RESULTATER

Numeriske Simuleringer og Eksperimenter med

STEKING AV BACON I MIKROBØLGEOVN

EiT - Matematikk Innen Anvendelser

gruppe 5 - FUÞARK - ᚠᚢᛏᚼᚾᚱᚴ -

Åsmund Ervik, Joakim Johnsen, Knut Halvor Skrede, Turid Schoonderbeek Solberg, Paul Vo

2011

Sammendrag

Text in abstract

Forord

Text in foreword

Åsmund Ervik, Joakim Johnsen, Paul Vo,
Knut Halvor Skrede, Turid Schoonderbeek Solberg
20. mars 2011

Innhold

Kapittel 1

Innledning

1.1 Innledning

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus

vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Kapittel 2

Theory

2.1 The physical system

The physical system to be modeled is a thin, rectangular strip of bacon which is subjected to approx. 750 watts of microwave radiation at the frequency of 2.4 GHz. To be modeled is the heat in the strip, phase transitions and transport.

2.2 Our approach

Initially, our approach to modelling was to use a staggered approach: first solve the heat equation for the three media meat, solid fat, and liquid fat, thus giving the temperature at step n. Then at step $n+1$, the temperature is regarded as known, and we solve the transport equation for liquid fat out of the bacon. Then we know the distribution of liquid fat at step $n+2$, where we solve the heat equations again, and we repeat ad nauseam.

2.3 Differential equations

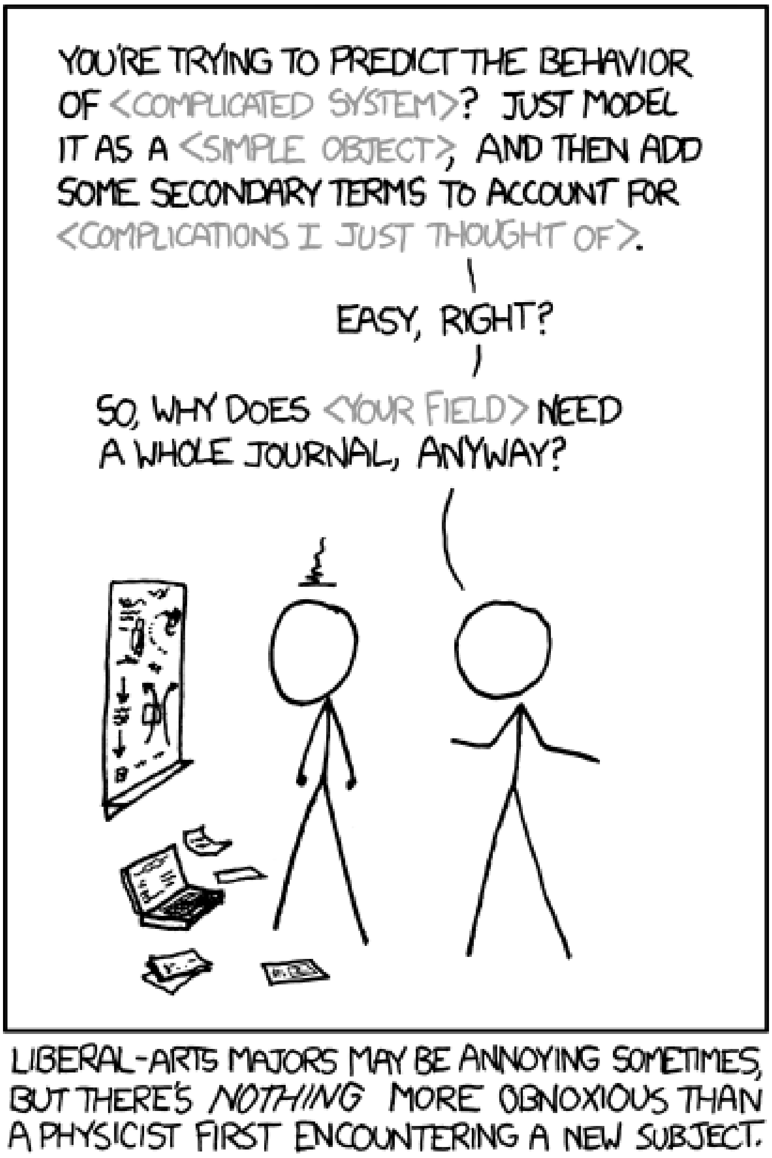
The differential equations governing the heat are all variations of the heat equation, with various source terms, **??** 2.1:

$$(\rho c_p)_m \frac{\partial T}{\partial t} - \alpha_m \boldsymbol{\nabla}^2 T = J^{MW} \quad \text{m}, \tag{2.1}$$

$$\eta_s (\rho c_p)_s \frac{\partial T}{\partial t} - \eta_s \alpha_s \boldsymbol{\nabla}^2 T = J^{MW} - J^{Melt} \quad \text{s}, \tag{2.2}$$

$$\eta_l (\rho c_p)_l \frac{\partial T}{\partial t} - \eta_s \alpha_l \boldsymbol{\nabla}^2 T - \eta_l (\rho c_p)_l (\mathbf{v} \cdot \boldsymbol{\nabla}) T = J^{MW} \quad \text{l}. \tag{2.3}$$

Here the subscript m denotes meat, s denotes solid fat, and l denotes liquid fat. J^{MW} is the source term representing the microwave oven. In the initial



Figur 2.1: Our initial approach to the problem

approach, this is modelled as a cylindrically symmetric term, with the radial power distribution on the form **??** 2.4,

$$J^{MW}(r) = 0.5+2.55008x-0.588013x^2+0.032445x^3+0.00124411x^4-0.0000973516x^5, \quad (2.4)$$

i.e. a fifth degree polynomial, interpolated from the results in [1].

Furthermore, there is the term J^{Melt} which represents heat loss due to latent heat in the solid-liquid phase transition. This can be written as

$$\frac{L\rho}{T_2 - T_1}\frac{\partial T}{\partial t}, \quad \mathrm{T} \in (\mathrm{T}_1, \mathrm{T}_2),$$

where the fat is melting between the temperatures T_1 and T_2. For fat these are typically around 30-50°C. For $T \notin (T1, T2)$ this term is zero, and it is readily seen that this is equivalent to a modification of the constant $c_{p,l}$ in **??** 2.1 when $T \in (T1, T2)$, thus it is not a serious complication.

In the last of the heat equations, there also appears what is called a convective derivative, on the form $(\mathbf{v} \cdot \boldsymbol{\nabla})T$. This is a somewhat more complicated term, and is essentially a modification due to transport of liquid fat implying an implicit transport of heat.

Kapittel 3

Numerikk

3.1 Von Neumann-analyse av Crank-Nicolson-skjemaet

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial^2 x} \tag{3.1}$$

Ved å anvende Crank-Nicolson oppnås følgende iterasjonsskjema

$$u_i^{n+1} = u_i^n (1-2D) + D\left(u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}\right) + D\left(u_{i+1}^n + u_{i-1}^n\right) \tag{3.2}$$

$$D = \frac{1}{2}\frac{\alpha \Delta t}{(\Delta x)^2} \tag{3.3}$$

Innfører tilnærmelsen $u_j^n \approx E_j^n$ hvor $E_j^n = G^n e^{i\beta x_j}$, her er n en potens. Innsatt i **??** 3.1:

$$G^{n+1}e^{i\beta x_j} = G^n e^{i\beta x_j}(1-2D) + G^{n+1}D\left(e^{i\beta x_{j+1}} - 2e^{i\beta x_j} + e^{i\beta x_{j-1}}\right) + G^n D\left(e^{i\beta x_{j+1}} + e^{i\beta x_{j-1}}\right)$$

Hvis vi deler (3) på $G^n e^{i\beta x_j}$ og benytter at $x_{j+1} \approx x_j + h$ får vi

$$\begin{aligned}
G\left[1 - D\left(2\cos\beta h - 2\right)\right] &= 1 - 2D\left(1 - \cos\beta h\right) \\
\cos\beta h &= \frac{1}{2}\left(1 - \sin^2\frac{\beta h}{2}\right) \\
G\left(1 + 4D\sin^2\frac{\beta h}{2}\right) &= 1 - 4D\sin^2\frac{\beta h}{2} \\
G &= \frac{1 - 4D\sin^2\frac{\beta h}{2}}{1 + 4D\sin^2\frac{\beta h}{2}}
\end{aligned}$$

Vi vet at den maksimale verdi av $\sin^2 x$ er 1, og at iterasjonsskjemaet er stabilt dersom $|G| \leq 1$, innsatt:

$$|G| = \left| \frac{1-4D}{1+4D} \right| \tag{3.4}$$

Verdt å observere at så lenge $D > 0 \;\Rightarrow\; |G| \leq 1$, slik at Crank-Nicolson-skjemaet er *ubetinget stabilt*

3.1.1 Truncation error

The Crank-Nicholson method is based on the trapezoidal rule. From the trapezoidal rule the truncation error is represented by

$$\int_0^k f(t)dt = \frac{1}{2}k(f(0) - f(k)) - \frac{1}{12}k^3 f''(\frac{k}{2}) + ...$$

Using the formula

$$u(x_m, t_{n+1}) - u(x_m, t_n) = \int_{t_n}^{t_{n+1}} u_t(x_m, t)dt$$

and approximating the integral by the trapezoidal rule, and abbreviate the notation $u_m^{n+1/2} = u(x_m, t_n + \frac{1}{2}k)$, one obtain

$$\begin{aligned} u_m^{n+1} &= u_m^n + \frac{1}{2}k(\partial_t u_m^n + \partial_t u_m^{n+1}) - \frac{1}{2}k^3 \partial_t^3 u_m^{n+\frac{1}{2}} \\ &\overset{\text{heat eq.}}{=} u_m^n + \frac{1}{2}k(\partial_x^2 u_m^n + \partial_y^2 u_m^n + \partial_z^2 u_m^n + \partial_x^2 u_m^{n+1} + \partial_y^2 u_m^{n+1} + \partial_z^2 u_m^{n+1}) - \frac{1}{2}k^3 \partial_t^3 u_m^{n+\frac{1}{2}} \end{aligned}$$

Discretizing the double derivative with central differences, and denoting the step sizes in x, y, z direction by h, f, g respectively, gives

$$\begin{aligned} u_m^{n+1} &= u_m^n + \frac{1}{2}k(\frac{1}{h^2}\delta_x^2 u_m^n + \frac{1}{f^2}\delta_y^2 u_m^n + \frac{1}{g^2}\delta_z^2 u_m^n + \frac{1}{h^2}\delta_x^2 u_m^{n+1} + \frac{1}{f^2}\delta_y^2 u_m^{n+1} + \frac{1}{g^2}\delta_z^2 u_m^{n+1}) \\ &\quad -\frac{1}{2}k(\frac{1}{12}h^2\partial_x^4 u_m^n + \frac{1}{12}g^2\partial_y^4 u_m^n \frac{1}{12}f^2\partial_z^4 u_m^n + \frac{1}{12}h^2\partial_x^4 u_m^{n+1} + \frac{1}{12}g^2\partial_y^4 u_m^{n+1}\frac{1}{12}f^2\partial_z^4 u_m^{n+1}) \\ &\quad -\frac{1}{2}k^3\partial_t^3 u_m^{n+\frac{1}{2}} \\ &= u_m^n + \frac{r}{2}(\delta_x^2 u_m^n + \delta_x^2 u_m^{n+1}) + \frac{p}{2}(\delta_y^2 u_m^n + \delta_y^2 u_m^{n+1}) + \frac{q}{2}(\delta_z^2 u_m^n + \delta_z^2 u_m^{n+1}) + \tau_m^n \end{aligned}$$

where $r = k/h^2$, $p = k/f^2$, $q = k/g^2$, and τ_m^n is the truncation error times k. The expression for τ_m^n is

$$\tau_m^n = -\frac{1}{12}k^3\partial_t^2 u_m^{n+\frac{1}{2}} - \frac{1}{12}kh^2\partial_x^4 u_m^{n+\frac{1}{2}} - \frac{1}{12}kg^2\partial_y^4 u_m^{n+\frac{1}{2}} - \frac{1}{12}kf^2\partial_z^4 u_m^{n+\frac{1}{2}}$$

where we have used $\frac{1}{2}(\partial_x^4 u_m^n + \partial_x^4 u_m^{n+}) = \partial_x^4 u_m^{n+\frac{1}{2}} + O(k^2)$. Hence, the truncation error is

$$
\begin{aligned}
\frac{\tau_m^n}{k} &= -\frac{1}{12}k^2\partial_t^2 u_m^{n+\frac{1}{2}} - \frac{1}{12}h^2\partial_x^4 u_m^{n+\frac{1}{2}} - \frac{1}{12}g^2\partial_y^4 u_m^{n+\frac{1}{2}} - \frac{1}{12}f^2\partial_z^4 u_m^{n+\frac{1}{2}} \\
&= O(k^2 + h^2 + g^2 + f^2)
\end{aligned}
$$

Kapittel 4

Eksperimenter

4.1 Eksperimenter

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus

vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Kapittel 5

Numerikkresultater

5.1 Numerikkresultater

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus

vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Kapittel 6

Konklusjon

6.1 Konklusjon

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus

vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Bibliografi

[1] Yi Huang and Xu Zhu. Microwave oven field uniformity analysis. In *Antennas and Propagation Society International Symposium, 2005 IEEE*, volume 3B, pages 217 – 220 vol. 3B, 2005.