Homework 7

1 A point moves in $\mathbf{E}^2$ along an ellipse with the law of motion $x = a\cos t$, $y = b\sin t$, $0 \le t < 2\pi$, $(0 < b < a)$. Find the velocity and acceleration vectors. Find the points of the ellipse where the angle between velocity and acceleration vectors is acute. Find the points where speed attains its maximum value.

2 Find a natural parameter for the following interval of the straight line:

$$C\colon \begin{cases} x = t \\ y = 2t + 1 \end{cases}, \quad 0 < t < \infty\,.$$

3 Consider the following curve (a helix): $\mathbf{r}(t)\colon \quad \begin{cases} x(t) = R\cos\Omega t \\ y(t) = R\sin\Omega t \\ z(t) = ct \end{cases}$.

Find velocity and acceleration vectors of this curve.

Find a natural parameter of this curve.

What can you say about the acceleration of this curve?

4 Calculate the curvature of the parabola $x = t, y = mt^2$ $(m > 0)$.

Let s be a natural parameter on this parabola. Show that $\int_0^\infty k(s)ds = \int_0^\infty k(t)|\mathbf{v}(t)|dt$ and calculate this integral.

5 Consider the parabola

$$\mathbf{r}(t)\colon \begin{cases} x = v_x t \\ y = v_y t - \frac{gt^2}{2} \end{cases}.$$

(It is path of the point moving under the gravity force with initial velocity $\mathbf{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$.)

Calculate the curvature at the vertex of this parabola.

6 Consider the ellipse $x = a\cos t, y = b\sin t$ $(a, b > 0,\ 0 \le t < 2\pi)$ in $\mathbf{E}^2$. Calculate the curvature $k(t)$ of this ellipse.

† Calculate $\int k(s)ds$ over the ellipse, where s is a natural parameter.

† Find the radius of a circle which has second order touching with the ellipse at the point $(0, b)$.

7 Find the curvature of the helix considered in Exercise 3.

8 Calculate the curvature of the following curve (latitude on the sphere)

$$\begin{cases} x = R\sin\theta_0\cos\varphi(t) \\ y = R\sin\theta_0\sin\varphi(t) \\ z = R\cos\theta_0 \end{cases}, \text{ where } \varphi(t) = t, 0 \le t < 2\pi\,.$$

9† Show that the curvature of an arbitrary curve on the sphere of the radius R is greater or equal to $\frac{1}{R}$.