Cartan algebras

We say that $\mathcal{H} \subseteq \mathcal{G}$ is Cartan subalgebra in $\mathcal{G}$ if $\mathcal{H}$ is nilpotent subalgebra and it coicides with tis normaliser.

Consider polynomial

$$P_x(z) = \det(z - \operatorname{ad} x) = a_0(x) + a_1(x)z + a_2(x)z^2 + \ldots + a_N(x)z^N ,$$

where $N = \dim \mathcal{G}$. It is evident that $a_0 \equiv 0$.

We say that algebra $\mathcal{G}$ has rank l if there exist $x \in \mathcal{G}$ such that this polynomial has non-zero coefficient a_l, but all a_k identically vanish for $k < l$.

We say that $x \in \mathcal{G}$ is *regular* element in $\mathcal{G}$ if $a_l(x) \neq 0$, where l is a rank of the algebra $\mathcal{G}$

Theorem Set of regualr elements in algebra Lie is open, dense and simply connected.

Theorem Let x be an arbitrary regular element in Lie algebra $\mathcal{G}$. Then

$$\mathcal{G}_x^0 = \{\xi\colon\ (\operatorname{ad} x)^n \xi = 0\}$$

is subalgebra, and this subalgebra is Cartan subalgebra.

Theorem The group G of inner automorphisms of Lie algebra $\mathcal{G}$ acts transitevely on set of Cartan algebras.

Corollary One can see that