Konspekt pervoj chasti doklada Karabegova
po ego lekcijam po Skaipu (kak ja ikh pnimaju)

Let M be a manifold with a point $\mathbf{p}$ on it.

Let $\rho = \rho(\nu)$ be a `formal` density on manifold M [1] and $\sigma = \sigma(\nu)$ be a `formal` function on M such that

$$\frac{d\rho(\nu)}{d\nu} = \sigma(\nu)\rho(\nu)\,. \tag{1}$$

We consider `formal` generalised functions $\Lambda(\nu)$ with support at the point $\mathbf{p}$

$$\Lambda(\nu) = \Lambda_0 + \nu\Lambda_1 + \dots\,, \quad \text{and } \Lambda_0(1) \neq 0\,. \tag{2}$$

We say that the generalised `formal` function Λ is associated with the density ρ if

$$\Lambda\left(\partial_{\mathbf{v}} f + f \mathrm{div}_\rho \mathbf{v}\right) = 0 \tag{3}$$

where $\mathbf{v}$ is an arbitrary vector field on M (not `formal`) and f is an arbitrary `formal` function on M.

We say that the generalised `formal` function Λ is *strongly associated* with the density ρ if this function is associanted with the density ρ and the following condtion holds also:

$$\frac{d}{d\nu}\Lambda(f) = -\frac{n}{2\nu}\Lambda + \Lambda\left(\frac{df}{d\nu} + f\sigma\right)\,. \tag{4}$$

where n is a dimension of manifold M, $f = f(\nu)$ is as usual an arbitrary `formal` function and a function σ is defined by equation (1) [2].

Notice that

a) generalised function associated with a `formal` density is defined up to a `formal` constant, and

b) generalised function stronlgly associated with a `formal` density is defined up to a constant (not `formal` just a complex number).

Theorem `(Karabegov) If a density is an oscillatory density (see the foot-note at the first page) then the inverse is also true.`

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[1] **Remark** The `formal` density $\rho(\nu)$ obeys also the condition that

$$\rho = e^{\varphi(\nu)}\rho_0(\nu)\,, \quad \text{where } \varphi(\nu) = \textstyle\sum_{k\geq -1} \nu^k \varphi_k\,.$$

I do not know how to codify it without the use of the phase function Later I will call density of this type *oscillatory* density.

[2] Equation (4) becomes natural if we apply it to generalised function $\Lambda_{\rho_0,\varphi} = \nu^{-\frac{n}{2}} \int f\rho$, where a `formal` density ρ is equal to $\rho = e^{\varphi}\rho_0$, ρ_0 is a density (non-vanishing at point $\mathbf{p}$) and φ is a `formal` function such that $\varphi = \frac{1}{\nu}\varphi_{-1} + \varphi_0 + \nu\varphi_1 + \dots$, and a function φ_1 has non-degenerate hessian at the point $\mathbf{p}$.