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CS 225 – Discrete Structures in CS

Homework 5, Part 1

Set 5.2: problems 12, 15

Set 5.3: problems 10, 18, 27

12. $\frac{1}{1\times2}+\frac{1}{2\times3}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}$, for every integer n≥ 1. Prove by induction:

Basis step: show P(1) is true.

$\frac{1}{1(1+1)}[left\ hand\ side]=\frac{1}{1+1}[right\ hand\ side]$

$LHS=\frac{1}{2}, RHS=\frac{1}{2}, therefore\ P(1) is\ true.$

Induction step:

Hypothesis: assume P(k) is true, so P(k+1) is true, for some integer k.

$\forall k\geq 1, P(k)\to P(k+1).$

$\frac{1}{(k+1)(k+2)}[left\ hand\ side], for\ some\ integer\ k.$ [suppose inductive hypothesis is true]

$we\ must\ show\ that\ \ldots\frac{1}{(k+1)(k+2)}=\frac{k+1}{k+2}$

$$\frac{k((k+1)+1)}{(k+1)((k+1)+1)}+\frac{1}{(k+1)((k+1)+1)}=\frac{k((k+1)+1)+1}{(k+1)((k+1)+1)}\equiv\frac{k(k+2)+1}{(k+1)((k+1)+1)}$$

$$\equiv\frac{k((k+1)+1)}{(k+1)((k+1)+1)}=\frac{k^2+2k}{(k+1)((k+1)+1)}=\frac{k(k+2)}{(k+1)(k+2)}=\frac{k}{k+1}$$

And this is the right side of P(k+1). Therefore the property of n=k+1 holds true.

15. $\sum_{i=1}^{n} i(i!)=(n+1)!-1$, for every integer n≥ 1. Prove by induction:

Basis step: show P(1) is true.

Left: 1(1!)= 1

Right: (1+1)!-1

2!-1=(1x2)-1=1

So P(1) is true

Inductive hypothesis: For all integers k≥ 1, and suppose P(k) is true.

Inductive step: We will show that for all integers k≥ 1, if P(k) is true then P(k+1) is true.

$\sum_{i=1}^{k+1} i(i!) = ((k+1)+1)! - 1$

Left: $\sum_{i=1}^{n} 1(1!) = 1$

$[(k+1)! - 1] + (k+1)[(k+1)!], \textit{this is the new left side.}$

k=1, left side is 5

Right: $((k+1)+1)! - 1$

(k+2)!-1

New right side: = (k+1)![1+(k+1)]-1

k=1, right side is 5

Therefore:

Since the new left is 5 when k+1 is substituted for 1+1 and the new right side is the same, then the property of n=k+1 is true.

$10. n^3 - 7n + 3 \; is \; divisible \; by \; 3, for \; each \; integer \; n \geq 0.$

Basis step: show P(0) is true.

$0^3 - 7(0) + 3 = 3, \quad \frac{3}{3} = 1, therefore \; P(0) \; is \; true$

$by \; definition \; of \; divisibility.$

$we \; must \; show \; that \; n^3 - 7n + 3 = 3q \; for \; some \; q \in Z$

Inductive step:

Hypothesis: For every integer k≥ 0, if P(k) is true then P(k+1) is true.

$We \; must \; show \; that \; (k+1)^3 - 7(k+1) + 3 \;\; is \; divisible \; by \; 3.$

LHS: $(k+1)^3 - 7(k+1) + 3 = 3q \; for \; some \; integer \; q.$

$(k^3 + 2k^2 + 3k + 1 - 7k - 7 + 3) \equiv ((k^3 - 7k + 3) + (3k^2 + 3k - 6))$

$(k^3 - 7k + 3) + 3k(k+1) - 6 \;\; [k(2k+3) is \; an \; even \; integer \; by \; product \; of \; two \; consecutive$

$integers.]$

$3q + (3 \times 2)r - 6, for \; some \; integers \; q \; and \; r.$

$This \; whole \; equationis \; divible \; by \; 3, \equiv 3|(3q + 6r - 6) \equiv q + 2r - 2$

Therefore: The property holds true for n=k+1.

$18. 5^n + 9 < 6^n, for \; each \; integer \; n \geq 2.$

Basis step: show P(2) is true

Inductive hypothesis: We must show that P(2) holds true.

$LHS: 5^2 + 9 = 34$

RHS: $6^2 = 36$

$therefore\ the\ statement\ hold\ true\ for\ P(2) because\ 34 < 36.$

Assuming P(k) is true, we must show that P(k+1) holds true for any integer k≥ 2.

$5^{k+1} + 9 < 6^{k+1}$

$$LHS: (5^k \times 5 + 9)$$

$$RHS: (6^k \times 6)$$

$$(5^k \times 5 + 9) < (6^k \times 6)$$

$Since\ the\ right\ side\ carries\ the\ inductive\ hypothesis, times\ another\ 6, P(k+1) holds$

Therefore: The property holds true.

27. for the sequence $d_1, d_2, d_3, \ldots$ is defined by $d_1 = 2, and\ d_k = \frac{d_{k-1}}{k} for\ each\ integer$

$$k \geq 2. Show\ that\ for\ every\ integer\ n \geq 1, d_n = \frac{2}{n!}$$

Basis step: show P(1) is true

Inductive hypothesis: The basis step was already proven true in the question or given information. P(1) holds true.

$\frac{2}{1!} = 2, so\ d_1 = 2, as\ was\ to\ be\ shown$

Inductive step: We must show that if P(1) is true then P(k+1) for any integer k≥1.

$d_{k+1} = \frac{d_k}{k+1} = \frac{2}{k!} \times \frac{1}{k+1} = \frac{2}{(k+1)!}$

Therefore: The property holds true for n=k+1.