Consider differential equation

$$\hat{P}(F) = G$$

where $\hat{P} = \hat{P}\left(\frac{d}{dx}\right)$ polynomial on derivative, f, g polynomials on x.

`Lemma`: On the space of polynomials

$$\left(\frac{d}{dx} + \lambda\right)^{-1} G(x) = G\left(x + \frac{d}{d\lambda}\right)\frac{1}{\lambda}\,.$$

It follows from this lemma that the solutions of differential equation

$$\hat{P}(F) = \prod \left(\frac{d}{dx} + \lambda_i\right) F = G$$

is the polynomial

$$F(x) = G\left(x + \sum_i \frac{d}{d\lambda_i}\right)\frac{1}{\lambda_1\lambda_2\ldots\lambda_n}\,.$$

`Proof of the lemma`

We have that the commutator $\left[\frac{d}{dx} + \lambda, x + \frac{d}{d\lambda}\right] = 0$, hence

$$\left(\frac{d}{dx} + \lambda\right) G\left(\left(\frac{d}{d\lambda} + x\right)\right)\frac{1}{l} = G\left(\left(\frac{d}{d\lambda} + x\right)\right)\left(\frac{d}{dx} + \lambda\right)\frac{1}{\lambda} = G\left(\left(\frac{d}{d\lambda} + x\right)\right)\mathbf{1} = G(x)\,. \blacksquare$$