THIS DOES NOT WORK.

HADAMARD MATRIX DOES NOT DIAGONALISE L

Hadamard matrices and interacting strings diagonalisation

The matrices that I wrote in the yesterday blog were Hadamard matrices. When diagonalising potential energy

$$U = M_{ik}x^i x^k, M = \begin{pmatrix} 2 & -1 & 0 & 0 & 0\dots & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0\dots & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0\dots & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1\dots & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2\dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots\dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & -1\dots & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0\dots & -1 & 2 & -1 \\ -1 & 0 & 0 & 0 & 0\dots & 0 & -1 & 2 \end{pmatrix},$$

for Lagrangian of interacting strings

$$L = \sum_k \frac{m\dot{x}_k^2}{2} + \sum \frac{k(x_k - x_{k+1})^2}{2} =$$

$$\frac{m\dot{x}_0^2}{2} + \frac{m\dot{x}_1^2}{2} + \frac{m\dot{x}_2^2}{2} + \dots \frac{m\dot{x}_{N-1}^2}{2} + \frac{m\dot{x}_N^2}{2} +$$

$$\frac{k(x_1 - x_0)^2}{2} + \frac{k(x_2 - x_1)^2}{2} + \dots \frac{k(x_{N-1} - x_N)^2}{2} r + = \frac{k(x_N - x_0)^2}{2}.$$

we come to the orthogonal operator P which transform the one basis vector of the initial basis $\{\mathbf{e}_i\}$ to the vector $\sum \mathbf{e}_i$ (this vector corresponds to zero mode of the osillations). Thus we see that calculations become elegant if the orthogonal matrix P contains ± 1. Thus we come to Hadamard matrices. There is the simple examples of Hadamard matrices which appear in dimensions $N + 1 = 2^k$ ($N + 1$ is number of particles)

There are two questions.

1) Why Hadamard matrices lead to the answer?

2) Calculate the eigenvalues of the matrix M of potential energy.

Here we produce claculations for $N + 1 = 8$ particles. The circulatn matrix M of potential energy has the form

$$M = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix},$$

and the orthongonal matrix which produces the diagonalisation has the form

$$\frac{1}{2\sqrt{2}}\begin{pmatrix} + & + & + & + & + & + & + & + \\ + & - & + & - & + & - & + & - \\ + & + & - & - & + & + & - & - \\ + & - & - & + & + & - & - & + \\ + & + & + & + & - & - & - & - \\ + & - & + & - & - & + & - & + \\ + & + & - & - & - & - & + & + \\ + & - & - & + & - & + & + & - \end{pmatrix}.$$

Here instead $\pm$ you have to put ± 1 (see the yesterday blog.)

The rows of these matrix for the vectors $\{\mathbf{f}_0, \mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3, \mathbf{f}_4, \mathbf{f}_5, \mathbf{f}_6, \mathbf{f}_7\}$ of the new orthonormal basis which diaognalises the Lagrangian.

THIS DOES NOT WORK: MISTAKE!!!

We have that

$$M\mathbf{f}_0 = M\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = 0\,, \quad M\mathbf{f}_1 = M\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \\ 4 \\ -4 \\ 4 \\ -4 \\ 4 \\ -4 \end{pmatrix} = 4\mathbf{f}_1\,,$$

$$M\mathbf{f}_2 = M\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -2 \\ -2 \\ 2 \\ 2 \\ -2 \\ -2 \end{pmatrix} = 2\mathbf{f}_2\,, \quad M\mathbf{f}_3 = M\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ -2 \\ 2 \\ 2 \\ -2 \\ -2 \\ 2 \end{pmatrix} = 2\mathbf{f}_3\,,$$

$$M\mathbf{f}_4 = M\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 2 \\ -2 \\ 0 \\ 0 \\ -2 \end{pmatrix} \neq 2\mathbf{f}_4\,, \quad M\mathbf{f}_5 = M\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 4 \\ -2 \\ -2 \\ 4 \\ -4 \\ 2 \end{pmatrix} \neq 2\mathbf{f}_5\,,$$

$$M\mathbf{f}_6 = \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -2 \\ 0 \\ 0 \\ -2 \\ 2 \\ 0 \end{pmatrix} \neq 2\mathbf{f}_6\,, \quad M\mathbf{f}_7 = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ -2 \\ 4 \\ -4 \\ 2 \\ 2 \\ -4 \end{pmatrix} \neq 2\mathbf{f}_6\,,$$

We see that $\{\mathbf{f}_0, \mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3\}$ are eigenvectors, however $\{\mathbf{f}_4, \mathbf{f}_5, \mathbf{f}_6, \mathbf{f}_7\}$ are not...
TRISTE....
However

$$M\mathbf{f}_4 = \mathbf{f}_4 - \mathbf{f}_7\,, \quad M\mathbf{f}_5 = 3\mathbf{f}_5 - \mathbf{f}_6\,, \quad M\mathbf{f}_6 = \mathbf{f}_6 - \mathbf{f}_5 \quad M\mathbf{f}_7 = \mathbf{f}_7 - \mathbf{f}_4 - 2\mathbf{f}_5 - 2\mathbf{f}_6\,.$$