Homework 7

1 A point moves in $\mathbf{E}^2$ along an ellipse with the law of motion $x = a\cos t$, $y = b\sin t$, $0 \le t < 2\pi$, $(0 < b < a)$. Find the velocity and acceleration vectors. Find the points of the ellipse where the angle between velocity and acceleration vectors is acute.

Find the points where speed attains its maximum value.

What is the direction of acceleration vector at these points?

2 Find a natural parameter for the following interval of the straight line:

$$C\colon \begin{cases} x = t \\ y = 2t+1 \end{cases}, \quad 0 < t < \infty\,.$$

Calculate a curvature of the straight line C.

3 Let C: $\mathbf{r} = \mathbf{r}(t)$, $0 \le t \le 2$ be a curve in $\mathbf{E}^2$ such that at an arbitrary point of this curve the velocity and acceleration vectors $\mathbf{v}(t)$ and $\mathbf{a}(t)$ are orthogonal to each other and

$$\mathbf{v}(t)\big|_{t=0} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}\,.$$

Find the length of this curve.

4 Consider the following curve (a helix): $\mathbf{r}(t)\colon \begin{cases} x(t) = R\cos\Omega t \\ y(t) = R\sin\Omega t \\ z(t) = ct \end{cases}.$

Find velocity and acceleration vectors of this curve.

Find the curvature of this curve.

5 Calculate the curvature of the parabola $x = t, y = mt^2$ $(m > 0)$.

Let s be a natural parameter on this parabola. Show that $\int_0^\infty k(s)ds = \int_0^\infty k(t)|\mathbf{v}(t)|dt$ and calculate this integral.

6 Consider the parabola

$$\mathbf{r}(t)\colon \begin{cases} x = v_x t \\ y = v_y t - \frac{gt^2}{2} \end{cases}.$$

(It is path of the point moving under the gravity force with initial velocity $\mathbf{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$.)

Calculate the curvature at the vertex of this parabola.

7 Consider the ellipse $x = a\cos t, y = b\sin t$ $(a, b > 0,\ 0 \le t < 2\pi)$ in $\mathbf{E}^2$. Calculate the curvature $k(t)$ of this ellipse.

Find the radius of a circle which has second order touching with the ellipse at the point $(0, b)$.

† Calculate $\int k(s)ds$ over the ellipse, where s is a natural parameter.

8 Calculate the curvature of the following curve (latitude on the sphere)

$$\begin{cases} x = R\sin\theta_0\cos\varphi(t) \\ y = R\sin\theta_0\sin\varphi(t) \\ z = R\cos\theta_0 \end{cases}, \text{ where } \varphi(t) = t, 0 \le t < 2\pi\,.$$

9† Show that the curvature of an arbitrary curve on the sphere of the radius R is greater or equal to $\frac{1}{R}$.