Equation $x = kx + 1$ has not a root if $k = 1$ and it has just one root if $k \neq 1$.

One can say that equation $x = kx + 1$ has one root on $\mathbf{RP}$ $(x = \infty)$ if $k = 1$ and it has two roots $x_1 = \infty, x_2 = \frac{1}{1-k}$ if $k \neq 1$.

It is diffiuclt to avoid temptation to compare it with quadratic equation:

$$\texttt{this equation has one root} \ \Leftrightarrow \texttt{if its discriminant vanishes} \ .$$

Make this vague statement exact.

For an arbitrary non-degenerate matrix $K = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$, $\det K = AD - BC \neq 0$, consider projective transfromations

$$F(x) = \frac{Ax + B}{Cx + D} \,.$$

of $\mathbf{RP}$* Equation $x = kx + 1$ can be considered as a fixed point of projective transformation with matrix $K = \begin{pmatrix} k & 1 \\ 0 & 1 \end{pmatrix}$:

$$x = F(x) \qquad (1a)\,.$$

Consider this eqution in new coordinates: let u be a new affine coordinate:

$$u = \frac{mx + n}{px + q}\,, x = \frac{au + b}{cu + d}\,, \left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} m & n \\ p & q \end{pmatrix} = 1 \right)$$

Then projective transformation $F(x) = kx + 1$ in new coordinate u will have the appearance

$$\tilde{F}(u) = K \circ F \circ K^{-1} = K \left(k \frac{au + b}{cu + d} + 1 \right) \frac{mk \frac{au+b}{cu+d} + n}{pk \frac{au+b}{cu+d} + q} = \frac{Au + B}{Cu + D}\,.$$

In the case if $C \neq 0$, then the number of fixed points of this transformation is defined by the discriminant of the equation

$$\frac{Au + B}{Cu + D} = u \Leftrightarrow Cu^2 - (A - D)u - BD = 0\,.$$

The discriminant of this equation is equal to

$$\mathcal{D} = (A - D)^2 + 4BC = \mathrm{Tr}\mathcal{F}'^{\,2} - 4 \det \mathcal{F}'\,.$$

For initial matrix $\mathcal{F} = \begin{pmatrix} k & 1 \\ 0 & 1 \end{pmatrix}$, and

$$k = 1 \Leftrightarrow \mathcal{D} = (k - 1)^2 = 0\,.$$

* In homogeneous coordinates: $[x : y] \mapsto [x' : y'] = F\left([x : y]\right) = [Ax + By : Cx + Dy]$.