

It is a little note about so called Jacobian problem.

Let  $P(x, y)$  and  $Q(x, y)$  be two polynomials on variables  $x, y$  such that

$$\det \begin{pmatrix} \frac{\partial P}{\partial x} & \frac{\partial P}{\partial y} \\ \frac{\partial Q}{\partial x} & \frac{\partial Q}{\partial y} \end{pmatrix} = 1 \quad (1)$$

Then the inverse functions are always polynomials too. For example to the polynomial transformation

$$\begin{cases} x \mapsto x + (x + y)^3 \\ y \mapsto x + y \end{cases}$$

corresponds inverse transformation

$$\begin{cases} x \mapsto x - y^3 \\ y \mapsto y + y^3 - x \end{cases}$$

As is stated in the book of Kirillov: ("Shto takojе chislo"—broshjura, 1993 god) this problem is unsolved till now. (Kak ja ponimaju net kontrprimera i ne dokazano eto utverzhdenije.) One can following to this book consider non-commutative version of this problem. Let  $W$  be associative algebra with unity, generated by two generators  $p$  and  $q$  which obey only to the constraint:

$$pq - qp = 1$$

(Weyl algebra) Let  $A$  and  $B$  be two polynomials on  $p$  and  $q$  such that

$$AB - BA = 1 \quad (2)$$

Then the endomorphism generated by the map

$$p \mapsto A, q \mapsto B$$

has to be an isomorphism!—i.e.  $p$  and  $q$  can be expressed as polynomials of  $A$  and  $B$ . You see that the first problem is quasiclassical limit of the second one (commutator (2)  $\rightarrow$  Poisson bracket = jacobian.)