Spirality of particles, Screw in mechanics and one invariant of motions in E^3

Consider affine space E^3, fix a point O (nachalo koordinat) we come to linear space IR^3. To arbitrary motion $\mathcal{A}$ in E^3 corresponds a pair (A, b) where A is orthogonal transformation of IR^3 and b is a vector: $x' = Ax + b$ If we consider another nachalo koordinat O' such that vector OO' is equal to r then to the motion $\mathcal{A}$ corresponds a pair (A, b'): $y' = Oy + b'$ where $x = y + r$ and

$$b' = b + Ar - r \tag{1}$$

At what extent vector b is defined iniquely for a given motion $\mathcal{A}$ under a changing of nachalo coordinate? Let $\mathbf{n}$ be axis corresponding to orthogonal operator A: linear transformation via A is rotation around axis $\mathbf{n}$, or in another words $\mathbf{n}$ is eigen vector of A: $A\mathbf{n} = \mathbf{n}$

It is easy to see from (1) that the component of vector b orthogonal to $\mathbf{n}$ can be vanished by suitable choice of r and component of b parallel to axis remains unchanged. Formally it follows from the fact that image of operator $A - \mathbf{id}$ is two-dimensional space ($\det(A - 1) = 0$) orthogonal to b.

We come to the invariant of motions: projection of b on $\mathbf{n}$.

Statement

Every motion of E^3 can be expressed as rotation over axis and translation along this axis, i.e. screw movement (vintovoje dvizhenije).

Shag vinta–projection of b on n is invariant.

Infinitezimally to this invariant corresponds invariant in affine algebra of rotations and translation: scalar product $\mathbf{Lp}$ of operators of angualr moment and moments.

This simple statement can be considered as geometrical interpretation of spirality...