iFlipper: Label Flipping for Individual Fairness

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ABSTRACT

As machine learning becomes prevalent, mitigating any unfairness present in the training data becomes critical. Among the various notions of fairness, this paper focuses on the well-known individual fairness where similar individuals must be treated similarly. While individual fairness can be improved when training a model (in-processing), we contend that fixing the data before model training (pre-processing) is a more fundamental solution. In particular, we show that *label flipping is an effective pre-processing technique for improving individual fairness*. Our system iFlipper solves the optimization problem of minimally flipping labels given a limit to the number of individual fairness violations, where a violation occurs when two similar examples in the training data have different labels. We first prove that the problem is NP-hard. We then propose an approximate linear programming algorithm and provide theoretical guarantees on how close its result is to the optimal solution in terms of the number of label flips. We also propose techniques for making the solution to the linear programming more optimal without exceeding the violations limit. Experiments on real datasets show that iFlipper significantly outperforms other pre-processing baselines in terms of individual fairness and accuracy on unseen test sets. In addition, iFlipper can be combined with in-processing techniques for even better results.

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1 INTRODUCTION

Machine learning (ML) impacts our everyday lives where applications include recommendation systems [9], job application [11], and face recognition [7]. Unfortunately, ML algorithms are also known to reflect or even reinforce bias in the training data and thus make unfair decisions [3, 38]. This issue draws concerns from both the public and research community, so algorithms have been proposed to mitigate bias in the data and improve the fairness of ML models.

There are several prominent notions of fairness, and we focus on individual fairness [13], which states that *similar individuals must be treated similarly*. Suppose that two applicants are applying to the same school. If the two applicants have similar application materials, then it makes sense for them to obtain the same or similar outcomes. Likewise if two individuals are applying for a loan and have similar financial profiles, it is fair for them to be accepted or rejected together as well. In addition to individual fairness, the other prominent fairness notions include group fairness and causal fairness. Group fairness [1, 42, 44] focuses on the parity between two different sensitive groups (e.g., male versus female), and causal fairness [23] looks at fairness from a causal perspective (e.g., does gender affect an outcome?). However, these notions do not capture the individual fairness we are interested in.

How can one improve a model's individual fairness? There are largely three possible approaches: fixing the data before model training (pre-processing), fixing the model training itself (in-processing), and fixing the model after training (post-processing). Among them, most of the literature focuses on in-processing [36, 40, 41] and more recently pre-processing techniques [24, 25, 43]. We contend that pre-processing is important because biased data is the root cause of unfairness, so fixing the data is the more fundamental solution rather than having to cope with the bias during or after model training. The downside of pre-processing is that one cannot access the model and has to address the bias only using the training data. Due to this challenge, only a few pre-processing techniques for individual fairness have been proposed, which we will compare with in the experiments.

We propose *label flipping as a way to mitigate data bias for individual fairness* and assume a binary classification setting where labels have 0 or 1 values. Given a training set of examples, the idea is to change the labels of some of the examples such that similar examples have the same labels as much as possible. Exactly which examples are considered similar is application dependent, and we assume the criteria is given as an input. For example, one may compute the Euclidean distance between two examples and consider them similar if the distance is within a certain threshold. As the training set becomes more individually fair, the trained model becomes fairer as well (see Section 4.2). Our label flipping approach is inspired by the robust training literature of learning from noisy labels [35]. The standard approach for handling such labels is to

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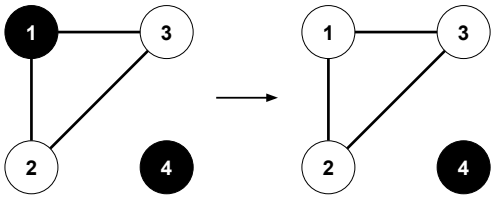


Figure 1: Label flipping example for individual fairness using a graph representation. Two similar nodes have an edge, and color indicates the label. By flipping the label of node 1, all pairs of similar nodes have the same label, which is considered individually fair.

ignore or fix them [34]. In our setting, we consider any label that reduces individual fairness as biased and would like to fix it.

We can use a graph representation to illustrate label flipping as shown in Figure 1. Each node represents a training example, and its color indicates the label (black indicates label 1, and white indicates 0). Two similar nodes are connected with an edge, and a violation occurs when the edge is connected with two nodes with different labels. In the figure, there are four nodes where only the nodes 1, 2, and 3 are similar to each other. In addition, nodes 1 and 4 have label 1, and nodes 2 and 3 have label 0. We can see that there are two "violations" of fairness in this dataset: (1,2) and (1,3) because there are edges between them, and they have different colors. After flipping the label of node 1 from 1 to 0, we have no violations.

Our key contribution is in formulating and solving a constrained label flipping problem for the purpose of individual fairness. One consequence of label flipping is an accuracy-fairness trade-off where the model's accuracy may diminish. As an extreme example, if we simply flip all the labels to be 0, then a trained model that only predicts 0 is certainly fair, but inaccurate to say the least. Even if we carefully flip the labels, we still observe a trade-off as we detail in Section 4.2. We thus formulate the optimization problem where the objective is to minimize the number of label flipping while limiting the number of individual fairness violations to an allowed number. The optimization can be formally stated as an instance of mixed-integer quadratic programming (MIQP) problem, and we prove that it is NP-hard. We then transform the problem to an approximate linear programming (LP) problem for efficient computation. Interestingly, we show that our LP algorithm has theoretical guarantees on how close its result is to the optimal solution in terms of the number of flips performed. We then further optimize the solution given by the LP algorithm to reduce the number of flips while ensuring the number of violations is not exceeding the given limit.

We call our approach iFlipper and empirically show how its label flipping indeed results in individually-fair models and significantly outperforms other pre-processing baselines on real datasets. In particular, the state-of-the-art baselines [24, 25] use representation learning to place similar examples closer in a feature space. We show that iFlipper has better accuracy-fairness trade-off curves and is also significantly more efficient. We also compare iFlipper with baselines (e.g., greedy approach) for solving the optimization problem and justify our techniques. Finally, we demonstrate how iFlipper can be integrated with in-processing techniques [40] for even better results.

The rest of the paper is organized as follows.

- We formulate the label flipping optimization as an MIQP problem and prove that it is NP-hard (Section 2).
- We propose iFlipper, which solves this problem by converting it into an approximate LP algorithm that has theoretical guarantees and present an additional optimization to further improve the solution given by the LP solver (Section 3).
- We evaluate iFlipper on real datasets and show how it outperforms other pre-processing baselines and can be integrated with in-processing techniques for better results (Section 4).

2 PROBLEM DEFINITION

2.1 Preliminaries

We focus on a binary classification setting, and assume a training dataset $D = \{(x_i, y_i)\}_{i=1}^{n}$ where x_i is an example, and y_i is its label having a value of 0 or 1. A binary classifier h can be trained on D, and its prediction on a test example x is $h(x)$.

Individual Fairness [13] states that similar individuals must be treated similarly. The criteria for determining if two examples are similar depends on the application, and we assume it is given as an input, though research on how to automatically discover such similarity measures exists [17, 28, 37]. For example, for each x, we may consider all the examples within a certain distance to be similar. Alternatively, we may consider the top-k nearest examples to be similar. In our work, regardless of the similarity measures used, we capture the Boolean similarity information among examples as a similarity matrix $W_{ij} \in \{0,1\}^{n \times n}$ where $W_{ij} = 1$ if and only if x_i and x_j are similar. In order to satisfy individual fairness, we introduce the notion of *individual fairness violations* as follows.

Definition 1. ***Individual Fairness Violation.*** *Given a similarity matrix W on a training set, an individual fairness violation occurs when $W_{ij} = 1$, but $y_i \neq y_j$.*

Our goal is to reduce the number of individual fairness violations to a pre-defined maximum allowed number.

Definition 2. ***m-Individually Fair Dataset.*** *Given a similarity matrix W, a dataset is considered m-individually fair if there are at most m individual fairness violations.*

A smaller m naturally translates to an m-individually fair dataset that is more likely to result in a trained model that is more individually fair on the unseen test set as we demonstrate in Section 4.2. To measure the final individual fairness of a trained model h, we compute the *consistency score* [25] on an unseen test set, which is defined as follows, where W_{ij} here is the similarity matrix on the unseen test set.

$$\text{Consistency Score} = 1 - \frac{\sum_i \sum_j |h(x_i) - h(x_j)| \times W_{ij}}{\sum_i \sum_j W_{ij}}$$

Intuitively, if the trained model is individually fair, its predictions on similar examples are the same, so the consistency score increases. In the extreme case, a consistency score of 1 indicates that all similar pairs of test examples get the same predictions from the model.

2.2 Label Flipping Optimization Problem

We define the label flipping optimization problem for individual fairness. Given a training dataset D and a limit m of violations allowed, our goal is to flip the minimum number of labels in D such that it becomes individually fair. This statement can be formalized as a mixed-integer quadratic programming (MIQP) problem:

$$\begin{aligned} \text{(MIQP)} \quad \min \quad & \sum_{i=1}^{n} (y_i - y_i')^2 \\ \text{s.t.} \quad & \sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij}(y_i - y_j)^2 \leq m \\ & y_i \in \{0,1\}, \forall i \end{aligned} \tag{1}$$

where y_i indicates an output label, and y_i' is its original value. Intuitively, we count the number of flips ($(y_i - y_i')^2 = 1$) while ensuring that the number of violations ($(y_i - y_j)^2 = 1$ and $W_{ij} = 1$) is within the limit m. We call a solution *feasible* if it satisfies the violations constraints in Equation 1, but may or may not be optimal.

MIQP is an NP-hard problem in general, and we prove that our specific instance of the MIQP problem is also NP-hard.

Theorem 1. *The MIQP problem in Equation 1 is NP-hard.*

Proof. We prove that the MIQP problem in Equation 1 is NP-hard by reducing it from the well-known *at most k-cardinality s-t cut problem*. Given an undirected graph $G = (V, E)$, the *at most k-cardinality minimum s-t cut problem* is to find the minimum sum of edge weights in the cut edge set C to divide the vertex set V into two sets V_1 and V_2, where $s \in V_1$ and $t \in V_2$, and the cut edge set $C = \{(v_1, v_2) \in E : v_1 \in V_1, v_2 \in V_2\}$ has at most k edges. This problem is known to be NP-hard even if all the edge weights are 1 [6, 19, 22]. Furthermore, the *existence of at most k-cardinality s-t cut problem* is also known to be NP-hard [6].

We will show the reduction as a three-step approach. First, given an instance of the *at most k-cardinality s-t cut problem* on $G = (V, E)$ with all edge weights equal to 1, we show how to create $k + 1$ label flipping problems in Equation 1 accordingly. Second, we show how a solution to any of the $k + 1$ MIQP problems represents an s-t cut to the original graph cut problem. Third, we establish that if there exists an *at most k-cardinality s-t cut*, it must be one of the $k + 1$ s-t cuts obtained in the previous step.

Step 1: We show how to create MIQP problems for a given *at most k-cardinality s-t cut problem* on $G = (V, E)$. We create a variable y_i for each $v_i \in V - \{s, t\}$. We make $W_{ij} = 1$ for each edge $(i, j) \in E$ in the graph, and $W_{ij} = 0$ for other pairs of (i, j). If a node v_i is only connected to s (t), we make $y_i' = 1(0)$ and add $(y_i - y_i')^2$ to the objective function. That is, we set the initial label of nodes connected to s (t) as 1 (0). If a node v_i is connected to both s and t, we add two terms to the objective function for this node, one with $y_i' = 0$ and the other one with $y_i' = 1$. If a node v_i is not connected to s or t, we do not add any terms to the objective. Now that we have defined all y_i, their initial assignments y_i', and the weight W_{ij}, we vary m from 0 to k to create $k + 1$ instances of MIQP problems with different number of allowed violations as the constraints.

The intuition for the above process of creating $k + 1$ MIQP problem is that an s-t cut is a binary partition of a graph and can be viewed as a binary-valued labeling process. The nodes that are connected to the source s have label 1 and the nodes that are connected to sink t have label 0. A cut consists of two types of edges: e_f and e_v. A flip edge $e_f : (v_1, v_2) : v_1 \in \{s, t\}, v_2 \in V - \{s, t\}$ is an edge between s (t) and other nodes. If an e_f edge exists in a cut, that means a node which was connected to s (t) is no longer in the same partition as s (t), which can be represented as a flip of label. See the $(s, 1)$ edge in Figure 2 as an example for flip edges. A violation edge $e_v : (v_1, v_2) : v_1 \in V - \{s, t\}, v_2 \in V - \{s, t\}$ is an edge between two nodes that are not s or t. If an e_v edge exists in a cut, that means two nodes that are supposed to be similar and have the same label ended up in two different partitions, which can be represented as a violation. See the $(1, 2)$ edge in Figure 2 as an example for violation edges. The cardinality of the cut equals to the sum of the counts of the two types of edges, which equals to the number of flips and violations in the MIQP solution. Our mapping from graph cutting to label flipping is inspired by a similar process used in image segmentation [21] that aims at using graph cutting to split an image into foreground and background.

Step 2: We show how a solution to any of the k+1 MIQP problems can be mapped to an s-t cut. After solving a corresponding MIQP, we put all nodes with $y_i = 1$ in V_1 and $y_i = 0$ in V_2 as the resulting s-t cut. The cardinality of the cut equals to the sum of label flips and violations in the label flipping problem.

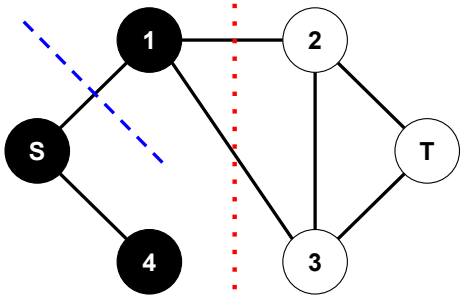


Figure 2: An example for the s-t cut. The cut represented by the blue dashed line consists of one e_f edge, and the cut represented by the red dotted line consists of two e_v edges.

In Figure 2, after removing s and t, we get the same graph representation as Figure 1. By solving the label flipping problem with m equal to 0 or 1, the resulting label assignment is $y_4 = 1$ and $y_1 = y_2 = y_3 = 0$. The corresponding cut is shown as the blue dashed line in Figure 2. The cut is $V_1 = \{s, 4\}, V_2 = \{1, 2, 3, t\}$, and $C = \{(s, 1)\}$. This label assignment has 1 flip (node 1 flips to label 0) and 0 violations. This flip is represented as an e_f edge (s,1) in the cut. If we solve the problem with $m = 2$, the result is $y_1 = y_4 = 1$ and $y_2 = y_3 = 0$. The corresponding cut is shown as the red dotted line in Figure 2. The cut is $V_1 = \{s, 1, 4\}, V_2 = \{2, 3, t\}$, and $C = \{(1, 2), (1, 3)\}$. This label assignment has 0 flips and 2 violations, represented by two e_v edges in the cut: (1,2) and (1,3).

Step 3: Since the cardinality of the cut equals to the sum of label flips and violations in the label flipping problem, finding an at most k-cardinality s-t cut can be solved by finding a label assignment where the sum of flips and violations equals to at most k. To find such a label assignment, we can repeatedly solve the MIQP problem

$k+1$ times, for all m values in $\{0, \ldots, k\}$, and check if the sum of flips and violations is less than or equal to k. The k-cardinality minimum s-t cut, if exists, must equal to the result of the MIQP problem with at least one of the possible m values. Therefore, if the label flipping MIQP problem is not NP-hard, then the k-cardinality minimum s-t cut problem would also not be NP-hard, which contradicts the known results. □

2.3 Baseline Approaches for Solving the Optimization Problem

Our key contribution is to propose algorithms for the label flipping problem that not only scales to large datasets, but also provides feasible and high-quality solutions. We first present two naïve algorithms that are efficient, but could fail to produce a feasible solution. These two baseline algorithms show that efficient algorithms that focus on local optimality are not sufficient. We then convert the problem to an approximate linear programming (LP) problem and solve it efficiently with theoretical guarantees in Section 3. Interestingly, iFlipper further optimizes the solution given by the LP solver using an idea similar to the greedy algorithm shown in this section.

Greedy Algorithm. The greedy algorithm repeatedly flips label of nodes that reduce the number of violations the most. The algorithm terminates if there are m or fewer violations or if we cannot reduce the violations anymore. For example, suppose we start from the graph in Figure 1 where there are initially two violations and set m = 1. We need to determine which label leads to the largest reduction in violations when flipped. We can see that flipping node 1 results in two fewer violations while flipping the other nodes does not change the number of violations. Hence, the greedy algorithm flips node 1 to reduce the number of violations to zero and then terminates. In general, the greedy algorithm does not always find an optimal result and may not produce a feasible solution even if one exists. Consider the example in Figure 3 where there is one violation between nodes 2 and 3. If we set m = 0, the feasible solution is to flip nodes 1 and 2 or flip nodes 3 and 4 together. Unfortunately, the greedy algorithm immediately terminates because it only flips one node at a time, and no single flip can reduce the number of violations.

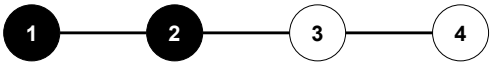


Figure 3: A graph where the greedy algorithm fails to find a feasible solution with zero violations.

The computational complexity of the greedy algorithm is $O(n^2)$ because we flip at most $O(n)$ labels, and each time a label is flipped, we update the number of violations of each neighbor of the node.

Gradient Based Algorithm. An alternative naïve approach is to use a Lagrangian multiplier to move the violations constraint into the objective function and solve the problem via gradient descent. This approach is common in machine learning, and the following equation shows the problem setup:

$$\text{(Lagrangian)} \ \min \ \sum_{i=1}^{n} (y_i - y_i')^2 + \lambda \sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij}(y_i - y_j)^2 \quad (2)$$
$$y_i \in [0,1], \forall i$$

where λ is a hyperparameter that controls the trade-off between fairness and accuracy. A higher λ favors better fairness. Although this gradient based algorithm is efficient, it shares the same problem as the greedy algorithm where it may get stuck in a local minima and thus fail to find a feasible solution. We demonstrate the limitations of the greedy and gradient based algorithms in Section 4.4.

3 IFLIPPER

We explain how iFlipper converts the MIQP problem into an approximate linear program (LP) problem and produces a feasible solution with theoretical guarantees. The conversion is done in two steps: from MIQP to an equivalent integer linear program (ILP) using linear constraints (Section 3.1) and from the ILP problem to an approximate LP problem (Section 3.2). We present iFlipper's algorithm to solve the approximate LP problem and show why its result always leads to a feasible solution (Section 3.3). We then provide theoretical guarantees on how far the result is from the optimal solution of the ILP problem, and propose a reverse-greedy approach to further optimize the solution (Section 3.4). Finally, we present iFlipper's overall workflow with a complexity analysis (Section 3.5).

3.1 From MIQP to Equivalent ILP

We convert the MIQP problem to an equivalent ILP problem. We first replace the squared terms in Equation 1 to absolute terms:

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} |y_i - y_i'| \\ \text{s.t.} \quad & \sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij}|y_i - y_j| \le m \\ & y_i \in \{0,1\}, \forall i \end{aligned} \quad (3)$$

The resulting formulation is equivalent to the original MIQP because y_i and y_i' have binary values.

To convert this problem into an equivalent ILP problem, we replace each absolute term with an XOR expression, which can be expressed as four linear constraints. For two binary variables x and y, one can easily see that $|x - y| = x$ XOR y. Also, each expression $z = x$ XOR y is known to be equivalent to the following four linear constraints: $z \le x + y$, $z \ge y - x$, $z \ge x - y$, and $z \le 2 - x - y$. For example, if $x = 1$ and $y = 1$, z is bounded by $z \le 2 - x - y$ and is thus 0. For other combinations of x and y, z will be bounded to its correct XOR value as well. We thus introduce the auxiliary variables z_i and z_{ij} to represent y_i XOR y_i' and y_i XOR y_j, respectively, and obtain the following integer linear programming (ILP) problem:

$$\begin{aligned} \text{(ILP)} \quad \min \quad & \sum_{i=1}^{n} z_i \\ \text{s.t.} \quad & \sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij} z_{ij} \le m \\ & y_i, z_i \in \{0,1\}, \forall i, \quad z_{ij} \in \{0,1\}, \forall i, j \\ & z_i - y_i \le y_i', \quad z_i - y_i \ge -y_i' \\ & z_i + y_i \ge y_i', \quad z_i + y_i \le 2 - y_i' \\ & z_{ij} - y_i - y_j \le 0, \quad z_{ij} - y_i + y_j \ge 0 \\ & z_{ij} + y_i - y_j \ge 0, \quad z_{ij} + y_i + y_j \le 2 \end{aligned} \quad (4)$$

Since the ILP problem is equivalent to the MIQP problem, we know it is NP-hard as well. In the next section, we convert the ILP problem to an approximate linear program (LP), which can be solved efficiently.

3.2 From ILP to Approximate LP

We now relax the ILP problem to an approximate LP problem. We first replace the integer constraints in Equation 1 with range constraints to obtain the following LP problem:

$$\begin{aligned}
\text{(LP)} \quad \min \quad & \sum_{i=1}^{n} z_i \\
\text{s.t.} \quad & \sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij} z_{ij} \le m \\
& y_i, z_i \in [0,1], \forall i, \quad z_{ij} \in [0,1], \forall i, j \\
& z_i - y_i \le y_i', \quad z_i - y_i \ge -y_i' \\
& z_i + y_i \ge y_i', \quad z_i + y_i \le 2 - y_i' \\
& z_{ij} - y_i - y_j \le 0, \quad z_{ij} - y_i + y_j \ge 0 \\
& z_{ij} + y_i - y_j \ge 0, \quad z_{ij} + y_i + y_j \le 2
\end{aligned} \tag{5}$$

Although this problem can be solved more efficiently than the ILP problem, its result cannot be used as is because of the continuous values. Hence, we next convert the result into a binary integer solution that is close to the optimal solution of the ILP problem. A naïve method is to round the continuous values to their nearest integers, but this does not guarantee a feasible solution.

Example 1. *Suppose we start from the graph in Figure 4a where there are four violations, and we set $m = 2$. Solving the LP problem in Equation 5 can produce the solution in Figure 4b where the labels of nodes 1 and 4 are flipped from 1 to 0.5 (gray color). One can intuitively see that the labels are minimally flipped while the total amount of violations is exactly 2. However, rounding the labels changes the two 0.5's back to 1's as shown in Figure 4c, which is not a feasible solution.*

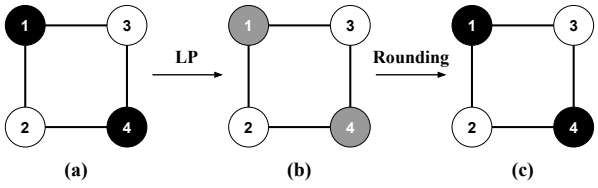


Figure 4: Performing simple roundings on the optimal solution's values of the LP problem may result in an infeasible solution for the ILP problem.

3.3 Constructing a Feasible Solution

We now explain how to construct a feasible integer solution for the ILP problem (Equation 4) from an optimal solution of the LP problem (Equation 5). We first prove a surprising result that any optimal solution y^* can be converted to another optimal solution $\check{y}$ where all of its variables have one of the three values: 0, 1, or some $\alpha \in (0,1)$. That is, α is between 0 and 1, but not one of them. Next, we utilize this property and propose an adaptive rounding algorithm that converts $\check{y}$ into a feasible solution whose values are only 0's and 1's.

Optimal Solution Conversion. We prove the following lemma for converting an optimal solution to our desired form.

Lemma 2. *Given an optimal solution y^* of Equation 5, we can always convert y^* to a new solution $\check{y}$ where $\check{y}$ is also optimal and only has 0, 1, or $\alpha \in (0,1)$ as its values.*

Proof. Suppose there are at least two distinct values α and β that are not 0 or 1 in y^*. We can combine all the y_i^* nodes whose value is α to one node while maintaining the same number of violations because an α-α edge has no violation (see Figure 5 for an illustration). We call this collection an α-cluster. Likewise, we can generate a β-cluster.

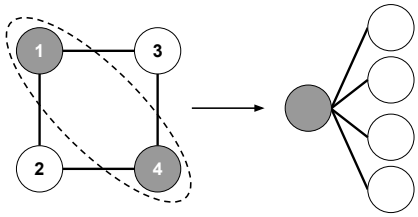


Figure 5: Continuing from Figure 4, recall that the optimal LP solution has nodes 1 and 4 with 0.5 labels. We can combine nodes 1 and 4 to construct a 0.5-cluster connected to nodes 2 and 3 with four edges as shown on the right. The number of violations is still 2.

Without loss of generality, let us assume that $0 < \alpha < \beta < 1$, and there are E edges between the two clusters. Suppose an α-cluster and a β-cluster have A_0 and B_0 nodes whose initial labels are 0, respectively, and A_1 and B_1 nodes whose initial values are 1, respectively. Let $N_\alpha = A_0 - A_1$ and $N_\beta = B_0 - B_1$. Now suppose there are U nodes connected to the α-cluster and V nodes connected to the β-cluster satisfying

$$0 \le a_1 \le \ldots \le a_k < \alpha < a_{k+1} \le \ldots \le a_U \le 1 \text{ and}$$
$$0 \le b_1 \le \ldots \le b_l < \beta < b_{l+1} \le \ldots \le b_V \le 1.$$

Note that there is no connected node with a value of α or β by construction. Let us also add the following nodes for convenience: $a_0 = 0$, $a_{U+1} = 1$, $b_0 = 0$, $b_{V+1} = 1$. We can then reduce at least one unique non-0/1 value in y^*, by either changing α to a_k or a_{k+1}, or changing β to b_l or b_{l+1}, or changing both α and β to the same value, while keeping the solution optimal using the following Lemmas 2.1 and 2.2. The key idea is that at least one of the conversions allows the solution to have the same number of label flippings and the same or even fewer number of violations, keeping the solution optimal. The complete proof and detailed conditions for each case are given in Appendix A and B.

Lemma 2.1. *For an α-cluster with $N_\alpha = 0$ in the optimal solution, we can always convert α to either a_k or a_{k+1} while maintaining an optimal solution. As a result, we can reduce exactly one non-0/1 value in the optimal solution.*

Lemma 2.2. *For an α-cluster with $N_\alpha \neq 0$ and a β-cluster with $N_\beta \neq 0$ in the optimal solution, we can always convert (α, β) to one*

Algorithm 1: iFlipper's LP solution conversion algorithm.

Input: Optimal solution y^* for the LP problem
Output: Transformed optimal solution $\check{y}$ where each value is one of $\{0, \alpha, 1\}$

1 $\check{y} = y^*$;
// There are T unique non-0/1 values in y^*
2 T = GetNumClusters(y^*);
3 **while** $T > 1$ **do**
4 $\quad$ *ZeroClusterList* = GetZeroClusters($\check{y}$);
5 $\quad$ **for** α *in ZeroClusterList* **do**
$\quad\quad$ // Details are in Algorithm 2
6 $\quad\quad$ $\check{y}$ = TransformWithOneCluster($\check{y}$, α);
7 $\quad\quad$ $T = T$ - 1;
8 $\quad$ **if** $T > 1$ **then**
9 $\quad\quad$ α, β = GetNonZeroTwoClusters($\check{y}$);
$\quad\quad$ // Details are in Algorithm 2
10 $\quad\quad$ $\check{y}$ = TransformWithTwoClusters($\check{y}$, α, β);
11 $\quad\quad$ T = GetNumClusters($\check{y}$);
// Now there is at most one cluster in $\check{y}$
12 **return** $\check{y}$;

of $(a_k, \beta + \frac{N_\alpha}{N_\beta}(a_k - \alpha))$, $(a_{k+1}, \beta - \frac{N_\alpha}{N_\beta}(a_{k+1} - \alpha))$, $(\alpha + \frac{N_\beta}{N_\alpha}(\beta - b_l), b_l)$, $(\alpha - \frac{N_\beta}{N_\alpha}(b_{l+1} - \beta), b_{l+1})$, *or* $(\frac{\alpha N_\alpha + \beta N_\beta}{N_\alpha + N_\beta}, \frac{\alpha N_\alpha + \beta N_\beta}{N_\alpha + N_\beta})$, *while maintaining an optimal solution. As a result, we can reduce at least one non-0/1 value in the optimal solution.*

We can repeat this adjustment until the solution $\check{y}$ has at most one unique value α that is neither 0 nor 1. □

Based on Lemma 2, Algorithm 1 shows how to convert the optimal LP solution y^* to another optimal solution $\check{y}$ whose values are in $\{0, \alpha, 1\}$. We first obtain all the unique non-0/1 values in y^* and for each value v construct a v-cluster. We then pick a cluster (say the α-cluster) with $N_\alpha = 0$ and change α using the TransformWithOneCluster function, which implements the converting process described in Lemma 2.1's proof, until there are no more such clusters (Lines 4-7). Among the rest of the clusters, we choose two (say the α-cluster and β-cluster) and transform them using the TransformWithTwoClusters function, which follows the combining process in Lemma 2.2's proof (Lines 8-11). We repeat these steps until there is at most one non-0/1 value in $\check{y}$.

Example 2. *To illustrate Algorithm 1, consider Figure 4a again where $m = 2$. Say we obtain an optimal solution from an LP solver as shown in Figure 6a where nodes 1 and 4 have the labels $\alpha = 0.1$ and $\beta = 0.9$, respectively, while the other nodes have 0 labels. This solution uses 1 flip and has 2 violations. We first construct an α-cluster and a β-cluster where each cluster only contains a single node, and there is no edge between them as shown in Figure 6b. We then convert (α, β) using the TransformWithTwoClusters because N_α and N_β are -1. Since $X > 0$ and $Y = 0$, we go to the third case in TransformWithTwoClusters. As a result, we set (α, β) to (0.5, 0.5) and reduce one non-0/1 unique value in the solution. The solution now has one non-0/1 value (i.e., 0.5) and still has 2 violations using 1 flip.*

Adaptive Rounding into a Binary Solution. We now convert $\check{y}$ into a feasible integer solution with only 0's and 1's. If we use simple rounding to change α to 0 or 1 as in the naïve method in Section 3.2, the resulting integer solution may not be feasible like Figure 4c. Fortunately, the fact that we only have three possible values allows

Algorithm 2: Transformation functions in Algorithm 1.

Function TransformWithOneCluster(y, α):
$\quad$ a, k, U = GetClusterInfo(y, α);
$\quad$ **if** $2k - U \leq 0$ **then**
$\quad\quad$ $\alpha \leftarrow a_{k+1}$;
$\quad$ **else**
$\quad\quad$ $\alpha \leftarrow a_k$;
$\quad$ // Now $\alpha \in \{a_k, a_{k+1}\}$
$\quad$ y = ConvertSolution(y, α);
$\quad$ **return** y;
Function TransformWithTwoClusters(y, α, β):
$\quad$ $a, N_\alpha, k, U, b, N_\beta, l, V, E$ = GetClustersInfo(y, α, β);
$\quad$ $X, Y = \frac{N_\alpha}{N_\beta}, \frac{(2k-U-E)N_\beta - (2l-V+E)N_\alpha}{N_\beta}$;
$\quad$ **if** $X < 0, Y \leq 0$ **then**
$\quad\quad$ **if** $X + 1 \leq 0$ **then**
$\quad\quad\quad$ $\alpha \leftarrow \alpha + min(a_{k+1} - \alpha, -\frac{N_\beta}{N_\alpha}(b_{l+1} - \beta))$;
$\quad\quad$ **else**
$\quad\quad\quad$ $\alpha \leftarrow \alpha + min(a_{k+1} - \alpha, -\frac{N_\beta}{N_\alpha}(b_{l+1} - \beta), \frac{N_\beta(\beta-\alpha)}{N_\alpha+N_\beta})$
$\quad\quad$ $\beta \leftarrow \beta - \frac{N_\alpha}{N_\beta}\epsilon_\alpha$;
$\quad$ **else if** $X < 0, Y > 0$ **then**
$\quad\quad$ **if** $X + 1 \geq 0$ **then**
$\quad\quad\quad$ $\alpha \leftarrow \alpha - min(\alpha - a_k, -\frac{N_\beta}{N_\alpha}(\beta - b_l))$;
$\quad\quad$ **else**
$\quad\quad\quad$ $\alpha \leftarrow \alpha - min(\alpha - a_k, -\frac{N_\beta}{N_\alpha}(\beta - b_l), -\frac{N_\beta(\beta-\alpha)}{N_\alpha+N_\beta})$;
$\quad\quad$ $\beta \leftarrow \beta + \frac{N_\alpha}{N_\beta}\epsilon_\alpha$;
$\quad$ **else if** $X > 0, Y \leq 0$ **then**
$\quad\quad$ $\alpha \leftarrow \alpha + min(a_{k+1} - \alpha, \frac{N_\beta}{N_\alpha}(\beta - b_l), \frac{N_\beta(\beta-\alpha)}{N_\alpha+N_\beta})$;
$\quad\quad$ $\beta \leftarrow \beta - \frac{N_\alpha}{N_\beta}\epsilon_\alpha$;
$\quad$ **else if** $X > 0, Y > 0$ **then**
$\quad\quad$ $\alpha \leftarrow \alpha - min(\alpha - a_k, \frac{N_\beta}{N_\alpha}(b_{l+1} - \beta))$;
$\quad\quad$ $\beta \leftarrow \beta + \frac{N_\alpha}{N_\beta}\epsilon_\alpha$;
$\quad$ // Now (α, β) is one of the cases in Lemma 2.2
$\quad$ y = ConvertSolution(y, α, β);
$\quad$ **return** y;

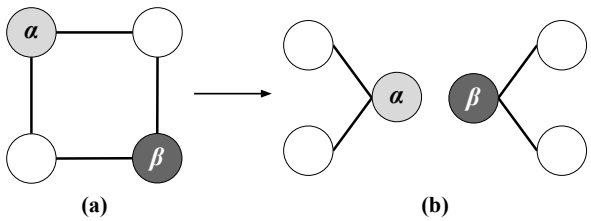


Figure 6: Cluster construction example for Algorithm 1.

Algorithm 3: iFlipper's adaptive rounding algorithm.

Input: Transformed optimal solution $\check{y}$ for the LP problem where each value is one of $\{0, \alpha, 1\}$
Output: Feasible binary integer solution $\overline{y}$

```
// Get the number of (0-α) and (1-α) label pairs
```
$N_{0\alpha}, N_{1\alpha}$ = GetNumberOfPairs($\check{y}$);
if $N_{0\alpha} \leq N_{1\alpha}$ **then**
  $\overline{\alpha} \leftarrow 1$;
else
  $\overline{\alpha} \leftarrow 0$;
```
// Replace α with ᾱ ∈ {0,1}
```
$\overline{y}$ = ApplyRounding($\check{y}, \overline{\alpha}$);
return $\overline{y}$;

us to propose an adaptive rounding algorithm (Algorithm 3) that guarantees feasibility. We first denote N_{ab} as the number of (a-b) label pairs with edges within the optimal solution $\check{y}$. For example, $N_{0\alpha}$ is the number of similar node pairs whose labels are 0 and α, respectively. Then the key idea is to round α to 1 if there are more edges between 1 and α (i.e., $N_{0\alpha} \leq N_{1\alpha}$), and round α to 0 otherwise. For example, consider Figure 4b again. Here $\alpha = 0.5$, $N_{0\alpha} = 4$, and $N_{1\alpha} = 0$. We have $N_{0\alpha} > N_{1\alpha}$, so rounding α to 0 will result in a feasible solution with 0 violations.

We prove why Algorithm 3 is correct in Lemma 3.

Lemma 3. *Given an optimal solution $\check{y}$ of Equation 5 where each value is one of {0, α, 1}, applying adaptive rounding (Algorithm 3) always results in a feasible solution.*

Proof. We continue using the notation N_{ab}, which is the number of (a-b) label pairs with edges in the optimal solution $\check{y}$. The number of violations in $\check{y}$ can be expressed as a function of α:

$$\begin{aligned} \#\text{ Violations} &= \sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij}\check{z}_{ij} = \sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij}|\check{y}_i - \check{y}_j| \\ &= f(\alpha) = N_{01} + \alpha N_{0\alpha} + (1-\alpha)N_{1\alpha} \leq m \end{aligned} \tag{6}$$

In addition, the violations constraint in Equation 5 is satisfied because $\check{y}$ is an optimal solution.

We now want to round α to either 0 or 1 while satisfying the violations constraint. If $N_{0\alpha} \leq N_{1\alpha}$, we have $f(1) = N_{01} + N_{0\alpha} \leq N_{01} + \alpha N_{0\alpha} + (1-\alpha)N_{1\alpha} \leq m$ ($\because 0 < \alpha < 1$), so setting $\alpha = 1$ guarantees a feasible solution. On the other hand, if $N_{1\alpha} < N_{0\alpha}$, we have $f(0) = N_{01} + N_{1\alpha} < N_{01} + \alpha N_{0\alpha} + (1-\alpha)N_{1\alpha} \leq m$ ($\because 0 < \alpha < 1$), so setting $\alpha = 0$ ensures feasibility.

We can also derive a difference in the number of violations between the rounded solution $\overline{y}$ and the optimal solution $\check{y}$, and the difference is non-negative due to the condition for each case:

$$\begin{aligned} &\alpha = 1 \rightarrow f(\alpha) - f(1) = (1-\alpha)(N_{1\alpha} - N_{0\alpha}) \\ &\alpha = 0 \rightarrow f(\alpha) - f(0) = \alpha(N_{0\alpha} - N_{1\alpha}) \end{aligned}$$

□

Lemma 3 shows that the rounding choice of α depends on the number of connected pairs and not on α's value. This result explains why simple rounding as in the naïve method does not necessarily lead to a feasible solution.

In the next section, we analyze how accurate the rounded solution is compared to the optimal solution of the ILP problem and investigate whether it can be further improved.

3.4 Optimality and Improvement

We analyze the optimality of Algorithm 3 and propose a technique called *reverse greedy* for further optimizing it without exceeding the violations limit.

Optimality Analysis. We prove the theoretical bounds of Algorithm 3 in Lemma 4:

Lemma 4. *For a given optimal solution $\check{y}$ of Equation 5, let us denote M_{ab} as the number of nodes in $\check{y}$ where the label a was flipped to b. Then the objective value of the output $\overline{y}$ from Algorithm 3 is at most C more than the optimal objective value of the original ILP problem where the value of C depends on α:*

$$\begin{aligned} &\alpha = 1 \rightarrow C = (1-\alpha)(M_{0\alpha} - M_{1\alpha}) \\ &\alpha = 0 \rightarrow C = \alpha(M_{1\alpha} - M_{0\alpha}) \end{aligned} \tag{7}$$

Proof. Since the optimal solution of the ILP problem is always one of the feasible solutions of the LP problem, the optimal objective values (OPT) of the two optimization problems should satisfy:

$$OPT_{LP} \leq OPT_{ILP} \tag{8}$$

The objective value of $\check{y}$ can then be expressed as follows:

$$OPT_{LP}(\check{y}) = \sum_{i=1}^{n} \check{z}_{ij} = (M_{01} + M_{10}) + \alpha M_{0\alpha} + (1-\alpha)M_{1\alpha} \tag{9}$$

We first consider the case where $\alpha = 1$. Here the objective value of $\overline{y}$ is $OPT_{LP}(\overline{y}) = (M_{01} + M_{10}) + M_{0\alpha}$. We can then derive the bound on $OPT_{LP}(\overline{y})$ from Equation 8 and Equation 9 as follows:

$$\begin{aligned} &OPT_{LP}(\check{y}) - OPT_{LP}(\overline{y}) \leq OPT_{ILP} - OPT_{LP}(\overline{y}) \\ &(1-\alpha)(M_{1\alpha} - M_{0\alpha}) \leq OPT_{ILP} - OPT_{LP}(\overline{y}) \\ &OPT_{LP}(\overline{y}) \leq OPT_{ILP} + (1-\alpha)(M_{0\alpha} - M_{1\alpha}) \end{aligned} \tag{10}$$

We note that $(1-\alpha)(M_{0\alpha} - M_{1\alpha})$ is always non-negative because $\check{y}$ is an optimal solution, i.e., $OPT_{LP}(\check{y}) \leq OPT_{LP}(\overline{y})$.

Similarly, if $\alpha = 0$, we can obtain the objective value bound of $\alpha(M_{1\alpha} - M_{0\alpha})$. □

Reverse Greedy Algorithm. Lemma 4 shows that the bound may sometimes be loose because it depends on α and the number of nodes whose labels are flipped to α. There is a possibility that Algorithm 3 flips nodes unnecessarily, which results in fewer than m violations (see Section 4.5 for empirical results). Hence, we propose an algorithm called *reverse greedy* (Algorithm 4), which unflips flipped labels as long as the number of violations does not exceed m. As the name suggests, we run the greedy algorithm in Section 2.2 in the reverse direction where for each iteration, we unflip nodes that increase the violations the least to recover the original labels as much as possible. Thus, reverse greedy can only improve the optimality of Algorithm 3.

Example 3. *Continuing from our example using Figure 4 and $m = 2$, suppose we run Algorithm 3 on Figure 4b and round the 0.5 labels to 0 to obtain the feasible solution in Figure 7a. A total of 2 flips are performed compared to the initial graph Figure 4a. However, there*

Algorithm 4: iFlipper's reverse greedy algorithm.

Input: Feasible binary integer solution $\overline{x}$, target number of violations m

Output: Improved binary integer solution $\tilde{x}$

$\tilde{x} = \overline{x}$;
$flipped\_labels$ = GetFlippedLabel($\tilde{x}$);
$violations$ = GetViolations($\tilde{x}$);
while $|violations| \leq m$ **do**
    // Unflip flipped labels of the nodes so that the violations are increased the least
    $\tilde{x}$ = FlipLabelLeastViolation($\tilde{x}$, $flipped\_labels$);
    $flipped\_labels$ = GetFlippedLabel($\tilde{x}$);
    $violations$ = GetViolations($\tilde{x}$);
return $\tilde{x}$;

exists an optimal solution like Figure 7b where only 1 flip is necessary. Running Algorithm 4 will unflip nodes 1 or 4, and unflipping node 1 produces this solution.

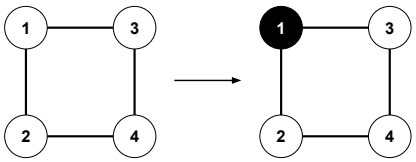


Figure 7: The reverse greedy algorithm can further optimize a rounded solution from Algorithm 3.

The time complexity of reverse greedy is the same as the greedy algorithm, i.e., $O(n^2)$, but in practice requires fewer iterations than greedy because the number of violations is relatively close to m.

3.5 Putting Everything Together

We now describe the overall workflow of iFlipper. For a given ILP problem, we first convert it into an approximate LP problem. We then solve the approximate LP problem and convert its optimal solution to another optimal solution that only has values in {0, α, 1} using Algorithm 1. We then apply adaptive rounding using Algorithm 3. Finally, we possibly improve the rounded solution by unflipping labels using Algorithm 4.

Complexity Analysis. iFlipper consists of four parts: solving the approximate LP problem, the converting algorithm, the adaptive rounding algorithm, and the reverse-greedy algorithm. The interior-points algorithm for solving LP has a time complexity of $O(n^{2+\varepsilon})$, where ε is smaller than 1. The converting algorithm has a time complexity of $O(n^2)$: in the worst case, we may need to combine clusters n times, and for each combining process, we need to check all neighbors of a particular cluster, which is up to n points. For the adaptive rounding algorithm, the time complexity is also $O(n^2)$, because we need to count the number of $(1, \alpha)$ edges and $(0, \alpha)$ edges, which is $O(n^2)$ in the worst case. The reverse greedy algorithm, as analysed in Section 3.4, is $O(n^2)$. Overall, the complexity of iFlipper is bounded by the LP solver, which is $O(n^{2+\varepsilon})$. This result may look slower than the greedy algorithm, but in practice, iFlipper runs much faster than greedy. The reason is that we use efficient LP solvers, and the empirical running times of the adaptive rounding and greedy algorithms are much faster than their theoretical worst case complexities. We show the empirical running time in Section 4.

4 EXPERIMENTS

In this section, we evaluate iFlipper on real datasets and address the following key questions:

- Is there an accuracy-fairness trade-off for iFlipper?
- How does iFlipper compare with various baselines in terms of model accuracy, individual fairness, and efficiency?
- How useful is each component of iFlipper?
- Can iFlipper be integrated with in-processing techniques?

We implement iFlipper in Python and use Scikit-learn [31] and PyTorch [30] libraries for model training. For performing optimization, we use two software packages: CPLEX [10] and MOSEK [27]. We run all experiments on Intel Xeon Gold 5115 CPUs.

4.1 Setting

Datasets. We experiment on the following three popular datasets in the fairness literature. We randomly split each dataset into training, test, and validation sets and use the same pre-processing as in IBM's AI Fairness 360 toolkit [4]. See Table 1 for more details.

- COMPAS [2]: Contains 6,167 people examples and is used to predict recidivism. The features include gender, race, and criminal history.
- AdultCensus [20]: Contains 45,222 people examples and is used to predict whether an individual's income exceeds $50K per year. The features include gender, age, and occupation.
- Credit [12]: Contains 1,000 examples and is used to predict an individual's credit risk. The features contain race, credit amount, and credit history.

We also considered two additional fairness datasets: Bank [26] and LSAC [39]. However, these datasets exhibit fewer individual fairness issues as shown in Appendix D. Hence, we do not include them in our main experiments because the fairness improvements are not as significant.

Similarity Matrices. We consider two similarity matrices using Euclidean distance (i.e., $d(x_i, x_j) = \|x_i - x_j\|^2$). When measuring distances, we follow previous approaches [24, 43] and exclude sensitive attributes like gender and race. The rationale is that individuals who have similar information other than the sensitive attributes values must be treated similarly. See Table 1 for the configurations.

- *kNN-based*: Considers (x_i, x_j) as a similar pair if x_i is one of x_j's nearest neighbors or vice versa:

$$W_{ij} = \begin{cases} 1 & \text{if } x_i \in NN_k(x_j) \text{ or } x_j \in NN_k(x_i) \\ 0 & \text{otherwise} \end{cases}$$

  where $NN_k(x_j)$ denotes the set of k nearest examples of x_j in a Euclidean space.
- *threshold-based*: Considers (x_i, x_j) as a similar pair if their distance is smaller than a threshold T:

$$W_{ij} = \begin{cases} 1 & \text{if } d(x_i, x_j) \leq T \\ 0 & \text{otherwise} \end{cases}$$

Dataset	# Train	# Test	# Valid	Sen. Attr	k	T
COMPAS	3,700	1,850	617	gender	20	3
AdultCensus	27,133	13,566	4,523	gender	20	3
Credit	700	201	199	age	20	7

Table 1: Settings for the three datasets.

Measures. We evaluate a model's accuracy by computing the portion of correctly-predicted data points. For individual fairness, we use the consistency score [25] defined in Section 2.1, which measures the consistency of predictions on the test set between similar individuals. For both accuracy and consistency score, higher values are better. We report mean values over 5 trials for all measures.

Methods Compared. We compare iFlipper with the following existing pre-processing algorithms for individual fairness:

- *LFR* [43]: LFR is a fair representation learning algorithm that optimizes between accuracy, group fairness, and individual fairness in terms of data reconstruction loss.
- *iFair* [24]: iFair is a fair representation learning algorithm that optimizes accuracy and individual fairness. Compared to LFR, iFair do not focus on group fairness, which enables the classifier to have better individual fairness. If two data points are close to each other in the input space, iFair aims to map them close to each other in the feature space as well.
- *PFR* [25]: PFR is a fair representation learning algorithm that tries to learn an embedding of the fairness graph. The fairness graph used in PFR is similar to the one used in iFlipper. Similar to iFair, it optimizes individual fairness while preserving the original data by mapping similar individual to nearby points in the learned representation. Among the three baselines, only PFR is able to support a general similarity matrix. PFR uses an efficient trace optimization algorithm, which can learn representations much faster than iFair.

For all baselines, there are multiple hyperparameters that can balance the competing objectives. We start with the same sets of hyperparameters as described in their papers, and tune them to provide the best results. For iFlipper, we vary the target number of violations m to balance between the model accuracy and fairness.

Optimization Solutions Compared. We compare iFlipper with the following optimization baselines described in Section 2.3 and 3.1.

- *Greedy*: Repeatedly flips labels that reduce the number of violations the most.
- *Gradient*: Solves an unconstrained optimization problem.
- *ILP Solver*: Solves the ILP problem exactly.

We use CPLEX (one of the state-of-the-art optimization solvers) to solve the ILP problem. For the LP problem in iFlipper, we use MOSEK because CPLEX take a long time to construct the LP problem, which makes the overall running time too long for our purposes. For *Gradient*, we tune λ to satisfy the violations limit.

An interesting behavior of MOSEK is that empirically all of its optimal solutions only contain values in $\{0, \alpha, 1\}$ without having to run our Algorithm 1. Note that this behavior is limited to the label flipping problems we are solving and does not necessarily occur for other LP problems. We suspect that MOSEK's algorithm effectively contains the functionality of Algorithm 1. We also make the same observations for CPLEX as well for our LP problems. We find these results encouraging because it means that Algorithm 1 is not a significant runtime overhead and can even be tightly integrated with the LP solving code. A detailed analysis of MOSEK's code is an interesting future work. For any other solver that does not have this behavior, one can always run Algorithm 1 on its solutions.

Models and Hyperparameter Tuning. We use logistic regression (LR), random forest (RF), and neural network (NN) for our experiments. We tune the model hyperparameters (e.g., learning rate and maximum depth of the tree) such that the model has the highest validation accuracy. Since we want to compare fair datasets from different pre-processing algorithms, we do not need to further improve the fairness during the model training.

4.2 Accuracy and Fairness Trade-off

Figure 8 shows the trade-off between fairness and accuracy for the COMPAS dataset when running iFlipper with different allowed number of violations. The x-axis is accuracy, and the y-axis is the consistency score. There are two curves, which correspond to two different similarity matrices: kNN-based and threshold-based. Each data point on the curve has two other results: (number of violations after repairing, number of flips). As we flip more labels, there are fewer violations in the repaired dataset, and the resulting model has a higher consistency score on the test set, which means it has better individual fairness. However, the accuracy drops as we flip more labels. The trade-off curves of the other datasets are similar and can be found in Appendix C.

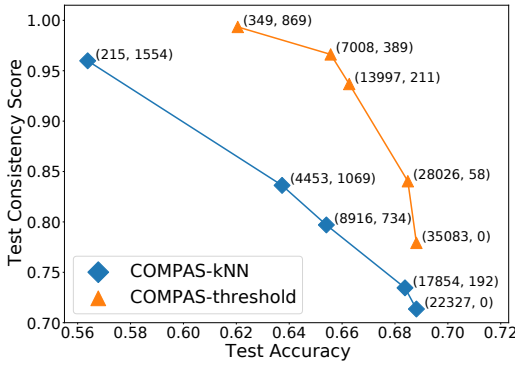


Figure 8: Accuracy-fairness trade-off curves for iFlipper. For each data point, we also show the number of violations after repairing and the number of flips in parentheses.

4.3 Performance and Runtime Results

We now compare iFlipper with other baselines using the three datasets and two similarity matrices.

4.3.1 Accuracy and Fairness Comparison. We first compare the accuracy and fairness results of iFlipper with the other methods. Figure 9 shows the trade-off results with logistic regression where the x-axis is the accuracy, and the y-axis is the consistency score on a test set. *Original* is where we train a model on the original data with no data pre-processing. For LFR, iFair, and PFR, we employ 20, 45, and 15 different hyperparameter sets, respectively, as described in their papers. For iFlipper, we use 10–11 different m values to

control accuracy and fairness. For a clear visualization, we omit some points in the bottom left region of the graph for methods other than iFlipper. As a result, iFlipper significantly outperforms the baselines in terms of both test accuracy and consistency score for all cases, which demonstrates that our label flipping approach is effective in improving individual fairness while maintaining high accuracy. We also observe that iFlipper performs better than the three baselines on both similarity matrices. This result is important because the similarity matrix may vary depending on the application. The results for a random forest and neural network are in Appendix G, and the key trends are similar to Figure 9 where iFlipper shows a better trade-off than the baselines.

Table 2 shows a more detailed comparison with the baselines using the AdultCensus dataset. For each ML model, we fix the consistency score (denoted as "C. Score") to be some value as shown below each model name in Table 2 in parentheses. Then for each method, we report the test accuracy when we tune its hyperparameter such that the trained model has the highest validation accuracy while achieving the fixed consistency score on the validation set. As a result, iFlipper consistently shows the highest test accuracy compared to the other baselines for all models.

Dataset	**Adult-kNN**			**Adult-threshold**		
Model (C. Score)	**LR** (0.92)	**RF** (0.85)	**NN** (0.90)	**LR** (0.95)	**RF** (0.80)	**NN** (0.95)
LFR	.794	.827	.806	.821	.827	.815
iFair	.775	.820	.805	.798	.820	.805
PFR	.812	.840	.829	.828	.820	.827
iFlipper	**.828**	**.842**	**.831**	**.845**	**.850**	**.845**

Table 2: Accuracy comparison of methods with similar individual fairness on different models. In order to align the individual fairness, for each model we make the methods have similar consistency scores on the validation data (C. Score) by tuning their hyperparameters.

In comparison to the other baselines, iFlipper's accuracy-fairness trade-off is also much cleaner as shown in Figure 9 where the other baselines have noisy trade-offs and even overlapping results for different hyperparameter sets. The benefit of having a clean trade-off is that iFlipper can easily balance between accuracy and fairness by varying the limit of violations m. In Figure 9, we observe that iFlipper is flexible and can obtain a wide range of test consistency scores, all the way up to nearly 1.0.

4.3.2 Runtime Comparison. We evaluate the efficiency of iFlipper and the three baselines in Table 3. For each method, we show the average runtime for all experiments in Figure 9. We note that the runtime of LFR and iFair does not depend on the similarity matrix. For the iFair experiments on the AdultCensus dataset, we lower the training data using uniform sampling to fit iFair because fitting iFair on the entire data takes more than 24 hours. We confirm that this relaxation does not hurt the original performance reported in the iFair paper [24]. As a result, iFlipper is much faster than LFR and iFair for all cases. Although PFR seems to be the fastest, it performs much worse than iFlipper in terms of accuracy and fairness as shown in Figure 9.

Datasets	**Sim. Matrix**	**Avg. Runtime (sec)**			
		LFR	**iFair**	**PFR**	**iFlipper**
COMPAS	kNN	786	29,930	0.48	5.69
	threshold	786	29,930	0.42	9.02
Adult-Census	kNN	700	21,321†	16.64	183
	threshold	700	21,321†	16.90	626
Credit	kNN	22.78	394	0.01	2.86
	threshold	22.78	394	0.01	2.52

Table 3: Runtime results of LFR, iFair, PFR, and iFlipper on the COMPAS, AdultCensus, and Credit datasets. For each method, we show the average runtime for all experiments in Figure 9. The symbol † indicates that we reduce the size of the training data for better efficiency.

4.4 Optimization Solution Comparison

We now compare iFlipper with other optimization solutions: *Greedy*, *Gradient*, and an ILP solver. Figure 10 makes a comparison in terms of optimality and efficiency on the AdultCensus dataset where we use the kNN-based similarity matrix. We set the target number of violations m to three different values for a detailed comparison. Note that all three subfigures within Figure 10 use the same legends.

We first compare the resulting number of violations of the methods in Figure 10a. Obviously, the optimal solution from the ILP solver is the closest to the target number of violations. We observe that iFlipper never exceeds the target. On the other hand, both *Greedy* and *Gradient* are unable to find a feasible solution in some cases. For example, *Greedy* and *Gradient* cannot reduce the number of violations to fewer than 6,373 and 50,535, respectively. This result is expected because these methods may fall into local minima as we explained in Section 2.3.

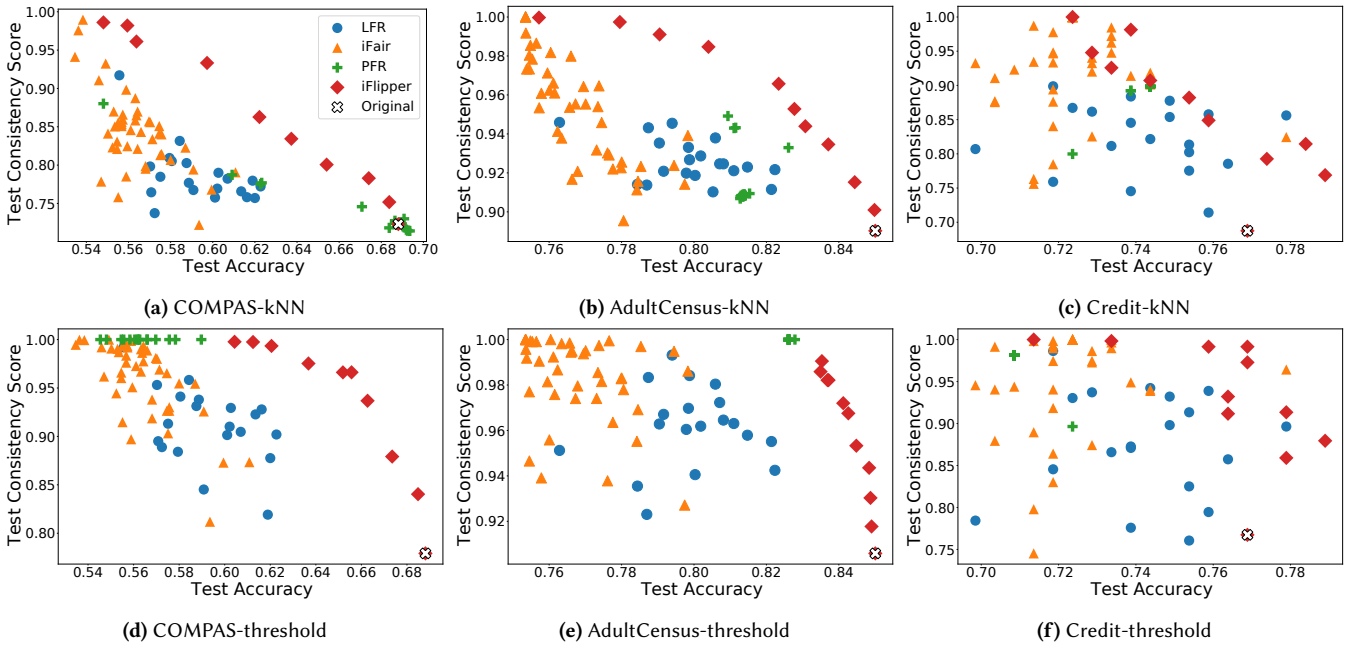
We also compare the number of flips in Figure 10b. As a result, iFlipper produces solutions that are close to the optimal ones. When m = 433 and 4,336, *Greedy* shows the fewest number of label flips because it fails to find a feasible solution. *Gradient* has the largest number of flips. We suspect this poor performance is due to the fact that *Gradient* (1) solves an unconstrained optimization problem that makes it hard to satisfy the limit on violations, and (2) has a rounding error as it provides continuous values.

Finally, we compare the average runtimes (i.e., wall clock times in seconds) of each method in Figure 10c. As a result, iFlipper is much faster than the ILP solver as it solves an approximate LP problem. In comparison, *Greedy* is sometimes even slower than the ILP solver because it uses more iterations to reduce the violations. *Gradient* is the most efficient, but the result is not meaningful because the numbers of violations and flips are even worse than *Greedy*.

We also perform the above experiments on the COMPAS dataset. The results are in Appendix E, and the trends are similar to Figure 10.

4.5 Ablation Study

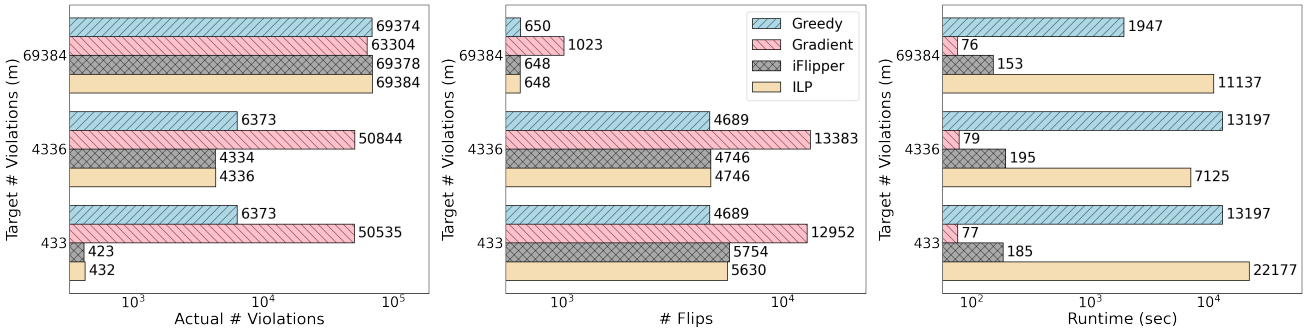
We perform an ablation study in Figure 11 to demonstrate the effectiveness of each component in iFlipper using the AdultCensus dataset and kNN-based similarity matrix. The results for the COMPAS dataset are similar and shown in Appendix F. As we explained



(a) COMPAS-kNN **(b)** AdultCensus-kNN **(c)** Credit-kNN

(d) COMPAS-threshold **(e)** AdultCensus-threshold **(f)** Credit-threshold

Figure 9: Accuracy-fairness trade-offs of logistic regression on the three datasets using the two similarity matrices. In addition to the four methods LFR, iFair, PFR, and iFlipper, we add the result of model training without any pre-processing and call it "Original." As a result, only iFlipper shows a clear accuracy and fairness trade-off.



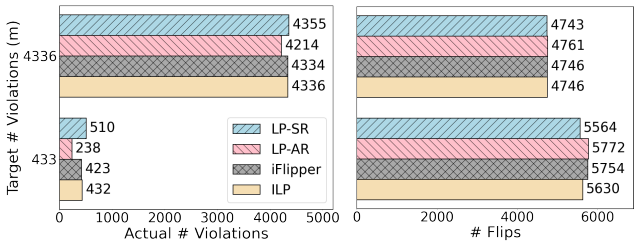
(a) Number of violations **(b)** Number of flips **(c)** Runtime (sec)

Figure 10: A detailed comparison of iFlipper against two naïve optimization solutions (*Greedy* and *Gradient*) and ILP solver on the AdultCensus dataset where we use the kNN-based similarity matrix. Here the initial number of violations is 86,730. We show the results for three different target violation limits (m).

in Section 4.1, MOSEK always produces an optimal LP solution that only has values in $\{0, \alpha, 1\}$, so we do not have an ablation study without the LP solution conversion. Again, the fact that MOSEK effectively contains the conversion functionality indicates that the conversion overhead is not significant and that a tight integration with the LP solving is possible. We thus compare iFlipper (LP solver with both adaptive rounding and reverse greedy algorithms) with the following variants: (1) LP solver with simple rounding (LP-SR); and (2) LP solver with adaptive rounding (LP-AR). We also consider an optimal solution from the ILP solver to compare the optimality of each solution.

Figure 11a shows that the adaptive rounding algorithm always provides a feasible solution while simple rounding to a fractional optimal solution results in an infeasible solution as we discussed in Lemma 3. However, the rounding algorithm does flip more labels than the optimal solution as shown in Figure 11b, resulting in an unintended fairness improvement and accuracy drop. In this case, the reverse greedy algorithm in iFlipper is used to reduce the optimality gap with the optimal solution by recovering as many original labels as possible without exceeding the limit of violations.

Table 4 shows the average runtime of each component (LP solver, adaptive rounding, and reverse greedy) in iFlipper in Figure 11. As a result, the runtime for solving the LP problem is dominant, which demonstrates that iFlipper is able to provide a near-exact solution with minimal time overhead.



(a) Number of violations **(b)** Number of flips

Figure 11: Ablation study for iFlipper on the AdultCensus dataset and kNN-based similarity matrix.

Method	Avg. Runtime (sec)
LP Solver (effectively includes Alg. 1)	179.85
+ Adaptive Rounding (Alg. 3)	0.48
+ Reverse Greedy (Alg. 4)	8.89

Table 4: Avg. runtimes of iFlipper's components in Figure 11.

4.6 Compatibility with In-processing Method

In this section, we demonstrate how iFlipper can be combined with an in-processing algorithm to further improve individual fairness. We evaluate iFlipper with SenSR [40], which is an in-processing fair training algorithm that is robust to sensitive perturbations to the inputs, on the AdultCensus dataset. For iFlipper, we use the same distance function in SenSR, which computes the distance using the features that are not correlated to sensitive attributes and use the threshold $T = 2$ to construct the similarity matrix. For a fair comparison, we report both our consistency score and the GR-Con. (gender and race consistency) / S-Con. (spouse consistency) metrics used by SenSR to evaluate the individual fairness. Here Con. measures the consistency between $h(x_i)$ and $h(x_i)$ when x_i and x_j are the same except the sensitive attributes. To evaluate Con., we use the same method in SenSR where the test data examples are duplicated, but assigned different sensitive attribute values. As a result, Table 5 shows that using both methods gives the best individual fairness for both metrics while having little accuracy drop. Thus, iFlipper complements in-processing algorithms by removing the bias inherent in the data during the pre-processing step.

Dataset	Method	Test Acc.	Con. Score	GR/S-Con.
Adult-Census	Original	0.854	0.904	0.919 / 0.869
	iFlipper	0.852	0.950	0.935 / 0.891
	SenSR	0.837	0.949	0.988 / 0.967
	Both	0.831	**0.962**	**0.993** / **0.970**

Table 5: Using iFlipper, SenSR, or both combined on the AdultCensus dataset.

5 RELATED WORK

Various fairness measures have been proposed to capture legal and social issues [29]. The prominent measures include individual fairness [13], group fairness [1, 42, 44], and causal fairness [23]. Individual fairness captures the notion that similar individuals should be treated similarly. Group fairness measures like equal opportunity [16], equalized odds [16], and demographic parity [14] ensure that two sensitive groups have similar statistics. Causal fairness identifies causal relationships among attributes. All these measures complement each other, and there is a known conflict between group fairness and individual fairness [5, 15] that they can not be satisfied at the same time because of their different assumptions. Our primary focus is on individual fairness.

Recently, many unfairness mitigation techniques for individual fairness [4] have been proposed where they can be categorized into pre-processing [24, 25, 43], in-processing [36, 40, 41], and post-processing [32] techniques depending on whether the fix occurs before, during, or after model training, respectively. Among the categories, we focus on pre-processing because fixing the data can solve the root cause of unfairness.

The recent pre-processing works LFR [43], iFair [24], and PFR [25] all propose fair representation learning algorithms that optimize accuracy and individual fairness. Using an adjacency matrix that represents a fairness graph, the goal is to optimize for a combined objective function with reconstruction loss and individual fairness loss based on the fairness graph. In comparison, iFlipper takes the alternative approach of flipping labels to fix bias, which results in better accuracy-fairness trade-offs and efficiency.

It is also important to understand the in-processing and post-processing techniques. SenSR [40] enforces the model to be invariant under certain sensitive perturbations to the inputs using adversarial learning. Several extensions have been proposed as well. SenSeI [41] enforces invariance on certain sensitive sets and uses a regularization-based approach for jointly optimizing accuracy and fairness, and BuDRO [36] extends the fairness loss used in SenSR to use gradient boosting. For post-processing, GLIF [32] formulates a graph smoothening problem and uses Laplacian regularization to enforce individual fairness. In comparison, iFlipper can complement all these techniques as we demonstrated with SenSR.

Pre-processing techniques for other fairness measures have been proposed as well. For group fairness, Kamiran et al. [18] and Calmon et al. [8] change features of the training data to remove dependencies between the sensitive attribute and label and thus achieve statistical parity. For causal fairness, Capuchin [33] makes a connection between multivalued dependencies (MVDs) and causal fairness, and proposes a data repair algorithm for MVDs using tuple inserts and deletes that also ensures causal fairness. In comparison, iFlipper uses label flipping, which has not been used in any of these works.

6 CONCLUSION

We proposed iFlipper, which flips labels in the training data to improve the individual fairness of trained models. iFlipper uses pairwise similarity information and minimizes the number of flipped labels to limit the number of individual fairness violations to be within a threshold. We proved that this MIQP optimization problem is NP-hard. We then converted the problem to an approximate LP problem, which can be solved efficiently. Our key finding is that the proposed LP algorithm has theoretical guarantees on how close its result is to the optimal solution in terms of the number of label flips. In addition, we further optimized the algorithm without exceeding the violations limit. We demonstrated on real datasets that iFlipper outperforms pre-processing unfairness mitigation baselines in terms of fairness and accuracy, and can complement existing in-processing techniques for better results.

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A PROOF FOR LEMMA 2.1

We continue from Section 3.3 and provide a full proof for Lemma 2.1. Here we restate Lemma 2.1 with the problem setup:

Lemma 2.1. *Suppose an α-cluster has A_0 points whose initial labels are 0 and A_1 points whose initial values are 1. Let $N_\alpha = A_0 - A_1$. Now suppose there are U nodes connected to the α-cluster (shown in Figure 12) satisfying:*

$$0 \le a_1 \le ... \le a_k < \alpha < a_{k+1} \le ... \le a_U \le 1.$$

Let us also add the following nodes for convenience: $a_0 = 0$, $a_{U+1} = 1$. For an α-cluster with $N_\alpha = 0$ in the optimal solution, we can always convert α to either a_k or a_{k+1} while maintaining an optimal solution. As a result, we can reduce exactly one non-0/1 value in the optimal solution.

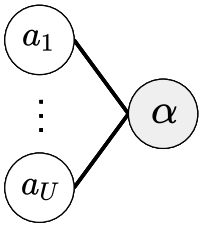


Figure 12: Problem setup for Lemma 2.1.

Proof. We compute the initial number of flips and violations in Figure 12 as follows:

$$\begin{aligned} &\#\text{ Flips} = \alpha A_0 + (1-\alpha)A_1 = A_1 + \alpha N_\alpha = A_1 (\because N_\alpha = 0) \\ &\#\text{ Vio.} = \sum_{i=1}^{U} |a_i - \alpha| = -\sum_{i=1}^{k} a_i + \sum_{i=k+1}^{U} a_i + (2k-U)\alpha \end{aligned} \tag{11}$$

We first observe that the number of flips is independent of the α value. Hence, even if we change α to an arbitrary value, the solution still has the same objective value.

Now consider a small positive value $\epsilon_\alpha > 0$. Suppose we change α by ϵ_α such that

$$0 \le a_1 \le ... \le a_k \le \alpha + \epsilon_\alpha \le a_{k+1} \le ... \le a_U \le 1 \tag{12}$$

Similarly, we also change α by $-\epsilon_\alpha$ such that

$$0 \le a_1 \le ... \le a_k \le \alpha - \epsilon_\alpha \le a_{k+1} \le ... \le a_U \le 1 \tag{13}$$

Note that such ϵ_α always exists because $a_k < \alpha < a_{k+1}$. If we change α to $\alpha + \epsilon_\alpha$ while satisfying Equation 12, the violations becomes $-\sum_{i=1}^{k} a_i + \sum_{i=k+1}^{U} a_i + (2k-U)(\alpha + \epsilon_\alpha)$. Similarly, if we change α to $\alpha - \epsilon_\alpha$ while satisfying Equation 13, the violations becomes $-\sum_{i=1}^{k} a_i + \sum_{i=k+1}^{U} a_i + (2k-U)(\alpha - \epsilon_\alpha)$. Hence, the change in the number of violations for each case can be computed as follows:

$$\begin{aligned} \alpha + \epsilon_\alpha &\rightarrow \Delta(\#\text{ Vio.}) = (2k-U)\epsilon_\alpha \\ \alpha - \epsilon_\alpha &\rightarrow \Delta(\#\text{ Vio.}) = -(2k-U)\epsilon_\alpha \end{aligned} \tag{14}$$

From Equation 14, we observe that one of the transformations always maintains or even reduces the violations according to the sign of $(2k-U)$, i.e., the solution is feasible. Specifically, if $(2k-U) \le 0$, we change α to a_{k+1} by setting $\epsilon_\alpha = a_{k+1} - \alpha$, otherwise we change α to a_k by setting $\epsilon_\alpha = \alpha - a_k$. As a result, we can remove one non-0/1 value α while maintaining an optimal solution.

□

B PROOF FOR LEMMA 2.2

We continue from Section 3.3 and provide a full proof for Lemma 2.2. Here we restate Lemma 2.2 with the problem setup:

Lemma 2.2. *Let us assume that $0 < \alpha < \beta < 1$, and there are E edges between the two clusters. Suppose an α-cluster and a β-cluster have A_0 and B_0 points whose initial labels are 0, respectively, and A_1 and B_1 points whose initial values are 1, respectively. Let $N_\alpha = A_0 - A_1$ and $N_\beta = B_0 - B_1$. Now suppose there are U nodes connected to the α-cluster and V nodes connected to the β-cluster satisfying*

$$\begin{aligned} &0 \le a_1 \le ... \le a_k < \alpha < a_{k+1} \le ... \le a_U \le 1 \text{ and} \\ &0 \le b_1 \le ... \le b_l < \beta < b_{l+1} \le ... \le b_V \le 1. \end{aligned}$$

Let us also add the following nodes for convenience: $a_0 = 0$, $a_{U+1} = 1$, $b_0 = 0$, $b_{V+1} = 1$. For an α-cluster with $N_\alpha \neq 0$ and a β-cluster with $N_\beta \neq 0$ in the optimal solution, we can always convert (α, β) to one of $(a_k, \beta + \frac{N_\alpha}{N_\beta}(a_k - \alpha))$, $(a_{k+1}, \beta - \frac{N_\alpha}{N_\beta}(a_{k+1} - \alpha))$, $(\alpha + \frac{N_\beta}{N_\alpha}(\beta - b_l), b_l)$, $(\alpha - \frac{N_\beta}{N_\alpha}(b_{l+1} - \beta), b_{l+1})$, or $(\frac{\alpha N_\alpha + \beta N_\beta}{N_\alpha + N_\beta}, \frac{\alpha N_\alpha + \beta N_\beta}{N_\alpha + N_\beta})$, while maintaining an optimal solution. As a result, we can reduce at least one non-0/1 value in the optimal solution.

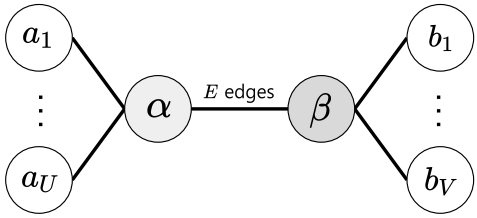


Figure 13: Problem setup for Lemma 2.2.

Proof. We compute the initial number of flips and violations in Figure 13 as follows:

$$\begin{aligned} &\#\text{ Flips} = \alpha A_0 + (1-\alpha)A_1 + \beta B_0 + (1-\beta)B_1 = A_1 + B_1 + \alpha N_\alpha + \beta N_\beta \\ &\#\text{ Vio.} = \sum_{i=1}^{U} |a_i - \alpha| + \sum_{i=1}^{V} |b_i - \beta| + E|\beta - \alpha| = (2k - U - E)\alpha + \\ &\qquad (2l - V + E)\beta + C \quad (C = -\sum_{i=1}^{k} a_i + \sum_{i=k+1}^{U} a_i - \sum_{i=1}^{l} b_i + \sum_{i=l+1}^{V} b_i) \end{aligned} \tag{15}$$

Now consider two small positive values $\epsilon_\alpha > 0$ and $\epsilon_\beta > 0$. Since both N_α and N_β are non-zero, there are only two cases: $\frac{N_\alpha}{N_\beta} < 0$ and $\frac{N_\alpha}{N_\beta} > 0$.

Case 1 $\frac{N_\alpha}{N_\beta} < 0$**:** Suppose we change α by ϵ_α and β by ϵ_β, respectively, such that

$$\begin{aligned} &0 \le a_1 \le ... \le a_k \le \alpha + \epsilon_\alpha \le a_{k+1} \le ... \le a_U \le 1 \\ &0 \le b_1 \le ... \le b_l \le \beta + \epsilon_\beta \le b_{l+1} \le ... \le b_V \le 1 \\ &\alpha + \epsilon_\alpha \le \beta + \epsilon_\beta \end{aligned} \tag{16}$$

We then compute the number of flips for $(\alpha+\epsilon_\alpha, \beta+\epsilon_\beta)$ as $A_1 + B_1 + (\alpha+\epsilon_\alpha)N_\alpha + (\beta+\epsilon_\beta)N_\beta$. In order to have the same number of flips as the initial value in Equation 15, $(\epsilon_\alpha, \epsilon_\beta)$ should satisfy $\epsilon_\alpha N_\alpha + \epsilon_\beta N_\beta = 0$.

Similarly, we also change α by $-\epsilon_\alpha$ and β by $-\epsilon_\beta$, respectively, such that

$$\begin{gathered} 0 \le a_1 \le \ldots \le a_k \le \alpha - \epsilon_\alpha \le a_{k+1} \le \ldots \le a_U \le 1 \\ 0 \le b_1 \le \ldots \le b_l \le \beta - \epsilon_\beta \le b_{l+1} \le \ldots \le b_V \le 1 \\ \alpha - \epsilon_\alpha \le \beta - \epsilon_\beta \end{gathered} \tag{17}$$

In this case, $(\alpha-\epsilon_\alpha, \beta-\epsilon_\beta)$ also maintains the same number of label flips if $\epsilon_\alpha N_\alpha + \epsilon_\beta N_\beta = 0$.

From now, we consider $(\epsilon_\alpha, \epsilon_\beta)$ that also satisfies $\epsilon_\alpha N_\alpha + \epsilon_\beta N_\beta = 0$, i.e., $\epsilon_\beta = -\frac{N_\alpha}{N_\beta}\epsilon_\alpha$. Note that such ϵ_α and ϵ_β always exist because $a_k < \alpha < a_{k+1}$, $b_l < \beta < b_{l+1}$, and $\frac{N_\alpha}{N_\beta} < 0$. Therefore, both $(\alpha+\epsilon_\alpha, \beta+\epsilon_\beta)$ and $(\alpha-\epsilon_\alpha, \beta-\epsilon_\beta)$ have the same number of label flipping as the initial number.

If we change (α, β) to $(\alpha+\epsilon_\alpha, \beta+\epsilon_\beta)$ while satisfying Equation 16 and $\epsilon_\alpha N_\alpha + \epsilon_\beta N_\beta = 0$, the violations becomes $(2k-U-E)(\alpha+\epsilon_\alpha) + (2l-V+E)(\beta+\epsilon_\beta) + C$. Similarly, if we change (α, β) to $(\alpha-\epsilon_\alpha, \beta-\epsilon_\beta)$ while satisfying Equation 17 and $\epsilon_\alpha N_\alpha + \epsilon_\beta N_\beta = 0$, the violations becomes $(2k-U-E)(\alpha-\epsilon_\alpha)+(2l-V+E)(\beta-\epsilon_\beta)+C$. Hence, the change in the number of violations for each case can be computed as follows:

$$\begin{aligned} (\alpha+\epsilon_\alpha, \beta+\epsilon_\beta) \rightarrow \Delta(\#\text{ Vio.}) &= (2k-U-E)\epsilon_\alpha + (2l-V+E)\epsilon_\beta \\ &= \frac{(2k-U-E)N_\beta - (2l-V+E)N_\alpha}{N_\beta}\epsilon_\alpha \\ (\alpha-\epsilon_\alpha, \beta-\epsilon_\beta) \rightarrow \Delta(\#\text{ Vio.}) &= -(2k-U-E)\epsilon_\alpha - (2l-V+E)\epsilon_\beta \\ &= -\frac{(2k-U-E)N_\beta - (2l-V+E)N_\alpha}{N_\beta}\epsilon_\alpha \end{aligned} \tag{18}$$

From Equation 18, we observe that one of the transformations always maintains or even reduces the violations according to the sign of $\frac{(2k-U-E)N_\beta-(2l-V+E)N_\alpha}{N_\beta}$, i.e., the solution is feasible.

- If $\frac{(2k-U-E)N_\beta-(2l-V+E)N_\alpha}{N_\beta} \le 0$, we change (α, β) to $(\alpha+\epsilon_\alpha, \beta+\epsilon_\beta)$ so that the solution is still optimal. Recall $(\epsilon_\alpha, \epsilon_\beta)$ satisfies the three inequalities and one condition: $\alpha+\epsilon_\alpha \le a_{k+1}$, $\beta+\epsilon_\beta \le b_{l+1}$, $\alpha+\epsilon_\alpha \le \beta+\epsilon_\beta$, and $\epsilon_\alpha N_\alpha + \epsilon_\beta N_\beta = 0$. Among the possible $(\epsilon_\alpha, \epsilon_\beta)$, we choose the upper bound of ϵ_α and the corresponding ϵ_β ($\epsilon_\beta = -\frac{N_\alpha}{N_\beta}\epsilon_\alpha$). To get an upper bound of ϵ_α, we find the equality conditions for each inequality and take the smallest value among them. If $1 + \frac{N_\alpha}{N_\beta} \le 0$, the last inequality ($\alpha+\epsilon_\alpha \le \beta+\epsilon_\beta$) always hold because $\epsilon_\alpha \le \epsilon_\beta$. Hence, we consider only the first two inequalities and set ϵ_α to $min(a_{k+1}-\alpha, -\frac{N_\beta}{N_\alpha}(b_{l+1}-\beta)$. On the other hand, if $1 + \frac{N_\alpha}{N_\beta} > 0$, we set ϵ_α to $min(a_{k+1}-\alpha, -\frac{N_\beta}{N_\alpha}(b_{l+1}-\beta), \frac{N_\beta(\beta-\alpha)}{N_\alpha+N_\beta})$ from the three inequalities. As a result, we can convert (α, β) to $(a_{k+1}, \beta - \frac{N_\alpha}{N_\beta}(a_{k+1}-\alpha))$, $(\alpha - \frac{N_\beta}{N_\alpha}(b_{l+1}-\beta), b_{l+1})$, or $(\frac{\alpha N_\alpha+\beta N_\beta}{N_\alpha+N_\beta}, \frac{\alpha N_\alpha+\beta N_\beta}{N_\alpha+N_\beta})$, which is one of the cases in Lemma 2.2.
- If $\frac{(2k-U-E)N_\beta-(2l-V+E)N_\alpha}{N_\beta} > 0$, we change (α, β) to $(\alpha-\epsilon_\alpha, \beta-\epsilon_\beta)$ so that the solution is still optimal. Recall $(\epsilon_\alpha, \epsilon_\beta)$ satisfies the three inequalities and one condition: $a_k \le \alpha-\epsilon_\alpha$, $b_l \le \beta-\epsilon_\beta$, $\alpha-\epsilon_\alpha \le \beta-\epsilon_\beta$, and $\epsilon_\alpha N_\alpha + \epsilon_\beta N_\beta = 0$. Among the possible $(\epsilon_\alpha, \epsilon_\beta)$, we choose the upper bound of ϵ_α and the corresponding ϵ_β ($\epsilon_\beta = -\frac{N_\alpha}{N_\beta}\epsilon_\alpha$). To get an upper bound of ϵ_α, we find the equality conditions for each inequality and take the smallest value among them. If $1 + \frac{N_\alpha}{N_\beta} \ge 0$, the last inequality ($\alpha-\epsilon_\alpha \le \beta-\epsilon_\beta$) always holds because $\epsilon_\alpha \ge \epsilon_\beta$. Hence, we consider only the first two inequalities and set ϵ_α to $min(\alpha-a_k, -\frac{N_\beta}{N_\alpha}(\beta-b_l)$. On the other hand, if $1 + \frac{N_\alpha}{N_\beta} < 0$, we set ϵ_α to $min(\alpha-a_k, -\frac{N_\beta}{N_\alpha}(\beta-b_l), -\frac{N_\beta(\beta-\alpha)}{N_\alpha+N_\beta})$ from the three inequalities. As a result, we can convert (α, β) to $(a_k, \beta + \frac{N_\alpha}{N_\beta}(\alpha-a_k))$, $(\alpha + \frac{N_\beta}{N_\alpha}(\beta-b_l), b_l)$, or $(\frac{\alpha N_\alpha+\beta N_\beta}{N_\alpha+N_\beta}, \frac{\alpha N_\alpha+\beta N_\beta}{N_\alpha+N_\beta})$, which is one of the cases in Lemma 2.2.

Case 2: $\frac{N_\alpha}{N_\beta} > 0$: The proof is similar to the proof for Case 1 except that we consider $(\alpha+\epsilon_\alpha, \beta-\epsilon_\beta)$ and $(\alpha-\epsilon_\alpha, \beta+\epsilon_\beta)$ instead of $(\alpha+\epsilon_\alpha, \beta+\epsilon_\beta)$ and $(\alpha-\epsilon_\alpha, \beta-\epsilon_\beta)$. We now write the full proof for Case 2 for completeness.

Suppose we change α by ϵ_α and β by $-\epsilon_\beta$, respectively, such that

$$\begin{gathered} 0 \le a_1 \le \ldots \le a_k \le \alpha+\epsilon_\alpha \le a_{k+1} \le \ldots \le a_U \le 1 \\ 0 \le b_1 \le \ldots \le b_l \le \beta-\epsilon_\beta \le b_{l+1} \le \ldots \le b_V \le 1 \\ \alpha+\epsilon_\alpha \le \beta-\epsilon_\beta \end{gathered} \tag{19}$$

We then compute the number of flips for $(\alpha+\epsilon_\alpha, \beta-\epsilon_\beta)$ as $A_1 + B_1 + (\alpha+\epsilon_\alpha)N_\alpha + (\beta-\epsilon_\beta)N_\beta$. In order to have the same number of flips as the initial value in Equation 15, $(\epsilon_\alpha, \epsilon_\beta)$ should satisfy $\epsilon_\alpha N_\alpha - \epsilon_\beta N_\beta = 0$.

Similarly, we also change α by $-\epsilon_\alpha$ and β by ϵ_β, respectively, such that

$$\begin{gathered} 0 \le a_1 \le \ldots \le a_k \le \alpha-\epsilon_\alpha \le a_{k+1} \le \ldots \le a_U \le 1 \\ 0 \le b_1 \le \ldots \le b_l \le \beta+\epsilon_\beta \le b_{l+1} \le \ldots \le b_V \le 1 \\ \alpha-\epsilon_\alpha \le \beta+\epsilon_\beta \end{gathered} \tag{20}$$

In this case, $(\alpha-\epsilon_\alpha, \beta+\epsilon_\beta)$ also maintains the same number of label flips if $\epsilon_\alpha N_\alpha - \epsilon_\beta N_\beta = 0$.

From now, we consider $(\epsilon_\alpha, \epsilon_\beta)$ that also satisfies $\epsilon_\alpha N_\alpha - \epsilon_\beta N_\beta = 0$, i.e., $\epsilon_\beta = \frac{N_\alpha}{N_\beta}\epsilon_\alpha$. Note that such ($\epsilon_\alpha$ and ϵ_β) always exist because $a_k < \alpha < a_{k+1}$, $b_l < \beta < b_{l+1}$, and $\frac{N_\alpha}{N_\beta} > 0$. Therefore, both $(\alpha+\epsilon_\alpha, \beta-\epsilon_\beta)$ and $(\alpha-\epsilon_\alpha, \beta+\epsilon_\beta)$ have the same number of label flipping as the initial number.

If we change (α, β) to $(\alpha+\epsilon_\alpha, \beta-\epsilon_\beta)$ while satisfying Equation 19 and $\epsilon_\alpha N_\alpha - \epsilon_\beta N_\beta = 0$, the violations becomes $(2k-U-E)(\alpha+\epsilon_\alpha) + (2l-V+E)(\beta-\epsilon_\beta) + C$. Similarly, if we change (α, β) to $(\alpha-\epsilon_\alpha, \beta+\epsilon_\beta)$ while satisfying Equation 20 and $\epsilon_\alpha N_\alpha - \epsilon_\beta N_\beta = 0$, the violations becomes $(2k-U-E)(\alpha-\epsilon_\alpha)+(2l-V+E)(\beta+\epsilon_\beta)+C$. Hence, the change in the number of violations for each case can be computed as follows:

$$
\begin{aligned}
(\alpha+\epsilon_\alpha, \beta-\epsilon_\beta) \rightarrow \Delta(\#\text{ Vio.}) &= (2k-U-E)\epsilon_\alpha - (2l-V+E)\epsilon_\beta \\
&= \frac{(2k-U-E)N_\beta - (2l-V+E)N_\alpha}{N_\beta}\epsilon_\alpha \\
(\alpha-\epsilon_\alpha, \beta+\epsilon_\beta) \rightarrow \Delta(\#\text{ Vio.}) &= -(2k-U-E)\epsilon_\alpha + (2l-V+E)\epsilon_\beta \\
&= -\frac{(2k-U-E)N_\beta - (2l-V+E)N_\alpha}{N_\beta}\epsilon_\alpha
\end{aligned} \tag{21}
$$

From Equation 21, we observe that one of the transformations always maintains or reduces the violations according to the sign of $\frac{(2k-U-E)N_\beta-(2l-V+E)N_\alpha}{N_\beta}$, i.e., the solution is feasible.

- If $\frac{(2k-U-E)N_\beta-(2l-V+E)N_\alpha}{N_\beta} \leq 0$, we can change (α, β) to $(\alpha+\epsilon_\alpha, \beta-\epsilon_\beta)$ so that the solution is still optimal. Recall $(\epsilon_\alpha, \epsilon_\beta)$ satisfies the three inequalities and one condition: $\alpha+\epsilon_\alpha \leq a_{k+1}$, $b_{l+1} \leq \beta-\epsilon_\beta$, $\alpha+\epsilon_\alpha \leq \beta+\epsilon_\beta$, and $\epsilon_\alpha N_\alpha - \epsilon_\beta N_\beta = 0$. Among the possible $(\epsilon_\alpha, \epsilon_\beta)$, we choose the upper bound of ϵ_α and the corresponding ϵ_β ($\epsilon_\beta = \frac{N_\alpha}{N_\beta}\epsilon_\alpha$). To get an upper bound of ϵ_α, we find the equality conditions for each inequality and take the smallest value among them. Specifically, we set ϵ_α to $min(a_{k+1}-\alpha, \frac{N_\beta}{N_\alpha}(\beta-b_l), \frac{N_\beta(\beta-\alpha)}{N_\alpha+N_\beta})$ and ϵ_β. As a result, we can convert (α, β) to $(a_{k+1}, \beta-\frac{N_\alpha}{N_\beta}(a_{k+1}-\alpha))$, $(\alpha+\frac{N_\beta}{N_\alpha}(\beta-b_l), b_l)$, or $(\frac{\alpha N_\alpha+\beta N_\beta}{N_\alpha+N_\beta}, \frac{\alpha N_\alpha+\beta N_\beta}{N_\alpha+N_\beta})$, which is one of the cases in Lemma 2.2.
- If $\frac{(2k-U-E)N_\beta-(2l-V+E)N_\alpha}{N_\beta} > 0$, we can change (α, β) to $(\alpha-\epsilon_\alpha, \beta+\epsilon_\beta)$ so that the solution is still optimal. Recall $(\epsilon_\alpha, \epsilon_\beta)$ satisfies the three inequalities and one condition: $a_k \leq \alpha-\epsilon_\alpha$, $\beta+\epsilon_\beta \leq b_{l+1}$, $\alpha-\epsilon_\alpha \leq \beta+\epsilon_\beta$, and $\epsilon_\alpha N_\alpha - \epsilon_\beta N_\beta = 0$. Among the possible $(\epsilon_\alpha, \epsilon_\beta)$, we choose the upper bound of ϵ_α and the corresponding ϵ_β ($\epsilon_\beta = \frac{N_\alpha}{N_\beta}\epsilon_\alpha$). To get an upper bound of ϵ_α, we find the equality conditions for each inequality and take the smallest value among them. In this case, the last inequality ($\alpha-\epsilon_\alpha \leq \beta+\epsilon_\beta$) always hold. Hence, we consider only the first two conditions and set ϵ_α to $min(\alpha-a_k, \frac{N_\beta}{N_\alpha}(b_{l+1}-\beta))$. As a result, we can convert (α, β) to either $(a_k, \beta+\frac{N_\alpha}{N_\beta}(a_k-\alpha))$ or $(\alpha-\frac{N_\beta}{N_\alpha}(b_{l+1}-\beta), b_{l+1})$, which is one of the cases in Lemma 2.2.

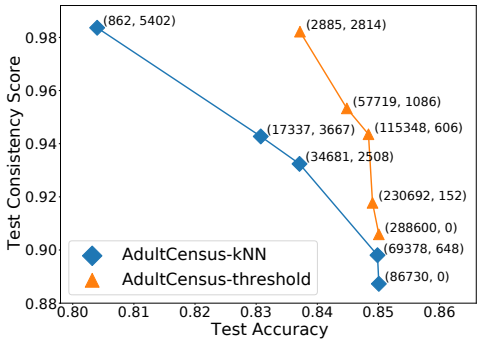
We summarize the main results for each case below and conclude that (α, β) can be transformed into one of the five cases in Lemma 2.2 while maintaining an optimal optimal. As a result, we remove at least one of α and β. If both converted values already exist in the solution, we can even reduce two non-0/1 values.

- If ($\frac{N_\alpha}{N_\beta} < 0$, $\frac{(2k-U-E)N_\beta-(2l-V+E)N_\alpha}{N_\beta} \leq 0$, $1+\frac{N_\alpha}{N_\beta} \leq 0$), we convert (α, β) to $(\alpha+\epsilon_\alpha, \beta+\epsilon_\beta)$ where $\epsilon_\alpha = min(a_{k+1}-\alpha, -\frac{N_\beta}{N_\alpha}(b_{l+1}-\beta)$ and $\epsilon_\beta = -\frac{N_\alpha}{N_\beta}\epsilon_\alpha$.
- If ($\frac{N_\alpha}{N_\beta} < 0$, $\frac{(2k-U-E)N_\beta-(2l-V+E)N_\alpha}{N_\beta} \leq 0$, $1+\frac{N_\alpha}{N_\beta} > 0$), we convert (α, β) to $(\alpha+\epsilon_\alpha, \beta+\epsilon_\beta)$ where $\epsilon_\alpha = min(a_{k+1}-\alpha, -\frac{N_\beta}{N_\alpha}(b_{l+1}-\beta), \frac{N_\beta(\beta-\alpha)}{N_\alpha+N_\beta})$ and $\epsilon_\beta = -\frac{N_\alpha}{N_\beta}\epsilon_\alpha$.
- If ($\frac{N_\alpha}{N_\beta} < 0$, $\frac{(2k-U-E)N_\beta-(2l-V+E)N_\alpha}{N_\beta} > 0$, $1+\frac{N_\alpha}{N_\beta} \geq 0$), we convert (α, β) to $(\alpha-\epsilon_\alpha, \beta-\epsilon_\beta)$ where $\epsilon_\alpha = min(\alpha-a_k, -\frac{N_\beta}{N_\alpha}(\beta-b_l)$ and $\epsilon_\beta = -\frac{N_\alpha}{N_\beta}\epsilon_\alpha$.
- If ($\frac{N_\alpha}{N_\beta} < 0$, $\frac{(2k-U-E)N_\beta-(2l-V+E)N_\alpha}{N_\beta} > 0$, $1+\frac{N_\alpha}{N_\beta} < 0$), we convert (α, β) to $(\alpha-\epsilon_\alpha, \beta-\epsilon_\beta)$ where $\epsilon_\alpha = min(\alpha-a_k, -\frac{N_\beta}{N_\alpha}(\beta-b_l), -\frac{N_\beta(\beta-\alpha)}{N_\alpha+N_\beta})$ and $\epsilon_\beta = -\frac{N_\alpha}{N_\beta}\epsilon_\alpha$.
- If ($\frac{N_\alpha}{N_\beta} > 0$, $\frac{(2k-U-E)N_\beta-(2l-V+E)N_\alpha}{N_\beta} \leq 0$), we convert (α, β) to $(\alpha+\epsilon_\alpha, \beta-\epsilon_\beta)$ where $\epsilon_\alpha = min(a_{k+1}-\alpha, \frac{N_\beta}{N_\alpha}(\beta-b_l), \frac{N_\beta(\beta-\alpha)}{N_\alpha+N_\beta})$ and $\epsilon_\beta = \frac{N_\alpha}{N_\beta}\epsilon_\alpha$.
- If ($\frac{N_\alpha}{N_\beta} > 0$, $\frac{(2k-U-E)N_\beta-(2l-V+E)N_\alpha}{N_\beta} > 0$), we convert (α, β) to $(\alpha-\epsilon_\alpha, \beta+\epsilon_\beta)$ where $\epsilon_\alpha = min(\alpha-a_k, \frac{N_\beta}{N_\alpha}(b_{l+1}-\beta))$ and $\epsilon_\beta = \frac{N_\alpha}{N_\beta}\epsilon_\alpha$.

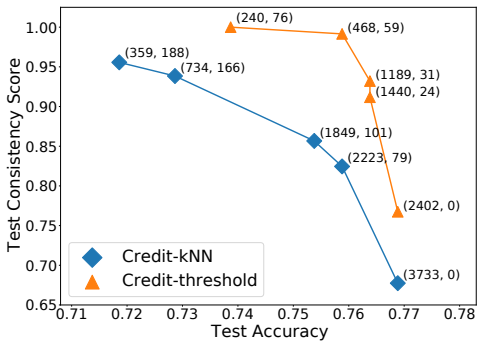
□

C TRADE-OFF FOR OTHER DATASETS

We continue from Section 4.2 and show the trade-off results on the AdultCensus and Credit datasets in Figure 14. The results are similar to the COMPAS dataset in Figure 8 where there is a clear trade-off between accuracy and fairness.



(a) AdultCensus

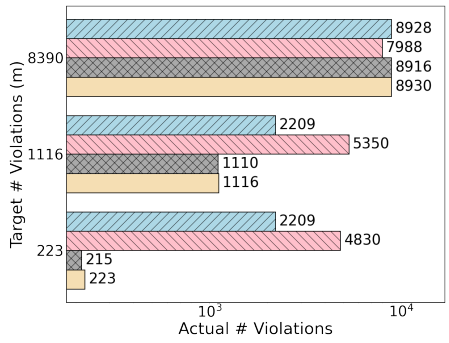


(b) Credit

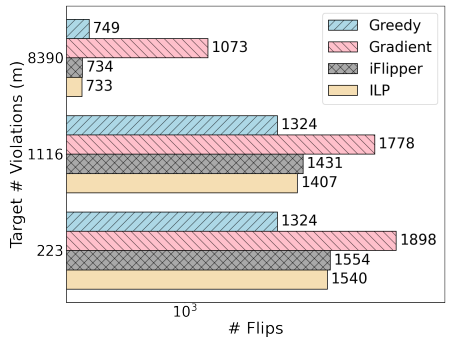
Figure 14: Trade-off curves on the AdultCensus and Credit datasets.

D BANK AND LSAC DATASETS RESULTS

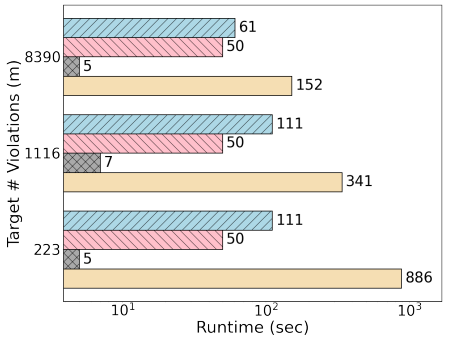
We continue from Section 4.1 and show experimental results on the Bank and LSAC datasets when training a logistic regression model.



(a) Number of violations

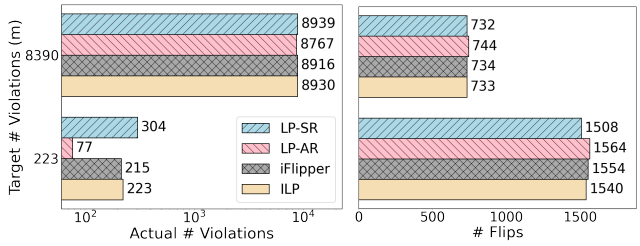


(b) Number of flips



(c) Runtime (sec)

Figure 15: A detailed comparison of iFlipper against two naïve solutions (*Greedy* and *Gradient*) and ILP solver on the COMPAS dataset where we use the kNN-based similarity matrix. Here the initial number of violations is 22,327. We show the results for three different target violation limits (m). All three subfigures use the same legends.



(a) Number of violations **(b)** Number of flips

Figure 16: Ablation study for iFlipper on the COMPAS dataset and the kNN-based similarity matrix.

Table 6 shows consistency scores with respect to the threshold-based similarity matrix. As a result, both datasets have almost 1.0 consistency scores, which means that they are already inherently fair in terms of individual fairness. The reason is that these datasets contain much fewer violations compared to other fairness datasets.

Dataset	Test Accuracy	Consistency Score
Bank	0.956	0.997
LSAC	0.825	0.986

Table 6: Accuracy and fairness results using logistic regression on the Bank and LSAC datasets.

E OPTIMIZATION SOLUTIONS FOR COMPAS

We continue from Section 4.4 and perform the same experiments on the COMPAS dataset where we use the kNN-based similarity matrix. In Figure 15, the key trends are still similar to Figure 10 where iFlipper (1) always satisfies the violations limit while both *Greedy* and *Gradient* result in an infeasible solution for some cases (Figure 15a), (2) provides the solution closest to the optimal in terms the number of label flips (Figure 15b), and (3) is much faster than other optimization solutions (Figure 15c). Compared to the Adult-Census results in Figure 10, iFlipper is the most efficient because the COMPAS dataset is relatively small and contains fewer violations.

F ABLATION STUDY FOR COMPAS DATASET

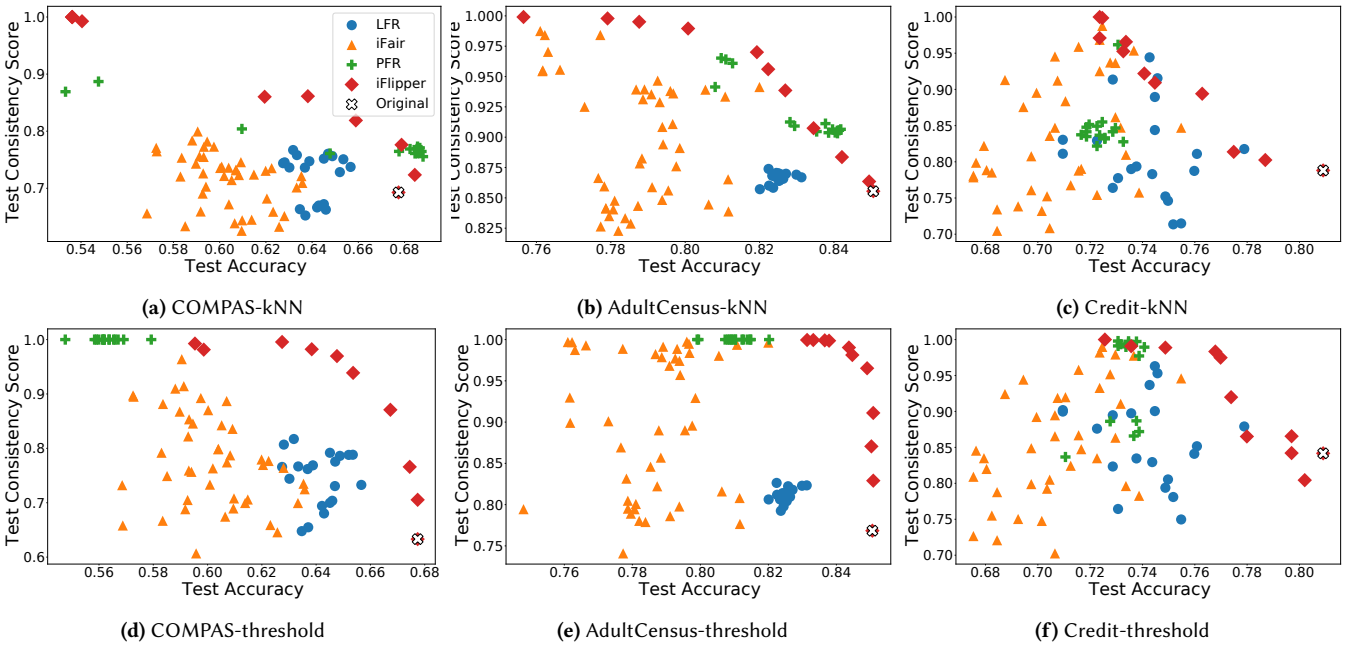
We continue from Section 4.5 and provide the ablation study for the COMPAS dataset where we use the kNN-based similarity matrix in Figure 16. The observations are similar to those of Figure 11 where both adaptive rounding and reverse greedy algorithms are necessary for iFlipper to provide a near-exact solution. In addition, Table 7 shows the average runtime of each component in iFlipper in Figure 16 and the results are similar to Table 4 where the proposed algorithms are efficient in practice.

Method	Avg. Runtime (sec)
LP Solver (effectively includes Alg. 1)	5.09
+ Adaptive Rounding (Alg. 3)	0.07
+ Reverse Greedy (Alg. 4)	0.32

Table 7: Avg. runtimes of iFlipper's components in Figure 16.

G COMPARISON WITH OTHER ML MODELS

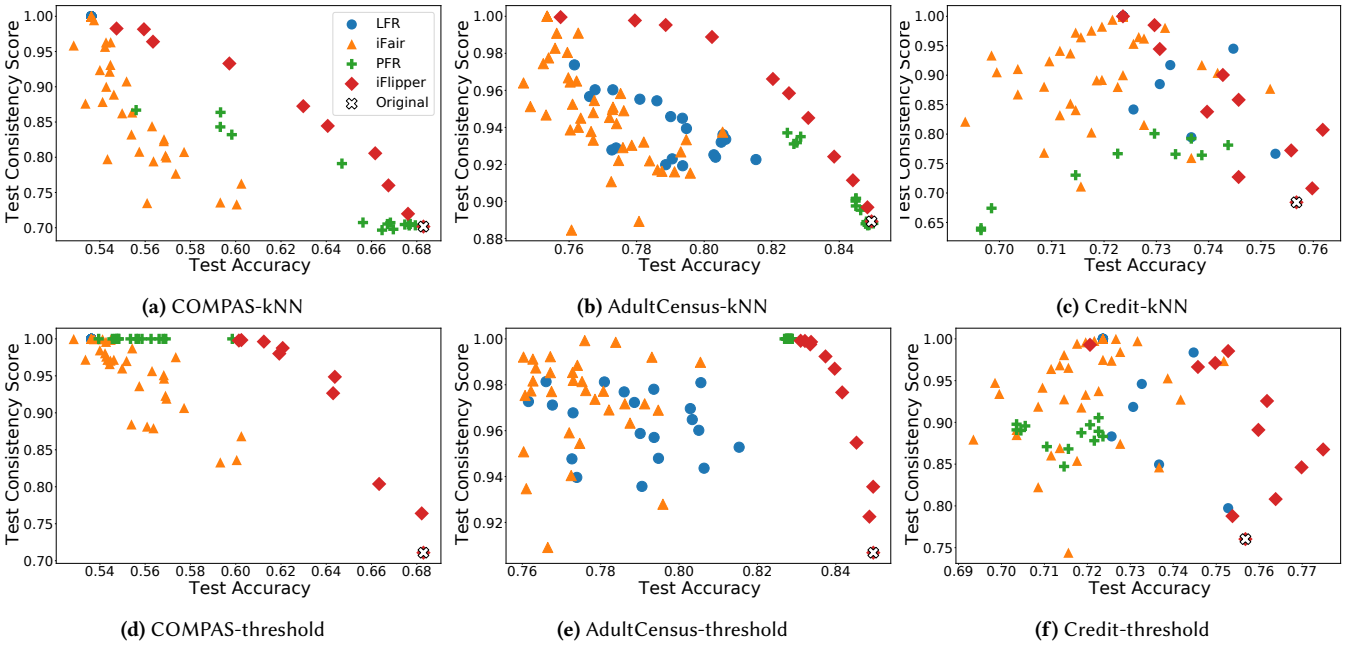
In Section 4.3, we compared iFlipper with the baselines using logistic regression. In this section, we perform the same experiments using random forest and neural network models. Figure 17 and Figure 18 are the trade-off results using the random forest and neural network, respectively. The key trends are still similar to Figure 9 where iFlipper consistently outperforms the baselines in terms of accuracy and fairness. The results clearly demonstrate that how iFlipper's pre-processing algorithm benefits various ML models.



(a) COMPAS-kNN (b) AdultCensus-kNN (c) Credit-kNN

(d) COMPAS-threshold (e) AdultCensus-threshold (f) Credit-threshold

Figure 17: Accuracy-fairness trade-offs of random forest on the three datasets using the two similarity matrices. In addition to the four methods LFR, iFair, PFR, and iFlipper, we add the result of model training without any pre-processing and call it "Original." As a result, only iFlipper shows a clear accuracy and fairness trade-off.



(a) COMPAS-kNN (b) AdultCensus-kNN (c) Credit-kNN

(d) COMPAS-threshold (e) AdultCensus-threshold (f) Credit-threshold

Figure 18: Accuracy-fairness trade-offs of neural network on the three datasets using the two similarity matrices. In addition to the four methods LFR, iFair, PFR, and iFlipper, we add the result of model training without any pre-processing and call it "Original." As a result, only iFlipper shows a clear accuracy and fairness trade-off.