Determinant and Berezinian. Their homological interpretation.

Let $\vee(x^a, \theta_b, \xi^c, y_d)$ be an algebra of pseudodifferential forms on ΠT^*M (x^a, θ_b are coordinates on ΠT^*M, $\xi^b \approx dx^a, y$ are odd and $y_b \approx d\theta_b$ are even)

The symplecti form is just $\omega = xi^m y_m$.

Consider the spectral sequence (Severra spectral sequence)

$$(E_r, d_r)$$

where $E_0 = C(\Pi(\Pi T^*M))$ is an algebra of pseudodifferential forms $F(x, \theta, \xi, y)$, and $d_0 = d + \mathbf{x}^m y_m$

To calculate E_1, consider homotpy oeprator $\Delta = \frac{\partial^2}{p y_m \partial \xi^m}$ and notice that

$$(d_0 \Delta + \Delta d_0) F = (N - l_\xi + l_y) F$$

where l_ξ and l_y is an order of ξ and y in F (more accurately one have to write down integrals.) Since $l_\xi \leq N$ The Right hand side has chance to vanish only if $l_\xi = n, l_y = 0$, i.e. only

$$F(x, \theta, \xi, y) = s(x, \theta) \xi^1 \xi^2 \ldots \xi^n$$

has chance to be not coboundary of d_0. One can see that this object is really non-trivial cocycle and its class transforms as half-density.