14 May 2016

We say that L^{ab} is $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ tensor of type $t = 0, 1$ if under changing of coordinates it transforms as

$$L^{ab} = (-1)^Z x^a_m L^{mn} x^b_n \,, \quad Z = Z(a, m) = am + (p(L) + t)a + (p(L) + t + 1)m \qquad (1)$$

Sure to see that this is well-defined one has to check that we do not fail in contradiction changing coordinates (we have to consider three coordinate systems...)

Fact If tensor L is of the type $t = 0$ then the decomposition

$$L^{ab} = \underbrace{\frac{1}{2}(L^{ab} + (-1)^{ab} L^{ba})}_{\text{symmetric}} + \underbrace{\frac{1}{2}(L^{ab} - (-1)^{ab} L^{ba})}_{\text{antisymmetric}}$$

is invariant under changing of coordinates.

In other words if type $t = 0$ tensor L^{ab} is symmetric in given coordinates

$$L^{ab} = (-1)^{ab} L^{ba}$$

then it remains symmetric in all coordinate systems, and the same for antisymmetric.

Respectively

If tensor L is of the type $t = 1$ then the decomposition

$$L^{ab} = \underbrace{\frac{1}{2}(L^{ab} - (-1)^{(a+1)(b+1)} L^{ba})}_{\text{“symmetric”}} + \underbrace{\frac{1}{2}(L^{ab} + (-1)^{(a+1)(b+1)} L^{ba})}_{\text{“antisymmetric”}} +$$

is invariant under changing of coordinates.

In other words if type $t = 1$ tensor L^{ab} is sheefted antisymmetric=“symmetric” in given coordinates

$$L^{ab} = -(-1)^{(a+1)(b+1} L^{ba} = (-1)^{ab+a+b} L^{ba}$$

then it remains sheefted antisymmetric=“symmetric” in all coordinate systems, and the same for “antisymmetric”.

Fact 2. If tensor L^{ab} belongs to class t then tensor $\mathcal{L}^{ab} = (-1)^a L^{ab}$ belongs to the class $t + 1$.

One can formulate the analogous statements for covaraint tensors:

We say that E_{ab} is $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ tensor of type $t = 0, 1$ if under changing of coordinates it transforms as

$$E_{ab} = (-1)^Z x^m_a E_{mn} x^n_b \,, \quad Z = Z(a, m) = am + (p(E) + t)a + (p(E) + t + 1)m \qquad (4)$$

Fact 1' If tensor E_{ab} is of type $t = 0$, and it is symmetric in given coordinates

$$E_{ab} = (-1)^{ab} E_{ba}$$

then it remains symmetric in all coordinate systems, and the same for antisymmetric.

Respectively

If tensor E is of the type $t = 1$, and it is sheefted antisymmetric="symmetric" in given coordinates

$$E_{ab} = -(-1)^{(a+1)(b+1)}(-1)^{ab+a+b} E_{ba}$$

then it remains "symmetric" in all coordinate systems, and the same for antisymmetric.

The transformation $E_{ab} \to (-1)^a E_{ab}$ chanes the type of the tensor.

And finally the **Fact 3**

If tensor L^{ab} belongs to class t and it is invertible, then the inverse tensor belongs to the class $t + p(L)$. Symmetrical tensor goes to symmetrical if it is even and to shifted symmetrical if it is odd)

In other words the tensor E^{ab} defining odd symplectic structure is an odd tensor of type $t = 0$. Its inverse is an odd "antisymmetrical" tensor of the type $t = 1$

Now revenons a nos moutons.

In our considerations we in fact cosnidered for Riemannian and sympelcti structure the tensor which looked the same, but the type was different!!!

We can to define Riemannian metric by the tensor $\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$ of the type $t = 1$

You was right when you told that 'you believe to formulae and do not believe to concrete expression $\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$.

For κ_{ab} we may take "symmetrical"=shifted antisymmetrical tensor $\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$ of the type $t = 1$ and using transformation $\kappa \to (-1)^a \kappa$ make from this the tensor $\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$ of the type $t = 0$.

We come to the fact that both symplectic and Riemannian structure are defined by the tensor which looks the same in special coordinates. For Riemannian structure it is the tensor of the type $t = 0$ and for ymplectic structure it is the tensor of the type $t = 1$.

Good?