Conformal map and in terms of Green function

This is the topic why I began to read the Lavrentiev inspired by French book. Now it is alright. If we know the Green function $G_U(z_0, z)$ then it defines in $U \backslash \{z_0\}$ holomorphic function $\hat{G} = G + iU$ where U is defined up to period $\Pi = \oint ...$

Thus we come to the map $e^{\frac{...G}{\Pi}}$.

(comment ca marche: the question of bijection????)

Example How to construct $\mathbf{H} \to D$ We know that

$$G_{\mathbf{H}} = ... (\log|z - z_0| - \log|z - \bar{z}_0|)$$

(coefficient $-\frac{1}{2\pi}$)

Hence $G(z_0, z) =\approx \frac{z-z_0}{z-\bar{z}_0}$

is it unique? Yes up to the coefficient $e^{i\varphi}$.

In the blog on 7 August we considered function

$$\frac{G(z_0, z) - G(z_0, \zeta)}{1 - G(\bar{z_0}.\zeta)G(z_0, z)}$$

This we see that this function is equal to $\frac{z-\zeta}{z-\bar{\eta}}$ up to a coefficient.

Remark We used here that every F holomorphic map of Disc $|z| < 1$ onto itself such that $F(\zeta) = 0$ is nothing but $\frac{z-\zeta}{1-\bar{\zeta}z} e^{i\varphi}$