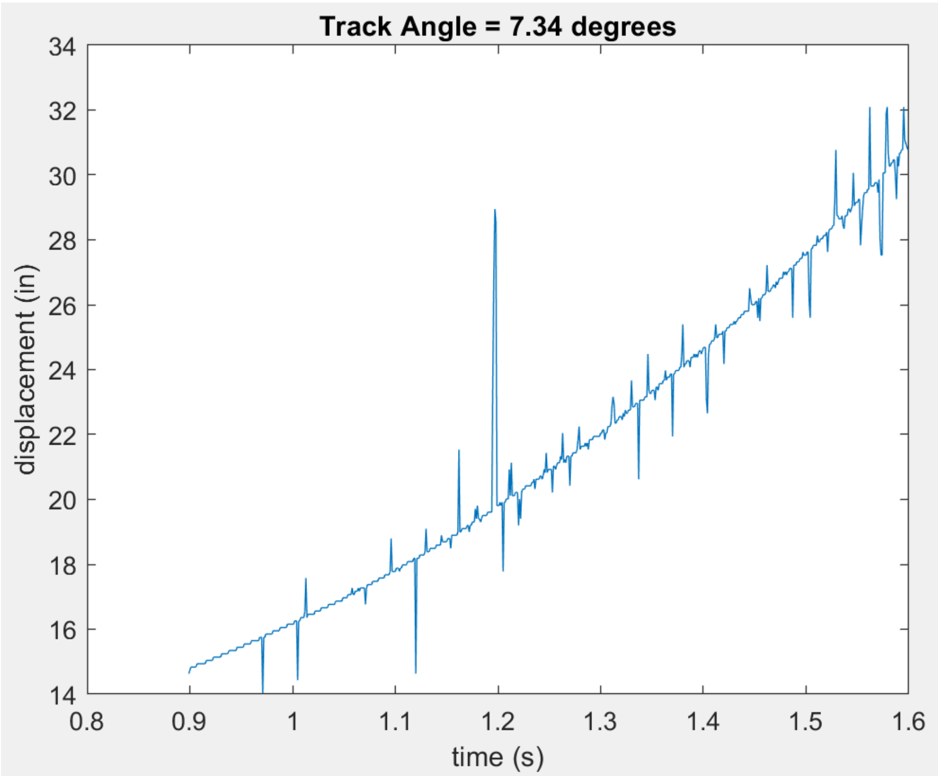


Figure 6. Displacement vs. Time for 7.34 degrees Track

Next, below is the plot of the linear fit of the displacement versus time squared. We can observe that the linear fit obtained is closely representative of the rolldown data disregarding the noise in the measurement. This comparison means that the gradient can be compared with the analytical model's equation to recover the R/r ratio. This is all done in the MATLAB script.

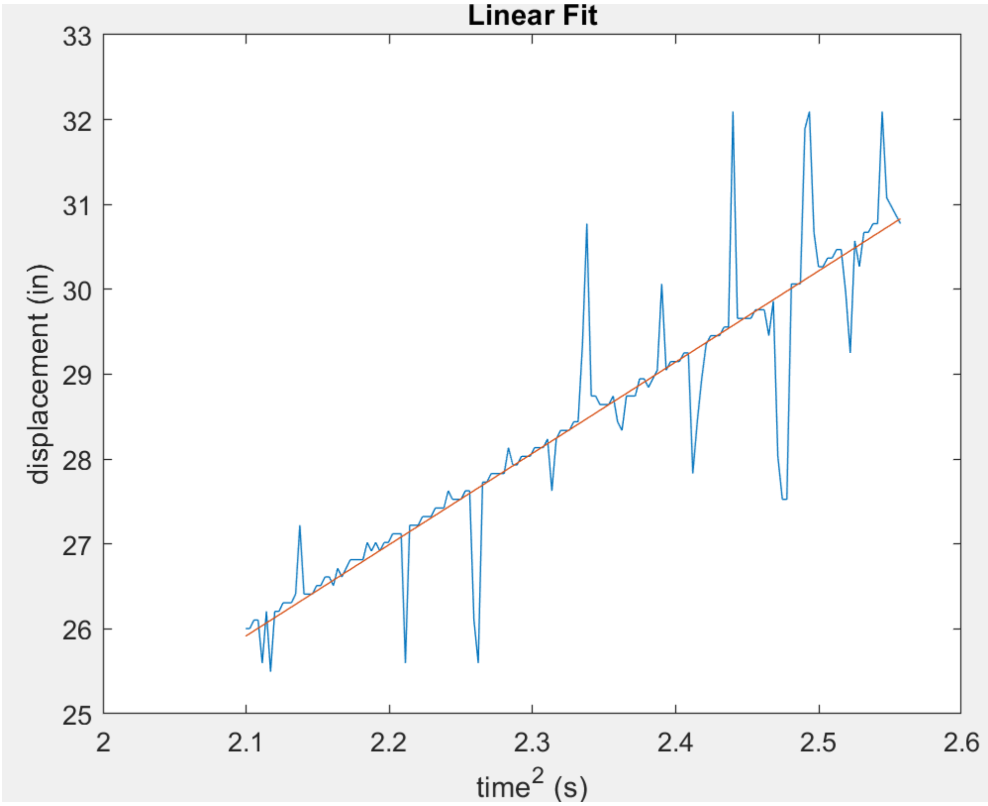


Figure 7. Linear fit of the Measured Data

From the resulting linear fit, we obtain:

$$R/r = 1.80$$

5.36 Degrees Track

The same procedure is done on the system with the track angle of 5.36 degrees. Below are the resulting plots.

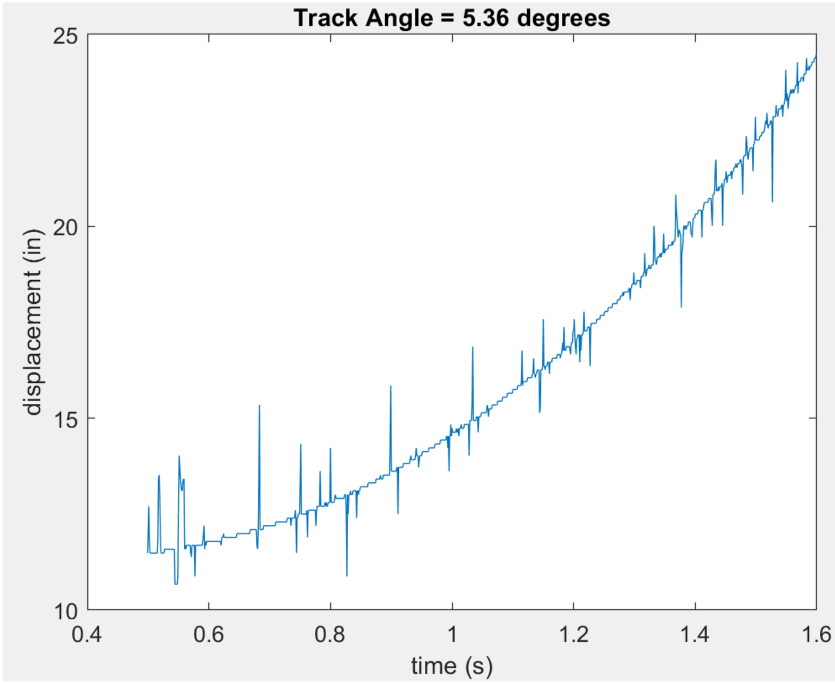


Figure 8. Displacement vs. Time for 5.36 degrees Track

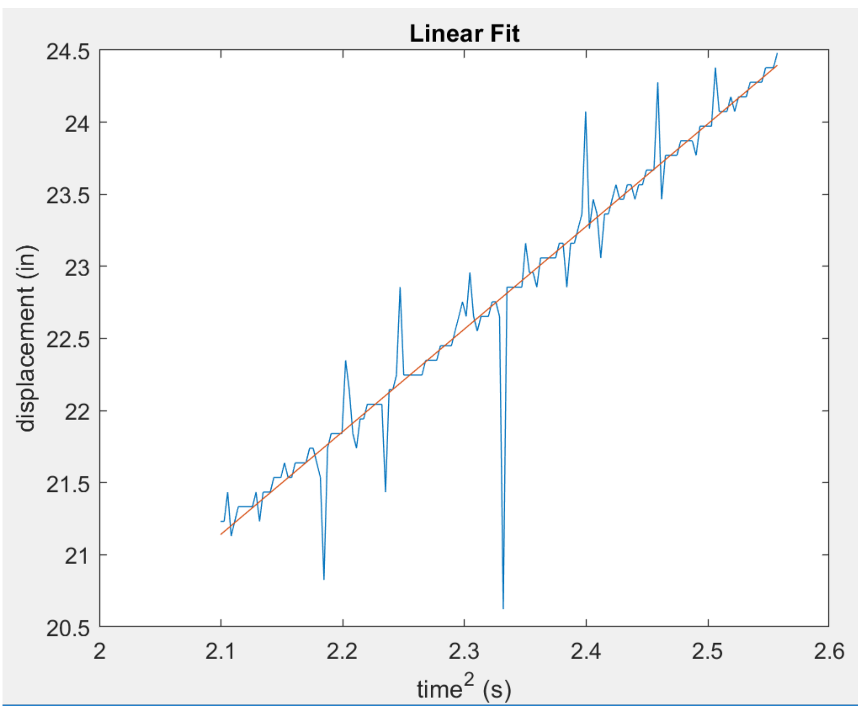


Figure 9. Linear fit of the Measured Data

The resulting R/r ratio obtain from this angle is:

$$R/r = 1.96$$

Plotting the Measured vs. Theoretical Displacement

The resulting R/r ratio can be used to calculate the theoretical coefficient in the analytical model equation. Below are the result of the measured versus theoretical displacement obtained from the R/r ratio of 1.8. The bold red line represents the theoretical displacement while the white dots represent the actual data.

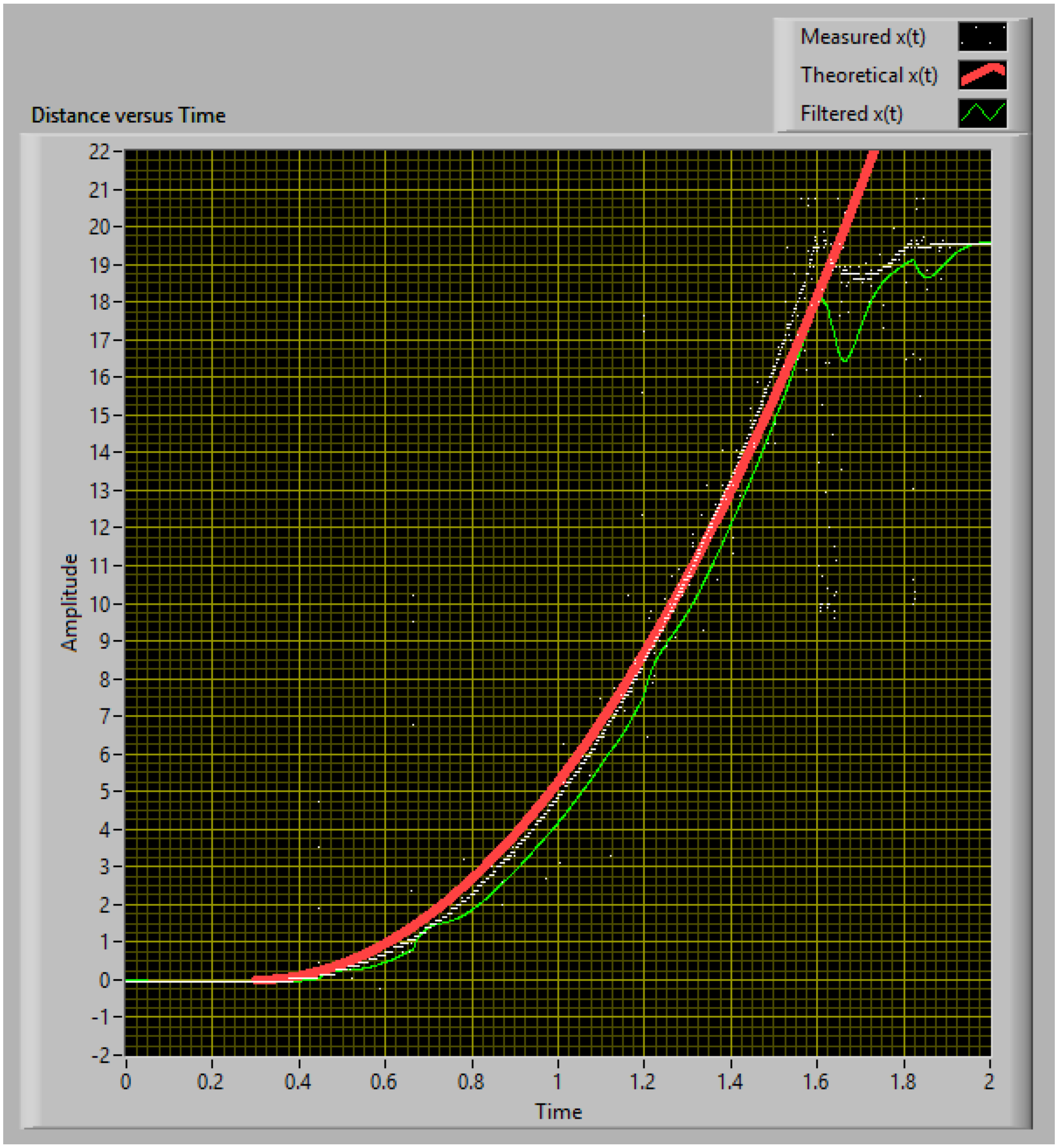


Figure 10. Measured vs. Theoretical Displacement for 7.34 degrees track

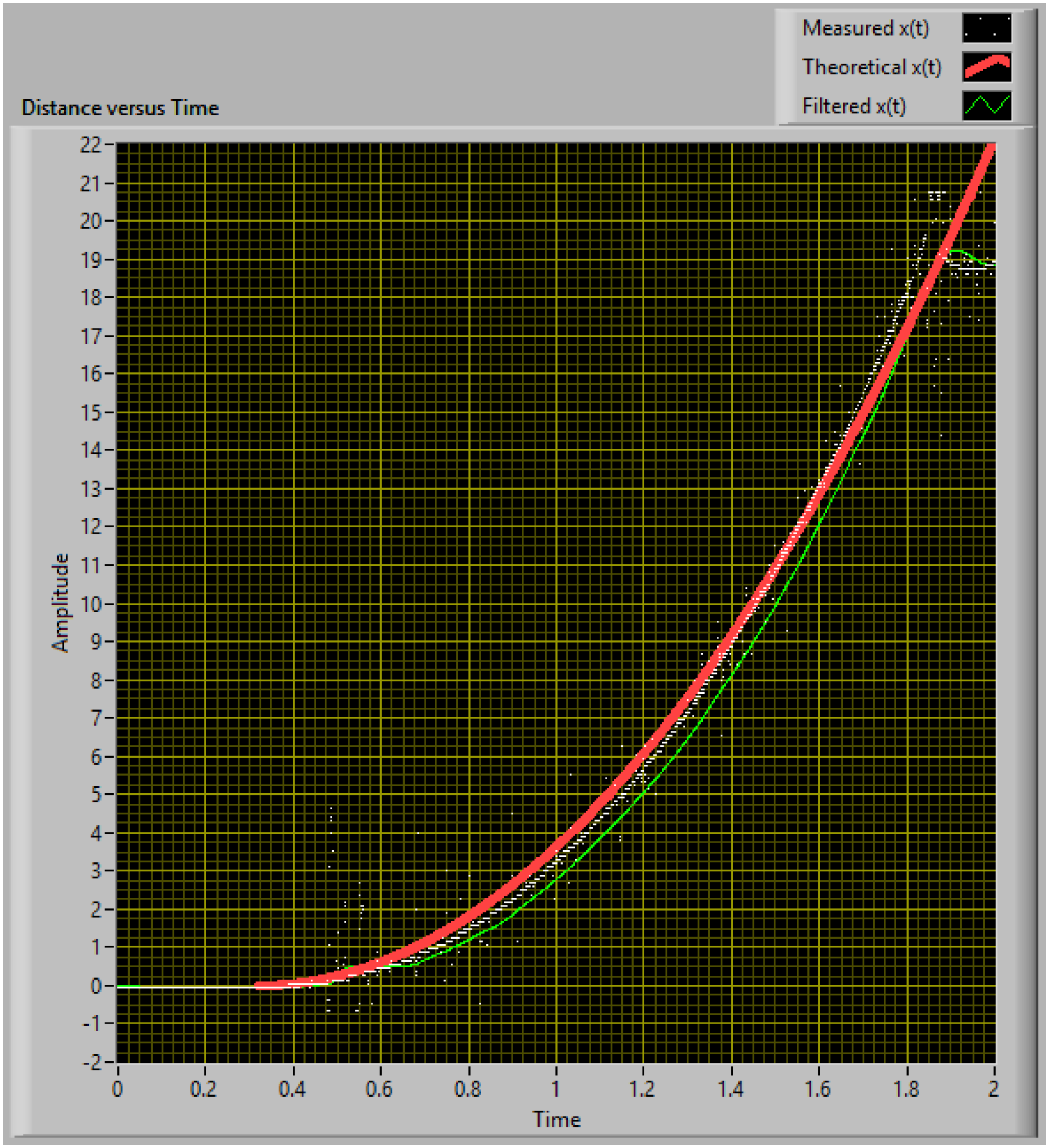


Figure 11. Measured vs. Theoretical Displacement for the 5.36 degrees track.

Part 2

In part 2, the previous data collected is exported into a csv file, which is exported by a VI to be plotted and filtered. The plots below represent the rolldown data of the 7.34 degrees track with variations of filter order and cutoff frequency.

In the plots below, I kept the filter order to be at 2 while changin the cutoff frequency.

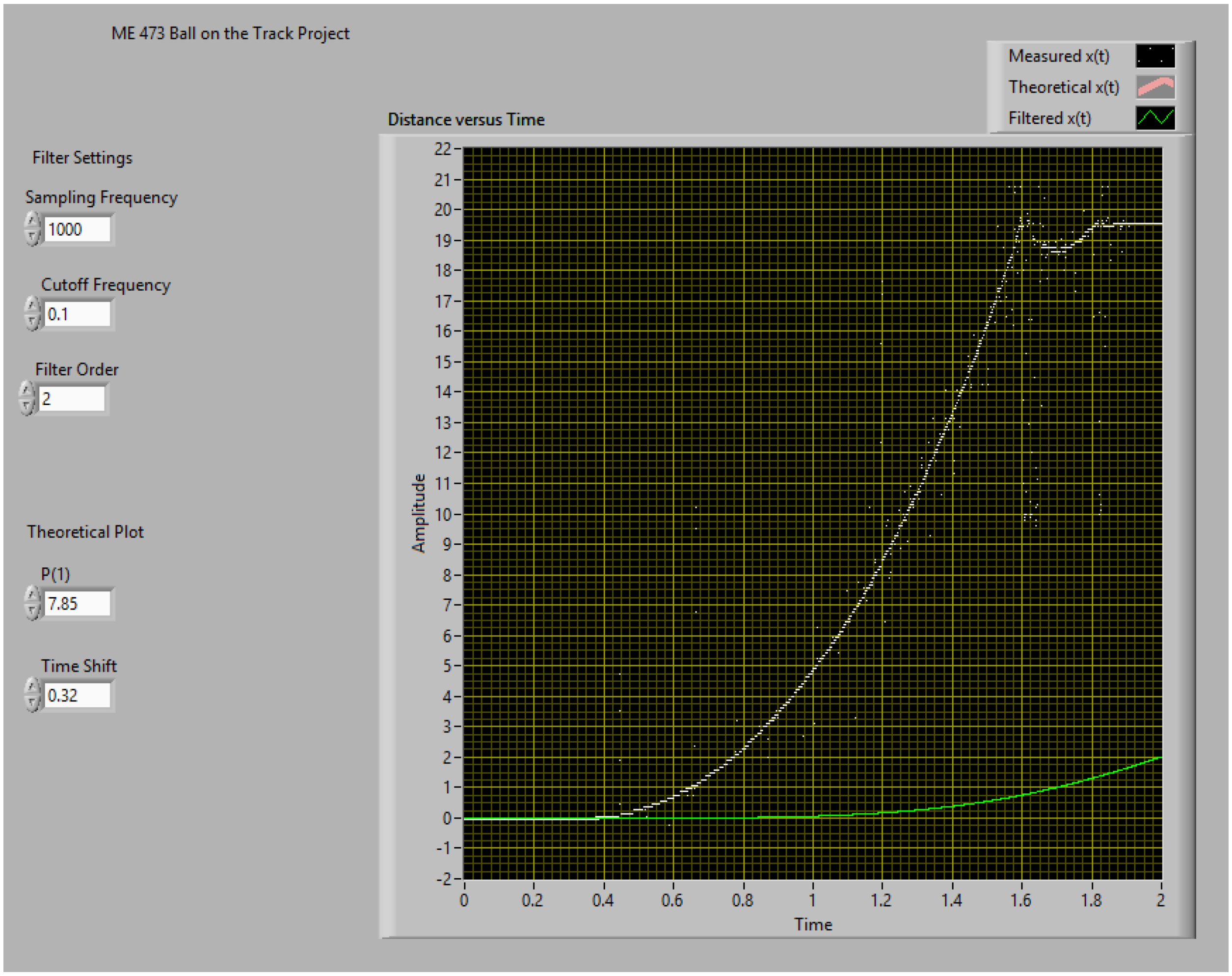


Figure 12. 2nd order Butterworth with 0.1 Hz Cutoff Frequency

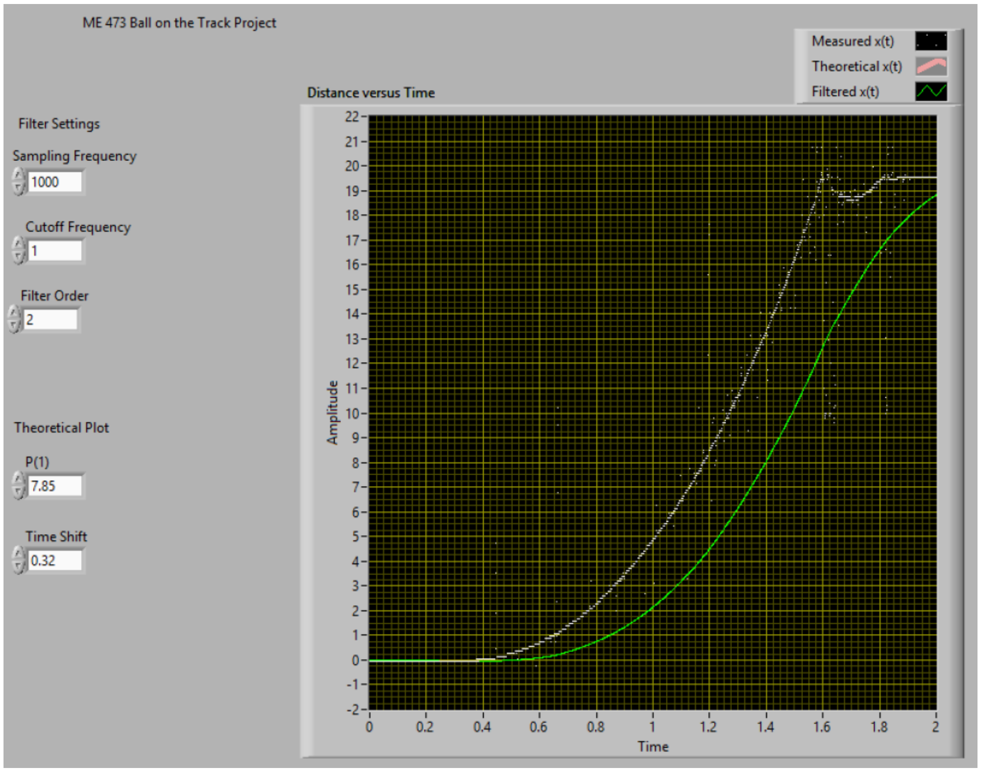


Figure 13. 2^{nd} order Butterworth with 1 Hz Cutoff Frequency

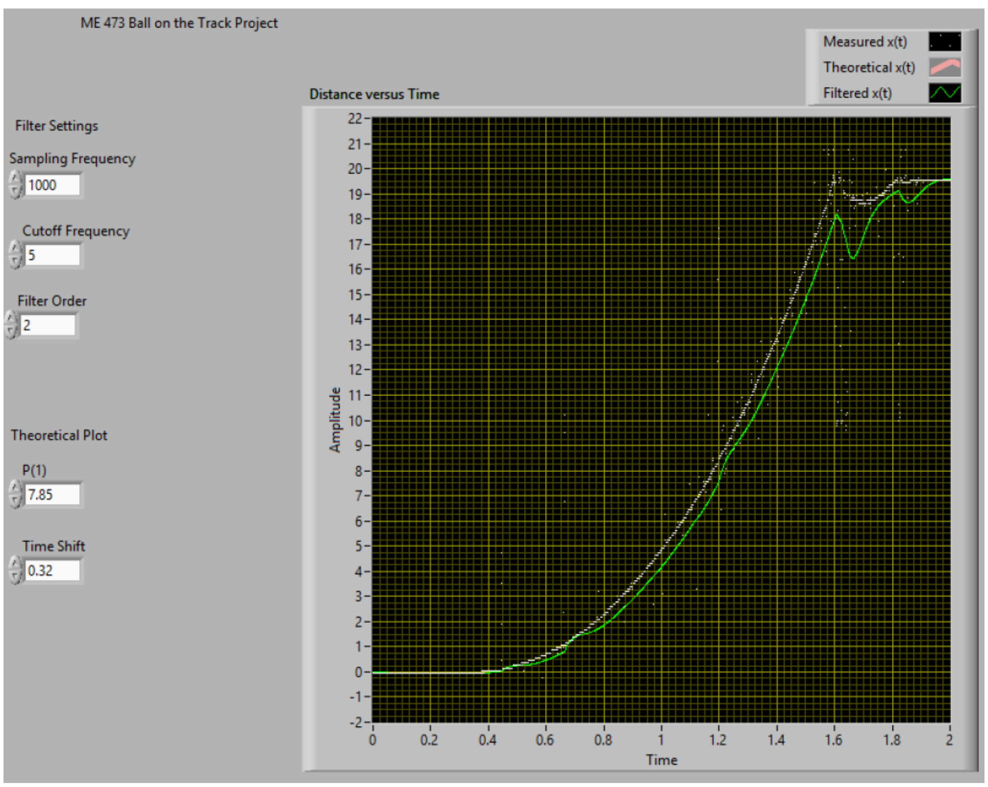


Figure 14. 2^{nd} order Butterworth with 5 Hz Cutoff Frequency

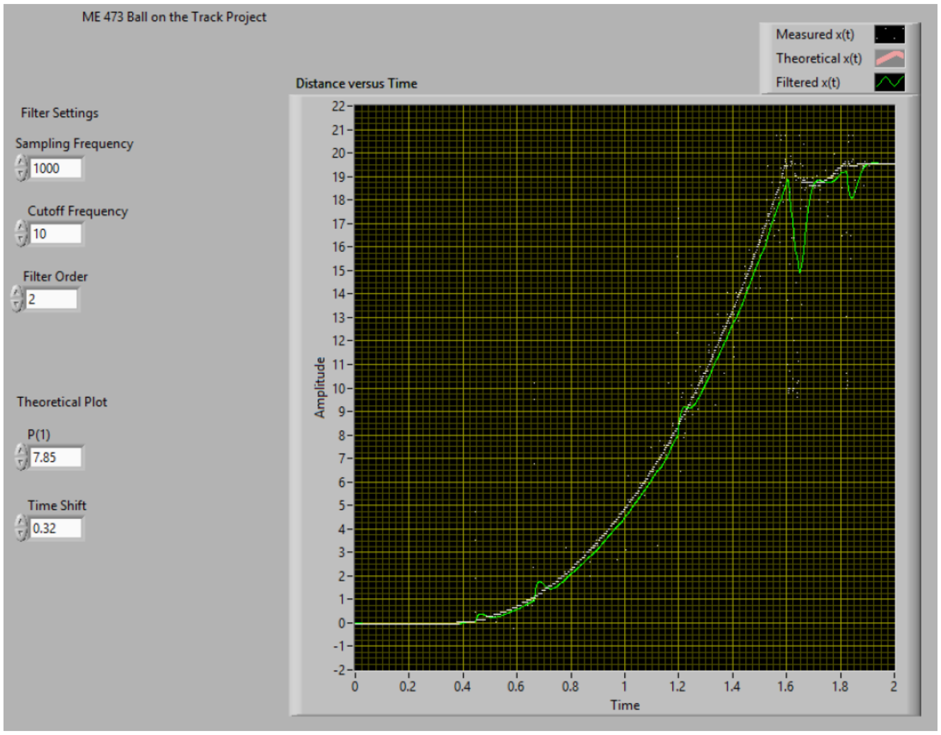


Figure 15. 2nd order Butterworth with 10 Hz Cutoff Frequency

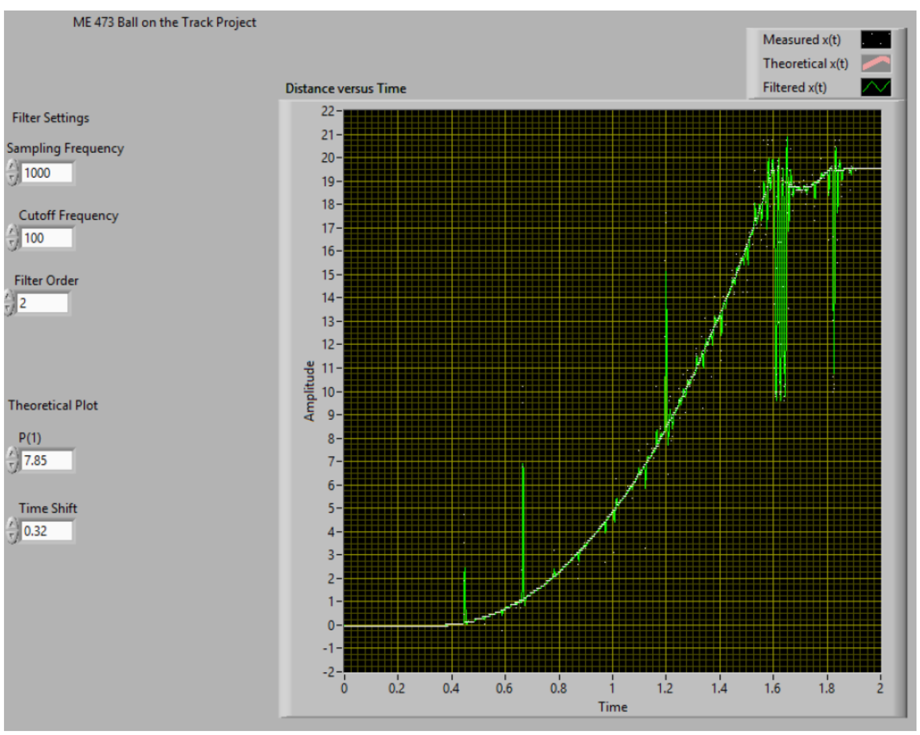


Figure 16. 2nd order Butterworth with 100 Hz Cutoff Frequency

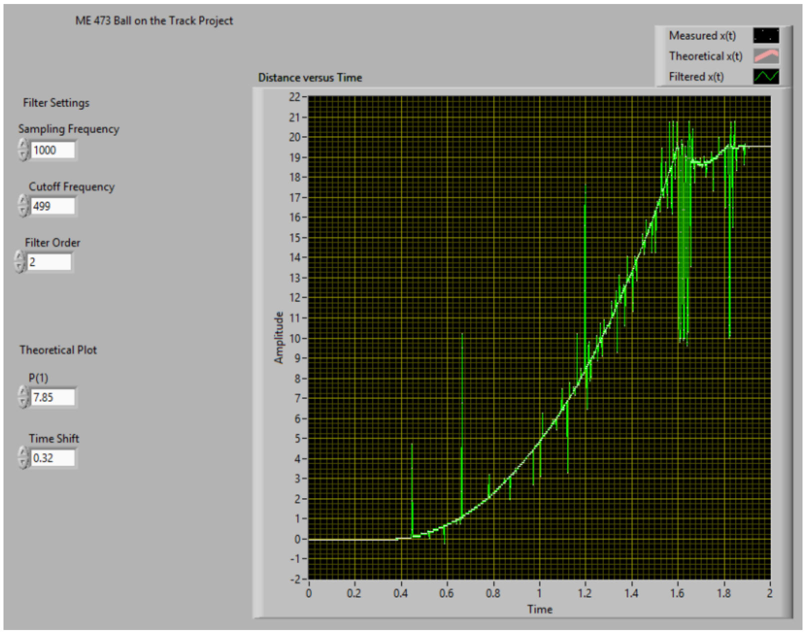


Figure 17. 2nd order Butterworth with 499 Hz Cutoff Frequency

In the plot below, I kept the cutoff frequency to 5 Hz (which I think is the best cutoff frequency) and lowered the filter order to be 1.

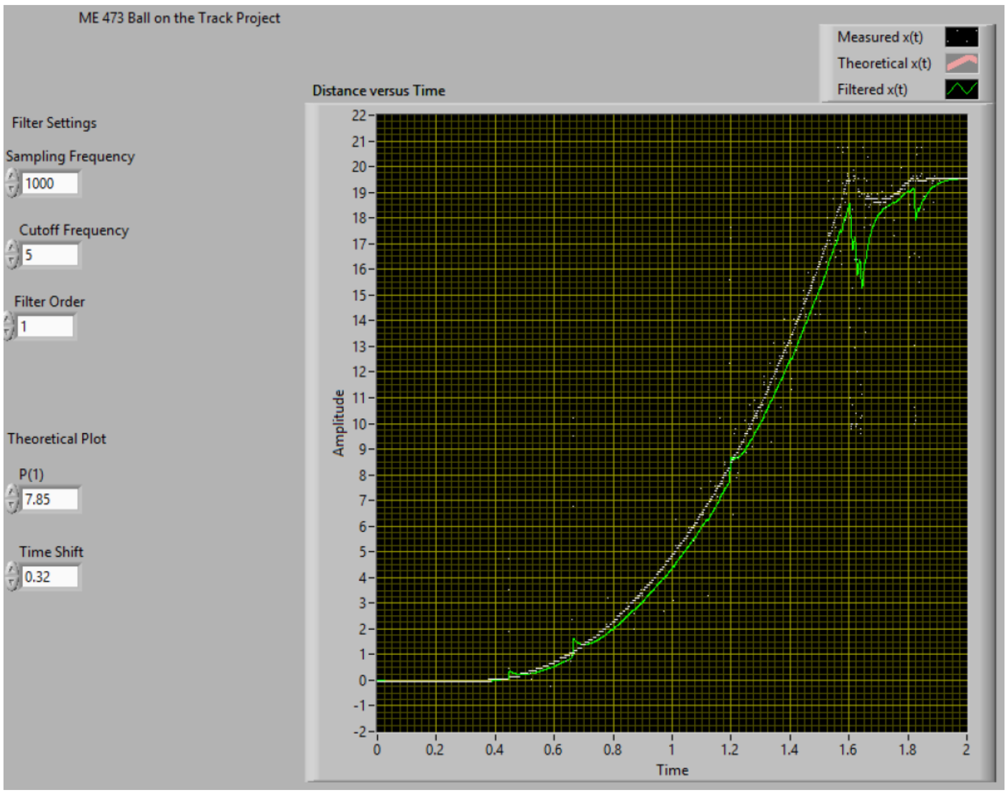


Figure 18. 1st order Butterworth with 5 Hz Cutoff Frequency

Discussion

System Identification

The calibration data recovered should be accurate upon comparing the results with classmates. However, problems arised when I compared my R/r ratio with my classmate's values. My technique to determine the R/r ratio is by implementing a MATLAB script that applies a linear regression to a range of measured value. I started with obtaining R/r values of 2.2, which are far higher in comparison to my classmate's. However, I realized that picking specific regions of the obtained data plays a huge roll to the variance of the R/r. Therefore, my solution to this problem is to iteratively decide the best range that provides the R/r value which will produce the most similar measured vs. theoretical plot in the LabView generated plot. The resulting R/r ratio I picked is 1.8, which if used in the analytical model, closely approximates the trajectory of the ball. (Fig. 10 and Fig. 11). This value is obtained by picking the right half region of the obtained data excluding the points when the ball crashes and in the 7.34 degrees track. I believe that these setup provides more accurate R/r because we the quadratic relationship is more accurate after the ball has rolled for a while.

Setting up the Filter

The resulting plots of the varying filter setup provides a lot of information on how filters work. Firstly, **setting the cutoff frequency too low would cause the filtered data to follow the general smoothed shape of the measurement, but with values highly off the measured data. This result is caused by the filter eliminating useful spectrum of the data**, which would make the Butterworth filter to be innacurate.

The ideal case we would like to achieve is to eliminate only the spectra referring to the noise in the measurement. Therefore, increasing the cutoff frequency into 5 shows the best observed outcome of the filter (Fig. 14). The result of this filter is a smoothed curve whose values are approximately equal to the measurement. The smoothed out curve result means that noise has been filtered out.

Moving on to increase the frequency to 100 Hz, this would render the filter to be obsolete since it includes high spectra pertaining to noise, which spits out the same filtered data as the measured data. I also included the result of the filter when the cutoff is near the Nyquist frequency (500 Hz) which would prevent the Butterworth filter to filter almost nothing at all.

Lastly, changing the filter order would also affect the output of the filter. Reducing the filter order to 1 will cause the filter output to be more similar to the measured data, while still capturing some of the noise as compared to the same cutoff frequency of a 2nd order filter. This is the result of a softer knee of the Butterworth filter which would not attenuate the noise fast enough that it is recaptured in the filter output. Increasing the filter order to higher that 2nd order will cause the filter to smooth out the result better. However, there would be a phase lag associated with the result that will provide approximately the same displacement for a sample time that is shifted to the right.

This experiment could be improved by adding optical encoder as the sensor.

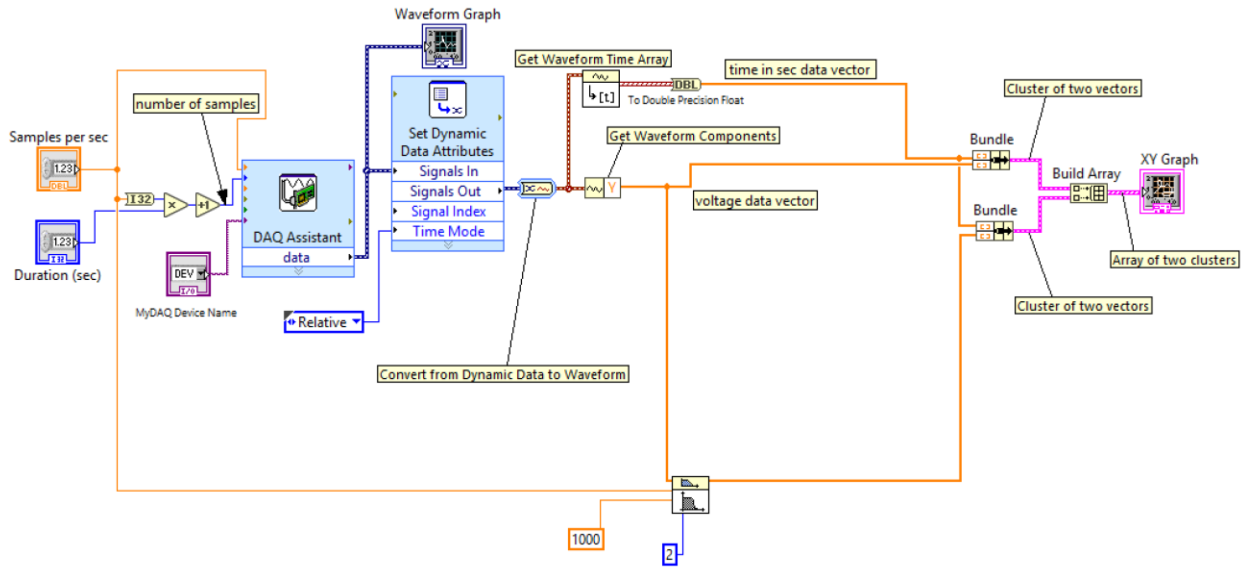
Appendix

MATLAB Code

```
% ME 473 Project 3
% Khrisna Kamarga
clear all; close all; clc;

rolldown = xlsread("Project3_K.xlsx");
calibrate = xlsread("Project3_Calibration.xlsx","Khrisna");

%%
close all; clc;

voltage = calibrate(:,1);
distance = calibrate(:,2);

fit = polyfit(voltage, distance, 1);
plot(voltage, polyval(fit, voltage));
hold on
plot(voltage, distance);
pause
hold off
%%
clc; close all;

cutaway = [1450, 1600];
time = rolldown(:,7);
time = time(cutaway(1):cutaway(2));
voltage = rolldown(:,8);
voltage = voltage(cutaway(1):cutaway(2));

displacement = fit(1)*voltage + fit(2);
P = polyfit(time.^2, displacement, 1);

trend =  P(1)* time.^2 + P(2);

syms rR
syms X

% X = solve((X*386.09*sind(7.34)*0.5)/(2/5 + X) == P(1), X);
X = solve((X*386.09*sind(5.36)*0.5)/(2/5 + X) == P(1), X);
X = vpa(X);
rR = sqrt(X);
Rr = 1/rR;

plot(time, displacement)
xlabel("time (s)");
ylabel("displacement (in)");
title("Track Angle = 5.36 degrees");
pause

plot(time.^2, displacement)
hold on
plot(time.^2, polyval(P, time.^2))
xlabel("time^2 (s)");
ylabel("displacement (in)");
title("Linear Fit");

pause

disp(Rr)

%%
close all; clc;
X = ((1.8+1.96)/2)^-2;
X = 1.8^-2;
coeff = (X*386.09*sind(5.36)*0.5)/(2/5 + X)
```

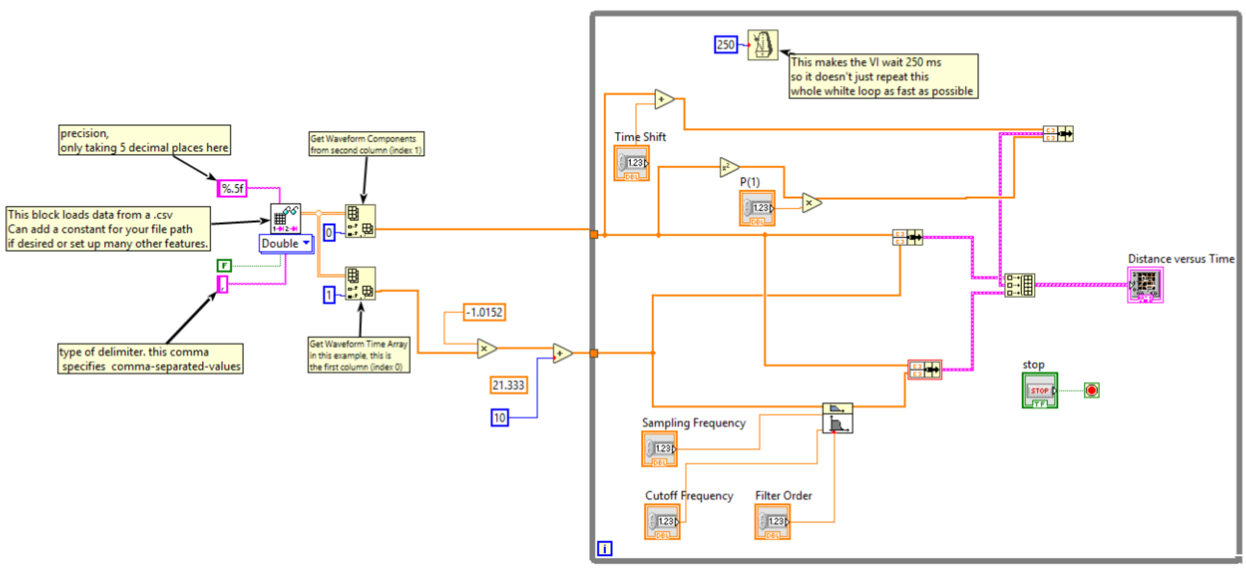
LabView VI

Calibration and Rolldown Data Collection



Kamarga_ME473_Project3_calibrationAndRolldown

Measurement Filtering and Comparison with Theoretical Value



Kamarga_ME473_Project3_csvProcessing