Homework 8

1) Let M be a surface embedded in Euclidean space $\mathbf{E}^3$. We say that the triple of vector fields $\{\mathbf{e},\mathbf{f},\mathbf{n}\}$ is adjusted to the surface M if $\mathbf{e},\mathbf{f},\mathbf{n}$ be three vector fields defined on the points of this surface such that they form an orthonormal basis at any point, so that the vectors $\mathbf{e},\mathbf{f}$ are tangent to the surface and the vector $\mathbf{n}$ is orthogonal to the surface.

Consider the derivation formulae for adjusted vector fields $\{\mathbf{e},\mathbf{f},\mathbf{n}\}$:

$$d\begin{pmatrix}\mathbf{e}\\ \mathbf{f}\\ \mathbf{n}\end{pmatrix}=\begin{pmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{pmatrix}\begin{pmatrix}\mathbf{e}\\ \mathbf{f}\\ \mathbf{n}\end{pmatrix}, \tag{1}$$

where a,b,c are 1-forms on the surface M.

Write down the explicit expression for connection, Weingarten operator, the mean curvature and the Gaussian curvature of M in terms of 1-forms a,b,c and vector fields $\{\mathbf{e},\mathbf{f},\mathbf{n}\}$.

2) Show that in derivation formulae $da+b\wedge c$=0.

3) Find explicitly a triple of vector fields $\{\mathbf{e},\mathbf{f},\mathbf{n}\}$ adjusted to the surface M if M is
a) cylinder
b) cone
c) sphere

4) Using results of the previous exercise find explicit expression for derivation formulae (1) in the case if the surface M is a) cylinder, b) cone, c) sphere

Deduce from these results the formulae for Gaussian and mean curvature for cylinder, cone and sphere

5*) Let $\{\mathbf{e},\mathbf{f},\mathbf{n}\}$ and $\{\tilde{\mathbf{e}},\tilde{\mathbf{f}},\tilde{\mathbf{n}}\}$ be two triples of vector fields adjusted to the surface M.

What is the relation between these triples?

How 1-forms a,b,c in derivation formulae will change if we will change $\{\mathbf{e},\mathbf{f},\mathbf{n}\}$ to $\{\tilde{\mathbf{e}},\tilde{\mathbf{f}},\tilde{\mathbf{n}}\}$?

Show that 2-form da and 2-form $b\wedge c$ are independent on the choice of the adjusted triple.

6)* What will be the result of the parallel transport of an arbitrary tangent vector along the closed curve C on the cone?

How this result correlates with the fact that the Gaussian connection of the cone equals to zero.