

Why the function multivalued function $\arctan \frac{y}{x}$ appeared often in calculations?
Because this function is conjugate to $\log \sqrt{x^2 + y^2}$:

$$\log z = \log(x + iy) \log \sqrt{x^2 + y^2} + \arctan \frac{y}{x}$$