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CS-225 Discrete Structures in CS

Homework 6

1. Use iteration to guess an explicit formula for the sequence - $e_k = 3e_{k-1} + 2$, for all integers $k \geq 2$, where $e_1 = 3$ (using the formulas from summation_formula_updated.pdf to simplify your answers whenever possible). (tower of Hanoi example?)

    - $Given\ e_1 = 3, e_k = 3e_{k-1} + 2, for\ all\ int\ k \geq 2.$
    - $e_1 = 3$
    - $e_2 = 3e_1 + 2 = 3^1(3) + 2(3^0) = 3^2 + 3^0(2) = 11$
    - $e_3 = 3e_2 + 2(1) = 3\left(3^2 + 3^0(2)\right) + 3^0(2) = 3^3 + 3^1(2) + 3^0(2) = 35$
    - $e_4 = 3e_3 + 2(1) = 3(3^3 + 3^1(2) + 3^0(2)) + 2(3^0) = 3^4 + 3^2(2) + 3^1(2) + 3^0(2) = 98$
    - $e_n = 3^n + 2(3^{n-2} + 3^{n-3} + \cdots 3^2 + 3^1 + 3^0)$
    - $e_n = 3^n + 2\sum_{i=0}^{n-2} 3^i$

$$e_n = 3^n + 2\left(\frac{3^{n-2+1} - 1}{3-1}\right) \equiv 3^n + \left(\frac{(2(3^{n-1}) - 2)}{2}\right) \equiv 3^n + 3^{n-1} - 1\ [ans]$$

2. Use mathematical induction to verify the correctness of the formula you obtained in the above problem.

    - $e_1 = 3, e_k = 3e_{k-1} + 2\ [for\ all\ integers\ k \geq 2]$
    - $P(n) \equiv e_n = 3^n + 3^{n-1} - 1, to\ prove\ for\ all\ integers\ n \geq 1, P(n) is\ true$
    - Basis Step: $P(1) is\ true.$
        1. $e_1 = 3^1 + 3^0 - 1 \equiv 3 + 1 - 1 = 3$
        2. We are given $e_1 = 3, hence\ P(1) is\ true.$
    - Inductive hypothesis: $P(k) is\ true\ for\ all\ integers\ k \geq 1$
        1. $P(k) \equiv e_k = 3^k + 3^{k-1} - 1$
    - Inductive step: $P(k+1) is\ true, for\ all\ integer\ k \geq 1.$
        1. $Show\ that\ P(k+1) \equiv e_{k+1} = 3^{k+1} + 3^{k+1-1} - 1 \equiv 3^{k+1} + 3^k - 1$
        2. $e_{k+1} = 3e_k + 2, from\ the\ given\ terms$
        3. $e_{k+1} = 3\left(3^k + 3^{k-1} - 1\right) + 2 \equiv 3^{k+1} + 3^k - 3 + 2$

4. $e_{k+1} = 3^{k+1} + 3^k - 1$, as was to be shown
5. $Hence, P(k+1)\ holds.$

3. Use iteration to guess an explicit formula for the sequence - $t_k = t_{k-1} + 3k + 1$, for all integers k ≥ 1, where $t_0 = 0$ (using the formulas from summation_formula_updated.pdf to simplify your answers whenever possible).

- Given: $t_k = t_{k-1} + 3k + 1$, for all integers k≥ 1
- Given: $t_0 = 0$
- $t_1 = t_0 + 3(1) + 1 = 0 + 3 + 1 = 4$
- $t_2 = t_1 + 3(2) + 1 = (0 + 3 + 1) + 3(2) + 1 = 3^1(3) + 3^0(2) = 11$
  1. $3^2 + 3^0(2)$ $[my\ guess\ is\ \cancel{t_n = 3^n + n}]$
- $t_3 = t_2 + 3(3) + 1 = (3^2 + 2) + 3^2 + 1 = 3^2(2) + 3^1 = 21$
  1. $3^2(2) + 3$ $[my\ guess =\ t_n = 3^2(n-1) + 2(n-2) + n^0(n - (n-1)]$
- $t_4 = t_3 + 3(4) + 1 = (3^2(2) + 3) + 3(4) + 1 = 3^2(2) + 3(5) + 1$
  1. $= 3^2(2) + \big(3(3) + 3(2)\big) + 1 = 3^2(3) + 3^1(2) + 3^0$
  2. $So\ \sum_{i=0}^{n}$

$$t_n = , for\ every\ n \geq 0.$$

4. Give a recursive definition of the set of all integers (both negative and positive) that are multiples of 3.

- $Define\ the\ set: S\ is\ a\ set\ of\ all\ integers(negative\ and\ positive).$
- $Recursion:$
- 1. $(r \in S \wedge 3 \in S) \to 3r \in S, for\ any\ integer\ r \in S.$
- 2. $any\ r\ in\ S\ is\ divisible\ by\ 3.$

5. Give a recursive definition for the set of all strings of a's and b's where all the strings are of odd lengths. (Assume, S is set of all strings of a's and b's where all the strings are of odd lengths. Then S = { a, b, a**a**a, a**b**a, a**a**b, a**b**b, b**a**a, b**b**a, b**a**b, b**b**b, a**aaa**a, ...).

- Base: a) $a \in S,\quad b)\ b \in S,\quad c)$
- Recursion: $If\ u \in S, then$
  1. $(a) aua \in S,\ (b) bub \in S,\ (c)\ aub \in S, (d) bua \in S$
  2. All string in S have an odd amount of characters.
  3. By our recursion the aforementioned strings are ∈ S
  4. $a\ and\ b\ are\ spelled\ the\ same\ forward\ and\ backward$