Homework 5

1. Calculate the differentials of the following 1-forms:

a) xdx ,

b) xdy

c) $xdx + ydy$,

d)$xdy + ydx$,

e) $xdy - ydx$

f) $x^4dy + 4x^3ydx$,

g) $xdy + ydx + dz$,

h) $xdy - ydx + dz$.

For each 1-forms listed above find a function f (0-form) such that $df = \omega$, if possible. If it is not possible, explain why.

2 Consider one-form

$$\omega = \frac{xdy - ydx}{x^2 + y^2} \tag{1}$$

This form is defined in $\mathbf{E}^2 \setminus 0$.

Calculate differential of this form.

Write down this form in polar coordinates

Find a function f such that $\omega = df$.

Is this function defined in the same domain as ω?

3* Let $\omega = a(x, y)dx + b(x, y)dy$ be a closed form in $\mathbf{E}^2$, $d\omega = 0$.

Consider the function

$$f(x, y) = x \int_0^1 a(tx, ty)dt + y \int_0^1 b(tx, ty)dt \tag{2}$$

Show that

$$\omega = df\,.$$

This proves that an arbitrary closed form in $\mathbf{E}^2$ is an exact form.

Why we cannot apply the formula (2) to the form ω defined by the expression (1)?