On one property of quadrics. 4 XII 2013 Upout two months ago Crabor Megyesi
suggested me the following problem:

Let C be a quadric in the plane.

Denote by C* a locus of the prints such in 1E 3 such that any of these points is a vertex of circular cone over the curue C.

(E. g. if C is a circle then C* is a line which is orthogonal pessing through the curue of this circle and or thogonal to the plane of circle). Show that in the case if C is not a circle then C* is a quedric too, and $(C^*)^* = C$. I rolved the problem, and my answer to beautiful.

On the other head my solution in some sense is "brute force" solution. I still canon findhown not more beautiful (and illuminating) volution. Here I will state an amuer and explein my calculations. First de formula ted détailed

answer;

Statement. Let C le a quadric in the plane I with foci Fi, Fz. Then C* or a quadric in the plane & wich is orthogonal to plene 2 and intersects with 2 by the line F.F. The quedric Cot penser via foci Fr and F2 Respectively the quadric C passes via foci of curve C...
If Let T, R be arbitrary paints on curves C, C* respectively, and v, V be largent velocs. (Vat a punt F of cured C, and Vat a point of a curve (*) Then is it directed along the axis of circuler come over (*)

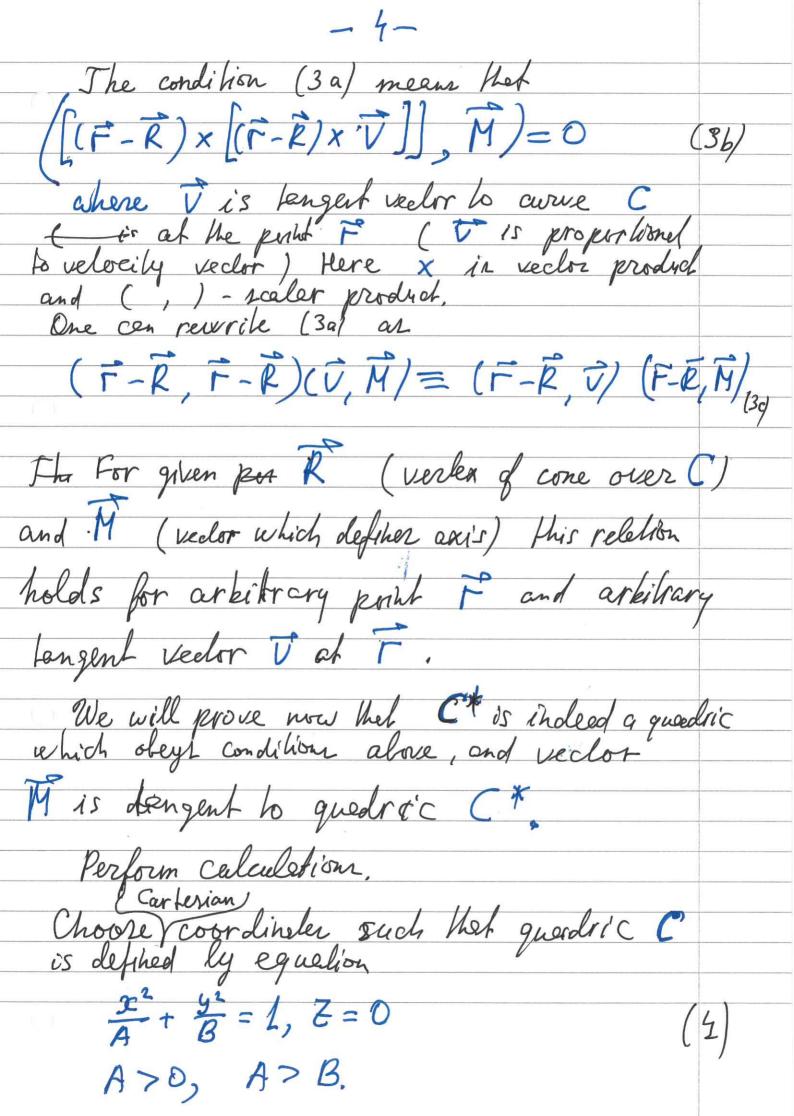
Vir directed along the axis of circuler come over (*) and the following relation holds. $(\vec{P}-\vec{R},\vec{F}-\vec{R})(\vec{v},\vec{V})=(\vec{F}-\vec{R},\vec{v})(\vec{F}-\vec{R},\vec{V})(2)$ If $C: \frac{x^2}{A} + \frac{y^2}{B} = 1 \cdot (A > 0, B < A), Z = 0$ then $C^*: \frac{x^2}{A-B} - \frac{Z^2}{B} = 1, y=0.$ (3) ((ellipse) = hyperbola, (hyperbola) = ellipse.

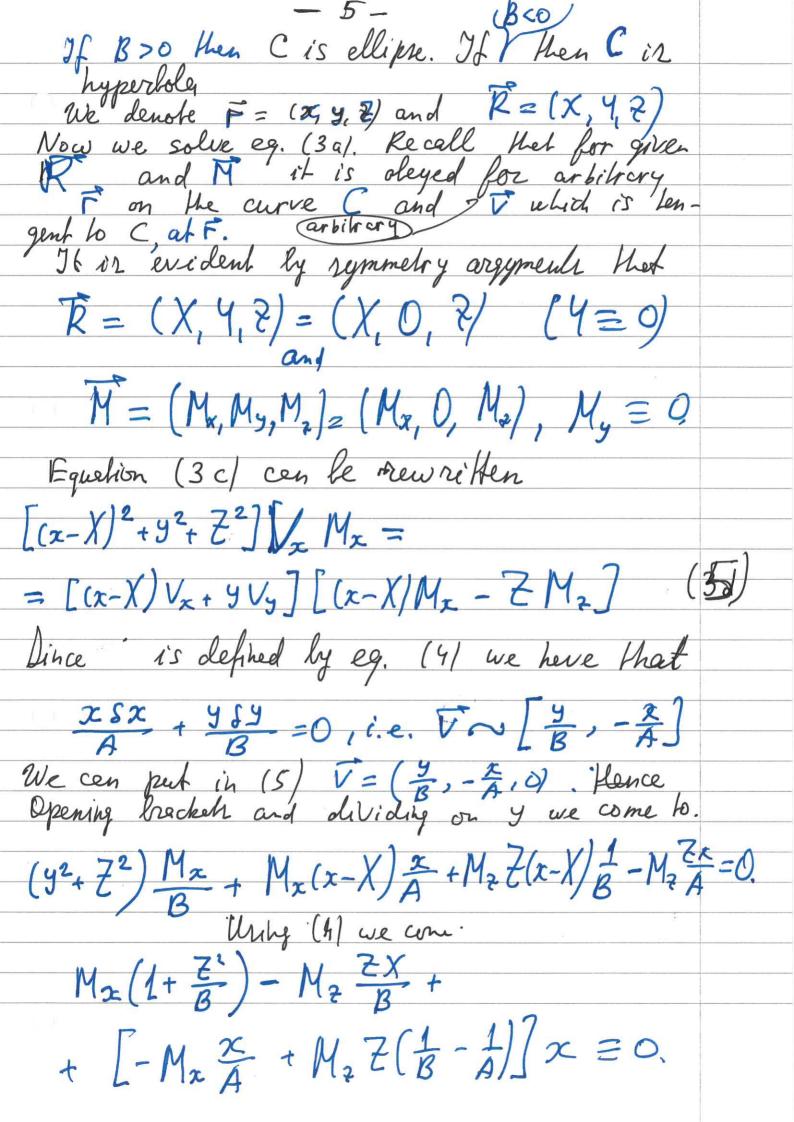
In the degenerale cere (C is perelola)
inver is analogous. The same:

Chir a perelole pranshy through Focus F

of perelola C. If C: 22 post x=py2, Z=0 $C^*: x = \frac{1}{4p} - p = 0$ mmmm Denote as in (2) by to an arlibrary push on quedric C and by R an arlibery publ on a curve C. [Elle Will do not know that

is a quedric). Let M be a vector
altached at the point R such that is paint
of poincular come over curve C. The condition that R is a verlex of circular cone and M is directed along exis means that for arhibrary F & C S F-R is orthogonal to M (3a)(where of it along curice C).





This relation holds for artificing x. (1+2) M2 - ZX My=0 1-XM2+Z(B-A)M=0. $\det \left(\frac{1+\frac{27}{B}}{-\frac{x}{A}}, \frac{2x}{2(\frac{1}{B}-\frac{1}{A})} \right) = 0$ $\left(1+\frac{2^{1}}{B}\right)\left(\frac{1}{B}-\frac{1}{A}\right)=\frac{X^{1}}{AB}$ $\frac{X^2}{A-R} - \frac{2^2}{B} = 1$ M = [Mx: 0: My] = [7x : 0:1+3) = $= \left[\frac{ZX}{B} : 0 : \frac{X^2}{A-B} \right] = \left[\frac{Z}{B} : 0 : \frac{X}{A-B} \right]$ We see that C'is quadric passing throug faci M is directed along largest Veclor I Tygle

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