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CS-225: Discrete Structures in CS

Homework 4, Part 1

Exercise Set 6.1 of the required textbook: Question #6, #12, #25(a, b, c), #27 (b, c) , #33(a, c), #35(c, d)

6. $a.$ $\boldsymbol{A \subseteq B}$ **[False] because every element of for x ∈ A is not in B**

Starting Point: Suppose x is a particular but arbitrarily chosen element of A.
To Show: Therefore, x is an element of B.
By definition of A, there is an integer a such that x=5a+2
[We must show that x=(10b-3)]
Let b=5a+2
[we must check that b is an integer]
Then b is an integer because of the definition of products and sums of integers.
Thus by definition x may be an element of B
Disprove by counterexample.

$x = 22$
$22 = 5a + 2$
$20 = 5a$
$4 = a$

$y = 22$
$22 = 10b - 3$
$25 = 10b$
$\frac{25}{10} = b = 2.5\ (not\ an\ integer)$

Thus 22∈A whereas 22∉ B, so $A \nsubseteq B$

$b.$ $\boldsymbol{B \subseteq A}$ **because every element in B is an A.**

Starting Point: Suppose x is a particular but arbitrarily chosen element of B .
To Show: Therefore, x is an element of A.
By definition of B, there is an integer such that y=10b-3.
[We must show that y=5a+2]
Let a=10b-3.
Then a is an integer because of the definition of products and sums of integers.
Thus by definition y can be an element of A

$y = 5(10b - 3) + 2$
$y = 50b - 15 + 2$
$y = 50b - 13$
$\frac{y+13}{50} = b$
$y = 5\left(\frac{y+13}{50} - 3\right) + 2$
$y = \frac{5y+65}{50} - 15 + 2$
$y = \frac{y+13}{10} - 13$

$10(y+13) = y+13$
$10y+117 = y$
By definition y is an integer therefore $B \subseteq A$

c. **B = C because $\boldsymbol{C \subseteq B}$**
By definition of C, z=10c+7
Let b=10c+7
Therefore $y = 10(10c+7) - 3$
$y = 100c + 70 - 3$
$y = 100c + 67$
$c = \frac{y-67}{100}$
$y = 10\left(\frac{10(y-67)}{100} + 7\right) - 3$
$y = y - 67 + 70 - 3$
$y = y - 67 + 67 = y \quad y = y$

And $\boldsymbol{B \subseteq C}$
By definition y=10b-3
Let c=10b-3
Therefore $z = 10(10b-3) + 7$
$z = 100b - 30 + 7$
$z = 100b - 23$
$\frac{z+23}{100} = b$
$z = 10\left(\frac{z+23}{100}(10) - 3\right) + 7$
$z = z + 23 - 30 + 7$
$z = z$

12. a. $A \cup B = \{x \in R | -3 \leq x < 2\}$
b. $A \cap B = \{x \in R | -1 < x \leq 0\}$
c. $A^c = \{x \in R | x \leq 6 \text{ } or \text{ } x > 8\}$
d. $A \cup C = \{x \in R | -3 \leq x \leq 8\}$
e. $A \cap C = \{x \in R | \emptyset\}$
f. $B^c = \{x \in R | x \leq -1 \text{ } or \text{ } x \geq 2\}$
g. .$A^c \cap B^c = \{x \in R | x \leq -1 \text{ } or \text{ } x > 8\}$
h. $A^c \cup B^c = \{x \in R | -\infty, \infty\}$
i.$(A \cap B)^c = \{x \in R | x \leq -1 \text{ } or \text{ } x > 0\}$
j. $(A \cup B)^c = \{x \in R | x < -3 \text{ } or \text{ } x \geq 2\}$

25. a. $\bigcup_{i=1}^{4} R_i = \{x \in R |\ x \text{ is in at least one of the intervals } (1,2), or \left(1,1\frac{1}{2}\right), or \left(1,1\frac{1}{3}\right), or \ (1, 1\frac{1}{4})\}$
b. $\bigcap_{i=1}^{4} R_i = \{x \in R | x \text{ is in all of the intervals } (1,2), and \ \left(1,1\frac{1}{2}\right), and \ \left(1,1\frac{1}{3}\right), and \ (1, 1\frac{1}{4})\}$
c. They are not mutually disjoint because they all have the element 1 in common.

27. b. Yes it is a partition of Z because it is mutually disjoint.

c. No it is not a partition because there are common elements making then not mutually disjoint.

33. a. $\{\emptyset\}$

c. $\{\emptyset, \{\emptyset, \{\emptyset\}\}\}$

35. c. $A \times (B \cap C) = \{a, b\} \times \{2\} = (\boldsymbol{a}, \mathbf{2})\ (\boldsymbol{b}, \mathbf{2})$

d. $(A \times B) \cap (A \times C)$

$A \times B = \{(a, 1), (b, 1), (\mathrm{a},2), (\mathrm{b},2)\}$

$A \times C = \{(a, 2), (b, 2), (a, 3), (b, 3)\}$

$(\boldsymbol{A} \times \boldsymbol{B}) \cap (\boldsymbol{A} \times \boldsymbol{C}) = \{(\boldsymbol{a}, \mathbf{2}), (\boldsymbol{b}, \mathbf{2})\}$