

Consider variables  $\eta^\alpha, 1 \leq \alpha \leq N$ , which commute with  $x^s$  and  $\partial/\partial x^s, 1 \leq s \leq n$ , but do not commute with each other. A tensor is a polynomial

$$T_{\alpha_1 \dots \alpha_k}(x) \eta^{\alpha_1} \dots \eta^{\alpha_k}.$$

A covariant derivative is

$$\nabla_i = \frac{\partial}{\partial x^i} - \Gamma_{i\sigma}^\alpha(x) \text{ad}(\zeta_\alpha^\sigma) = \text{ad} \left( \frac{\partial}{\partial x^i} - \Gamma_{i\sigma}^\alpha(x) \zeta_\alpha^\sigma \right),$$

where the variables  $\zeta_\alpha^\sigma$  commute with  $x^s$  and  $\partial/\partial x^s$  and satisfy

$$[\zeta_\alpha^\sigma, \eta^\gamma] = \delta_\alpha^\gamma \eta^\sigma \text{ and } [\zeta_\alpha^\sigma, \zeta_\beta^\tau] = \delta_\alpha^\tau \zeta_\beta^\sigma - \delta_\beta^\sigma \zeta_\alpha^\tau.$$

If the variables  $\eta^\alpha$  commute, then

$$\zeta_\alpha^\sigma = \eta^\sigma \frac{\partial}{\partial \eta^\alpha}.$$

If we set  $\Gamma_i = \Gamma_{i\sigma}^\alpha(x) \zeta_\alpha^\sigma$ , then

$$\nabla_i = \frac{\partial}{\partial x^i} - \text{ad}(\Gamma_i) \text{ and } [\Gamma_i, \eta^\alpha] = \Gamma_{i\sigma}^\alpha(x) \eta^\sigma.$$

The curvature of the connection  $\nabla$  is

$$R_{ij} = [\nabla_i, \nabla_j] = R_{ij\tau}^\sigma \text{ad}(\zeta_\sigma^\tau),$$

where

$$R_{ij\tau}^\sigma = -\frac{\partial}{\partial x^i} \Gamma_{j\tau}^\sigma + \frac{\partial}{\partial x^j} \Gamma_{i\tau}^\sigma + \Gamma_{i\tau}^\alpha \Gamma_{j\alpha}^\sigma - \Gamma_{j\tau}^\alpha \Gamma_{i\alpha}^\sigma.$$