RESULTATER

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Numeriske Simuleringer og Eksperimenter med STEKING AV BACON I MIKROBØLGEOVN

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EiT - Matematikk Innen Anvendelser
gruppe 5 - FUÞARK - 『↑↑*∤℟Լ - ፲ン፲ィ፲

ÅSMUND ERVIK, JOAKIM JOHNSEN, KNUT HALVOR SKREDE, TURID SCHOONDERBEEK SOLBERG, PAUL VO

Sammendrag

Text in abstract

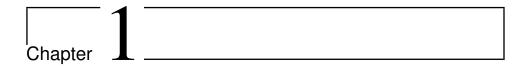
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Text in foreword

Åsmund Ervik, Joakim Johnsen, Paul Vo, Knut Halvor Skrede, Turid Schoonderbeek Solberg March 20, 2011

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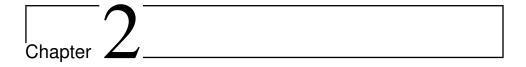
Innledning

1.1 Innledning

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Theory

2.1 The physical system

The physical system to be modeled is a thin, rectangular strip of bacon which is subjected to approx. 750 watts of microwave radiation at the frequency of 2.4 GHz. To be modeled is the heat in the strip, phase transitions and transport.

2.2 Our approach

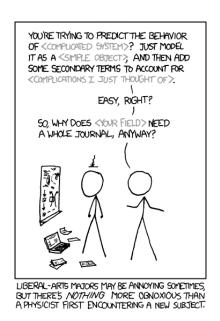


Figure 2.1: Our initial approach to the problem

Initially, our approach to modelling was to use a staggered approach: first solve the heat equation for the three media meat, solid fat, and liquid fat, thus giving the temperature at step n. Then at step n+1, the temperature is regarded as known, and we solve the transport equation for liquid fat out of the bacon. Then we know the distribution of liquid fat at step n+2, where we solve the heat equations again, and we repeat ad nauseam.

2.3 The heat equations

The differential equations governing the heat transport are all variations of the heat equation, with various source terms, eq. (2.1):

$$(\rho c_p)_m \frac{\partial T}{\partial t} - \alpha_m \nabla^2 T = J^{MW} \quad m,$$
 (2.1)

$$\eta_s(\rho c_p)_s \frac{\partial T}{\partial t} - \eta_s \alpha_s \nabla^2 T = J^{MW} - J^{Melt} \quad \text{s}, \qquad (2.2)$$

$$\eta_l(\rho c_p)_l \frac{\partial T}{\partial t} - \eta_s \alpha_l \nabla^2 T - \eta_l(\rho c_p)_l (\mathbf{v} \cdot \nabla) T = J^{MW} \quad 1.$$
(2.3)

Here the subscript m denotes meat, s denotes solid fat, and l denotes liquid fat. J^{MW} is the source term representing the microwave oven. In the initial approach, this is modelled as a cylindrically symmetric term, with the radial power distribution on the form eq. (2.4),

$$J^{MW}(r) = 0.5 + 2.55008x - 0.588013x^2 + 0.032445x^3 + 0.00124411x^4 - 0.0000973516x^5,$$
(2.4)

i.e. a fifth degree polynomial, interpolated from the results in [1].

Furthermore, there is the term J^{Melt} which represents heat loss due to latent heat in the solid-liquid phase transition. This can be written as

$$\frac{L\rho}{T_2 - T_1} \frac{\partial T}{\partial t}, \quad T \in (T_1, T_2),$$

where the fat is melting between the temperatures T_1 and T_2 . For fat these are typically around 30-50°C. For $T \notin (T1,T2)$ this term is zero, and it is readily seen that this is equivalent to a modification of the constant $c_{p,l}$ in eq. (2.1) when $T \in (T1,T2)$, thus it is not a serious complication.

In the last of the heat equations, there also appears what is called a convective derivative, on the form $(\mathbf{v} \cdot \nabla)T$. This is a somewhat more complicated term, and is essentially a modification due to transport of liquid fat implying an implicit transport of heat. However, in the staggered approach used here, the velocity field is considered as known when it comes to the heat equations.

2.4 The transport equations

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 $_{ ext{Chapter}}^{ extstyle 3}$

Numerikk

3.1 Von Neumann-analyse av Crank-Nicolsonskjemaet

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial^2 x} \tag{3.1}$$

Ved å anvende Crank-Nicolson oppnås følgende iterasjonsskjema

$$u_i^{n+1} = u_i^n (1 - 2D) + D \left(u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1} \right) + D \left(u_{i+1}^n + u_{i-1}^n \right)$$
 (3.2)

$$D = \frac{1}{2} \frac{\alpha \Delta t}{(\Delta x)^2} \tag{3.3}$$

Innfører tilnærmelsen $u_j^n \approx E_j^n$ hvor $E_j^n = G^n e^{i\beta x_j}$, her er n en potens. Innsatt i eq. (3.1):

$$G^{n+1}e^{i\beta x_j} = G^n e^{i\beta x_j} (1-2D) + G^{n+1}D \left(e^{i\beta x_{j+1}} - 2e^{i\beta x_j} + e^{i\beta x_{j-1}} \right) + G^n D \left(e^{i\beta x_{j+1}} + e^{i\beta x_{j-1}} \right)$$

Hvis vi deler (3) på $G^n e^{i\beta x_j}$ og benytter at $x_{j+1} \approx x_j + h$ får vi

$$G\left[1 - D\left(2\cos\beta h - 2\right)\right] = 1 - 2D\left(1 - \cos\beta h\right)$$

$$\cos\beta h = \frac{1}{2}\left(1 - \sin^2\frac{\beta h}{2}\right)$$

$$G\left(1 + 4D\sin^2\frac{\beta h}{2}\right) = 1 - 4D\sin^2\frac{\beta h}{2}$$

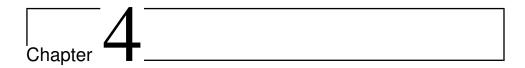
$$G = \frac{1 - 4D\sin^2\frac{\beta h}{2}}{1 + 4D\sin^2\frac{\beta h}{2}}$$

3.1. VON NEUMANN-ANALYSE AV CRANK-NICOLSON-SKJEMAET

Vi vet at den maksimale verdi av $\sin^2 x$ er 1, og at iterasjonsskjemaet er stabilt dersom $|G| \leq 1$, innsatt:

$$|G| = \left| \frac{1 - 4D}{1 + 4D} \right| \tag{3.4}$$

Verdt å observere at så lenge $D>0 \ \Rightarrow \ |G|\leq 1,$ slik at Crank-Nicolsonskjemaet er $ubetinget\ stabilt$

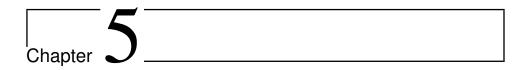


Eksperimenter

4.1 Eksperimenter

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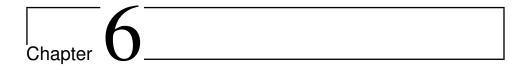


Numerikkresultater

5.1 Numerikkresultater

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Konklusjon

6.1 Konklusjon

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