**Thick morphisms and Green function**

Fact: action $\mathcal{S} = \mathcal{S}(x, y)$ is classics of Green function

Let a function $S = S(x, q)$ defines classical thick morphism and $S_\hbar = S_\hbar(x, q)$ defines corresponding quantum thick morphism then

$$G(t, x, y) = \int dq e^{\frac{i}{\hbar} S(x,q) - yq} \tag{1}$$

defines Green function such that

$$\partial_t G + \Delta G = 0? \tag{1a}$$

The classical analog of formula (1) is the formula

$$W(t, x, y) = S(x, q) - yq \quad \text{at } y = S_q \,.$$

It is just classical action.

One can ask the question: how to calculate (1a) To do it we use fantastic formula from de Witt: $W(t, x, y) = \sigma(t, x, y)$ is the length of geodesic One can see that $\sigma$ and $\partial_\mu \sigma$ tend to zero if $y \to x$. This implies that

$$\partial^2 \sigma(x, y) \partial x^u \partial x^v \big|_{x \to y} = g_{\mu\nu}$$

and so on (see de Witt)