Dear Pierre you remember about 15 years ago you told me about your teacher in Mathematics, (his name was something like Gudar...) who told you about relations between functions $\tan\theta$ and *sinhtheta*:

$$1+\tan^2\theta=\cos\theta \qquad\qquad 1+sinh^2\theta=cosh^2\theta \tag{1}$$

Why I recall this?

A week ago I considered not very common a realisation of hyperbolic plane: R^2 with induced Riemannian metric

$$G=(dx^2+dy^2-dz^2)\big|_{z=sqrt1+x^2+y^2}=\frac{(1+y^2)dx^2-2xydxdy+(1+x^2)dy^2}{1+x^2+y^2}$$

(this is upper sheet of hyperboloid $z^2-x^2-y^2=1$ with metric induced by metric $dx^2+dy^2-dz^2$ in $\mathbf{E}^{2.1}$)

One can see that

$$\text{lines } y=kx\,,\quad \text{hyperbolas } \frac{y^2}{p^2}-x^2=1$$

are geodesics of this metric.

One can see that these geodesics are rotations of geodesic $\begin{pmatrix} x=t\\ y=0\\ z=\sqrt{1+t^2}\end{pmatrix}$ on hyperboloid

Consider two geodesics

$$y=x\tan\theta\,,\quad \text{and}\quad \frac{y^2}{\text{sh}^2\theta}-x^2=1 \tag{1a}$$

These geodesics coincide on absolute—they are asimptotically the same.

The geodesic $y=x\tan\theta$ ($k=\tan\theta$) is rotation of geodesic $\begin{pmatrix} x=t\\ y=0\\ z=\sqrt{1+t^2}\end{pmatrix}$ on the angle θ:

$$\begin{pmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} t\\ 0\\ \sqrt{1+t^2}\end{pmatrix}$$

In the same way the geodesic $\frac{y^2}{\text{sh}^2\theta}-x^2=1$ is rotation of geodesic $\begin{pmatrix} x=t\\ y=0\\ z=\sqrt{1+t^2}\end{pmatrix}$ on the hyperbolic angle θ:

$$\begin{pmatrix}1 & 0 & 0\\ 0 & \text{cosh}\theta & \text{sinh}\theta\\ 0 & \text{sinh}\theta & \text{cosh}\theta\end{pmatrix}\begin{pmatrix} t\\ 0\\ \sqrt{1+t^2}\end{pmatrix}$$

We see the meaning of relation (1).

May be it is commonplace, I just wanted to mention it.