, .

Pencil of self-conjugate oeprators and conformal Laplacian

Consider pencil of Beltrami Laplace operators of weight δ,

$$\{\Delta_\mu\}\,,\Delta_\mu\colon \mathcal{F}_\mu \mapsto \mathcal{F}_{\mu+\delta}\,, \text{for an arbitrary } \mathbf{s}\in\mathcal{F}_\mu\,, \Delta_\mu(\mathbf{s}) = \rho^{\mu+\delta}\left(\Delta_{L.B.}\left(\frac{\mathbf{s}}{\rho^\mu}\right)\right)$$

We concetrate later on the case $\delta=\frac{2}{n}$, when principal symbol is invariant of conformal symmetries, but for BV it will be interesting to see the general....

$$\{\Delta_\mu\}\,,\Delta_\mu\colon \mathcal{F}_\mu \mapsto \mathcal{F}_{\mu+\frac{2}{n}}\,, \text{for an arbitrary } \mathbf{s}\in\mathcal{F}_\mu\,, \Delta_\mu(\mathbf{s}) = \rho^{\mu+\frac{2}{n}}\left(\Delta_{L.B.}\left(\frac{\mathbf{s}}{\rho^\mu}\right)\right)$$

where $\Delta_{L.B}$ is usual Laplace-Beltrami oeprator on functions

Now according general scheme, thake the singular point of this penci. We come to Laplacian of weight $\sigma=\frac{2}{n}$ acting on densities of weight $\lambda=\frac{1}{2}-\frac{1}{n}$. This is the Laplacian

$$\Delta_{\frac{1}{2}-\frac{1}{n}}(\mathbf{s}) = \rho^{\frac{1}{2}+\frac{1}{n}}\left(\Delta_{L.B.}\left(\frac{\mathbf{s}}{\rho^{\frac{1}{2}-\frac{1}{n}}}\right)\right)\,.$$

According the general scheme, this Laplacian has the form:

$$\Delta_{\frac{1}{2}-\frac{1}{n}}(\mathbf{s}) = \rho^{\frac{2}{n}}\left[\left(\partial_a\left(g^{ab}\partial_b s(x)\right)\right)+U(x)\right]\,, \text{where } \rho=\sqrt{\det G}|Dx|\,.$$

Theorem?? Consider cocyle

$$C(\tilde g, g) = \Delta(\tilde g) - \Delta(g)\,,$$

where $\tilde g = e^\sigma g$ is Weyl transformation of Riemannian metric

This cocycle takes values in scalar densities, and it is coboundary if $n>1$.

$$C(\tilde g, g) =$$

$$U(x) = U_G(x) = c\frac{2-n}{n-1}\left(\rho(\tilde g)R(\tilde g) - \rho(g)R(g)\right)\,. \qquad (c=4???)$$

In the case if $n=1$ we come to Schwarzian deriddvative.