

A vertical orange line is positioned to the left of the text. A small blue dot is located at the top of this line.

WEEK3: DYNAMIC PROGRAMMING

Agenda

Survey Comments

- Can you go over calculating **time complexity** of dynamic programming problems?
- can we go over more examples of analyzing time complexity of more difficult algorithms?
- Can we **go over the program examples** presented in this weeks explorations?
- If we could cover a Longest Common Sequence problem that would be really helpful!

• how to find a recurrence relation?

formula

rec for

$$\underline{\underline{T(n)}} = \underline{T(n_{\text{small}})} + \underline{\underline{\theta()}}$$

$$F(n) = \begin{cases} 1 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F(n-1) + F(n-2) & \text{otherwise} \end{cases}$$

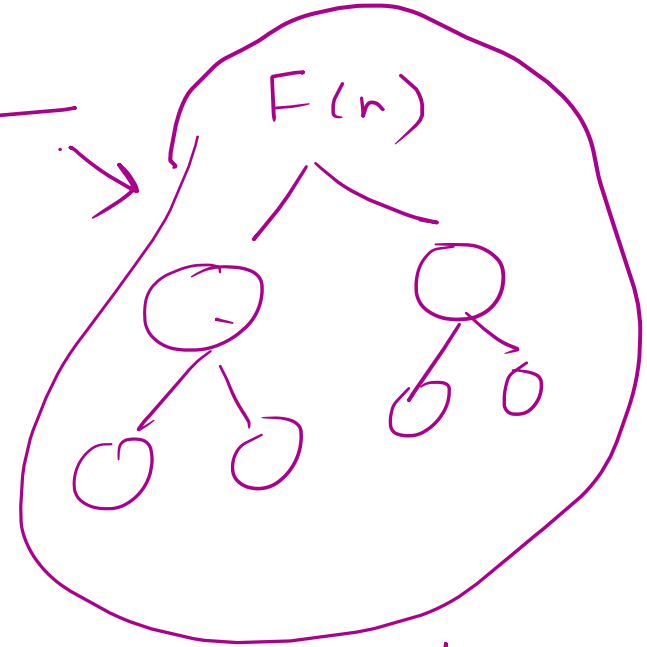
top ?

$$F(n) = \begin{cases} 1 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F(n-1) + F(n-2) & \text{otherwise} \end{cases}$$

$$\rightarrow \underline{\underline{T(n)}} = \frac{T(n-1)}{1}$$

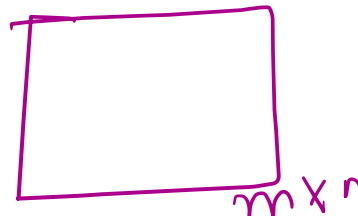
DP (m, n)

O()



Exponential

loop(i, j)



O(m.n)

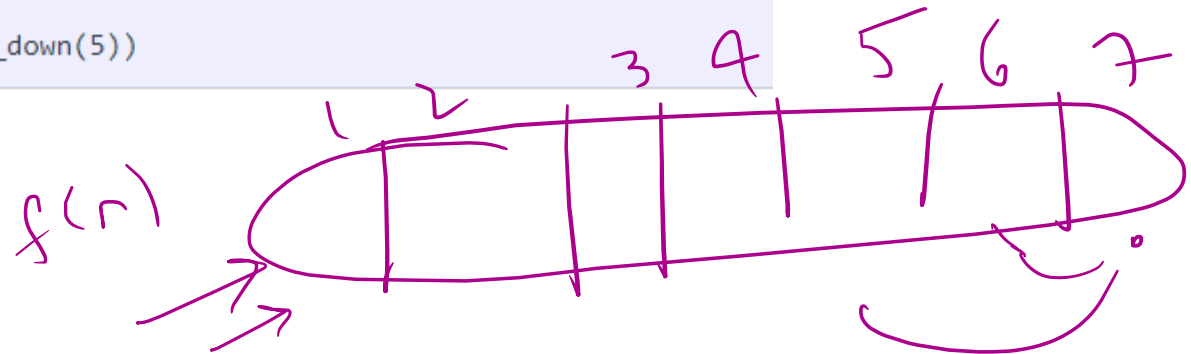
Linear F(n)

[n] → O(n)

```

FibMemo = {0:1, 1:1}
def Fib_top_down(n):
    if n in FibMemo:
        return FibMemo[n]
    FibMemo[n] = Fib_top_down(n-1) + Fib_top_down(n-2)
    return FibMemo[n]

print(Fib_top_down(5))
  
```



DP algorithms – Running Time

- Running time for dynamic programming algorithms.

Fibonacci number


```
import sys
def makechange_topdown( coins, amount):
    if amount == 0:
        return 0
```

```
    return makechange_topdown_helper(coins, amount, [0] * (amount + 1))
```

```
def makechange_topdown_helper( coins, amount, countmemo):
```

```
    if (amount < 0):
        return -1
```

```
    if (amount == 0):
        return 0
```

```
    if (countmemo[amount] != 0):
        return countmemo[amount]
```

```
    inf = sys.maxsize
```

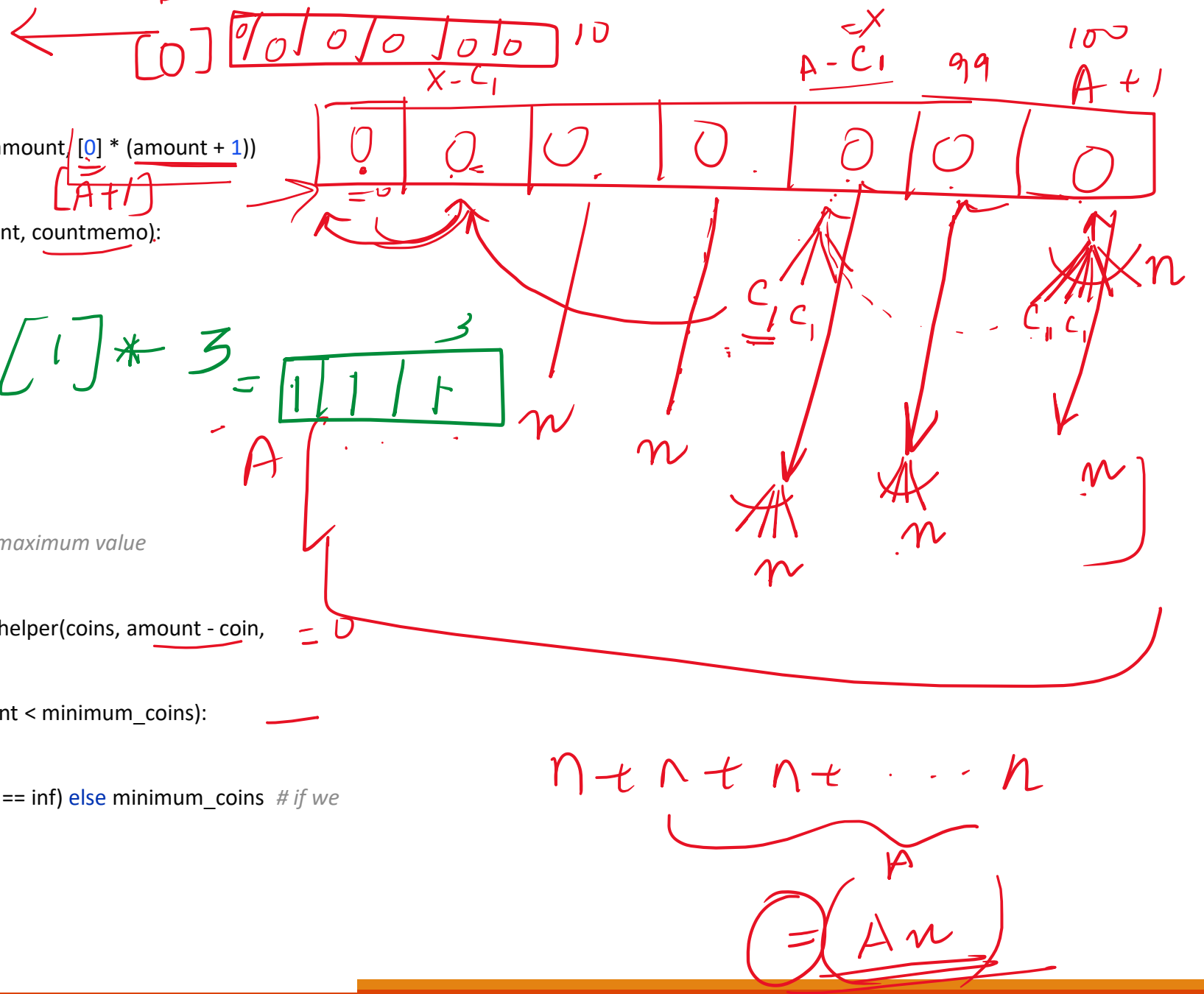
```
    minimum_coins = sys.maxsize # set to some maximum value
```

```
    for coin in coins:
        temp_coincount = makechange_topdown_helper(coins, amount - coin,
        countmemo)
```

```
        if (temp_coincount >= 0 and temp_coincount < minimum_coins):
            minimum_coins = 1 + temp_coincount
```

```
    countmemo[amount] = -1 if (minimum_coins == inf) else minimum_coins # if we
found a new minimum use it
    print(countmemo)
    return countmemo[amount]
```

```
print(makechange_topdown([1,2,3], 4))
```




```
import sys
def makechange_topdown( coins, amount):
    if amount == 0:
        return 0

    return makechange_topdown_helper(coins, amount, [0] * (amount + 1))
```

```
def makechange_topdown_helper( coins, amount, countmemo):
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```
    if (amount < 0):
        return -1.
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    if (amount == 0):
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    if (countmemo[amount] != 0):
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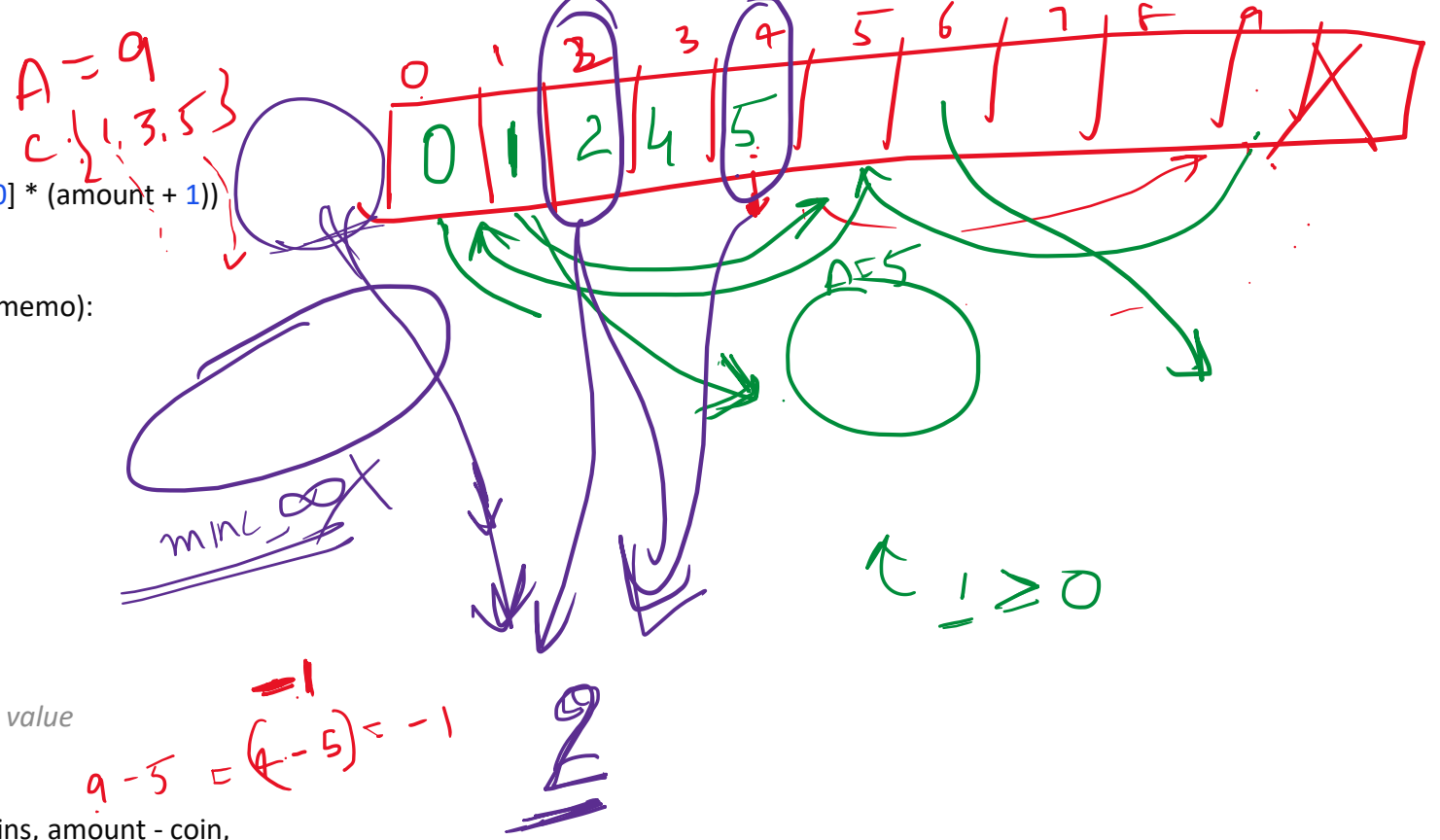
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    inf = sys.maxsize
    minimum_coins = sys.maxsize # set to some maximum value
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```
    for coin in coins:
        temp_coincount = makechange_topdown_helper(coins, amount - coin,
countmemo)
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        if (temp_coincount >= 0 and temp_coincount < minimum_coins):
            minimum_coins = 1 + temp_coincount
```

```
        countmemo[amount] = -1 if (minimum_coins == inf) else minimum_coins # if we
found a new minimum use it
        print(countmemo)
        return countmemo[amount]
```

```
print(makechange_topdown([1,2,3], 4))
```



min coins A

```

def makechange_bottomup(coins, amount):
    min_count_table = [amount + 1] * (amount + 1) # setting array elements to some large value that is not possible answer

    min_count_table[0] = 0 # setting the base case

    for i in range(1, amount + 1): # iterate through all possible amount values from base case
        for j in range(0, len(coins)): # find the number of coins needed for each coin denomination
            coin_val = coins[j]
            if (coin_val <= amount and (
                i - coin_val >= 0): # if denomination value is less than amount then we can use the coin
                # replace min_count_table[i] with mininum value of coins possible
                min_count_table[i] = min(min_count_table[i], min_count_table[i - coin_val] + 1)

    # we have a valid count of coins if min_count_table[amount] is valid
    if min_count_table[amount] > amount:
        result = -1
    else:
        result = min_count_table[amount]

    return result

print(makechange_bottomup([1, 3, 5], 8))

```

$O(A \times n)$

LCS

$sm =$ _____

abcde
↑↑

↑ $[0] * [n] =$

Ex 1/p 20p

ba

sn _____

ab ~~f~~

a f b
1 1

bca

ba

L.C. Sub

- How can we implement an algorithm using dynamic programming?
- DP Approaches : Bottom-up approach and Top-Down approach
- algorithm implementation
 - Debug existing code / your code on pycharm
 - Visualize code on <https://pythontutor.com/visualize.html#mode=edit>

General

- More dynamic programming examples
- <https://www.geeksforgeeks.org/top-20-dynamic-programming-interview-questions/>
- <https://www.interviewbit.com/courses/programming/topics/dynamic-programming/>

★ Form Groups / Find friends to go discuss code