Homework 3a.

Dear Geometry students. I would like to slow down a little bit on the tutorials. So next week we will continue Homework 3, and we also consider some extra material in Homework 3a. (Homework 4 we will consider during week 6.)

1 Let $\{\mathbf{e}, \mathbf{f}, \mathbf{g}\}$ be a basis in 3-dimensional vector space V.
Consider in the space V the following ordered triples
I)—- $\{\mathbf{e}+2\mathbf{f}+3\mathbf{g}\,, 2\mathbf{f}+\mathbf{g}\,, \mathbf{e}+2\mathbf{f}+\mathbf{g}\}$
II)—- $\{\mathbf{e}+\mathbf{f}-2\mathbf{g}\,, 2\mathbf{f}+\mathbf{g}\,, \mathbf{e}+\mathbf{f}+\mathbf{g}\}$
III)—- $\{\mathbf{e}+2\mathbf{f}+4\mathbf{g}\,, \mathbf{e}+3\mathbf{f}+9\mathbf{g}\,, \mathbf{e}+4\mathbf{f}+16\mathbf{g}\}$
Show that all these oredered triples are bases.
Show that I-st and II-nd bases have opposite orientations.
Show that II-nd and III-d bases have the same orientations.
Show that I-st and III-nd bases have opposite orientations.

2 Consider an operator P on $\mathbf{E}^3$ such that P is an orthogonal operator preserving the orientation of $\mathbf{E}^3$ and

$$P(\mathbf{e}_x) = \mathbf{e}_y, P(\mathbf{e}_z) = -\mathbf{e}_z\,.$$

Find an action of the operator P on an arbitrary vector $\mathbf{x} = x\mathbf{e}_x + y\mathbf{e}_y + z\mathbf{e}_z$.
Why P is a rotation operator? Find an angle and axis of the rotation.
(We assume that $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\}$ is an orthonormal basis.)

3 Consider an operator P on $\mathbf{E}^3$ such that

$$P(\mathbf{e}) = \frac{2}{3}\mathbf{e} + \frac{2}{3}\mathbf{f} + \frac{1}{3}\mathbf{g}\,, P(\mathbf{f}) = -\frac{1}{3}\mathbf{e} + \frac{2}{3}\mathbf{f} - \frac{2}{3}\mathbf{g}\,, P(\mathbf{g}) = -\frac{2}{3}\mathbf{e} + \frac{1}{3}\mathbf{f} + \frac{2}{3}\mathbf{g}\,.$$

Show that this is an orthogonal operator preserving the orientation of $\mathbf{E}^3$.
Find eigenvectors of this operator.
(We assume that $\{\mathbf{e}, \mathbf{f}, \mathbf{g}\}$ is an orthonormal basis in $\mathbf{E}^3$.)