

Even In most refined representation of Clifford algebra, it is difficult to bury the traces of supermathematics, i.e. Z_2 grading. The following lemma

$$\mathbf{Cliff}((Q_1, V_1)) \oplus (Q_2, V_2) = \mathbf{Cliff}(Q_1, V_1) \hat{\otimes} \mathbf{Cliff}(Q_2),$$

where $\mathbf{Cliff}(Q, V)$ is the Clifford algebra corresponding to vector space V equipped with bilinear form Q , and $\hat{\otimes}$ is Z_2 graded tensor product, (not usual!) It is very important for the classification theorem

Clifford algebra $\mathbf{Cliff}(Q, V)$ is isomorphic to wedge product of p algebras of double numbers $(a + b\varepsilon, \varepsilon^2 = 1)$, $r - p$ algebras of complex numbers $(a + b\varepsilon, \varepsilon^2 = -1)$, and $n - r$ algebras of dual numbers $(a + b\varepsilon, \varepsilon^2 = 0)$, where r is the rank of the form Q , $(p, r - p)$ is the signature of the form, n is the dimension of vector space V , i.e. p, q, r are integers such that quadratic form Q in V can be reduced to the form

$$Q = x_1^2 + \dots + x_p^2 - x_{p+1}^2 - \dots - x_r^2$$

in some linear coordinates

This lemma reveals Z_2 grading.

When proving this lemma we use that element $\mathbf{x} \oplus \mathbf{y}$ goes to the element $\mathbf{x} \otimes 1 + 1 \otimes \mathbf{y}$

$$(\mathbf{x} \otimes 1)(1 \otimes \mathbf{y}) = -(\mathbf{x} \otimes 1)(1 \otimes \mathbf{y})$$

hence

$$((\mathbf{x} \otimes 1) + (1 \otimes \mathbf{y}))^2 = (\mathbf{x} \otimes 1)^2 + (1 \otimes \mathbf{y})^2.$$