Comments on some questions

General comment: It is very useful to use Homeworks and Solutions of Homeworks.

Comments to the question 2b *

1. 3-dimensional sphere is a sphere $x^2 + y^2 + z^2 + t^2 = R^2$ in $\mathbf{E}^4$. (Do not confuce with 3-dimensional ball $x^2 + y^2 + z^2 \leq 3$ in $\mathbf{E}^3$!!!).

To calculate its volume you may use one of the following methods:

a) calculate volume in stereographic coordinates, where Riemannian metric is given by formula

$$G = \frac{4R^2(du^2 + dv^2 + dt^2)}{(R^2 + u^2 + v^2 + t^2)^2}$$

u, v, t are cartesian coordinates on $\mathbf{R}^3$, (see Homework 2). Calculations are very similar to those in the Exercise 6 of Homework 3, just now you need to calculate the integral $\int \sqrt{\det G} dudvdt$ over 3-dimensional space. In the case of calculation of area of 2-dimensional sphere in stereographic coordinates you calculated integral $\int \sqrt{\det G} dudv$ in polar coordinates $dudv = rdrd\varphi$ (see solutions of Homework 3). Now you may do calculations of integral $\int \sqrt{\det G} dudvdt$ in spherical coordinates: $dudvdt = r^2 \sin^2 \theta d\theta d\varphi$. In this exercise it is most important to reduce the answer to the standard integral. Do not worry if you cannot calculate this standard integral.

b) You may calculate the volume of 3-dimensional sphere using exercise 10 (and its Solution) from the Homework 3.

Comments to the question 2c

In the solution of this question it may be useful to use the fact that if plane is given by the equation $ax + by + cz = \rho$ and $a^2 + b^2 + c^2 = 1$, then the distance between origin and the plane is equal to $|\rho|$.

Comments to the question 4b)

You may use without proof the fact that shortest distance between two points on the sphere is an arc of the great circle.