

If  $(p_1, p_2, \dots, p_n)$  are prime numbers then for every integers  $(a_1, a_2, \dots, a_n)$  there exists  $N$  such that  $N \equiv a_i \pmod{p_i}$ ,  $(i = 1, 2, \dots, n)$ . This is famous Chinese residual Theorem.

If  $(x_1, x_2, \dots, x_n)$  are  $n$  points on the line  $\mathbf{R}$  then for every real numbers  $(y_1, y_2, \dots, y_n)$  there exists polynomial  $P(x)$  such that  $P(x_i) = y_i$ ,  $(i = 1, \dots, n)$ . In fact one can choose unique polynomial of order  $\leq n$  obeying these conditions. This polynomial is:

$$P(x) = \sum_{i=1}^n \frac{\prod_{m \neq i} (x - x_m)}{\prod_{m \neq i} (x_i - x_m)} y_i$$

(We suppose that the points  $x_1, \dots, x_n$  are distinct)

E.g. for sets  $(x_1, x_2, x_3)$  and  $(y_1, y_2, y_3)$

$$P(x) = \frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)} y_1 + \frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)} y_2 + \frac{(x - x_1)(x - x_2)}{(x_3 - x_1)(x_3 - x_2)} y_3$$

(we suppose that  $x_1, x_2, x_3$  are distinct points

*what happens if they are not distinct?*)

One can see the striking resemblance between these two formulae considering the following idea which revolutionised the mathematics of XX century. A commutative algebra (ring) can be considered as algebra (ring) of functions on some set. An arbitrary integer  $N$  can be considered as a function on prime numbers with values in reminders. Using this idea one can see the striking resemblance between the Chinese formula and Lagrange polynomial approximation formula

In the same way we can write:

Let  $N$  be integer. We consider a function  $N(p_i) = \text{remainder when divided on } p_i$   
For example the number  $N = 124$  defines a function

$$N(2) = 0 \pmod{2}, \quad N(3) = 1 \pmod{3}, \quad N(5) = 4 \pmod{5}, \quad N(7) = 5 \pmod{7}, \quad N(11) = 3 \pmod{11}, \blacksquare$$