Second attempt

It is well-known that on Riemannian manfiold, the Laplacian

$$\Delta_g = \Delta_g^{(B)} + c_n R \tag{1}$$

is invariant with respect to conformal transformations

$$\tilde{g}_{ik} = e^{\sigma} g_{ik} \tag{2}$$

Here $\Delta_g^{(B)}$ is standard Beltrami-Laplace:

$$\Delta_g^{(B)} = \frac{1}{\rho_g} \frac{\partial}{\partial x^i} \left(\rho_g g^{ik} \frac{\partial}{\partial x^k} \right) , \quad \rho_q = \det g^{\frac{1}{2}} \,.$$

is a scalar curvature of metric g, c_n is conast depending on the dimension of the space M.

What is the exact statement?

The standard answer is:

$$\Delta_{\tilde{g}} = e^{a_n \sigma} \Delta_g e^{b_n \sigma} \,, \quad \text{if } \Gamma_{ik} = e^{\sigma} g_{ik}$$

where a_n, b_n are constants. They measure so called conformal weight.

Sure this is much better to writhe down invartiant operator, without mentioning conformal weight. I prefer to tell this in the following way: Consider on Riemannian manifold, a tensorial density:

$$S_g^{ik} \partial_i \otimes \partial_k = \rho_g^{\frac{1}{n}} g^{ik} \partial_i \otimes \partial_k = \left(|Dx| \sqrt{\det g} \right)^{\frac{1}{n}} g^{ik} \partial_i \otimes \partial_k$$

This tensorial density depends on *conformal class* of Riemannian metric. In particular it *does not change* under Weyl transformations (2):

$$\left(|Dx| \sqrt{\det \tilde{g}} \right)^{\frac{1}{n}} \tilde{g}^{ik} \partial_i \otimes \partial_k = \left(e^{n\sigma} |Dx| \sqrt{\det g} \right)^{\frac{1}{n}} e^{-\sigma} g^{ik} \partial_i \otimes \partial_k = \left(|Dx| \sqrt{\det g} \right)^{\frac{1}{n}} g^{ik} \partial_i \otimes \partial_k \,.$$