One construction

We consider two fibre bundles $E_1 = \Pi T^*M \to M$ and $E_2 = \Pi TM \to M$.

Coordinates on E_1 are (x^a, θ_b), coordinates on E_2 are (y^a, ξ^b).

We consider also $T^*E_1 = T^*(\Pi T^*M)$ with coordinates

$$(x^a, \theta_b; p_a, \eta^b)$$

and $T^*E_2 = T^*(\Pi TM)$ with coordinates

$$(y^a, \xi^b; q_a, \pi_b)$$

We consider canonical Hamiltonian

$$h = \xi^a q_a \quad \text{on } \Pi T^*M$$

This is linear Hamiltonian, and $(h, h) = 0$ where $(\ ,\ )$ is canonical Poisson bracket in $T^*(\Pi TM)$.

The image of this linear Hamiltonian under MX symplectomorphism $\kappa\colon T^*(\Pi T^*M) \to T^*(\Pi TM)$ is quadratic Hamiltonian

$$H = k^*h = p_a \eta^a$$

Hamiltonian h generates degenerate homotopy Schouten bracket on ΠT^*M—de Rham differential and Hamiltonian H generates canonical Schoutten bracket on ΠT^*M:

$$\text{for arbitrary function } F \text{ on } \Pi T^*M\,, (h, F) = dF$$
$$\text{for arbitrary functions } G, H \text{ on } \Pi TM\,, ((H, G), H) = [G, H]$$

all other brackets vanish.

Let as usual P be a function on ΠT^*M such that

$$[P, P] = ((H, P), P) = 0\,.$$

This master-function defines on ΠT^*M Lichnerowicz differential

$$d_P F = [P, F] = ((H, P), F)$$

Consider on ΠT^*M *linear Hamiltonian H_P such that the Lichneroitch differential is just its homotopy Schouten bracket*, i.e.

$$(H_P, F) = d_P F$$

Hence

$$H_P=(P,H)=(P,H)=(P,p_a\eta^a)=p_a\frac{\partial P(x,\theta)}{\partial\theta_a}+\eta^a\frac{\partial P(x,\theta)}{\partial x^a}\,.$$

Now consider the Hamiltonian h_P in ΠTM, which is image of H_P under MX symplectomorphism:

$$h_p=(\kappa^{-1})^*H_p=q_a\frac{\partial P(x,\pi)}{\partial_a}+\xi^a\frac{\partial P(x,\pi)}{\partial x^a}$$

(znaki ne proverial)

We just come to Hamiltonian which generates homotopy Schoutem nracket on ΠTM—higher Koszul bracket.

We have pair of Hamiltonians (H,H_P) on $T^*(\Pi T^*M)$, $H_P=(H,P)$ and pair of Hamiltonians (h,h_P) on $T^*(\Pi TM)$ which are their MX image.

The quadratic Hamiltonian H (linear Hamiltonian h) generates canonical odd Poisson bracket on ΠT^*M (de Rham differential on ΠTM).

The linear Hamiltonian H_P (Hamiltonian h_P) generates Lichnerowicz differential on ΠT^*M (Higher Koszul bracket on ΠTM)

Now go to the map $\varphi_P\colon\Pi T^*M\to\Pi TM$

$$\phi_P\colon x^a=y^a,\xi^a=\frac{\partial P(x,\theta)}{\partial x^a}.$$

We know that

$$`d_P(\varphi^*\omega)=(H_P,\varphi^*\omega)=\varphi^*(d\omega)=\varphi^*(h,\omega)$$

in other words *morphism $\varphi_P\colon\Pi T^*M\to\Pi TM$ (more precisely, lifting of this morphism, canonical transformation of $T^*(\Pi TM)$ $\hat{\varphi_P}$) interwins Hamiltonians H_P and h:*

Conjugate thick morphism $\Phi_P^{(thick)}$ interwins Hamiltonians H and h_P:

We see that we have **three symplectomorphisms** *One MX symp,ectomrophism and two symoplectomorphisms thick and usual related with the map P. The later both are conjugate with each other via MX symplectomorpshim.*