Example of normal gauge

Consider curve

$$y = ax + \frac{1}{2}bx^2$$

Then

$$y' = x\sin\theta + y\cos\theta = x\sin\theta + ax\cos\theta + \ldots = x(\sin\theta + a\cos\theta) + \ldots$$

We fix:

$$\theta\text{:} \quad \sin\theta + a\cos\theta = 0\,,$$

then

$$y' = x\sin\theta + y\cos\theta = x\sin\theta + \left(ax + \frac{1}{2}bx^2\right)\cos\theta + \ldots == \frac{1}{2}bx^2\cos\theta =$$

$$\frac{1}{2}b(x'\cos\theta + y'\sin\theta)^2\cos\theta = \frac{1}{2}bx'^2\cos^3\theta\,.$$

We have

$$\mathrm{tg}\,\theta = -a \Rightarrow \cos\theta = \left(1 + a^2\right)^{-\frac{1}{2}}\,.$$

Hence

$$y' = \frac{1}{2}bx'^2\cos^3\theta = \frac{1}{2}\frac{b}{(1+a^2)^{\frac{3}{2}}}x'^2$$

Thus curvature is equal to

$$k = \frac{b}{(1+a^2)^{\frac{3}{2}}} = \frac{f_{xx}}{(1+f_x^2)^{\frac{3}{2}}}\,.$$