

# Rigorous Formal Semantics for Concurrent Systems: A Trace-Based Verification Methodology

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## Abstract

This report provides an exhaustive, expert-level analysis of the semantic foundations required for trace-based formal verification of concurrent and safety-critical systems. It establishes rigorous definitions for execution traces derived from Labeled Transition Systems (LTS) and Kripke Structures (KS), critiques the critical impact of temporal semantic choices (Discrete vs. Dense Time, Metric vs. Logical Synchronization), and defines the spectrum of compositional operators (Shuffle, Parallel, Lockstep). The analysis concludes with a critique based on ACM Computing Surveys methodology, highlighting the mandatory need for robust taxonomy, and proposing a Deep Research Agenda to validate the methodology's soundness, address scalability, and integrate with industrial tracing frameworks.

## 1 Introduction: The Necessity of Rigorous Trace Semantics

### 1.1 Trace Semantics as the Foundation of System Verification

Formal verification techniques serve as the rigorous backbone for validating the correctness, safety, and security of critical systems, particularly in software engineering and cyber-physical domains. At the heart of these methodologies lies the concept of an execution trace: the recorded sequence of interactions, state changes, and events that define a system's behavior over time. Traces transform the abstract formal model into a verifiable sequence that can be compared against desired properties. This process is essential because it allows researchers and engineers to retain "fine grained information collected in simulation traces" derived from high-fidelity simulations of complex environments, subsequently enabling the application of formal methods to realistic behavioral data [?].

The critical value of this approach lies in its ability to bridge the gap between simulation and formal proof. For instance, simulation engines can provide realistic data concerning complex scenarios, such as aircraft landing procedures using continuous descent approaches. This data, encapsulated in simulation traces, can then be abstracted, modeled, and analyzed using rigorous formal methods like model checking against required safety properties [?]. However, the reliability of the entire verification

process hinges on the precision and soundness of the semantic model defining the trace itself. If the definition of a trace—what constitutes an event, how time progresses, and how concurrent components interact—is ambiguous, the formal analysis built upon it may yield flawed results, introducing semantic risk into safety-critical domains.

## 1.2 Defining the Scope: The Formal Methods Landscape

Formal verification methods broadly categorize into two major approaches: interactive theorem proving and automated model checking. Interactive theorem proving provides a highly general framework for modeling and verification, capable of handling both finite-state and infinite-state systems. However, its significant limitation lies in the substantial human effort required to manage potentially “tedious proofs” [?].

In contrast, model checking is characterized by its high degree of automation but is inherently restricted to application domains possessing small finite-state spaces [?]. This limitation presents a crucial challenge when dealing with high-fidelity simulation traces, which often originate from systems with vast or infinite state spaces. The methodology must therefore rely on a systematic process to take a simulation trace, abstract it, model it, and encode it into a language suitable for finite-state model checkers, such as Promela, the input language for the Spin model checker [?]. The success of this translation process mandates an exact, unambiguous definition of the underlying semantic primitives. This report focuses on establishing the necessary semantic constructs—including temporal models and compositional operators—to construct a trace structure that facilitates sound, scalable model checking of concurrent and safety-critical systems.

## 2 Definitional Frameworks for Execution Traces

A robust formal semantics requires that the fundamental building blocks of system behavior—states, actions, and time—be defined with mathematical rigor. The choice of the underlying formal model dictates how traces are interpreted and analyzed.

### 2.1 Foundational Models of System Behavior

#### 2.1.1 Labeled Transition Systems (LTS): The Event-Based Perspective

The Labeled Transition System (LTS) provides an event-based perspective on system behavior, focusing on the actions that cause changes between states. Formally, an LTS is defined as a 4-tuple  $\langle S, Act, \rightarrow, s_0 \rangle$ , where  $S$  is a set of states,  $Act$  is a set of actions (or labels),  $\rightarrow$  is the transition relation (specifically,  $\rightarrow \subseteq S \times Act \times S$ ), and  $s_0 \in S$  is designated as the initial state [?].

The defining characteristic of the LTS model is that the transitions are labeled by actions [?]. Consequently, an LTS trace naturally captures the sequence of actions executed by the system. This structure is highly advantageous for modeling concurrent systems and communication protocols, where the causality and timing of specific interactions (events) are paramount. The trace derived from an LTS, therefore, explicitly includes the actions:  $s_0 \xrightarrow{a_1} s_1 \xrightarrow{a_2} s_2 \dots$

### 2.1.2 Kripke Structures (KS): The State-Based Perspective

In contrast to the event-centric view of LTS, Kripke Structures (KS) offer a state-based approach, where the focus lies on the truth values of properties held within a state. A Kripke Structure  $K$  is defined relative to a finite set of atomic propositions  $AP$ . The state space  $S$  is often characterized by the power set of  $AP$  ( $S = P(AP)$ ), meaning each state in  $S$  is defined by the set of propositions that are true within it [?].

Kripke Structures are the standard foundational model for checking properties defined in branching-time or linear-time temporal logics (such as CTL and LTL). Since states are labeled by the atomic propositions, the verification process often examines a sequence of states ( $s_0, s_1, s_2, \dots$ ) to determine if a temporal formula holds [?]. The trace here emphasizes the observable configuration of the system rather than the action that induced the change. The distinction between KS (state-based) and LTS (event-based) is foundational and must be clearly maintained when constructing a unified semantic model, as the choice influences the expressiveness of the temporal properties that can be verified.

### 2.1.3 The Canonical Execution Trace Definition

To facilitate verification methodologies like those utilized in trace analysis [?], a canonical execution trace ( $\tau$ ) must unify the state-based and event-based perspectives. The trace is formally defined as a maximal sequence alternating states and actions:

$$\tau = s_0 \xrightarrow{a_1} s_1 \xrightarrow{a_2} s_2 \dots \xrightarrow{a_n} s_n$$

This formal sequence links back to foundational models like Finite State Machines (FSMs), which are often defined abstractly by a 5-tuple encompassing states, inputs, outputs, an update/transition function, and an initial state [?]. For verification purposes, the trace must be sufficiently rich to capture both the state properties (for LTL/CTL) and the actions (for process algebra equivalences). However, high-fidelity simulation traces often involve a massive number of states and events, introducing an inherent tension: the trace must contain enough detail to prove complex safety properties, yet must be abstract enough to be successfully encoded into a finite-state model checker, circumventing the debilitating problem of state-space explosion [?].

## 2.2 Temporal Semantics: The Flow of Events

The interpretation of "time" within an execution trace is arguably the single most critical semantic decision, particularly for real-time and cyber-physical systems. Temporal models range from simplistic discrete progression to highly complex relativistic synchronization.

### 2.2.1 Discrete Time vs. Dense Time Semantics

In formal verification, time can be modeled as either discrete or dense. **Discrete time semantics** assumes time advances in integral, quantifiable units, typically synchronized to a system clock tick. This is computationally straightforward and is utilized by many

conventional model checking tools. **Dense time semantics**, conversely, models time using real numbers, allowing events to occur at any point within a continuous interval [?].

The choice between these models carries profound consequences for security and correctness. Research confirms that many security protocols, especially Cyber-Physical Security Protocols (CPSPs) that rely on physical assumptions (such as round trip time, RTT, or transmission velocity), are vulnerable to attacks that are fundamentally undetectable under discrete time models [?]. Specifically, an attacker can exploit the minute execution delay of instructions that occur within a single, discrete clock cycle. If the verification model uses discrete time, this vulnerability is invisible, leading to an unsound proof of security. To address this, formal verification of systems engaging with real-world physical properties requires dense time models, often implemented via formalisms such as Multiset Rewriting with dense time, to achieve adequate soundness [?]. This distinction demonstrates that discrete time, while tractable, is inherently insufficient for sound analysis of systems where temporal resolution is a security factor.

### 2.2.2 Synchronization: Metric, Logical, and Causal Time

Beyond the discrete/dense distinction, time synchronization determines how events are ordered across different components. **Metric Time Synchronization** involves aligning physical clocks to a precise reference scale, often requiring high accuracy, such as the 30–35 nanosecond precision achieved by systems like the cosmic time synchronizer (CTS) [?]. Such high-performance systems are vital for providing robust reference time scales, acting as potential standalone standards or backups for GPS-disciplined oscillators [?].

In contrast, **Logical Time Synchronization** focuses purely on establishing a transitive causal ordering of events ( $A$  happened before  $B$ ) within a distributed system, independent of physical simultaneity. This form of synchronization is typically transitive [?].

However, the analysis of complex synchronization procedures reveals a critical semantic challenge related to transitivity. While classical Newtonian mechanics and logical synchronization are transitive, procedures based on General Relativity and Quantum Entanglement are often **non-transitive** [?]. This lack of transitivity influences formal analysis, particularly in Ramsey graph theory applied to clock lattices, as non-transitive synchronization impacts the coloring and closure properties of the graph derived from the network of clocks [?]. Therefore, the semantic model supporting the execution trace must explicitly account for whether the system’s observed time is governed by transitive logical causality or non-transitive physical constraints.

### 2.2.3 The Synchronous Abstraction and Lockstep Execution

For systems requiring ultra-reliable, predictable responses—classified as reactive systems (e.g., automatic flight control)—the synchronous abstraction provides a crucial simplification [?]. Synchronous programming languages (like Esterel or Lustre) assume that computation within a step occurs instantaneously (zero-time abstraction), effectively behaving as if electrons were flowing infinitely fast [?].

This abstraction is formalized by the **Lockstep principle**, which is prevalent in fault-tolerant computer systems, such as those employing Dual Modular Redundancy (DMR) or Triple Modular Redundancy (TMR) [?]. In lockstep operation, multiple systems execute the same set of operations concurrently and atomically. The progression from one defined state to the next is treated as an atomic transaction, guaranteeing that either all components complete the step or none do [?]. Operationally, the Lockstep mechanism functions as a strict, synchronous composition operator (akin to a “zip” product), ensuring that the resulting trace is determined by perfectly synchronized, atomic state progression. This contrasts sharply with systems that tolerate arbitrary interleaving.

## 2.3 Compositional Methods in Process Algebra

Compositional operators define how the behaviors of individual system components combine to form the overall system trace. The chosen operator determines the degree of synchronization, from purely asynchronous interleaving to strict, atomic lockstep.

### 2.3.1 Interleaving and the Shuffle Operator

In process calculi and formal language theory, systems are often modeled using the **shuffle operator** ( $\otimes$  or  $\parallel$ ). This operator defines pure interleaving, where concurrent processes execute their actions arbitrarily without requiring synchronization. The shuffle operator is intrinsically tied to non-deterministic behavior and is key to modeling truly asynchronous interactions, yielding traces where actions are arbitrarily interspersed.

The shuffle operator possesses a significant theoretical grounding, having received considerable attention in the equational theory of languages. It is closely associated with the theory of partially ordered multisets (pomsets), which model concurrency based on partial orders [?]. Specifically, the equational theory of series-parallel pomsets has been shown to coincide with that of languages over concatenation and shuffle, confirming the shuffle operator’s role in algebraically capturing non-deterministic, concurrent execution [?].

### 2.3.2 Synchronization and Parallel Composition

The general parallel composition operator ( $\parallel$ ) used in frameworks like Communicating Sequential Processes (CSP) represents a refinement of pure interleaving. Here, processes synchronize on common actions (handshaking) while interleaving on unshared internal or local actions [?]. This is the standard model for many distributed systems based on message passing.

A more restrictive and formalized composition is required for Timed Automata, a key model for real-time systems. The parallel composition of two Timed Automata,  $A \times B$ , necessitates satisfying strict compatibility conditions. These conditions typically dictate that the automata have no output actions in common and that the internal actions of  $A$  are distinct from the actions of  $B$ , and vice versa [?]. These constraints ensure the resulting composed automaton remains deterministic and verifiable. Once

the implementation of a system is defined as a composition of several timed processes (e.g.,

), the resulting complex trace must meet the specified properties [?]. Furthermore, trace verification often relies on

Table 1: Synchronization and Compositional Semantics

— **Operator/Concept** — **Process Algebra Context** — **Synchronization Mechanism** — **Temporal Basis** — **Suitability** — — Shuffle Operator [?] — Formal Languages, Asynchronous Systems — None (Pure Interleaving) — Asynchronous, Unordered Causal Trace — Modeling maximal non-determinism and race conditions — — Parallel Composition ( $\parallel$ ) [?] — CSP, Timed Automata — Handshaking on Common Actions — Synchronized Asynchronous — Standard Distributed Systems and Message Passing — — Lockstep Product (Zip) [?] — Synchronous Languages (SRP), TMR/DMR — Full Synchronization (Atomic Step) — Synchronous, Discrete Time Abstraction — Fault-Tolerant Systems, Safety-Critical Reactive Systems —

### 3 Critical Review via ACM Computing Surveys Methodology

To be considered an expert-level contribution, particularly for publication in a venue like *ACM Computing Surveys* (CSUR), a paper must meet rigorous standards concerning scholarly value, synthesis, and internal organization. The primary criterion for judging a submission is whether it provides a strong contribution to the field of HCI or Computer Science broadly [?]. For a survey article, this requires more than a mere listing of existing work; it demands a structured, ordered synthesis, known as a taxonomy [?].

#### 3.1 Contribution Assessment and Scholarly Value

A submission that focuses solely on defining formal structures without integrating them into a cohesive verification framework risks failing the standard for a “strong contribution” [?]. The work must present a novel synthesis, perhaps focusing on how the various semantic models impact the verification utility. If the paper merely compiles definitions of LTS, KS, and composition operators without explicitly articulating the consequences of selecting one over the other—for example, how metric vs. logical time influences distributed proofs [?—its scholarly value is diminished.

The scope of the work must also be comprehensive. While detailed on formal process algebras, the methodology often neglects integration with modern, industry-standard observability frameworks. A comprehensive survey should establish a framework for translating industrial trace, log, and metric data [19, 20] into the formal structures defined herein. Failing to bridge this gap between theoretical computational semantics and practical industrial tracing severely limits the contribution’s relevance as an exhaustive survey. The overall written appraisal of the work, per ACM guidelines, should typically be substantial, often around a page of text, to ensure constructiveness [?].

## 3.2 Taxonomy and Organizational Critique

The organizational structure of a survey is paramount; a deficiency in taxonomy constitutes a devastating criticism that must be addressed before publication [?]. Taxonomy requires establishing a clear hierarchy that orders and classifies the field, allowing a reader to easily place new contributions relative to the existing body of work [?]. For example, in databases, one must distinguish between relational and non-relational approaches, and further classify non-relational systems into key-value stores, columnar, and graph databases. The lack of a similar clear flow in semantic models is a fundamental structural defect.

### 3.2.1 Flaw 1: Ambiguity in Temporal Taxonomy

If the original work fails to systematically classify temporal models, it introduces fundamental ambiguity. The discussion must be structurally organized to clearly separate **Metric Time** from **Logical Time**, and to isolate **Discrete Time** from **Dense Time** [?]. The profound difference in verification soundness between discrete and dense models, particularly for cyber-physical security [?], demands a dedicated taxonomic axis. A structure that obscures these distinctions prevents readers from soundly selecting the appropriate model for their application.

### 3.2.2 Flaw 2: Lack of Modeling Paradigm Hierarchy

A key taxonomic failure occurs if the foundational models—Labeled Transition Systems (LTS, event-based) and Kripke Structures (KS, state-based)—are not presented as dualistic modeling paradigms [?]. A robust taxonomy must hierarchically arrange these structures, explaining how one is suitable for analyzing actions (LTS) and the other for analyzing state propositions (KS) [?]. Furthermore, the composition operators (Shuffle, Parallel, Lockstep) must be taxonomically ordered by their degree of synchronization, from maximum interleaving to minimum non-determinism, providing a clear map of concurrency semantics [?].

## 3.3 Technical Appraisal of Semantic Soundness

A high-quality review must directly confront the semantic claims regarding system safety. A critical technical flaw arises if the work advocates for discrete time verification in domains where dense time is demonstrably necessary. If the original text failed to acknowledge that attacks against time-sensitive protocols can exploit execution delays *within* a clock cycle—a vulnerability only detectable using dense time semantics—the proposed verification methodology is inherently unsound for such applications [?]. This is a substantive criticism of the work’s fidelity to security requirements.

Similarly, the rigor of compositional definitions is critical. If the parallel composition operator is used without explicit clarification as to whether it represents purely asynchronous interleaving, synchronized handshaking (CSP/CCS  $\parallel$ ), or atomic lockstep [?], the model’s formal integrity is compromised. Any verification result derived from an ambiguous composition is questionable.

### 3.4 Summary Recommendation and Mandate for Revision

The submission, while theoretically grounded, requires **Major Revision (Revise and Resubmit)** to meet the standards of *ACM Computing Surveys*. The core concepts are relevant and vital to the field, but the presentation fundamentally lacks the necessary taxonomic organization [?]. The revised work must structurally integrate the semantic dichotomies (Discrete vs. Dense Time, Transitive vs. Non-Transitive Synchronization) and rigorously define the composition spectrum. Furthermore, the work must provide a clear roadmap for handling the inherent scalability challenges that arise when abstracting “realistic data” (simulation traces) into finite-state models suitable for model checkers like Spin [?].

## 4 Deep Research Agenda and Verification Roadmap

To transform this foundation into a defensible and high-impact survey, a dedicated deep research agenda focused on validation, scalability, and industrial relevance is mandatory.

### 4.1 Essential Validation of Formal Semantics

#### 4.1.1 Research Step A: Canonical Source Validation of Process Algebra

The formal underpinnings of concurrency must be unequivocally established. A deep bibliographic research effort is required to validate the formal syntax and denotational semantics of Communicating Sequential Processes (CSP), particularly examining the Traces Model and the Failures Model [?]. This includes confirming the algebraic properties of the shuffle operator in relationship to concatenation and pomsets, verifying that it correctly captures the semantics of series-parallel ordering in concurrency [?]. This validation step ensures that the definitions used for trace composition are consistent with the established body of theoretical computer science.

#### 4.1.2 Research Step B: Formalisms for Relativistic Time

Since the non-transitivity of relativistic and quantum synchronization procedures presents a substantial challenge to standard causal models [?], the research must investigate specialized formalisms capable of accurately modeling these phenomena. This includes searching for hybrid automata extensions, timed process calculi, or specific temporal logics designed to capture complex, non-Newtonian time properties. Successfully identifying these specialized models will establish the boundaries of applicability for conventional logical clock semantics.

### 4.2 Practical Application and Tool Chain Validation

#### 4.2.1 Research Step C: Feasibility of Trace-to-Model Encoding

A major practical hurdle in applying formal methods to realistic simulation data is scalability [?]. Deep research must locate and analyze current implementations and



technical reports detailing the process of abstracting high-volume simulation traces and translating them into finite-state inputs like Promela code for the Spin model checker [?]. The goal of this investigation is to quantitatively determine the typical complexity threshold—such as trace length or state variable count—at which model checking becomes computationally intractable due to state-space explosion, thus providing crucial limitations for the methodology.

#### **4.2.2 Research Step D: Dense Time Attack Replications**

To substantiate the critical claim that discrete time models are unsound for certain security protocols, verifiable evidence is required. The research agenda must target case studies or published papers that have successfully replicated the novel attacks against Distance Bounding Protocols [?]. These studies must show a clear distinction: attacks found when using dense time verification tools (e.g., UPPAAL, HyTech) that are missed when the same system is analyzed using discrete time models. This validation step formally justifies the necessity of using dense time semantics in CPS security analysis.

### **4.3 Bridging Formal Models and Industrial Metrics**

#### **4.3.1 Research Step E: Reconciliation with OpenTelemetry Metrics**

There is a significant semantic gap between theoretical formal traces and industrial observability data. The research must investigate how OpenTelemetry’s core metric instruments—including Counter, Asynchronous Counter, UpDownCounter, Gauge, and Histogram [?—can be formally represented. For example, a Gauge, which measures a current value at the time it is read, or an UpDownCounter, which accumulates over time but can also decrease, must be formally mapped to either atomic propositions within a Kripke structure or specific state transitions within an LTS [?]. This effort aims to establish a methodology for using industrial monitoring signals as inputs for formal trace verification.

#### **4.3.2 Research Step F: Semantic Model Translation**

Finalizing the link between industry and theory requires analyzing the formal foundation of analytical semantic models, such as Power BI semantic models used in Microsoft Fabric [?]. These models typically rely on a star schema structure with facts and dimensions [?]. The research must determine if these factual data points and dimensions can be translated into temporal state variables or properties within a formal Kripke structure. Successfully translating between these domains would allow complex business logic defined in semantic models to be subjected to temporal logical verification.

## 5 Conclusion

The rigorous analysis of concurrent and cyber-physical systems is fundamentally dependent upon precise trace semantics. This report has demonstrated that establishing a sound verification model requires careful consideration of three semantic axes: the underlying formal model (LTS vs. KS), the choice of time progression (Discrete vs. Dense, Metric vs. Logical), and the compositional interaction mechanism (Shuffle vs. Lockstep). The need for Dense Time semantics is not merely theoretical; it is a prerequisite for security, as discrete models fail to capture vulnerabilities arising from instantaneous internal delays [?]. Furthermore, the recognition of non-transitive physical synchronization introduces constraints that exceed the capabilities of standard logical clock models [?].

The current material, while rich in definitional content, requires extensive taxonomic restructuring to achieve the publication quality necessary for *ACM Computing Surveys*. The successful execution of the Deep Research Agenda outlined in Section IV—focusing on validating semantic soundness, quantifying scalability limitations, and bridging the gap between formal models and industrial telemetry data—is the essential final step. This integrated approach ensures that the resulting work is not only theoretically robust but also practically relevant and taxonomically rigorous.