PURE AND LOGIC GRADUATE LECTURES

This course counts as 33 hours of the taught component of a PGR programme.

Total Time: 22 lectures over 11 weeks (+ up to 11 tutorial sessions)
Academic Year: *2016/2017*
Course leader: *Tuomas Sahlsten*
Informal title: Additive combinatorics and ergodic methods in fractals

Unit co-ordinator: *Tuomas Sahlsten*
Teaching assistant (if applicable):

Purpose of the course

This course is an introduction to the sum-product theory in additive combinatorics by presenting the recently developed measure theoretical analogues by Kaimanovich, Vershik, Tao and Hochman. The theory has wide range of applications in the theory of random walks on groups, ergodic theory, geometric measure theory and fractal geometry. The methods presented here are actively studied in the field and so the course will present potential opportunities for new research directions.

We will concentrate on Hochman's inverse theorem for the entropy of convolutions (Annals of Maths. 180 (2014), no. 2, pp. 773–822) and discuss how Hochman's work leads to the solution of Furstenberg's projection problem of the 1-dimensional self-similar Sierpinski gasket, improvements on the Erdős problem on Bernoulli convolutions and Sinai's problem on iterated function systems contracting on average.

Potential audience

Graduate students in pure mathematics and logic.

Prerequisites

- *Mostly the understanding of basic measure theory is required. Moreover, knowledge of ergodic theory can be helpful but not necessary. Previous background on additive combinatorics may help in understanding the topic.*

Structure of the course

Lectures - 22 hours
Tutorials - 11 hours

Reading list

Most of the material is based on:

- *- M. Hochman: On self-similar sets with overlaps and inverse theorems for entropy. Annals of Mathematics 180 (2014), no. 2, pp. 773–822*
- *- M. Hochman: Self similar sets, entropy and additive combinatorics. Geometry and Analysis of Fractals, Springer Proceedings in Mathematics & Statistics Volume 88, 2014, pp. 225-252*
- ***Moreover, we will also explain briefly background and related topics from:***
- *- J. Bourgain: The discretized sum-product and projection theorems. Journal d'Analyse Mathématique 112 (2010), no. 1, pp. 193-236*
- *- H. Furstenberg: Ergodic fractal measures and dimension conservation. Ergodic Theory Dynam. Systems 28 (2008), no. 2, pp.405-422*

- *- P. Shmerkin: On the exceptional set for absolute continuity of Bernoulli convolutions. Geometric and Functional Analysis 24 (2014), no. 3, pp. 946-958*
- ***Helpful textbooks for some background:***
- *- K. Falconer: Fractal Geometry, John Wiley & Sons, Ltd, 2003.*
- *- P. Mattila: Geometry of Sets and Measures in Euclidean Spaces, Cambridge University Press, 1995*
- *- T. Tao, V. Vu: Additive Combinatorics, Cambridge University Press, 2006*

Assessment

2 assignments

Syllabus

1. Short overview of additive combinatorics
2. Entropy and its use in the study of uniformity
3. Additive combinatorics analogues for entropy (convolutions)
4. Multiscale analysis and convolution powers (Berry-Esseen theorem)
5. Growth of entropy of convolution powers (Kaimanovich-Vershik lemma)
6. Hochman’s inverse theorem
7. Applications of Hochman's inverse theorem
8. Random walks on groups (Bourgain-Gamburd theory) and related topics