Let E_1 be vector bundle with coordinates (x^a, u^i), and $(x^a, u^i; p_a, p_i)$ coordinates on T^*E_1. Let E_2 be vector bundle with coordinates (y^a, w^α), and $(y^a, w^\alpha; q_a, q_\alpha)$ are coordinates on T^*E_1.

Consider 'action' defining the thick morphism which preserves fibres:

$$S(x, u; q_a, q_\alpha) = x^a q_a + S_{red}(x, u, q)$$

Any function $g = g(y, w) \in C(E_2)$ will transform to the function $f(x, u) \in C(E_1)$ such that

$$\exp \frac{i}{\hbar} f(x, u^i) = \int \exp \frac{i}{\hbar} (g(y, w^\alpha) + S(x, u; q_a, q_\alpha) - y^a q_a - w^\alpha q_\alpha) DqDyDw\,, \quad (\hbar \to 0)\,.$$

Calculate:

$$\exp \frac{i}{\hbar} f(x, u^i) = \int \exp \frac{i}{\hbar} (g(y, w^\alpha) + x^a q_a + S_{red}(x, u, q_\alpha) - y^a q_a - w^\alpha q_\alpha)\, DqDyDw =$$

$$= \int \exp \frac{i}{\hbar} (g(y, w^\alpha) + S_{red}(x, u, q_\alpha) - w^\alpha q_\alpha)\, \delta(x - y) Dq_\alpha DyDw =$$

$$= \int \exp \frac{i}{\hbar} \left(g(x, w^\alpha) + S_{red}(x, u^i, q_\alpha) - w^\alpha q_\alpha\right) Dq_\alpha Dw =$$

(Delta-function is because thick morphism preserves fibres)

This is pull back generated by thich morphism.

$$C(E_2) \ni g(y, w^\alpha) \to f(x, u^i) \in C(E_1)$$

Now write down the map from $C(E_1^*)$ to $C(E_2^*)$ which corresponds to adjoint thick morphism:

$$C(E_1^*) \ni g(y, w_\alpha) \to f(x, u_i) \in C(E_1)$$

where

$$\exp \frac{i}{\hbar} g(y, w_\alpha) = \int \exp \frac{i}{\hbar} \left(f(y, u_i) + S^*_{red}(y, w_\alpha, p^i) - p^i u_i\right) Dp^i du_j\,,$$

where in S^*_{red} we just transpose variables.

The idea is of these caluclations is that if we have matrix R^i_α then

$$\int e^{R^i_\alpha u^\alpha q_i - y^i q_i} Dq = \delta(y^i - R^i_\alpha u^\alpha)$$

and we come to adjoint:

$$\int e^{R^i_\alpha u^\alpha q_i - u^\alpha p_\alpha} Du = \delta(p_\alpha - R^i_\alpha q_i)$$

and