Action for harmonic osillator: Misprint in Feynman book?

26 October 2018

Consider Lagrangian of harmonic osillator:

$$L = \frac{m\dot{x}^2}{2} - \frac{mw^2x^2}{2}\,,$$

and calculate $S(x_1, t_1; x_0, t_0)$

First write down the path $x(t)$ which

i) obeys the differential equation $\frac{d^2x}{dt^2} + \omega^x = 0$

ii) and it obeys the boundary condition $x(t_0) = x_0$ and $x(t_1) = x_1$.

One can see that this path is

$$x(t) = \frac{x_1 \sin\omega(t - t_0) - x_0 \sin\omega(t - t_1)}{\sin\omega(t_1 - t_0)}\,.$$

Respectively for velocity we have

$$v(t) = \omega\frac{x_1 \cos\omega(t - t_0) - x_0 \cos\omega(t - t_1)}{\sin\omega(t_1 - t_0)}\,.$$

Thus we have for Lagrangian

$$L(t) = \left(\frac{mv^2}{2} - \frac{mw^2x^2}{2}\right)_{x=x(t),v=v(t)} = \frac{m\omega^2}{2\sin^2\omega(t_1 - t_0)}\times$$

$$\left[\left(x_1\cos\omega(t - t_0) - x_0\cos\omega(t - t_1)\right)^2 - \left(x_1\sin\omega(t - t_0) - x_0\sin\omega(t - t_1)\right)^2\right] =$$

$$\frac{m\omega^2}{2\sin^2\omega(t_1 - t_0)}\left[x_1^2\cos 2\omega(t - t_0) + x_0^2\cos 2\omega(t - t_1) - 2x_1x_0\cos 2\omega\left(t - \frac{t_0 + t_1}{2}\right)\right]\,.$$

Finally we have that

$$S(x_1, t_1; x_0, t_0) = \int_{t_0}^{t_1}\left(\frac{mv^2}{2} - \frac{mw^2x^2}{2}\right)_{x=x(t),v=v(t)} = \int_{t_0}^{t_1} L(t)dt =$$

$$\frac{m\omega^2}{2\sin^2\omega(t_1 - t_0)}\int_{t_0}^{t_1} dt\left[x_1^2\cos 2\omega(t - t_0) + x_0^2\cos 2\omega(t - t_1) - 2x_1x_0\cos 2\omega\left(t - \frac{t_0 + t_1}{2}\right)\right] =$$

$$\frac{m\omega^2}{2\sin^2\omega(t_1 - t_0)}\left[\left(x_1^2 + x_0^2\right)\frac{\sin 2\omega(t_1 - t_0)}{2\omega} - 2x_1x_0\frac{\sin\omega(t_1 - t_0)}{\omega}\right] =$$

$$\frac{m\omega}{2\sin\omega(t_1 - t_0)}\left[\left(x_1^2 + x_0^2\right)\cos\omega(t_1 - t_0) - 2x_1x_0\right]\,.$$

Comparing with the book of Feynman we see the typos???