Homework 7

1 Let C be an ellipse in the plane $\mathbf{E}^2$ such that its foci are at the points $F_1 = (-1, 0)$ and $F_2 = (1, 0)$ and it passes through the point $K = (0, 2)$.

Write down the analytical formula which defines this ellipse in Cartesian coordinates (x, y), i.e. the equation defining this ellipse in Cartesian coordinates (x, y).

Find the area of this ellipse.

2 Let C be an ellipse in the plane $\mathbf{E}^2$ such that its foci are at the points $F_1 = (-5, 0)$, $F_2 = (16, 0)$. It is known that the point $K = (0, 12)$ belongs to the ellipse.

Write down the equation which defines this ellipse in Cartesian coordinates (x, y).

Find intersection of this ellipse with OX and OY axis.

Find the area of this ellipse.

3 Let H be hyperbola in the plane $\mathbf{E}^2$ such that it passes through the point $P = (2, 3)$, and its foci are at the points $F_{1,2} = (\pm 2, 0)$,

Find the intersection points of the hyperbola with OX axis.

Write down the analytical formula which defines this hyperbola, i.e. the equation defining this hyperbola in Cartesian coordinates (x, y).

Explain why this hyperbola does not intersect the axis OY.

b) Let H be hyperbola in the plane $\mathbf{E}^2$ such that it passes through the point $P = (3, 2)$, and its foci are at the points $F_{1,2} = (0, \pm 2)$,

Compare this question with the previous one.

Write down the analytical formula which defines this hyperbola.

4 Consider in the plane the curves C_1, C_2 and C_3 which are given in some Cartesian coordinates (x, y) by equations $C_1\colon \quad 4x^2 + 4x + y^2 = 0$, $C_2\colon \quad 4x^2 + 4x - y^2 = 0$,

$C_3\colon \quad 4x^2 + 4x + y = 0$.

Show that C_1 is an ellipse, C_2 is a hyperbola, and C_3 is a parabola.

5 Let H be hyperbola considered in the exercise **3**.

Consider in the plane $\mathbf{E}^2$ the ellpise such that it passes through the foci of the hyperbola H, and its foci are at the points where the hyperbola H intersects axis OX. Write down the equation of this ellipse.

6 The ellipse C on the plane $\mathbf{E}^2$ has foci at the vertices $A = (-1, -1)$ and $C = (1, 1)$ of the square $ABCD$, and it passes through the other two vertices $B = (-1, 1)$ and $D = (1, -1)$ of this square.

Find new Cartesian coordinates (u, v) (express them via initial coordinates (x, y)) such that the ellipse C has canonical form $C\colon \quad \frac{u^2}{a^2} + \frac{v^2}{b^2} = 1$ in these coordinates.

Write down the equation of ellipse C in initial Cartesian coordinates (x, y)

Calculate the area of this ellipse.

7 Consider a curve defined in Cartesian coordinates (x, y) by the equation

$$C: \quad px^2 + py^2 + 2xy + \sqrt{2}(x + y) = 0\,,$$

where p is a parameter.

How looks this curve

if $p > 1$? if $p = 1$? if $-1 < p < 1$? if $p = -1$? if $p < -1$?

Find an affine transformation

$$\begin{cases} x = au + bv + e \\ y = cu + dv + f \end{cases}, \qquad \left(\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} \neq 0\right) \tag{1}$$

which transforms this curve to the circle $u^2 + v^2 = 1$ in the case if $p > 1$

8* (`pursuit problem`) Consider two point in the plane $\mathbf{E}^2$, A, and B. Let point A starts moving at the origin, and moves along OY with constant velocity v: $\begin{cases} x = 0 \\ y = vt \end{cases}$.

Let point B starts moving at the point $(L, 0)$, its speed is equal also to v, and velocity vector is directed in the direction to the particle A.

Of course the particle B never will reach the particle A because their speeds are the same. On the other hand the particle B asymptotically will be tended to vertical axis. What is the distance between these particles at $t \to \infty$?

Hint: Consider the reference frame in which particle A is not moving, i.e. consider coordinates $\begin{cases} x' = x \\ y' = y + vt \end{cases}$.

Show that in these coordinates the trajectory of particle B will be a parabola.