Let $S(x,q) = S^a(x)q_a + \ldots$, then for arbitrary function $g(y)$, the inverse image under the thick morphsim Φ_S, $f(x) = \Phi_S^*(g)$ is equal to

$$f(x) = \lim_{\hbar\to 0} \frac{\hbar}{i} \log \left[\int e^{\frac{i}{\hbar}(g(y)+S(x,q)-y^a q_a)} \mathcal{D}(y,q) \right] .$$

Thus

$$\Phi_S^*(g+\varepsilon h) = \lim_{\hbar\to 0} \frac{\hbar}{i} \log \left[\int e^{\frac{i}{\hbar}(g(y)+S(x,q)-y^a q_a+\varepsilon \mathbf{h}(\mathbf{y}))} \mathcal{D}(y,q) \right] =$$

$$= \lim_{\hbar\to 0} \frac{\hbar}{i} \log \left[\int e^{\frac{i}{\hbar}(g(y)+S(x,q)-y^a q_a)} \left(1+\varepsilon h(y)\right) \mathcal{D}(y,q) \right] =$$

$$\Phi_S^*(g) + \varepsilon \lim_{\hbar\to 0} \frac{\hbar}{i} \left[\frac{\int e^{\frac{i}{\hbar}(g(y)+S(x,q)-y^a q_a)} h(y) \mathcal{D}(y,q)}{\int e^{\frac{i}{\hbar}(g(y)+S(x,q)-y^a q_a)} \mathcal{D}(y,q)} \right] .$$

We come to

$$\Phi_S^*(g+\varepsilon h) - \Phi_S^*(g+\varepsilon h) = \varepsilon \lim_{\hbar\to 0} \frac{\hbar}{i} \left[\frac{\int e^{\frac{i}{\hbar}(g(y)+S(x,q)-y^a q_a)} h(y) \mathcal{D}(y,q)}{\int e^{\frac{i}{\hbar}(g(y)+S(x,q)-y^a q_a)} \mathcal{D}(y,q)} \right] .$$

Here we have to be carefull with "concurrence" between small ε and big $\frac{1}{\hbar}$. We hae to care about the cobntribution of amplitudes...

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