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CS 225 – Discrete Structures in CS

Homework 5, Part 2

Sect 5.4, problems 2, 3 , 7 and the problem from canvas

2. We must show that $4|b_n \; for \; every \; integer \; n \geq 1$

$$P(n) \equiv 4|b_n \; or \; b_n = 4m, for \; some \; integer \; m.$$

$$Basis \; step: Prove \; P(a), P(a+1), \ldots P(b) are \; all \; true$$

$$Inductive \; step: Prove \; \forall k \geq a, P(a) \wedge P(1) \wedge \ldots, P(k) \to P(k+1)$$

$$Basis \; step: Show \; that \; P(1) and \; P(2) are \; true.$$

$$P(1) = b_1 = 4 = 4 \times 1$$

$$P(2) = b_2 = 12 = 4 \times 3$$

$$Both \; hold \; true \; by \; definition \; of \; division.$$

$$Inductive \; Hypothesis: P(i) is \; true \; for \; all \; i \; with \; 1 \leq i \leq k$$

$$Let \; k \geq 2 \; be \; any \; integer, so \; 4|b_i for \; all \; integers \; 1 \leq i \leq k$$

$$b_{k+1} = 4r, b_k = 4s \; for \; some \; integers \; r \; and \; s$$

$$Inductive \; step: Show \; that \; P(k+1) is \; true.$$

$$P(k+1) \equiv 4|b_{k+1}$$

$$b_{k+1} = b_{k-1} + b_k = 4r + 4s = 4(r+s) \quad [this \; is \; due \; to \; the \; inductive \; hypothesis]$$

$$Therefore, by \; basic \; algebra \; r + s \; is \; an \; integer, and \; the \; end \; case \; is \; divisible \; by \; 4.$$

3. $Suppose \; that \; c_0, c_1, c_2, \ldots is \; defined$

$$c_0 = 2, c_1 = 2, c_2 = 6, c_k = 3c_{k-3} \;\; for \; every \; integer \; k \geq 3$$

$$Prove \; that \; c_n is \; even \; for \; each \; integer \; n \geq 0$$

$$c_n = 2r, for \; any \; integer \; r.$$

$$Basis \; step: show \; that \; P(0), P(1) and \; P(2) are \; true.$$

$$P(0) = 2 \; which \; is \; divisible \; by \; 2 \; making \; it \; even$$

$$P(1) = 2 \; per \; above \; reasoning$$

$$P(2) = 6 \; is \; divisible \; by \; 2 \; making \; it \; an \; even \; integer \; also.$$

$$So\ each\ of\ these\ hold\ true.$$

$$Inductive\ Hypothesis: P(i) is\ true\ for\ all\ i\ within\ 0 \leq i \leq k$$

$$Let\ k \geq 3\ for\ all\ integer, so\ the\ outcome\ is\ an\ even\ integer\ for\ P(k+1).$$

$$c_{k+1} = 3c_{k-2}$$

$$c_k = 2r, per\ basis\ step.$$

$$Inductive\ Step: Show\ that\ P(k+1) is\ true, assuming\ the\ hypothesis\ holds\ true.$$

$$By\ replacement\ c_2 - c_0 = 6 - 2 = 4.$$

$$4 = 2.2$$

$$3 \times 4 = 12. which\ id\ divisible\ by\ 2.$$

$$Therefore\ P(k+1) holds\ true.$$

7. $Basis\ step: show\ that\ P(1), P(2)\ hold\ true.$

$$g_1 = 3, g_2 = 3, both\ hold\ true.$$

$Induction\ Hypothesis: If\ the\ hypothesis\ holds\ true\ for\ any\ integer\ k, then\ g_k = 2^k + 1\ is\ true.$

$$Induction\ step:$$

Canvas question: Let P(n) be the statement that a postage of n cents can be formed using just 4-cent and 5-cent stamps." Use strong mathematical induction to prove that P(n) is true for

$n \geq 12$. Answer the following questions to show a complete proof -

a. Show that the statements P(12), P(13), P(14), and P(15) are true, completing the basis step of the proof.
    a. P(12): 12=3(4)
    b. P(13): 13=2(4)+1(5)
    c. P(14): 14=1(4)+2(5)
    d. P(15): 15=3(5)
b. What is the inductive hypothesis of the proof?
    a. We can form i cents of postage using just 4-cent or 5-cent stamps, for all

       $k \geq 15$ cents of postage. That means $P(i)$ is true, for all $12 <= i <= k$.

c. What do you need to prove in the inductive step?
    a. Prove that P(k+1) holds true.
d. Complete the inductive step for $k \geq 15$.'
    a. Not sure what you want here