Homework 3

In all exercises we assume by default that Riemannian metric on embedded surfaces is induced by the Euclidean metric.

1 a) Show that surface of the cone $\begin{cases} x^2+y^2-k^2z^2=0 \\ z>0 \end{cases}$ in $\mathbf{E}^3$ is locally isometric to Euclidean plane.

2 a) Consider the domain D on the conic surface $x^2+y^2-z^2$ defined by the conditions

$$\begin{cases} 0<z<H \\ y\neq 0 \quad \text{if } x>0 \end{cases}.$$

(The second condition means that the line $x=z, y=0$ is removed from the surface of the cone)

Find a domain D' in Euclidean plane such that it is isometri4 to the surface D.

b) Find a shortest distance between points $A=(0,1,1)$ and $B=(0,-1,1)$ for an ant living on the surface D (we assume that $H>1$).

3 Consider plane with Riemannian metric given in cartesian coordinates (x,y) by the formula

$$G=\frac{4(dx)^2+4(dy)^2}{(1+x^2+y^2)^2}.$$

Show that this Riemannian manifold is locally isometric to the sphere.

4 Consider catenoid: $x^2+y^2=\cosh^2 z$ and helicoid: $y-x\tan z=0$.

Find induced Riemannian metrics on these surfaces.

Show that these surfaces are locally isomorphic.

5 a) Consider the domain D on the cone $x^2+y^2-k^2z^2$ defined by the condition $0<z<H$. Find an area of this domain using iduced Riemannian metric. Compare with the answer when using standard formulae.

6 a) Find an area of 2-dimensional sphere of radius R using explicit formulae for induced Riemannian metric in stereographic coordinates.

b)† Find a volume of n-dimensional sphere of radius R. (You may use Riemannian metric in stereographic coordinates, or you may do it in other way... You just have to calculate the answer.)