

I will define metaplectic in a way similar to spinor group.

Consider the vector space  $V \oplus V^*$  with canonical symplectic form  $<, >$ .

Let  $\hat{a} = \hat{a}_{\mathbf{X}}$  be a linear operator which is canonically assigned to the vector  $\mathbf{X}$ , such that

$$[\hat{a}_{\mathbf{X}}, \hat{a}_{\mathbf{Y}}] = \langle \mathbf{X}, \mathbf{Y} \rangle. \quad (1)$$

We consider transformations on the space of functions on  $V$  such that for an arbitrary  $\mathbf{X} \in V \oplus V^*$ ,

$$S^{-1} a_{\mathbf{X}} S = a_{F(\mathbf{X})} \quad (2)$$

One can see that for an arbitrary vectors  $\mathbf{X}, \mathbf{Y} \in V \oplus V^*$ ,

$$\langle \mathbf{X}, \mathbf{Y} \rangle = \langle F(\mathbf{X}), F(\mathbf{Y}) \rangle.$$

Thus we define the group of transformations of space of the functions on  $V$ , metaplectic group, and its epimorphism on symplectic group.

Now instead  $V$  consider the space  $C^n$ , and assign to every vector  $\mathbf{X}$ , the matrix  $M_{\mathbf{X}} = X^m \gamma_m$ .

We have instead equation (1) the following relation:

$$[M_{\mathbf{X}}, M_{\mathbf{Y}}]_+ = 2 \langle \mathbf{X}, \mathbf{Y} \rangle. \quad (1a)$$

where  $[\ , \ ]_+$  is the anticommutator, and  $\langle \ , \ \rangle$  now is the scalar product.

Now instead equation (2) we come to consider transformations  $O$  on the space matrices such that for an arbitrary  $\mathbf{X} \in C^n$

$$O^{-1} M_{\mathbf{X}} O = M_{F(\mathbf{X})} \quad (2a)$$

One can see that for an arbitrary vectors  $\mathbf{X}, \mathbf{Y}$ ,

$$\langle \mathbf{X}, \mathbf{Y} \rangle = \langle F(\mathbf{X}), F(\mathbf{Y}) \rangle.$$

Thus we define the group of spinor transformations and its epimorphism on orthogonal group.

This I already know