On one property of anchor

Consider algebroid:i.e. it is given vector bundle $E \to M$, commutator $[[\,,\,]]$ on sections obeying Jacobi idenity, and the anchor map ρ from the vector bundle $E \to M$ to the tangent bundle $TM \to M$ such that for two arbitrary sections s_1, s_2 and arbitrary function f the following relation holds:

$$[[fs_1, s_2]] = f[[s_1, s_2]] - (\rho(s_2)f)\, s_1$$

One can show that anchor map is obeying the natural relation

$$\rho[[s_1, s_2]] = [\rho s_1, \rho s_2]$$

where $[\,,\,]]$ is an ordinary commutator of vector fiels.

Examples of algebroids:

1) usual tangent bundle $TM \to M$, anchor ρ is idenity map

2) Lie algebra as a fibre bundle over point.

3) M—Poisson manifold. Then on sections of $T^*M \to M$ one has natural algeborid stricture defined by the relation that $[[df, dg]] = df, g$ and anchor is defined by the Poisson map from T^*M to TM.

4) Atihay algebroid.

We give the proof of ().

Often the property () is included in axioms.

First of all note that algebroid struture on $E \to M$ defines the Q-manifold structure on $\Pi E \to M$ by the follwoing formula: vector field Q is equal to

$$Q = c\xi\xi\frac{\partial}{\partial\xi} + \xi\rho\frac{\partial}{\partial x}$$