Thick morphisms, and Lie algebra and bialgebra

Consider on Lie algebra $\mathcal{G}$ Hamiltonian

$$H = c^m_{ik}\xi^i\xi^k\pi_m$$

where ξ are coordinates on $\Pi\mathcal{G}$ and π on $\Pi\mathcal{G}^*$

It is quadratic in ξ and linear in π.

The same and one Hamiltonian H is quadratic in momenta in $T^*(\Pi\mathcal{G}^*)$ and it is linear in momenta in $T^*(\Pi\mathcal{G})$. It generates differential ∂ in $\Pi\mathcal{G}$ and bracket in $\Pi\mathcal{G}^*$:

$$[f(\pi, g(\pi))] = ((H, f), g)\,, \quad \partial f(\xi) = (H, f)$$

Notice that here MX symplectomorphism is identical map– just momenta become coordinates and vice versa.

Let $P(\pi)$ be function on $\Pi\mathcal{G}^*$ (linear combination of multivectors) such that $[P, P] = 0$ Consider new hamiltonian

$$Q_P = (H, P) = \xi^i c^m_{ik}\pi_m \frac{\partial P(\pi)}{\partial \pi_k}$$

This Hamiltonian is linear over ξ and is quadratic and more over π (if $P(\pi)$ is arbitrary polynomial)

It defines differential ∂_P on $\Pi\mathcal{G}^*$:

$$\partial_P F = [P, F] = ((H, P), F) = (H_P, F)$$

and homotopy brackets on $\Pi\mathcal{G}^*$

Notice that thick morphism adjoint to map

$$\varphi_P\colon \quad \xi^i = \frac{\partial P(\pi)}{\partial \pi_i}$$

is finite series, i.e. polynomial since we have anticommuting variables

The morphism $\xi^i = \frac{\partial P(\pi)}{\partial \pi_i}$ intertwines differentials ∂ and ∂_P, thick morphism interwins odd Poisson bracket [,] on $\Pi\mathcal{G}$ and homotopy bracket on $\Pi\mathcal{G}^*$