

1. What is meant by each of the following terms: modulation, modulating signal, carrier signal, modulated signal, and demodulation? Explain the following types of signal modulation giving an application for each case: AM, FM, PM, PWM, PFM. How could the sign of the modulating signal be accounted for during demodulation in each of these types of modulation? Explain, in your own words, how AM modulation and demodulation works. When and why the demodulation process can fail?

Modulation:

According to the textbook, “modulation” refers to signals of which its properties (such as amplitude or frequency) have been varied. According to WikiBooks, modulation is a process of mixing a signal with a sinusoid to produce a new signal.

Modulating Signal:

Modulating Signal is also known as the data signal or the message signal. It is used to vary a property of a carrier signal.

Carrier Signal:

Carrier Signal is used for subsequent handling. The carrier signal is the sinusoidal signal that is used in the modulation.

Modulated Signal: Modulated signal is the combination of the carrier signal and the data signal, which is used for subsequent handling.

Demodulation: Demodulation is the process to recover the data signal from the modulated signal by removing the carrier signal.

AM: Amplitude Modulation.

This is a type of signal modulation in which the amplitude of the carrier signal is modulated in proportion to the message signal while the frequency and phase are kept constant. AM is used in AM radio (long distance radio transmission)

FM: Frequency Modulation

This is a type of signal modulation in which the frequency of the carrier signal is modulated in proportion to the message signal while the amplitude and phase are kept constant. FM is used in FM radio transmission (shorter distance radio transmission with higher quality due to its higher bandwidth).

PM: Phase Modulation

This is a type of signal modulation in which the phase of the carrier signal is varied according to the low frequency of the message signal. PM is used in digital transmission coding schemes that underlie a wide range of technologies like WiFi, GSM and satellite television.

PWM: Pulse Width Modulation

This is a type of signal modulation in which the carrier signal is a pulse sequence of constant amplitude. The pulse width is changed in proportion to the amplitude of the data signal while keeping the pulse

spacing constant. PWM signals are extensively used for controlling electric motors and other mechanical devices such as valves and machine tools.

PFM: Pulse frequency modulation

This is a type of signal modulation in which the carrier signal is a pulse sequence of constant amplitude. In this method, it is the frequency of the pulses that is changed in proportion to the value of the data signal, while keeping the pulse width constant. PFM can also be used in controlling motors, with better response because it is less susceptible to noise.

Accounting the sign:

In PCM (Pulse Code Modulation): an extra sign bit is added to represent the sign of the transmitted data sample.

In AM and FM: a phase-sensitive demodulator is used to extract the original signal with the correct algebraic sign.

In PWM and PFM: A sign change in the modulating signal can be represented by changing the sign of the pulses.

AM in depth:

AM is achieved by multiplying the data signal by a high-frequency (periodic) carrier signal.

$$x_a(t) = x(t)x_c(t)$$

The carrier could be any periodic signal such as harmonic (sinusoidal), square wave, or triangular. The main requirement is that the fundamental frequency of the carrier signal be significantly large (by a factor of 5 or 10) than the highest frequency of interest (bandwidth) of the data signal.

Assuming that the carrier signal is a cosine wave, the modulated signal is:

$$\tilde{x}(t) = \frac{2}{a_c} x_a(t) \cos 2\pi f_c t$$

Taking the Fourier Transform, the spectrum of the modulated signal can be recovered as:

$$\tilde{X}(f) = X(f) + \frac{1}{2}X(f - 2f_c) + \frac{1}{2}X(f + 2f_c)$$

Where the spectrum of the modulated signal is represented as  $X(f)$ , and this can be recovered by using a low pass filter to the modulated signal.

Demodulation failure:

As discussed previously, the spectrum of the modulated signal can be obtained by applying a low pass filter to only recover  $X(f)$ . However, if the frequency band of  $1/2X(f - 2f_c)$  overlaps the spectrum of the data signal, the signal recovered would not consist of purely the data signal, which results in failure.

Problem 2

8-bit ADC with FSV = 10V

What is the resolution and quantization error?

8 bit =  $2^8 = 256$  possible values

$$\text{resolution} = \frac{10 \text{ V}}{256 \text{ bits}} = \boxed{0.0390625 \text{ V/bit} \Leftarrow \text{resolution}}$$

$$QE = \pm \times \text{resolution}$$

$$\boxed{0.01953125 \text{ V} \Leftarrow \text{quantization error}}$$

Problem 3

Compare: Constant voltage bridge, constant current bridge, and half bridge for non-linearity, temp. effect, and cost

Constant Voltage Bridge

non-linearity:  $\frac{\delta V_o}{V_{ref}} = \frac{SR/R}{(4 + 2SR/R)}$ ,  $N_p = 50 \frac{SR}{R} \% \Leftarrow \text{ranked 2nd overall}$

temperature effect: Temperature will slightly affect the resistance of the resistors (minimal effect)

Cost: This circuit is cheap to build due to the usage of only resistors and voltage source

Constant Current Bridge

non-linearity:  $\frac{\delta V_o}{R_i \text{ref}} = \frac{SR/R}{(4 + SR/R)}$ ,  $N_p = 25 \frac{SR}{R} \% \Leftarrow \text{ranked 1st overall}$

temperature effect: Similar to that of constant voltage bridge (minimal effect)

Cost: expensive because ideal current source is hard to make

## Half Bridge

Non-linearity :  $\frac{\delta V_o}{V_{ref}} = \frac{R_F}{R} \frac{\delta R/R}{(1 - \delta R/R)}$  ,  $N_p = 100 \frac{\delta R}{R} \% \Rightarrow$  worst among all

temperature effect : This circuit uses op-amp, which is temperature sensitive

Cost : op-amp is more expensive than typical resistors and voltage source, but cheaper than current source

Calculate  $N_p$  for half bridge due to  $\delta V_{ref}$  in the  $V_{ref}$ .

$$\text{if } \frac{\delta V_{ref}}{V_{ref}} = 1\%$$

$$N_p = 100 \frac{\delta R}{R} \% = 100 \frac{\frac{\delta V_{ref}}{V_{ref}}}{1} \Rightarrow \boxed{N_p = 1\%}$$

Problem 4



### Problem 2

8-bit ADC with  $F_{SV} = 10V$ .

What is the resolution and quantization error of the ADC?

$$8 \text{ bit} = 2^8 = 256 \text{ possible values}$$

$$\text{resolution} = \frac{10}{256} = 0.0390625 \text{ V/bit} = \text{resolution}$$

$$QE = \frac{1}{2} \times \text{resolution} = 0.01953125 \text{ V}$$

### Problem 3

Compare: constant voltage bridge, constant current bridge, and half bridge for non-linearity, temperature effect, and cost.

Bridge Type	Non-linearity	Temperature Effect	Cost
Const. Voltage bridge	$V_o = \frac{SR/R}{4R + 2SR/R} V_s$ $N_r = 50 \frac{SR}{R} \%$	minimal effect	Cheapest
Const. Current bridge	$N_p = 25 \frac{SR}{R} \%$ <p>the non-linearity is half of that of the constant voltage</p>	minimal effect	Current source is much costlier than voltage source
Half Bridge	$N_p = 100 \frac{SR}{R} \%$ <p>worst / most non-linear</p>	Op amp is highly affected by temperature variation	

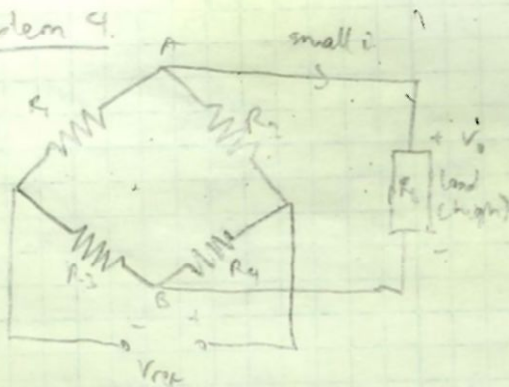
$N_r$  for half bridge due to  $\delta V_{ref}$  in the  $V_{ref}$ . Compute  $N_r$  for 1% error

$$N_p = 100 \frac{SR}{R} \%$$

$$N_p = 100 \times 0.01$$

$$N_p = 1\%$$

## Problem 4.



$$R_1 = R_2 = R_3 = R_4 = R$$

$R_2 \rightarrow$  strain gauge  $\rightarrow$  tensile

$R_3 \rightarrow$  strain gauge  $\rightarrow$  compressive

$$R_1 = R + \Delta R$$

$$R_3 = R - \Delta R$$

Find: Bridge output, show that it's non-linear  
what is the result if  $R_2 \rightarrow$  tensile,  $R_3 \rightarrow$  compressive

$$\Delta V_0 = V_0 |_{R_2=R_4=R} - V_0 |_{R_2=R_4=R, R_1=R+\Delta R, R_3=R-\Delta R}$$

$$R_1 = R + \Delta R$$

$$R_3 = R - \Delta R$$

$$\text{and } V_0 = \left( \frac{R_1}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \right) V_s$$

$$\Delta V_0 = \left( \frac{(R + \Delta R)R - (R - \Delta R)R}{(R + \Delta R + R)(R - \Delta R + R)} \right) V_s$$

$$= \frac{R^2 + \Delta R R - R^2 + \Delta R R}{4R^2 - \Delta R^2}$$

$$\Delta V_0 = \frac{2\Delta R}{4R - \Delta R/R} V_s$$

nonlinearity is reduced

non linear

If  $R_2 \rightarrow$  tensile,  $R_3 \rightarrow$  compressive

$$\Delta V_0 = \left( \frac{R}{R + R + \Delta R} - \frac{R}{R + R - \Delta R} \right) V_s$$

$$= \frac{R(R - \Delta R) - (R + \Delta R)R}{(R + R + \Delta R)(R + R - \Delta R)} V_s$$

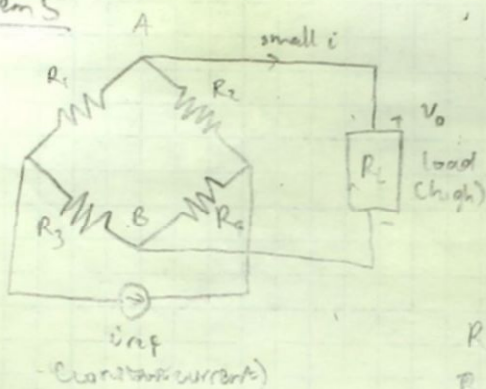
$$= \frac{R^2 - \Delta R R - R^2 - \Delta R R}{4R^2 - \Delta R^2}$$

$$= \frac{-2\Delta R R}{4R^2 - \Delta R^2}$$

there is a minus sign

$$\Delta V_0 = \frac{-2\Delta R}{4R - \frac{\Delta R}{R}} V_s$$



Problem 5

$$R_1 = R_2 = R_3 = R_4 = R$$

$R_1$  &  $R_2 \rightarrow$  strain gauge on a rotating shaft symmetrical along axis of rotation

$$R_1 = R + \delta R$$

$$R_2 = R - \delta R$$

Find: bridge output, show that it's linear

$$V_o = \frac{R_1 R_4 - R_2 R_3}{R_1 + R_2 + R_3 + R_4} i_s$$

$$= \frac{(R + \delta R)R - (R - \delta R)R}{R + \delta R + R - \delta R + R + R} i_s = \frac{R^2 + \delta R R - R^2 + R \delta R}{4R} i_s$$

$$V_o = \frac{\delta R}{2} i_s \leftarrow \text{linear since } V_o \propto i_s$$

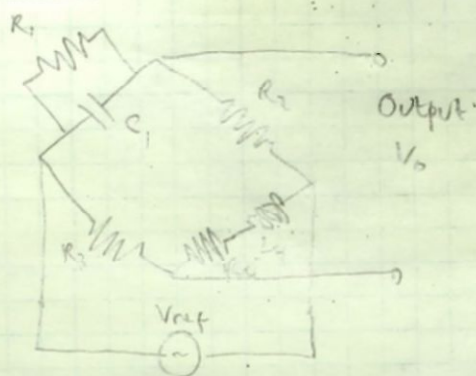
if  $R_4 \rightarrow$  tension,  $R_3 \rightarrow$  compression

$$V_o = \frac{R(R + \delta R) - R(R - \delta R)}{R + R + R + \delta R + R - \delta R}$$

$$= \frac{R^2 + \delta R R - R^2 + \delta R R}{4R} i_s$$

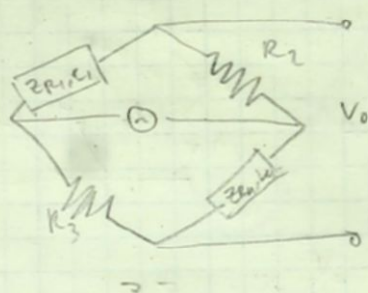
$$V_o = \frac{\delta R}{2} i_s \quad \text{no difference}$$

## Problem 6



Find conditions for a balanced Maxwell bridge

Explain how this circuit can be used to measure variations in  $C_1$  or  $L_4$



Substitute resistance with impedance

$$Z_{R_1, C_1} = \left( \frac{1}{R_1} + sC_1 \right)^{-1}$$

$$= \frac{R_1}{1 + R_1 C_1 s}$$

$$Z_{L_4} = R_4 + L_4 s$$

Equation derived from class

$$\frac{Z_1}{Z_1 + Z_2} = \frac{Z_3}{Z_3 + Z_4} \Rightarrow Z_1(Z_3 + Z_4) = Z_3(Z_1 + Z_2)$$

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

$$\frac{\frac{R_1}{1 + R_1 C_1 s}}{R_2} = \frac{R_3}{R_4 + L_4 s} \Rightarrow$$

$$\frac{R_1}{R_2 + R_1 C_1 R_2 s} = \frac{R_3}{R_4 + L_4 s}$$

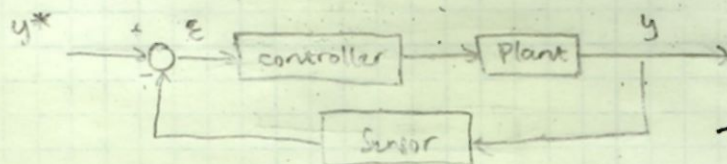
condition for a balanced bridge

To measure inductance, we have to know the capacitance and all the resistances.

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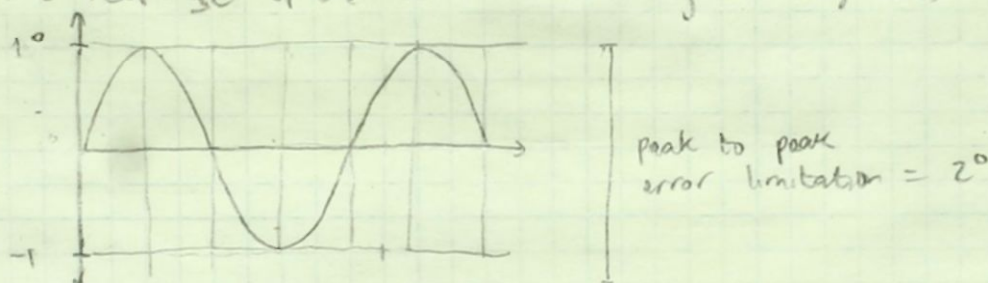
We can calculate the inductance or capacitance by balancing the bridge by adjusting one of the resistances. Then, we can take measurements of  $V_o$  at steady state, so we can calculate the capacitance or inductance by plugging  $s = 0$  to the equation derived.



Problem 7

Find the worst tolerable resolution for  $2^\circ$  steady state maximum error

The response of the servo motor  $y$  (in degrees)



The sensor needs to be at least within half the peak to peak deviation to provide correct feedback to the system that will cause the controller to fix its output to the plant.

worst resolution =  $1^\circ$