Homework 8

1. Find coordinate basis vectors, first quadratic form and unit normal vector field for

a) sphere of the radius R: $x^2 + y^2 + z^2 = R^2$,

$$\mathbf{r}(\theta,\varphi) \qquad \begin{cases} x = R\sin\theta\cos\varphi \\ y = R\sin\theta\sin\varphi \\ z = R\cos\theta \end{cases} \qquad (0 \le \varphi < 2\pi, 0 \le \theta \le \pi)\,, \qquad (1)$$

b) cylinder $x^2 + y^2 = R^2$,

$$\mathbf{r}(h,\varphi) \qquad \begin{cases} x = R\cos\varphi \\ y = R\sin\varphi \\ z = h \end{cases} \qquad (0 \le \varphi < 2\pi, -\infty < h < \infty) \qquad (2)$$

c) cone $x^2 + y^2 - k^2z^2 = 0$,

$$\mathbf{r}(h,\varphi) \qquad \begin{cases} x = kh\cos\varphi \\ y = kh\sin\varphi \\ z = h \end{cases} \qquad (0 \le \varphi < 2\pi, -\infty < h < \infty) \qquad (2)$$

d) graph of the function $z = F(x,y)$,

$$\mathbf{r}(u,,v) \qquad \begin{cases} x = u \\ y = v \\ z = F(u,v) \end{cases} \qquad (-\infty < u < \infty, -\infty < v < \infty) \qquad (3)$$

in the case if $F(u,v) = F = Au^2 + 2Buv + Cv^2$. Consider coordinate basis vectors, first quadratic form and unit normal vector field at origin, i.e. at the point $u = v = 0$.

Put down the special case of saddle when $F = uv$.

2. Consider helix $\mathbf{r}(t)$: $\begin{cases} x(t) = a\cos t \\ y(t) = a\sin t \\ z(t) = ct \end{cases}$.

Show that this helix belongs to cylinder surface $x^2+y^2 = a^2$. Using first quadratic form on the surface of cylindre calculate length of the helix $(0 \le t \le t_0)$. (Compare with problem 3 from Homework 7.)

3 Show that the curve $x = t\cos t, y = t\sin t, z = t$ belongs to the cone $x^2+y^2-z^2 = 0$. Find the length of this curve $(0 \le t \le t_0)$.

4 On the sphere of the radius R consider two points $\mathbf{r}_A$ with spherical coordinates $\{\theta_A, \varphi_A\}$ and $\mathbf{r}_B$ with spherical coordinates $\{\theta_B, \varphi_B\}$.

a) In the case if $\varphi_A = \varphi_B$ write down the parametric equation of the arc of the meridian C_{AB} which joins these points and calculate its length.

b) In the case if $\theta_A = \theta_B$ write down the parametric equation of the arc of the latitude which joins these points and calculate its length.

† Is the length of the arc of the great circle joining the points A, B shorter than the length of the arc of latitude? (You may consider only the case if $\varphi_A = 0, \varphi_B = \pi$.)

c†) Calculate the length of the arc of the great circle joining the points $\mathbf{r}_A = (\theta_A, \varphi_A)$ and $\mathbf{r}_B = (\theta_B, \varphi_B)$.