Geometry of Fadeev-Popov trick and BV formalism

H.M. Khudaverdian
The University of Manchester. School of Mathematics.

Manhester February 2018—2

Abstract

I try to return to considerations in papers [**?**], [**?**]...

Contents

1 Lecture I. Elements of Fadeev -Popov trick

1.1 Very elementary example

Let $\rho(\mathbf{r})$ be a function on $\mathbf{E}^2$ such that

- it is invariant with respect to rotation:

$$\rho(\mathbf{r}) = f(r)\,.$$

- it increases rapidly:

$$\int \rho(x,y)dx \wedge dy = 2\pi \int_0^\infty f(t)tdt < \infty\,.$$

Let C be a curve beginning at origin, and going to infinity such that for arbitrary positive number R, every circle $x^2 + y^2 = R^2$ intersects the curve C exactly at one point.

Suppose the curve C is defined by equation $F(x,y) = 0$. We say that function F is a gauge function.

Define function $B(x,y)$ such that

$$B(x,y) \int_0^{2\pi} \delta\left((F(x^\varphi, y^\varphi))\right) d\varphi = 1\,.$$

where

$$\begin{pmatrix} x^\varphi \\ y^\varphi \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \cos\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x\cos\varphi - y\sin\varphi \\ x\sin\varphi + y\sin\varphi \end{pmatrix}$$

We see that function $B(x,y)$ is invariant with respect to rotation:

$$B(x^\varphi, y^\varphi) = B(x,y)\,, \quad \text{i.e. } B(x,y) = B(r)$$

Thus we have:

$$\int_{\mathbf{E^2}} \rho(x,y)dx \wedge dy = \int_{\mathbf{E^2}} 1 \cdot \rho(x,y)dxdy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B(x,y)\rho(x,y)dxdy =$$

$$\int_0^{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\varphi dxdy\rho(x,y)B(x,y)\delta\left((F(x^\varphi, y^\varphi))\right) d\varphi =$$

$$\int_0^{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\varphi dx^\varphi dy^\varphi \rho(x^\varphi, y^\varphi)B(x^\varphi, y^\varphi)\delta\left((F(x^\varphi, y^\varphi))\right) =$$

$$\int_0^{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\varphi dxdy\rho(x,y)B(x,y)\delta\left((F(x,y))\right) =$$

$$2\pi \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dxdy\rho(x,y)B(x,y)\delta\left((F(x,y))\right) =$$