

Homework 2

1 Let $\{\mathbf{e}, \mathbf{f}\}$ be an orthonormal basis in \mathbf{E}^2 . Consider the following ordered pairs:

- a) $\{\mathbf{f}, \mathbf{e}\}$,
- b) $\{\mathbf{f}, -\mathbf{e}\}$,
- c) $\{\frac{\sqrt{2}}{2}\mathbf{e} + \frac{\sqrt{2}}{2}\mathbf{f}, -\frac{\sqrt{2}}{2}\mathbf{e} + \frac{\sqrt{2}}{2}\mathbf{f}\}$,
- d) $\{\frac{\sqrt{3}}{2}\mathbf{e} + \frac{1}{2}\mathbf{f}, \frac{1}{2}\mathbf{e} - \frac{\sqrt{3}}{2}\mathbf{f}\}$.

Show that all these ordered pairs are orthonormal bases in \mathbf{E}^2 .

Find amongst them the bases which have the same orientation as the orientation of the basis $\{\mathbf{e}, \mathbf{f}\}$.

Find amongst them the bases which have the orientation opposite to the orientation of the basis $\{\mathbf{e}, \mathbf{f}\}$.

2 Let $\{\mathbf{e}, \mathbf{f}\}$ be a basis in two-dimensional vector space V . Consider an ordered pair $\{\mathbf{a}, \mathbf{b}\}$ such that

$$\mathbf{a} = \mathbf{f}, \quad \mathbf{b} = \gamma\mathbf{e} + \mu\mathbf{f},$$

where γ, μ are arbitrary real numbers.

Find values γ, μ such that an ordered pair $\{\mathbf{a}, \mathbf{b}\}$ is a basis and this basis has the same orientation as the basis $\{\mathbf{e}, \mathbf{f}\}$.

3 Let $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\}$ be an orthonormal basis in \mathbf{E}^3 and let $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ be an arbitrary basis in \mathbf{E}^3 . Show that the basis $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ either has the same orientation as the basis $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\}$, or the same orientation as the basis $\{\mathbf{e}_y, \mathbf{e}_x, \mathbf{e}_z\}$.

4 Let $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\}$ be an orthonormal basis in \mathbf{E}^3 . Consider the following ordered triples:

- a) $\{\mathbf{e}_x, \mathbf{e}_x + 2\mathbf{e}_y, 5\mathbf{e}_z\}$,
- b) $\{\mathbf{e}_y, \mathbf{e}_x, 5\mathbf{e}_z\}$,
- c) $\{\mathbf{e}_y, \mathbf{e}_x, -5\mathbf{e}_z\}$,
- d) $\{\frac{\sqrt{3}}{2}\mathbf{e}_x + \frac{1}{2}\mathbf{e}_y, -\frac{1}{2}\mathbf{e}_x + \frac{\sqrt{3}}{2}\mathbf{e}_y, \mathbf{e}_z\}$,
- e) $\{\mathbf{e}_y, \mathbf{e}_x, \mathbf{e}_z\}$,
- f) $\{\mathbf{e}_y, \mathbf{e}_x, -\mathbf{e}_z\}$.

Show that all ordered triples a), b), c), d), e), f) are bases.

Show that the bases a), c), d) and f) have the same orientation as the basis $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\}$, and the bases b) and e) have the orientation opposite to the orientation of the basis $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\}$.

Show that bases d), e) and f) are orthonormal bases and bases a), b) and c) are not orthonormal bases.

5 Let $\{\mathbf{e}, \mathbf{f}, \mathbf{g}\}$ be a basis in vector space V . Show that ordered triples $\{\mathbf{f}, \mathbf{e} + 2\mathbf{f}, 3\mathbf{g}\}$ and $\{\mathbf{e}, \mathbf{f}, 2\mathbf{f} + 3\mathbf{g}\}$ are bases and these bases have opposite orientations.

6 Let $\{\mathbf{e}, \mathbf{f}, \mathbf{g}\}$ be a basis in 3-dimensional vector space V .

Consider in the space V the following ordered triples

I) — $\{\mathbf{e} + 2\mathbf{f} + 3\mathbf{g}, 2\mathbf{f} + \mathbf{g}, \mathbf{e} + 2\mathbf{f} + \mathbf{g}\}$

II) — $\{\mathbf{e} + \mathbf{f} - 2\mathbf{g}, 2\mathbf{f} + \mathbf{g}, \mathbf{e} + \mathbf{f} + \mathbf{g}\}$

III) — $\{\mathbf{e} + 2\mathbf{f} + 4\mathbf{g}, \mathbf{e} + 3\mathbf{f} + 9\mathbf{g}, \mathbf{e} + 4\mathbf{f} + 16\mathbf{g}\}$

Show that all these ordered triples are bases.

Show that I-st and II-nd bases have opposite orientations.

Show that II-nd and III-d bases have the same orientations.

Show that I-st and III-nd bases have opposite orientations.