Summary of Forster et al. (2016): On–Manifold Preintegration for Real-Time Visual–Inertial Odometry

1. Problem Overview

Visual–Inertial Odometry (VIO) estimates a platform's pose using a camera and an Inertial Measurement Unit (IMU). IMUs generate high-rate measurements (100–1000 Hz), while camera frames arrive far more slowly (10–60 Hz). Incorporating every IMU reading directly into an optimization back-end makes real-time inference computationally prohibitive.

Goal: Efficiently summarize all IMU data between camera frames into a *single* relative motion factor while preserving accuracy and correct manifold geometry.

2. Main Contributions

- A rigorous IMU preintegration theory carried out directly on the rotation manifold **SO(3)**.
- Closed-form expressions for preintegrated measurements, covariance propagation, and Jacobians.
- A bias-correctable formulation: optimization can adjust biases without re-integrating IMU data.
- Integration into a factor-graph VIO framework (e.g., GTSAM) using a structureless vision model.
- Demonstrated real-time performance with high accuracy on synthetic and real datasets.

3. State & Sensor Model

State at time k:

- Rotation $R_k \in SO(3)$
- Position $p_k \in \mathbb{R}^3$
- Velocity $v_k \in \mathbb{R}^3$
- Gyro bias b_k^g
- Accel bias b_k^a

IMU measurement model

- Gyroscope: $\tilde{\omega}(t) = \omega(t) + b^g(t) + \eta^g$
- Accelerometer: $\tilde{a}(t) = R(t)^T(a(t) - g) + b^a(t) + \eta^a$
- Biases follow random walk; noises are zero-mean white.

4. What is Preintegration?

Purpose: Replace hundreds of raw IMU readings between camera frames i and j with a single *relative* measurement consisting of: - Relative rotation ΔR_{ij} - Relative velocity change Δv_{ij} - Relative position change Δp_{ij}

These are computed using only IMU data and the current bias estimate at frame i.

5. Why On-Manifold?

Rotations live on **SO(3)**, not in Euclidean space. The authors use: - **Exponential map** for integrating angular velocity. - **Log map** for expressing rotational residuals and perturbations.

This avoids singularities (e.g., gimbal lock) and ensures consistent linearization.

6. Preintegration Mathematics

Rotation increment per IMU sample

$$\delta R = \exp((\tilde{\omega} - b^g - \eta^g)\,\Delta t)$$

Velocity and position increments

Accelerometer readings are rotated using the current rotation estimate to accumulate: - Integrated acceleration $\rightarrow \Delta v_{ij}$ - Integrated velocity $\rightarrow \Delta p_{ij}$

Noise terms and bias perturbations are propagated through first-order perturbation models.

7. Bias Correction

Preintegrated quantities depend on the bias at time i. During optimization, if bias updates by δb, the preintegrated values can be corrected **without recomputing the integration**: - Linear bias-correction terms are stored during integration. - Greatly improves efficiency in iterative optimization.

8. Factor Graph Formulation

A single **IMU factor** connects states at frames i and j. It encodes: - Preintegrated mean $(\Delta R_{ij}, \Delta v_{ij}, \Delta p_{ij})$ - Covariances - Jacobians wrt $R_i, v_i, p_i, b_i^g, b_i^a$ and similarly at j

Structureless visual factors eliminate 3D landmarks analytically, leaving only pose/velocity/bias variables in the graph.

9. Algorithmic Notes

- Preintegrate IMU readings between keyframes.
- Maintain covariance and Jacobian propagation during integration.
- Apply bias correction during optimization.
- Use analytic Jacobians for stable, fast convergence.
- Implemented using iSAM2 / GTSAM for incremental optimization.

10. Experimental Results

- Validated on synthetic sequences and real datasets (e.g., EuRoC MAV).
- Achieves real-time performance.
- More accurate or competitive with state-of-the-art VIO systems of the time.
- Reduced computation due to compact factor graphs and preintegration.

11. Limitations

- Relies on first-order approximations; very long intervals or extremely high dynamics may accumulate linearization error.
- Requires correct IMU noise modeling and time synchronization.
- Keyframe rate influences numerical stability.

12. Impact

This work became the standard preintegration method adopted in: - GTSAM (official implementation) - Many VIO systems (VINS-Mono, OKVIS variants, etc.) - Research on multi-sensor SLAM, event-based VIO, and modern tightly coupled estimators.

It is now foundational in visual-inertial estimation literature.