Homework 9

1. Calculate the shape operator for the sphere $x^2 + y^2 + z^2 = R^2$:

$$\mathbf{r}(\varphi, \theta) \qquad \begin{cases} x = R \sin\theta \cos\varphi \\ y = R \sin\theta \sin\varphi \\ z = R \cos\theta \end{cases} .$$

Calculate principal curvatures, Gaussian and mean curvature for this sphere.

2. Calculate the shape operator for the cylinder $x^2 + y^2 = R^2$:

$$\mathbf{r}(h, \varphi) \qquad \begin{cases} x = R \cos\varphi \\ y = R \sin\varphi \\ z = h \end{cases} .$$

Calculate principal curvatures, Gaussian and mean curvature for this cylinder.

† What values the normal curvature of an arbitrary curve on the cylinder of radius R can take? (You may consider first horizontal circle, vertical line and helix.)

3. Calculate the shape operator for the cone $x^2 + y^2 - k^2 z^2 = 0$:

$$\mathbf{r}(h, \varphi) \qquad \begin{cases} x = kh \cos\varphi \\ y = kh \sin\varphi \\ z = h \end{cases} .$$

Calculate principal curvatures, Gaussian and mean curvature for this cone.

4 † Calculate the shape operator, Gaussian and mean curvature for the surface

$$\mathbf{r}(u, v) \qquad \begin{cases} x = u \\ y = v \\ z = F(u, v) \end{cases} . \tag{1}$$

at the point $u = v = 0$ in the case if $F(u, v) = Au^2 + 2Buv + Cv^2$.

Consider the special case if $F(u, v) = uv$ (the surface is "saddle").

5 Assume that the action of the shape operator at the tangent coordinate vectors $\mathbf{r}_u = \partial_u$, $\mathbf{r}_v = \partial_v$ at the given point $\mathbf{p}$ of the surface $\mathbf{r} = \mathbf{r}(u, v)$ is defined by the relations: $S(\partial_u) = 2\partial_u + 2\partial_v$ and $S(\partial_v) = -\partial_u + 5\partial_v$. Calculate principal curvatures, Gaussian and mean curvatures of the surface at this point.

6 † Let A and B be two given points on the sphere with radius R with spherical coordinates $\{\theta_A, \varphi_A\}$ and $\{\theta_B, \varphi_B\}$.

a†) Show that the shortest curve joining these points is the arc of the great circle.

b†) Find the length of this curve.