Khrystian Clark
CS-225: Discrete Structures in CS
Homework 3, Part 2

Instruction: Use the method of proof by contraposition or/and by contradiction **only** if proving the problem statements given below -

Exercise Set 4.7 of the required textbook: Question #13, #24, #29
Exercise Set 4.8 of the required textbook: Question #7, #18 (a)

13. Let S be the statement: "The product of any irrational number and any nonzero rational number is irrational."
∀ irrational number(r) and nonzero rational number(s), the product is irrational

a. Write a negation of S: There exists an irrational number and a nonzero rational number whose product is rational.

b. Prove S by contradiction: Proof: Suppose not. That is, suppose there is an irrational number and a nonzero rational number whose product is rational.

$r = \frac{a}{b}$ *for some integers a and b, where* $b \neq 0$

$rs = \frac{c}{d}$ *for some integers c and d, where* $d \neq 0$

$s\left(\frac{a}{b}\right) = \frac{c}{d}$

$s = \frac{\frac{c}{d}}{\frac{a}{b}}$

$s = \frac{ad}{bc}$

$$rs = \frac{a}{b}\left(\frac{ad}{bc}\right) = \frac{a^2 d}{b^2 c}$$

Since the integer c above is not defined as nonzero, the denominator has the potential to equal zero due to the rules of multiplication. With the rules of division, a zero denominator cannot equal a rational number. Thus shows rs≠ rational.

24. The reciprocal of any irrational number is irrational. (the reciprocal of any nonzero real number x is 1/x)
Formal: ∀ irrational number x, 1/x is irrational.

a. Contraposition: Suppose you have any fraction with 1 as the numerator and any rational number (a) as the denominator. [We must show that a is rational]
    a. ¼ = .25 (rational due to a definitive end to the decimal)
    b. a=4 which is a rational number. Therefore a is rational by definition of rational.

b. Contradiction: Suppose there exists an irrational number (x) whose nonzero reciprocal (1/x) is rational.
    a. $x = \pi$ (*most commonly known irrational number*)
    b. $\frac{1}{x} = \frac{1}{\pi}$
    c. $\frac{1}{\pi} = 0.3183 \ldots \ldots$

    Therefore an irrational number whose nonzero reciprocal cannot be rational due to the definition of irrational and the counterexample [as was shown].

29. For all integers m and n, if m+n is even, then m and n are both even or m and n are both odd.

Contradiction: Suppose not. Suppose that for some integers m and n, if m+n is even then m is even and n is odd, or n is even and m is odd. m+n produces an even number s. By definition of even and odd, m=2k and n=2k+1 and s=2k., for some integer k

$s = m + n$

$2k = (2k) + 2k + 1 \; by\ substitution$

$2k = 4k + 1$

$-2k = 1$

$k = -\frac{1}{2}$ by basic algebra

Since k does not equal an integer by definition, then m cannot be even when n is odd in the formula m+n [as was to be shown above]

7. $3\sqrt{2} - 7 \;\; is\ irrational$

Suppose not. Suppose that $3\sqrt{2} - 7$ is rational. Then by definition of rational:

$3\sqrt{2} - 7 = \frac{a}{b} \; where\ b \neq 0$

$3\sqrt{2} = \frac{a}{b} + 7$ by adding 7 to each side

$= \frac{a}{b} + \frac{7b}{b}$ By the rule for adding fractions

$\sqrt{2} = \frac{a+7b}{3b}$ by adding fractions with a common denominator and dividing each side by 3.

Then a+7b and 3b are integers by the laws of integers. 3b ≠ 0 by the zero product property. Hence √ 2 is a quotient of two integers (a+7b and 3b, where 3b≠ 0). By the definition of rational this makes √ 2 rational which contradicts the fact that √ 2is rational. Hence $3\sqrt{2} - 7\ is, in\ fact, irrational.$

18. a. Prove that for every integer a, if a^3 is even then a is even. [This statement is true]

Suppose not. Suppose that there is an integer 'a' such that if $a^3$is even then a is odd.

An odd integer: a=2k+1

$(2k + 1)^3 = (8k^3 + 4k^2 + 2k + 1)$

$= 2(4k^3 + 2k^2 + k) + 1$

$4k^3 + 2k^2 + k = r, for\ any\ integer\ r.$

$a = 2r + 1$

By definition of odd, a is an odd number [as was to be shown]. Hence the initial statement is true.