**Gamma function**

We know very well the integral representation of Gamma function

$$\Gamma(x) = (x-1)! = \int_0^\infty t^x e^{-t} dt \tag{1}$$

We also know very well that

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}\,, \tag{2}$$

where

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1} dt\,.$$

In fact Eiuler worked with a different definition of Gamma-function, which was natural continuation on all $x$ of factorial. Using this definition he came to formulae (1) and (2).

I cannot avoid temptation to recall here this topic.

In fact Euler observed gamma-function in the following way: He noted that

$$k! = \frac{(N+k)!}{(k+1)_N} = \frac{N!(N+1)_k}{(k+1)_N} = \lim_{N\to\infty} \frac{N!N^k}{(k+1)_N}\,,$$

where $(A)_r = A(A+1)\ldots(A+r-1)$. Hence one can define

$$x! = \lim_{N\to\infty} \frac{N!N^x}{(x+1)_N}$$

and respectively

$$\Gamma(x) = (x-1)! = \lim_{N\to\infty} \frac{N!N^{x-1}}{(x)_N}\,. \tag{3}$$

This definition looks not very beautiful, but one can easily imply equations (1), (2) and anbtoher identites.

E.g. one can easy to see that equation (3) implies that

$$\frac{1}{\Gamma(x)} = xe^{\gamma x} \prod_{k=1}^\infty \left(\left(1+\frac{x}{k}\right)e^{-\frac{k}{n}}\right)\,,$$

where $\gamma = \lim_{N\to\infty}(1+\frac{1}{2}+\ldots+\frac{1}{N} - \log N)$ is Euler constant.

Indeed

$$\frac{1}{\Gamma(x)} =$$