Spectral sequences 2

Spectral sequence. What is it ? — it is a sequence (E_r, d_r) of complexes such that every subsequent space is cohomology of the previous one:

$$E_{r+1} = H(E_r, d_r)$$

They naturally arise if we calculate cohomology of complex using perturbation theory. It is powerful tool to compare two cohomologies.

We consider here spectral sequence which arises if we consider linear relations.

Image, kernel, Indeterminancy and

Let R be a relation on the linear space V. We say that this relation is *linear* if R is a linear subspace of $V \times V$. Consider the following fours subspaces in V:

Kernel of relation R

$$\ker R = \{x \in V\colon\ (x, 0) \in R\}\,.$$

Indeterminacy of relation R

$$\mathrm{Ind}\, R = \{y \in V\colon\ (0, y) \in R\}\,.$$

Domain of definition of relation R

$$\mathrm{Def}\ R = \{x \in V\colon\ \exists y, (x, y) \in R\}$$

and an image of the relation R:

$$\mathrm{Im}\ R = \{y \in V\colon\ \exists x, (x, y) \in R\}$$

Example Let A be linear operator on M, then consider a linear relation

$$R = R_A = \{(x, Ax)\}$$

It is easy to see that $\ker R_A = \ker A$, $\mathrm{Ind} R_A = 0$, $\mathrm{Def} R_A = V$ and $\mathrm{Im} R_A = \mathrm{Im} A$.

Let ∂_r a sequence of linear relations such that