Homework 8

1. Find coordinate basis vectors, first quadratic form, unit normal vector field, shape operator and Gaussian and mean curvatures for

a) sphere of the radius R: $x^2+y^2+z^2=R^2$,

$$\mathbf{r}(\theta,\varphi)\qquad \begin{cases} x=R\sin\theta\cos\varphi \\ y=R\sin\theta\sin\varphi \\ z=R\cos\theta \end{cases}\qquad (0\leq\varphi<2\pi, 0\leq\theta\leq\pi)\,,$$

b) cylinder $x^2+y^2=R^2$,

$$\mathbf{r}(h,\varphi)\qquad \begin{cases} x=R\cos\varphi \\ y=R\sin\varphi \\ z=h \end{cases}\qquad (0\leq\varphi<2\pi, -\infty<h<\infty)$$

c) cone $x^2+y^2-k^2z^2=0$,

$$\mathbf{r}(h,\varphi)\qquad \begin{cases} x=kh\cos\varphi \\ y=kh\sin\varphi \\ z=h \end{cases}\qquad (0\leq\varphi<2\pi, -\infty<h<\infty)$$

d) saddle $F=xy$ (you may perform the calculations only at origin).

$$\mathbf{r}(u,,v)\qquad \begin{cases} x=u \\ y=v \\ z=uv \end{cases}\qquad (-\infty<u<\infty, -\infty<v<\infty)$$

2. Consider helix $\mathbf{r}(t)$: $\begin{cases} x(t)=a\cos t \\ y(t)=a\sin t \\ z(t)=ct \end{cases}$. Show that this helix belongs to cylinder surface $x^2+y^2=a^2$. Using first quadratic form on the surface of cylindre or in a different way calculate length of the helix $(0\leq t\leq t_0)$.

3 Assume that the action of the shape operator at the tangent coordinate vectors $\mathbf{r}_u=\partial_u$, $\mathbf{r}_v=\partial_v$ at the given point $\mathbf{p}$ of the surface $\mathbf{r}=\mathbf{r}(u,v)$ is defined by the relations: $S(\partial_u)=2\partial_u+2\partial_v$ and $S(\partial_v)=-\partial_u+5\partial_v$. Calculate principal curvatures, Gaussian and mean curvatures of the surface at this point.

4 On the sphere of the radius $x^2+y^2+z^2=R^2$ in E^3 consider the triangle ABC with vertices at the North Pole and at Equator: $A=(0,0,R)$, $B=(R,0,0)$ and $C=(R\cos\varphi, R\sin\varphi, 0)$. The edges of this triangle are arcs of the meridians and the arc of the Equator.

Find the result of the parallel transport of vector $\mathbf{X}=\mathbf{e}_x$ attached at the North pole along the edges of the triangle ABC.

5† On the sphere $x^2+y^2+z^2=R^2$ in $\mathbf{E}^3$ consider the closed curve $\theta=\theta_0, \varphi=t$, $0\leq t<2\pi$ (latitude) Find the result of parallel transport of the vector tangent to the sphere along this curve.