Some amasing formulas

These days I study Hilbnert Theorem of non-immersion of Lobachevsky plane in $\mathbf{E}^3$*. To my surprise it has so many mathphysical applications. Here I just give some very preliminary results. (see the file in Etudes/Geometry/Lobachevsky)*

Consider on $\mathbf{R}^2$ Riemannian metric

$$G = dx^2 + 2cos\Theta(x,y)dxdy + dy^2$$

This metric naturally appears in the proof of Hilbert Theorem.

Performing routine calculations we come to

$$\Gamma^x_{xx} = \Theta_x \mathrm{cotan}\,\Theta\,, \quad \Gamma^y_{xx} = -\Theta_x \frac{1}{\sin\Theta}\,,$$

$$\Gamma^y_{yy} = \Theta_y \mathrm{cotan}\,\Theta\,, \quad \Gamma^x_{yy} = -\Theta_y \frac{1}{\sin\Theta}\,,$$

and

$$\Gamma^x_{xy} = \Gamma^x_{yx} = \Gamma^y_{xy}\Gamma^y_{yx} = 0$$

and

$$K = \frac{R}{2} = R_{1212}\det g = \frac{-\Theta_{xy} sin\Theta}{1-\cos^2\Theta} = \frac{-\Theta_{xy}}{\sin\Theta}\,,$$

and in particular the condition

$$\Theta_{xy} = \sin\Theta$$

means that this manifold is (whole?) Lobachesky plane (this function has to exist)

E.g. function

$$\theta = arctan Ce^(x+y)$$