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CS-225: Discrete Structures in CS

Homework 4, Part 2

Exercise Set 6.2 of the required textbook : Problem #6 (part 2), #11, #17 and
Exercise Set 6.3 of the required textbook: Problem #15, #38, # 42

6 (part 2).

a. Or
b. And
c. $x \in A \text{ and } x \in B \text{ and } x \in C$
d. Subsets

11. $Suppose\ A, B, and\ C\ are\ any\ set. We\ must\ prove\ that\ A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$

Suppose x is any element of:

$$Set\ Difference: A \cap B \cap C^c \subseteq (A \cap B) \cap (A \cap C)^c$$

$$De\ Morgans: A \cap B \cap C^c \subseteq (A \cap B) \cap A^c \cup C^c$$

$$Distributive: A \cap B \cap C^c \subseteq \left((A \cap A^c) \cap (B \cap A^c)\right) \cup C^c$$

$$Complement: A \cap B \cap C^c \subseteq \left(\emptyset \cap (B \cap A^c)\right) \cup C^c$$

$$Universal\ Bound\ Laws: A \cap B \cap C^c \subseteq (\emptyset \cup C^c)$$

$$Identity: A \cap B \cap C^c \subseteq C^c$$

Since the subset has an intersect of all common elements of A, B and C^c. Therefore it is a subset of $C^c as\ in\ itself.$

17. $Suppose\ A, B\ and\ C\ are\ any\ set. We\ must\ prove\ that\ if\ A \subseteq B\ then\ A \cup C \subseteq B \cup C$

We know that all elements of A are in B because A is a subset of B.

$$A \cup C\ contains\ all\ elements\ of\ A\ and\ C.$$

$$B \cup C\ contains\ B[B = \{A, and\ any\ other\ elements\}] and\ the\ whole\ set\ of\ C$$

Due to the fact that A is contained in B and C is contained in itself as an element,

$A \cup C\ is\ a\ subset\ of\ B \cup C.$

15. The statement is false because the order of operations makes the statement:

For any arbitrary element x,

$$x \in A \; and \; x \notin B, or \; x \in A \; and \; x \notin C. This \; a \; subset \; of \; x \in A \; or \; x \in C \; and \; x \notin B.$$

$$By \; definition \; of \; the \; subsets \; of \; universals$$

By definition of inclusion in union, x cannot ∉ C as a subset of x ∈ C. Thus the statement cannot be true.

38. $(A \cap B)^C \cap A = A - B$ [False]

$$De \; Morgans: A^C \cup B^C \cap A = A - B$$

$$Complement \; laws: B^C \cup \emptyset = A - B$$

$$Set \; difference: B^c \cup \emptyset = A \cap B^c$$

$$Identity: B^c = A \cap B^c$$

$$Therefore: B^c \neq A \cap B^c$$

42. $\left(A - (A \cap B)\right) \cap \left(B - (A \cap B)\right)$

$$Set \; difference: (A \cap (A \cap B)^c) \cap (B \cap (A \cap B)^c)$$

$$De \; Morgans: (A \cap A^c \cup B^c) \cap (B \cap A^c \cup B^c)$$

$$Complement: (\emptyset \cup B^c) \cap (U \cap A^c)$$

$$Identity: B^c \cap A^c$$

$$De \; Morgans: (B \cup A)^c$$