RESULTS

Numerical Simulations and Experiments with

BACON PREPARATION IN A MICROWAVE

EiT - Matematikk Innen Anvendelser

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2011

Abstract

Text in abstract

Preface

Text in foreword

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Contents

Chapter 1

Introduction

1.1 Problem formulation

The problem considered in this project is the preparation of bacon in a microwave oven. Bacon is defined as cured meat from side and back cuts of a pig. A microwave oven in this context is a household appliance typically delivering 750 W of microwave effect at a frequency of 2.1 GHz to a Faraday cage of volume 40 liters.

The motivation for the problem formulation was clear. As soon as modeling of food preparation was brought on the table, bacon seemed a prime choice. A microwave oven may seem an odd choice for bacon, but previous experiments have shown that optimal bacon can consistently be attained with this preparation method. Obvious advantages over traditional bacon preparation include less cleaning up to do afterwards and a shorter time-to-plate. The caveat is that microwave preparation is less suitable for feedback during the cooking process, as the bacon is obscured from view. To the traditional bacon chef, used to a touch-and-go approach, this is an obstacle to implementation, as overcooking bacon in a microwave oven yields inedible results. The purpose of this work, then, is to establish reliable numerical simulations that can serve as a basis for estimating cooking time for an arbitrary slice of bacon.

As the project considers the optimal preparation of bacon in a microwave oven, two natural questions were formulated:

- How is preparation of bacon in a microwave oven modelled numerically?
- How does water and fat content affect the preparation, and what preparation time is optimal?

1.2 Reasoning behind the problem formulation

The first question is obvious, as any attempt at predicting the preparation time necessitates a numerical model of bacon in a microwave oven; a simple glance at the relevant equations tells the experienced numerics person that no analytical solution exists. With a numerical solution of the heat and mass transport equations, the optimal preparation time can be predicted. But this presumes an understanding of what makes good bacon. So what does?

Work done by [1] and [3] suggest that two factors are important in determining good bacon. The first is a high enough temperature that the Maillard reaction can take place, which happens around 140^{circ}C. The second is that a significant proportion of the fat has melted and has been transported out of the bacon. Both of these factors are naturally affected by the composition of the bacon; especially water and fat content will play a major role.

Having identified these two factors as the important criteria for an optimal solution, experimental work is needed to determine the correlation between temperature and fat loss, and how finished bacon is. This is, of course, also a subjective question - different people prefer different grades of crispness.

Another experimental question is how much of the microwave effect that is absorbed by the bacon. The second law of thermodynamics dictates that some of the effect is lost in the microwave magnetron, the question is how much effect is dissipated in the bacon. This may include losses besides that of the magnetron, which has a typical efficiency of 65% [4]. It turns out that common household microwave ovens are rated to include this loss, with standardized calibration procedures published by the IEC's SC 59K, so a 750 W microwave oven will typically draw 1.1 kW of electrical power from the mains socket. This means using the rated power will be good enough for our purposes.

Finally, experiments are needed to verify the accuracy of the numerical results. As heat and mass transport is a Complicated Problem, this is not guaranteed a priori.

Chapter 2

Theory

2.1 The physical system

The physical system to be modeled is a thin, rectangular strip of bacon which is subjected to approx. 750 watts of microwave radiation at the frequency of 2.4 GHz. To be modeled is the heat in the strip, phase transitions and transport.

2.2 Our approach

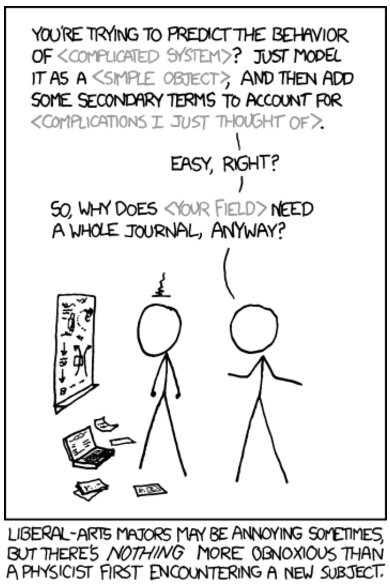

Figure 2.1: Our initial approach to the problem

Initially, our approach to modelling was to use a staggered approach: first solve the heat equation for the three media meat, solid fat, and liquid fat, thus giving the temperature at step n. Then at step $n+1$, the temperature is regarded as known, and we solve the transport equation for liquid fat out of the bacon. Then we know the distribution of liquid fat at step $n+2$, where we solve the heat equations again, and we repeat ad nauseam.

2.3 The heat equations

The differential equations governing the heat transport are all variations of the heat equation, with various source terms, eq. (2.1):

$$(\rho c_p)_m \frac{\partial T}{\partial t} - \alpha_m \boldsymbol{\nabla}^2 T = J^{MW} \quad \text{m}, \tag{2.1}$$

$$\eta_s (\rho c_p)_s \frac{\partial T}{\partial t} - \eta_s \alpha_s \boldsymbol{\nabla}^2 T = J^{MW} - J^{Melt} \quad \text{s}, \tag{2.2}$$

$$\eta_l (\rho c_p)_l \frac{\partial T}{\partial t} - \eta_s \alpha_l \boldsymbol{\nabla}^2 T - \eta_l (\rho c_p)_l (\mathbf{v} \cdot \boldsymbol{\nabla}) T = J^{MW} \quad \text{l}. \tag{2.3}$$

Here the subscript m denotes meat, s denotes solid fat, and l denotes liquid fat. J^{MW} is the source term representing the microwave oven. In the initial approach, this is modelled as a cylindrically symmetric term, with the radial power distribution on the form eq. (2.4),

$$J^{MW}(r) = 0.5 + 2.55008x - 0.588013x^2 + 0.032445x^3 + 0.00124411x^4 - 0.0000973516x^5, \tag{2.4}$$

i.e. a fifth degree polynomial, interpolated from the results in [2].

Furthermore, there is the term J^{Melt} which represents heat loss due to latent heat in the solid-liquid phase transition. This can be written as

$$\frac{L\rho}{T_2 - T_1} \frac{\partial T}{\partial t}, \quad \mathrm{T} \in (\mathrm{T}_1, \mathrm{T}_2),$$

where the fat is melting between the temperatures T_1 and T_2. For fat these are typically around 30-50°C. For $T \notin (T1, T2)$ this term is zero, and it is readily seen that this is equivalent to a modification of the constant $c_{p,l}$ in eq. (2.1) when $T \in (T1, T2)$, thus it is not a serious complication.

In the last of the heat equations, there also appears what is called a convective derivative, on the form $(\mathbf{v} \cdot \boldsymbol{\nabla})T$. This is a somewhat more complicated term, and is essentially a modification due to transport of liquid fat implying an implicit transport of heat. But, we're in luck boys! As the bacon strip is essentially two-dimensional, transport will almost exclusively happen in the vertical (small) direction. In our approach, the source terms don't vary in the vertical direction, this term will be a transport of heat in the vertical direction, and we're not really interested in the vertical temperature distribution. This means that we can happily neglect this term, in the spirit of Richard Feynmann: "It doesn't give any new physics".

2.4 The transport equations

Chapter 3

Numerics

3.1 Von Neumann analysis of the Crank-Nicolson-scheme

Consider the heat equation,

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial^2 x} \tag{3.1}$$

By applying Crank-Nicolson the following iteration scheme is attained,

$$u_i^{n+1} = u_i^n (1 - 2D) + D \left(u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}\right) + D \left(u_{i+1}^n + u_{i-1}^n\right) \tag{3.2}$$

$$D = \frac{1}{2} \frac{\alpha \Delta t}{(\Delta x)^2} \tag{3.3}$$

Using the approximation $u_j^n \approx E_j^n$ where $E_j^n = G^n e^{i\beta x_j}$, and n is a power, inserted into eq. (3.1), this gives

$$G^{n+1} e^{i\beta x_j} = G^n e^{i\beta x_j} (1 - 2D) + G^{n+1} D \left(e^{i\beta x_{j+1}} - 2e^{i\beta x_j} + e^{i\beta x_{j-1}}\right) + G^n D \left(e^{i\beta x_{j+1}} + e^{i\beta x_{j-1}}\right)$$

Dividing eq. (3.3) by $G^n e^{i\beta x_j}$, and using $x_{j+1} \approx x_j + h$ one obtains

$$\begin{aligned}
G \left[1 - D (2 \cos \beta h - 2)\right] &= 1 - 2D (1 - \cos \beta h) \\
\cos \beta h &= \frac{1}{2} \left(1 - \sin^2 \frac{\beta h}{2}\right) \\
G \left(1 + 4D \sin^2 \frac{\beta h}{2}\right) &= 1 - 4D \sin^2 \frac{\beta h}{2} \\
G &= \frac{1 - 4D \sin^2 \frac{\beta h}{2}}{1 + 4D \sin^2 \frac{\beta h}{2}}
\end{aligned}$$

As the maximum value of $\sin^2 x$ is 1, and since the iteration scheme is stable if $|G| \leq 1$, this implies stability,

$$|G| = \left|\frac{1-4D}{1+4D}\right| \tag{3.4}$$

A typical **C**ourant-**F**riedrichs-**L**ewy condition is an inequality of the form $\frac{\Delta t}{\Delta x} < C$, with C a constant.

It is noteworthy that $D > 0 \;\Rightarrow\; |G| \leq 1$, so the Crank-Nicolson scheme is *unconditionally stable*. There is, however, a caveat: if the time steps and space steps do not fulfill a CFL-condition, the scheme may present oscillations.

3.2 Truncation error in the Crank-Nicolson scheme

The Crank-Nicholson method is based on the trapezoidal rule. From the trapezoidal rule the truncation error is represented by

$$\int_0^k f(t)dt = \frac{1}{2}k(f(0)-f(k)) - \frac{1}{12}k^3 f''(\frac{k}{2}) + ...$$

Using the formula

$$u(x_m, t_{n+1}) - u(x_m, t_n) = \int_{t_n}^{t_{n+1}} u_t(x_m, t)dt$$

and approximating the integral by the trapezoidal rule, and abbreviate the notation $u_m^{n+1/2} = u(x_m, t_n + \frac{1}{2}k)$, one obtain

$$\begin{aligned} u_m^{n+1} &= u_m^n + \frac{1}{2}k(\partial_t u_m^n + \partial_t u_m^{n+1}) - \frac{1}{2}k^3\partial_t^3 u_m^{n+\frac{1}{2}} \\ &\overset{\text{heat eq.}}{=} u_m^n + \frac{1}{2}k(\partial_x^2 u_m^n + \partial_y^2 u_m^n + \partial_z^2 u_m^n + \partial_x^2 u_m^{n+1} + \partial_y^2 u_m^{n+1} + \partial_z^2 u_m^{n+1}) - \frac{1}{2}k^3\partial_t^3 u_m^{n+\frac{1}{2}} \end{aligned}$$

Discretizing the double derivative with central differences, and denoting the step sizes in x, y, z direction by h, f, g respectively, gives

$$\begin{aligned} u_m^{n+1} &= u_m^n + \frac{1}{2}k(\frac{1}{h^2}\delta_x^2 u_m^n + \frac{1}{f^2}\delta_y^2 u_m^n + \frac{1}{g^2}\delta_z^2 u_m^n + \frac{1}{h^2}\delta_x^2 u_m^{n+1} + \frac{1}{f^2}\delta_y^2 u_m^{n+1} + \frac{1}{g^2}\delta_z^2 u_m^{n+1}) \\ &\quad -\frac{1}{2}k(\frac{1}{12}h^2\partial_x^4 u_m^n + \frac{1}{12}g^2\partial_y^4 u_m^n \frac{1}{12}f^2\partial_z^4 u_m^n + \frac{1}{12}h^2\partial_x^4 u_m^{n+1} + \frac{1}{12}g^2\partial_y^4 u_m^{n+1}\frac{1}{12}f^2\partial_z^4 u_m^{n+1}) \\ &\quad -\frac{1}{2}k^3\partial_t^3 u_m^{n+\frac{1}{2}} \\ &= u_m^n + \frac{r}{2}(\delta_x^2 u_m^n + \delta_x^2 u_m^{n+1}) + \frac{p}{2}(\delta_y^2 u_m^n + \delta_y^2 u_m^{n+1}) + \frac{q}{2}(\delta_z^2 u_m^n + \delta_z^2 u_m^{n+1}) + \tau_m^n \end{aligned}$$

where $r = k/h^2$, $p = k/f^2$, $q = k/g^2$, and τ_m^n is the truncation error times k. The expression for τ_m^n is

$$\tau_m^n = -\frac{1}{12}k^3\partial_t^2 u_m^{n+\frac{1}{2}} - \frac{1}{12}kh^2\partial_x^4 u_m^{n+\frac{1}{2}} - \frac{1}{12}kg^2\partial_y^4 u_m^{n+\frac{1}{2}} - \frac{1}{12}kf^2\partial_z^4 u_m^{n+\frac{1}{2}}$$

where we have used $\frac{1}{2}(\partial_x^4 u_m^n + \partial_x^4 u_m^{n+}) = \partial_x^4 u_m^{n+\frac{1}{2}} + O(k^2)$. Hence, the truncation error is

$$\begin{aligned}\frac{\tau_m^n}{k} &= -\frac{1}{12}k^2\partial_t^2 u_m^{n+\frac{1}{2}} - \frac{1}{12}h^2\partial_x^4 u_m^{n+\frac{1}{2}} - \frac{1}{12}g^2\partial_y^4 u_m^{n+\frac{1}{2}} - \frac{1}{12}f^2\partial_z^4 u_m^{n+\frac{1}{2}} \\ &= O(k^2 + h^2 + g^2 + f^2)\end{aligned}$$

Chapter 4

Implementation

4.1 Tools

The programming language chosen to implemented the system was c++. The arguments leading to this choice was:

- Most of the group members had some earlier experience with the language.
- It allows for a high level of abstraction without loosing control of the underlying hardware.
- The code can be easily parallelized for shared memory machines using OpenMP.
- Many libraries exist for c++ that implements highly optimized linear algebra functionality.

Other languages cosidered where Matlab and c.

The tool chosen to visualize the resulting data was gnuplot and ffmpeg. Writing a program using OpenGL was considered, but dismissed on the grounds of beeing too time consuming.

4.2 Program design

The program mainly consist of three parts:

- C++ implementation of Linear algebra functionality of the discrete equations.
- C++ implementation of the conjugate gradient method for solving the equations described.

- Bash script for generating plots and and movie of the generated data.

The Bash script is not discussed in further detail as it can be considered simple.

4.3 The physical system

The main task of the physical system is to implement a matrix-vector multiplication procedure for the heat equation and a method of calculating the heating properties of a microwave.

4.3.1 Partitioning of the problem

4.3.2 Updating alpha and beta values

4.3.3 Calculating the distribution of the microwave effect

4.3.4 The matrix-vector multiplication procedure

4.4 The conjugate gradient method as an iterative algorithm

Chapter 5

Experiments

5.1 Experiments

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Chapter 6

Conclusion

6.1 Conclusion

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