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Hidden hyperbolicity

Let C be a circle in the plane and A, a point. Consider the locus K of the points K such that the length of the tangent from K to circle C is equal to the length of the segment KA:

$$M = \{K\colon \quad KA = \text{lenght of the tangent from } K \text{ to the circle.}\}$$

Then M is the line which is in the distance $\rho = \frac{1}{2}\left(a + \frac{1}{a}\right)$ from the centre of the circle C. (We suppose that the radius of the circle is equal to 1, and the point K is at the distance a and)

This is elementary problem, and it possesses the hidden hyperbolicity.

Indeed suppose first that $a < 1$.

Then conisder line l such that the circle is in the upper-half plane, and the point A is the centre of the hyperbolic circle C. Then due to lemma all the geodesics interesect the circle C under the right angle, i.e. the points of l belong to this locus.

If the $a > 1$ we take the point $a' = \frac{1}{a}$

Then one can see that for every point K of the locus M