I began to reread Sfarevitch-Nikilin about set of locally euclidean geometries. The following lemma: let $AB, A'B'$ are two segments of the same length. Then there exists a rotation or translation F such that $A' = F(A), B' = F(B)$. In the special cases the solution is obvious. (E.g. if $A = A'$ then this is just rotation of the segment)

This is why it is interesting to see how solution changes under the transformation

$$A \mapsto A + \mathbf{x}, \qquad B \mapsto B = \mathbf{x} \tag{1}$$

and less obvious under the transformation

$$A \mapsto B, B \mapsto A \tag{2}$$

The second is not trivial, e.g. if AB, $A'B'$ are paralell, then the parallel transport F transforms AB into $A'B'$, however if we replace $A' \leftrightarrow B'$ then we have to do the rotation on the angle π around the vertex D , where D is the centre of the parallelogram $ABA'B'$

This is childish exercise, but......

Let A, B be two points, and A', B' be another two points. Does there exist a rotation F such that $F\colon F(A) = A', F(B) = B'$?

This is evident that

$$|AB| = |A'B'|\,.$$

is necessary condition. Is it sufficient. Yes, it is.

This is not absolutely obvious in the case if directions $AB, A'B'$ are not the 'same', e.g. if $A = (1,0), B = (2,0), A' = (0,2), B' = (0,1)$.

One can clear the question geometrically, but let us try to do it using *brute force.*

WLOG suppose that $A = (0,0), B = (1,0)$, and let A', B' be two points, $A' = (a_1, a_2), B' = (b_1, b_2)$. Let $O = (x, y)$ is the centre of rotation F on the angle φ.

Then

$$\begin{cases} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} 0-x \\ 0-y \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \\ \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} 1-x \\ 0-y \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \end{cases}$$

i.e.

$$\begin{cases} \begin{pmatrix} 1-\cos\varphi & \sin\varphi \\ -\sin\varphi & 1-\cos\varphi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \\ \begin{pmatrix} 1-\cos\varphi & \sin\varphi \\ -\sin\varphi & 1-\cos\varphi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b_1 - \cos\varphi \\ b_2 - \sin\varphi \end{pmatrix} \end{cases}$$

i.e. if $\varphi \neq 0, \pi$ then

$$b_1 = a_1 + \cos\varphi, b_2 = a_2 + \sin\varphi$$

THus we define the angle.

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -\mathrm{cotan}\,\frac{\varphi}{2} \\ \mathrm{cotan}\,\frac{\varphi}{2} & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -\mathrm{cotan}\,\frac{\varphi}{2} \\ \mathrm{cotan}\,\frac{\varphi}{2} & 1 \end{pmatrix} \begin{pmatrix} b_1 - \cos\varphi \\ b_2 \end{pmatrix}$$