Homework 7

1 Let C be an ellipse in the plane $\mathbf{E}^2$ such that its foci are at the points $F_1 = (-1, 0)$ and $F_2 = (1, 0)$ and it passes through the point $K = (0, 2)$.

Write down the analytical formula which defines this ellipse.

Find the area of this ellipse.

2 Let C be an ellipse in the plane $\mathbf{E}^2$ such that its foci are at the points $F_1 = (-5, 0)$, $F_2 = (16, 0)$. It is known that the point $K = (0, 12)$ belongs to the ellipse.

Write down the analytical formula which defines this ellipse.

Find intersection of this ellipse with OX and OY axis.

Find the area of this ellipse.

3 Let H be hyperbola in the plane $\mathbf{E}^2$ such that it passes through the point $P = (3, 2)$, and its foci are at the points $F_1 = (0, 2)$, $F_2 = (0, -2)$. Find the intersection points of the hyperbola with OY axis.

Explain why this hyperbola does not intersect the axis OX.

4 Consider in the plane the curves C_1, C_2 and C_3 which are given in some Cartesian coordinates (x, y) by equations $C_1\colon\quad 4x^2 + 4x + y^2 = 0$, $C_2\colon\quad 4x^2 + 4x - y^2 = 0$,

$C_3\colon\quad 4x^2 + 4x + y = 0$.

Show that C_1 is an ellipse, C_2 is a hyperbola, and C_3 is a parabola.

5 Let H be hyperbola considered in the exercise **3**.

Consider in the plane $\mathbf{E}^2$ the ellpise such that it passes through the foci of the hyperbola H, and its foci are at the points where the hyperbola H intersects axis OY. Write down equation of this ellipse.

6 The ellipse C on the plane $\mathbf{E}^2$ has foci at the vertices $A = (-1, -1)$ and $C = (1, 1)$ of the square $ABCD$, and it passes through the other two vertices $B = (-1, 1)$ and $D = (1, -1)$ of this square.

Find new Cartesian coordinates (u, v) (express them via initial coordinates (x, y)) such that the ellipse C has canonical form $C\colon\quad \frac{u^2}{a^2} + \frac{v^2}{b^2} = 1$ in these coordinates.

Write down the equation of ellipse C in initial Cartesian coordinates (x, y)

Calculate the area of this ellipse.

7 Consider a curve defined in Cartesian coordinates (x, y) by the equation

$$C\colon\quad px^2 + py^2 + 2xy + \sqrt{2}(x + y) = 0\,,$$

where p is a parameter.

How looks this curve

if $p > 1$? if $p = 1$? if $-1 < p < 1$? if $p = -1$? if $p < -1$?

Find an affine transformation

$$\begin{cases} x = au + bv + e \\ y = cu + dv + f \end{cases}, \qquad \left(\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} \neq 0\right) \tag{1}$$

which transforms this curve to the circle $u^2 + v^2 = 1$ in the case if $p > 1$

8^* (`pursuit problem`) Consider two point in the plane $\mathbf{E}^2$, A, and B. Let point A starts moving at the origin, and moves along OY with constant velocity v: $\begin{cases} x = 0 \\ y = vt \end{cases}$.

Let point B starts moving at the point $(L, 0)$, its speed is equal also to v, and velocity vector is directed in the direction to the particle A.

Of course the particle B never will reach the particle A because their speeds are the same. On the other hand the particle B asymptotically will be tended to vertical axis. What is the distance between these particles at $t \to \infty$?

Hint: Consider the reference frame in which particle A is not moving, i.e. consider coordinates $\begin{cases} x' = x \\ y' = y + vt \end{cases}$.

Show that in these coordinates the trajectory of particle B will be a parabola.