Today I have began to understand the synthetic definition of horocycle:

Let Π be a pencil of lines which are parallel to each other in the sqme direction. Consider an isometry F such that it sends every line of this pencil to another (maybethe smae?) line of this pencil. Let l be an arbitrary line of the pencil Π , and let A be an arbitrary point on l Since F is isometry then the line between the point A and its image the point A' has the SAME ANGLES with lines l and l' = F(l).

Let a line r does not change under this transformation:

$$\forall A \in r, F(A) \in r$$

The line r is called HOROCYCLE