The map:

$$w = \frac{z-\zeta}{1-\bar{\zeta}z} \tag{1}$$

maps disc $|z|<1$ onto itself, and the point ζ on the point 0.

We take Green function $G(0;w) = -\frac{1}{2\pi}\log|w|$ for $w_0 = 0$ we come to

$$G(\zeta,z) - \frac{1}{2\pi}\log|w|\big|_{w=\frac{z-\zeta}{1-\bar{\zeta}z}} = -\frac{1}{2\pi}\log\left|\frac{z-\zeta}{1-\bar{\zeta}z}\right| = -\frac{1}{2\pi}\log|z-\zeta| - \frac{1}{2\pi}\log\left|1-\bar{\zeta}z\right| =$$

$$-\frac{1}{2\pi}\log|z-\zeta| + \frac{1}{2\pi}\log\left|z-\frac{1}{\bar{\zeta}}\right| - \log|\zeta| -$$

This is a Green function which we come to considering : the potential on the points of the circle is equal to

$$U(e^{i\varphi}) = -\frac{1}{2\pi}\log\left|e^{i\varphi}-\zeta\right| + \frac{1}{2\pi}\log\left|e^{i\varphi}-\frac{1}{\bar{\zeta}}\right| - \log|\zeta| = 0\,.$$

On the other hand this Green function possesses the information about the map (1).

the map

$$w = \frac{iz+1}{i+z}$$

maps hlapf plane onto the disc