Let $S = S(x, y, t)$ be a function such that

$$S_x = -p\,, \quad S_y = q\,, \quad S_t = -H(y, S_y)\,.$$

Consider Legendre of this function

$$\mathcal{S}(x, q, t) = y(x, q, t)q - S(x, y = y(x, q, t), t)\,, \quad \text{such that}\,, \quad q = S_y(x, y(x, q, t), t)\,.$$

Then

$$\mathcal{S}_x(x, q, t)\big|_{q,\,t \text{ are fixed}} = -S_x(x, y, t)\big|_{y,\,t \text{ are fixed}} = p\,,$$
$$\mathcal{S}_q(x, q, t)\big|_{x,\,t \text{ are fixed}} = y\,,$$

and

$$\mathcal{S}_t(x, q, t)\big|_{x,\,q \text{ are fixed}} = -S_t(x, y, t)\big|_{x,\,y \text{ are fixed}} = H(y, S_y) = H(\mathcal{S}_q, q)\,.$$