Consider the following example.

Let f_a $a = 1, \dots, n$ be a set of functions on $(0, 1)$ and let

1) Λ be an algebra of smooth functions on R

2) Λ_0 an algebra of smooth functions f such that *supp* $f \subseteq [0, 1]$

Consider a module P (P_0) generated by f_a with coefficients in Λ (Λ_0)

Then P and P_0 are projective modules and P is free.

Sophisticated explanation why P is free is following: P (P_0) corresponds to module of global sections of fibre bundle that is subbundle of trivial fibre bundle $[0, 1] \times R^n$ $(S^1 \times R^n)$. All bundles over discs are trivial.

It is really funny exercise to construct embedding ι of module P (P_0) in free module E^n with generators $\{e_a\}$ such that this embedding splits module E^n on projective modules:

$$\iota P \sum \ker \Pi = E$$

where $\Pi f_a = e_a$

In general case projective module P_0 is not free.

For example if f_a $(a = 1, 2)$ are two functions such that $f_a(0) + f_a(1) = 0$ and these functions are independent (e.g. $f_1 = \cos \pi x$ $f_2 = \sin \pi x$) then P_0 is projective not free module of global sections of Mobius strip.