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CS-225: Discrete Structures in CS

Homework 2, Part 1

Exercise Set 3.1 of the required textbook: Question #16(b, d, f) , #22(b), #29(a, b, c)

16.

b) ∀ real numbers x, x is a positive, negative or zero.

d) ∀ logicians x, x is not lazy.

f) ∀ real numbers x, x squared is not equal to -1

22.

b) ∀ valid arguments x, if x has true premises then x has a true conclusion.

29.

a) There is at least one rectangle that is a square. (true because the definition of a rectangle is a shape with four 90° angles, and a square has four 90° angles, therefore a square is a rectangle)

b) There is a rectangle that is not a square. (true because a sw=quare has four equal length sides, whereas a rectangle has at least two sets of equal sides)

c) If a geometric figure is a square then the geometric figure is a rectangle. (true because the definition of a rectangle is a shape with four 90° angles, and a square has four 90° angles, therefore a square is a rectangle)

Exercise Set 3.2 of the required textbook: Question #4(b), #5(b), #8, #21, #29, #40, #44

4.

b) Some graphs are not connected.

5.

b) There is at least one real number that is not positive, negative or zero.

8. Negation: Some of life’s problems have a simple solution.

Formal of the original: ∀ life’s problems x, ~ Q(x)

Formal of negation: ∃ life’s problems x, Q(x)

21. ∃ integer n, such that n is divisible by 6 and not divisible by 2 or 3

29.

Contrapositive: $\forall$ n $\in$ Z, if n is even and n≠2, then n is not prime.

(True)

Converse: $\forall$ n $\in$ Z, if n is odd or n=2, then n is prime.

(False: n cannot be odd and equal to 2, therefore its value is invalid. 9 is odd, 9≠2, and 9 is not a prime number)

Inverse: $\forall$ n $\in$ Z, if n is not prime, then n≠2 and is even.

(True)

Extra problem:

Let P(x) be the statement “x can speak French” and let Q(x) be the statement “x knows the computer language Python” and let R(x) be the statement "x is smart". Express each of the following sentences in terms of P(x), Q(x), R(x), quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

(1) All smart students at your school either can speak French or know the computer language Python.

  a. $\forall$ x $\in$ D, $R(x) \rightarrow (P(x) \vee Q(x))$

(2) Some smart students at your school do not speak French.

  a. $\exists$ x $\in$ D, such that $R(x) \wedge {\sim}P(x)$

(3) No one at your school knows the computer language Python.

  a. $\forall$ x $\in$ D, ${\sim}Q(x)$

(4) There is a student at your school who can neither speak French nor knows the computer language Python.

  a. $\exists$ x $\in$ D, such that ${\sim}P(x) \wedge {\sim}Q(x)$

(5) A student at your school is smart only if she/he can speak French.

  a. $\exists$ x $\in$ D, such that $P(x) \rightarrow R(x)$