Dear Martyn

I am sure that you know or almost know what I want to speak here about, but I think that better to fix it again

You understand well why every lattice defines the point $(x, y), y > 0$ in Lobachevsky plane. The are two things that have to be done.

1. (See Nikulin, Shafarevitch §14)

Two lattices $\Gamma_{\mathbf{a},\mathbf{b}}$, $\Gamma_{\mathbf{a}',\mathbf{b}'}$ are the same if

$$\begin{pmatrix} \mathbf{a}' \\ \mathbf{b}' \end{pmatrix} = \begin{pmatrix} p & q \\ m & n \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix}, \quad \text{where } p, q, m, n, \text{ are integers such that } pn - qm = \pm 1 .$$

This means that the group $SL_2(\mathbf{Z})$ (2×2 unimodular matrices with integer entires) acts on the Lobachevsky plane, and the distance has to be invariant under the action of this group.

Going from this group to Mobius group, we just change the coefficients:

$$SL_2(\mathbf{Z}) = \left\{ \begin{pmatrix} p & q \\ m & n \end{pmatrix}, \text{such that } p, m, q, n \in \mathbf{Z} \text{ and } pn - pq = 1 \right\} \Rightarrow$$

$$\Rightarrow SL_2(\mathbf{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \text{such that } a, b, c, d \in \mathbf{R} \text{ and } ad - bc = 1 \right\} .$$

We come to the problem to find the metric which is invariant with respect to this group.

2. We have to show that vertical lines and half-circles with centre at absolute $(y = 0)$ are geodesics on Lobachevsky plane. This is vital for our considerations.

Lemma *Let $F \neq \mathbf{id}$ be an isometry of Riemannian manifold, and let $F = \mathbf{id}$ at the points of curve C. Then curve C is geodesic!*

This lemma is founded on the remark of the book Nikulin, Shafarevich ($\S 16, p.2$)

It will be great tomorrow to discuss this.

I will be all the day in office.

HMK