RESULTATER

Numeriske Simuleringer og Eksperimenter med

STEKING AV BACON I MIKROBØLGEOVN

EiT - Matematikk Innen Anvendelser

gruppe 5 - FUÞARK - ᚠᚢᛏᚼᚾᚱᚴ -

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Sammendrag

Text in abstract

Forord

Text in foreword

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Chapter 1

Introduction

1.1 Introduction

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Chapter 2

Theory

2.1 The physical system

The physical system to be modeled is a thin, rectangular strip of bacon which is subjected to approx. 750 watts of microwave radiation at the frequency of 2.4 GHz. To be modeled is the heat in the strip, phase transitions and transport.

2.2 Our approach

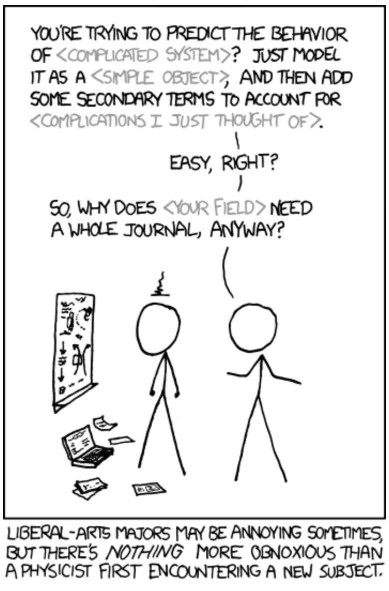

Figure 2.1: Our initial approach to the problem

Initially, our approach to modelling was to use a staggered approach: first solve the heat equation for the three media meat, solid fat, and liquid fat, thus giving the temperature at step n. Then at step $n+1$, the temperature is regarded as known, and we solve the transport equation for liquid fat out of the bacon. Then we know the distribution of liquid fat at step $n+2$, where we solve the heat equations again, and we repeat ad nauseam.

2.3 The heat equations

The differential equations governing the heat transport are all variations of the heat equation, with various source terms, eq. (2.1):

$$(\rho c_p)_m \frac{\partial T}{\partial t} - \alpha_m \boldsymbol{\nabla}^2 T = J^{MW} \quad \text{m}, \tag{2.1}$$

$$\eta_s (\rho c_p)_s \frac{\partial T}{\partial t} - \eta_s \alpha_s \boldsymbol{\nabla}^2 T = J^{MW} - J^{Melt} \quad \text{s}, \tag{2.2}$$

$$\eta_l (\rho c_p)_l \frac{\partial T}{\partial t} - \eta_s \alpha_l \boldsymbol{\nabla}^2 T - \eta_l (\rho c_p)_l (\mathbf{v} \cdot \boldsymbol{\nabla}) T = J^{MW} \quad \text{l}. \tag{2.3}$$

Here the subscript m denotes meat, s denotes solid fat, and l denotes liquid fat. J^{MW} is the source term representing the microwave oven. In the initial approach, this is modelled as a cylindrically symmetric term, with the radial power distribution on the form eq. (2.4),

$$J^{MW}(r) = 0.5 + 2.55008x - 0.588013x^2 + 0.032445x^3 + 0.00124411x^4 - 0.0000973516x^5, \tag{2.4}$$

i.e. a fifth degree polynomial, interpolated from the results in [1].

Furthermore, there is the term J^{Melt} which represents heat loss due to latent heat in the solid-liquid phase transition. This can be written as

$$\frac{L\rho}{T_2 - T_1} \frac{\partial T}{\partial t}, \quad \mathrm{T} \in (\mathrm{T}_1, \mathrm{T}_2),$$

where the fat is melting between the temperatures T_1 and T_2. For fat these are typically around 30-50°C. For $T \notin (T1, T2)$ this term is zero, and it is readily seen that this is equivalent to a modification of the constant $c_{p,l}$ in eq. (2.1) when $T \in (T1, T2)$, thus it is not a serious complication.

In the last of the heat equations, there also appears what is called a convective derivative, on the form $(\mathbf{v} \cdot \boldsymbol{\nabla})T$. This is a somewhat more complicated term, and is essentially a modification due to transport of liquid fat implying an implicit transport of heat. However, in the staggered approach used here, the velocity field is considered as known when it comes to the heat equations.

2.4 The transport equations

Chapter 3

Numerikk

3.1 Von Neumann analysis of the Crank-Nicolson-scheme

Consider the heat equation,

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial^2 x} \tag{3.1}$$

By applying Crank-Nicolson the following iteration scheme is attained,

$$u_i^{n+1} = u_i^n (1 - 2D) + D\left(u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}\right) + D\left(u_{i+1}^n + u_{i-1}^n\right) \tag{3.2}$$

$$D = \frac{1}{2}\frac{\alpha \Delta t}{(\Delta x)^2} \tag{3.3}$$

Using the approximation $u_j^n \approx E_j^n$ where $E_j^n = G^n e^{i\beta x_j}$, and n is a power, inserted into eq. (3.1), this gives

$$G^{n+1} e^{i\beta x_j} = G^n e^{i\beta x_j}(1 - 2D) + G^{n+1} D\left(e^{i\beta x_{j+1}} - 2e^{i\beta x_j} + e^{i\beta x_{j-1}}\right) + G^n D\left(e^{i\beta x_{j+1}} + e^{i\beta x_{j-1}}\right)$$

Dividing eq. (3.3) by $G^n e^{i\beta x_j}$, and using $x_{j+1} \approx x_j + h$ one obtains

$$\begin{aligned}
G\left[1 - D\left(2\cos\beta h - 2\right)\right] &= 1 - 2D\left(1 - \cos\beta h\right) \\
\cos\beta h &= \frac{1}{2}\left(1 - \sin^2\frac{\beta h}{2}\right) \\
G\left(1 + 4D\sin^2\frac{\beta h}{2}\right) &= 1 - 4D\sin^2\frac{\beta h}{2} \\
G &= \frac{1 - 4D\sin^2\frac{\beta h}{2}}{1 + 4D\sin^2\frac{\beta h}{2}}
\end{aligned}$$

As the maximum value of $\sin^2 x$ is 1, and since the iteration scheme is stable if $|G| \leq 1$, this implies stability,

$$|G| = \left|\frac{1-4D}{1+4D}\right| \tag{3.4}$$

A typical **C**ourant-**F**riedrichs-**L**ewy condition is an inequality of the form $\frac{\Delta x}{\Delta t} < C$, with C a constant.

It is noteworthy that $D > 0 \;\Rightarrow\; |G| \leq 1$, so the Crank-Nicolson scheme is *unconditionally stable*. There is, however, a caveat: if the time steps and space steps do not fulfill a CFL-condition, the scheme may present oscillations.

3.2 Truncation error in the Crank-Nicolson scheme

The Crank-Nicholson method is based on the trapezoidal rule. From the trapezoidal rule the truncation error is represented by

$$\int_0^k f(t)dt = \frac{1}{2}k(f(0) - f(k)) - \frac{1}{12}k^3 f''(\frac{k}{2}) + ...$$

Using the formula

$$u(x_m, t_{n+1}) - u(x_m, t_n) = \int_{t_n}^{t_{n+1}} u_t(x_m, t)dt$$

and approximating the integral by the trapezoidal rule, and abbreviate the notation $u_m^{n+1/2} = u(x_m, t_n + \frac{1}{2}k)$, one obtain

$$\begin{aligned}
u_m^{n+1} &= u_m^n + \frac{1}{2}k(\partial_t u_m^n + \partial_t u_m^{n+1}) - \frac{1}{2}k^3\partial_t^3 u_m^{n+\frac{1}{2}} \\
&\overset{\text{heat eq.}}{=} u_m^n + \frac{1}{2}k(\partial_x^2 u_m^n + \partial_y^2 u_m^n + \partial_z^2 u_m^n + \partial_x^2 u_m^{n+1} + \partial_y^2 u_m^{n+1} + \partial_z^2 u_m^{n+1}) - \frac{1}{2}k^3\partial_t^3 u_m^{n+\frac{1}{2}}
\end{aligned}$$

Discretizing the double derivative with central differences, and denoting the step sizes in x, y, z direction by h, f, g respectively, gives

$$\begin{aligned}
u_m^{n+1} &= u_m^n + \frac{1}{2}k(\frac{1}{h^2}\delta_x^2 u_m^n + \frac{1}{f^2}\delta_y^2 u_m^n + \frac{1}{g^2}\delta_z^2 u_m^n + \frac{1}{h^2}\delta_x^2 u_m^{n+1} + \frac{1}{f^2}\delta_y^2 u_m^{n+1} + \frac{1}{g^2}\delta_z^2 u_m^{n+1}) \\
&\quad -\frac{1}{2}k(\frac{1}{12}h^2\partial_x^4 u_m^n + \frac{1}{12}g^2\partial_y^4 u_m^n \frac{1}{12}f^2\partial_z^4 u_m^n + \frac{1}{12}h^2\partial_x^4 u_m^{n+1} + \frac{1}{12}g^2\partial_y^4 u_m^{n+1}\frac{1}{12}f^2\partial_z^4 u_m^{n+1}) \\
&\quad -\frac{1}{2}k^3\partial_t^3 u_m^{n+\frac{1}{2}} \\
&= u_m^n + \frac{r}{2}(\delta_x^2 u_m^n + \delta_x^2 u_m^{n+1}) + \frac{p}{2}(\delta_y^2 u_m^n + \delta_y^2 u_m^{n+1}) + \frac{q}{2}(\delta_z^2 u_m^n + \delta_z^2 u_m^{n+1}) + \tau_m^n
\end{aligned}$$

where $r = k/h^2$, $p = k/f^2$, $q = k/g^2$, and τ_m^n is the truncation error times k. The expression for τ_m^n is

$$\tau_m^n = -\frac{1}{12}k^3\partial_t^2 u_m^{n+\frac{1}{2}} - \frac{1}{12}kh^2\partial_x^4 u_m^{n+\frac{1}{2}} - \frac{1}{12}kg^2\partial_y^4 u_m^{n+\frac{1}{2}} - \frac{1}{12}kf^2\partial_z^4 u_m^{n+\frac{1}{2}}$$

where we have used $\frac{1}{2}(\partial_x^4 u_m^n + \partial_x^4 u_m^{n+}) = \partial_x^4 u_m^{n+\frac{1}{2}} + O(k^2)$. Hence, the truncation error is

$$\begin{aligned}\frac{\tau_m^n}{k} &= -\frac{1}{12}k^2\partial_t^2 u_m^{n+\frac{1}{2}} - \frac{1}{12}h^2\partial_x^4 u_m^{n+\frac{1}{2}} - \frac{1}{12}g^2\partial_y^4 u_m^{n+\frac{1}{2}} - \frac{1}{12}f^2\partial_z^4 u_m^{n+\frac{1}{2}} \\ &= O(k^2+h^2+g^2+f^2)\end{aligned}$$

Chapter 4

Eksperimenter

4.1 Eksperimenter

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Chapter 5

Numerikkresultater

5.1 Numerikkresultater

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Chapter 6

Konklusjon

6.1 Konklusjon

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