GRADUATE COURSE FOR PHD STUDENTS

Elements of Quantum Mechanics for mathematicians

This course counts as 33 hours of the taught component of a PGR programme.

Total Time: 22 lectures over 11 weeks (+ 11 tutorial sessions)
Academic Year: 2018/2019

Course leader: Hovhannes Khudaverdyan

Unit co-ordinator: Hovhannes Khudaverdian

Purpose of the course

From very beginning, the development of Quantum Mechanics had very strong interrelation with the development of mathematics in XX century. Nowdays the knowledge of quantum mechanics is indispensable in many areas of mathematics. This course is an attempt to deliver the main aspects of Quantum Mechanics paying special attention to arising mathematical constructions. These include theory of spin and angular momentum, represenations of the group $SO(3)$, uncertainty principle, Fourier transform, and generalized functions, quasi-classical approximation, and elements of quantum logic.

Potential audience

Graduate students in pure applied mathematics and logic.

Prerequisites

A clear understanding of linear algebra and calculus is required. Some knowledge of elements of functional analysis and the Lagrangian and Hamiltonian formalism in classical mechanics is desirable.

Structure of the course

Lectures - 22 hours

Tutorials - 11 hours

Reading list

There are many excellent textbooks in Quantum Mechanics such as

— L.D. Landau, E.M. Lifshitz *Quantum Mechanics: Non-Relativistic Theory (Volume 3)*

— Leonard I. Schiff *Quantum Mechanics*

— Enrico Fermi *Notes on Quantum Mechanics.*

Books for further reading and special topics:

— C.Piron *Méchanique Quantique. Bases et applications*

—Leon A. Takhtajan: Stony Brook University, Stony Brook, NY, *Quantum Mechanics for Mathematicians*

Assessment

2 assignments. Every assignment will contain about 8-10 problems for two weeks.

Also at the end of every lecture it will be formulated 3-4 questions, and there solutions will be discussed during tutorials.

Syllabus

- Unitary space: complex linear space with Hermitian metric. Self-adjoint operators in a unitary space. States and observables in Quantum Mechanics. Measurement: commuting and non-commuting observables. Cauchy-Bunyakovsky-Schwarz inequality and Heisenberg uncertainty principle.
- Wave-function in Quantum Mechanics and action in Classical Mechanics. Schroedinger equation. Coordinate and momentum representations. Harmonic oscillator.
- Rotation and angular momentum. Spin of a particle. Irreducible representations of the group $SO(3)$
- Perturbation theory: abrupt and adiabatic perturbations. Adiabatic invariants in Quantum Mechanics and in Classical Mechanics.
- Quasiclassical approximation in Quantum Mechanics and Hamilton-Jacobi equation in Classical Mechanics. Fourier transform and Legendre transform. The idea of Maslov index.
- Elements of Quantum Logic. Modular lattice of questions in Quantum Mechanics and distributive lattice of questions in Classical Mechanics.