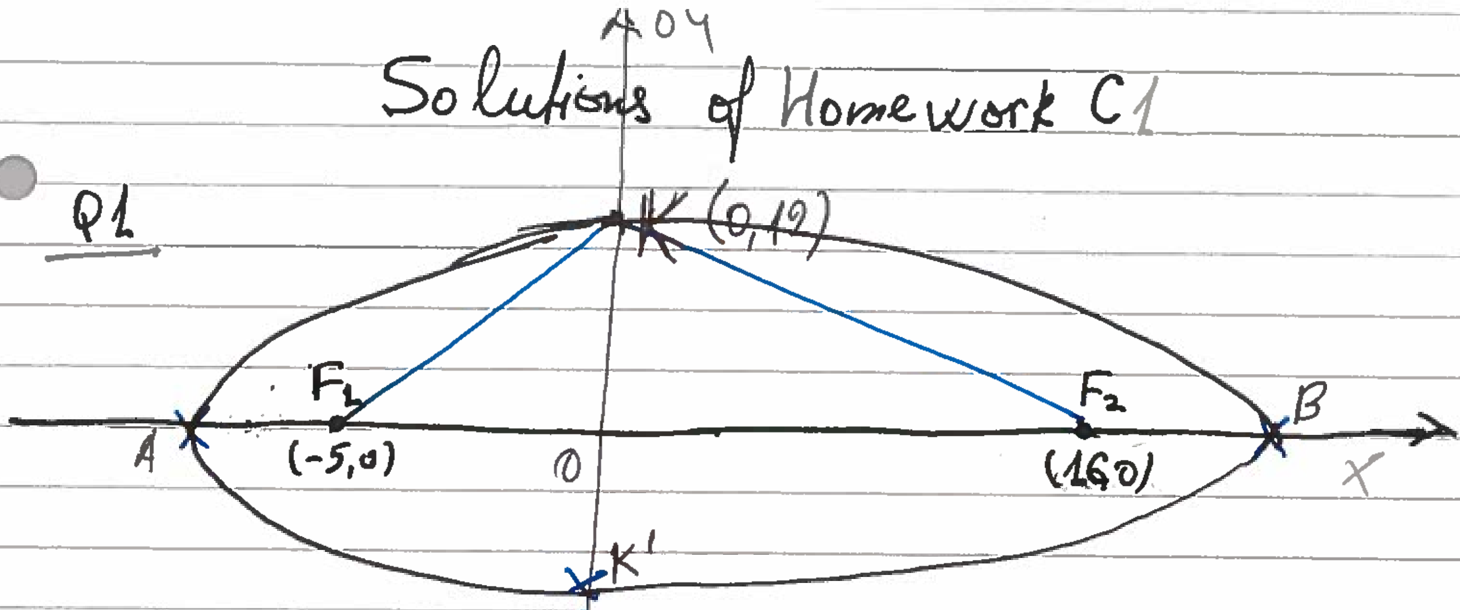
Solutions of Homework C1

Q1



Find intersection of ellipse with axis OX, OY

Solution

$$|KF_1| + |KF_2| = \sqrt{12^2 + 5^2} + \sqrt{12^2 + 16^2} =$$

$$= \sqrt{144 + 25} + \sqrt{4^2(3^2 + 4^2)} = \sqrt{169} + 4\sqrt{25} = 33$$

K' has coordinates $(0, -12)$

Let $B = (x, 0)$ then

$$|BF_1| + |BF_2| = |x - (-5)| + |x - 16| = |x + 5| + |x - 16| = 33$$

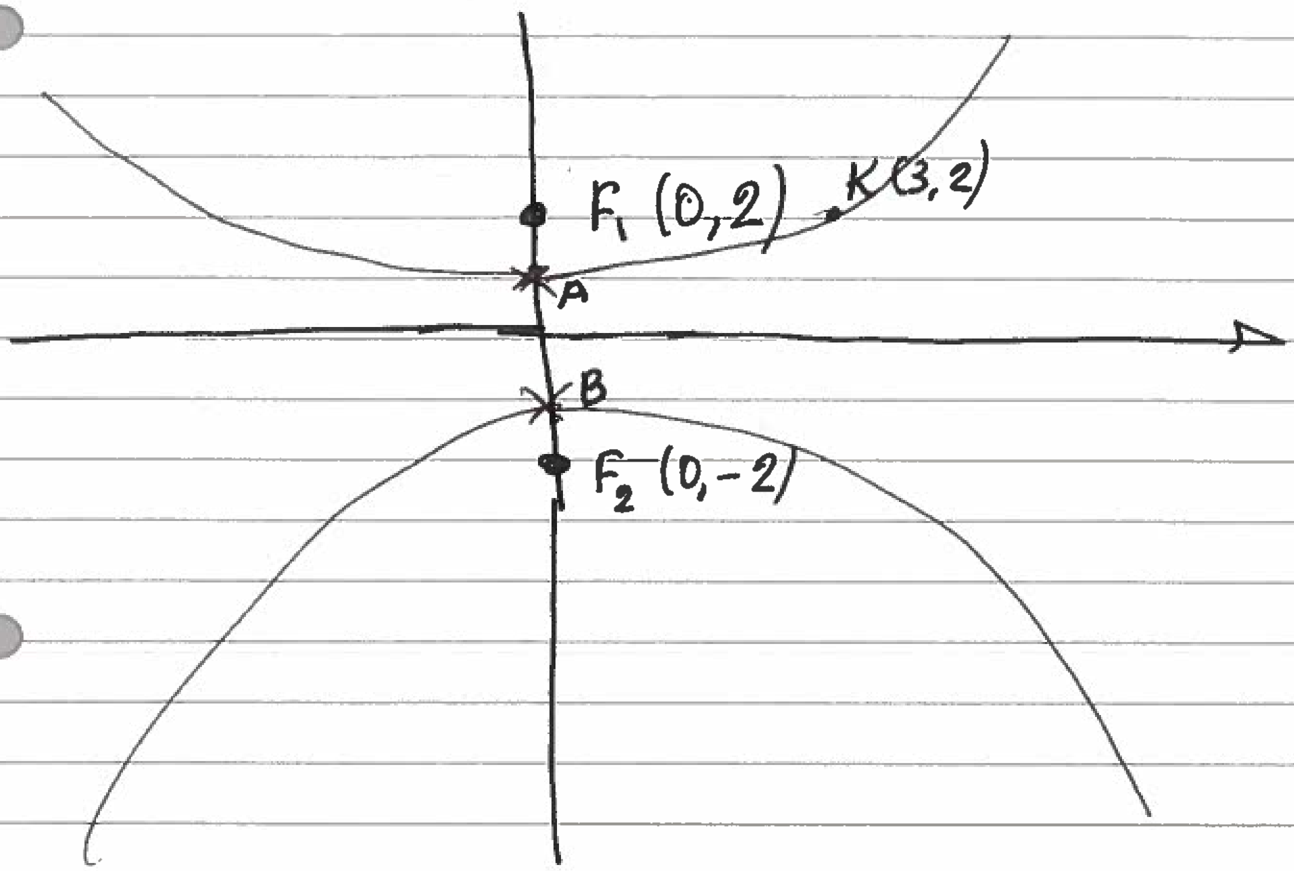
a) $x \geq 16$, $|x+5| + |x-16| = 2x - 11 = 33$. $\boxed{x = 22}$

b) $x \leq -5$ $\quad -x - 5 - x + 16 = 33$ $\quad -2x = 22$ $\boxed{x = -11}$

Intersection points $\boxed{A = (-11, 0)}$, $\boxed{B = (22, 0)}$, $\boxed{K' = (0, -12)}$

Solution of Homework K1

Q2



Hyperbola foci – $F_1(0,2)$, $F_2(0,-2)$

It passes through $K=(3,2)$

Find the intersection with OY axis.

Solution.

$$\big|\,|KF_1|-|KF_2|\,\big| = \big|\,|3-0|-\sqrt{3^2+4^2}\,\big| = |3-5| = 2.$$

If intersects at the point $(0,X)$ then

$$\big|\,|x-2|-|x-(-2)|\,\big| = \big|\,|x-2|-|x+2|\,\big| = 2.$$

$$|x-2|-|x+2| = \pm 2$$

1) $|x-2|-|x+2|=2 \Rightarrow \boxed{x=-1}$

2) $|x-2|-|x+2|=-2 \Rightarrow \boxed{x=1}$

Intersects at points $\boxed{(0,1)}$ and $\boxed{(0,-1)}$

Solution of Homework C1

Q3

- C_1: $4x^2+4x+y^2=0$

$(4x^2+4x+1)+y^2=1$

$4\left(x+\frac{1}{2}\right)^2+y^2=1$

Choose new Cartesian coordinates

$$\begin{cases} x = x' - \frac{1}{2} \\ y = y' \end{cases} \qquad \cancel{4x'^2+4y'} \qquad \underline{4x'^2+y'^2=1} \text{ Ellipse}$$

- or

$$\begin{cases} x = y' - \frac{1}{2} \\ y = x' \end{cases} \qquad \underline{4y'^2+x'^2=1} \text{ Ellipse.}$$

C_2: $4x^2+4x-y^2=0$

$(4x^2+4x+1)-y^2=1$

$4\left(x+\frac{1}{2}\right)^2-y^2=1$

- Choose new Cartesian coordinates

$$\begin{cases} x = x' - \frac{1}{2} \\ y = y' \end{cases} \Longrightarrow \underbrace{4x'^2-y'^2=1}_{\text{Hyperbola}} \quad \text{or} \quad \begin{cases} x = y' - \frac{1}{2} \\ y = x' \end{cases} \quad \underbrace{4y'^2-x'^2=1}_{\text{Hyperbola}}$$

C_3: $4x^2+4x+y=0$

$(4x^2+4x+1)+(y-1)=0 \qquad 4\left(x+\frac{1}{2}\right)^2+(y-1)=0$

Choose new Cartesian coordinates

- $$\begin{cases} x = y' - \frac{1}{2} \\ y = x' + 1 \end{cases}$$

$4y'^2+x'=0$

$\underbrace{x'=-4y'^2}_{\text{parabola}}$

Solution of Homework C1.

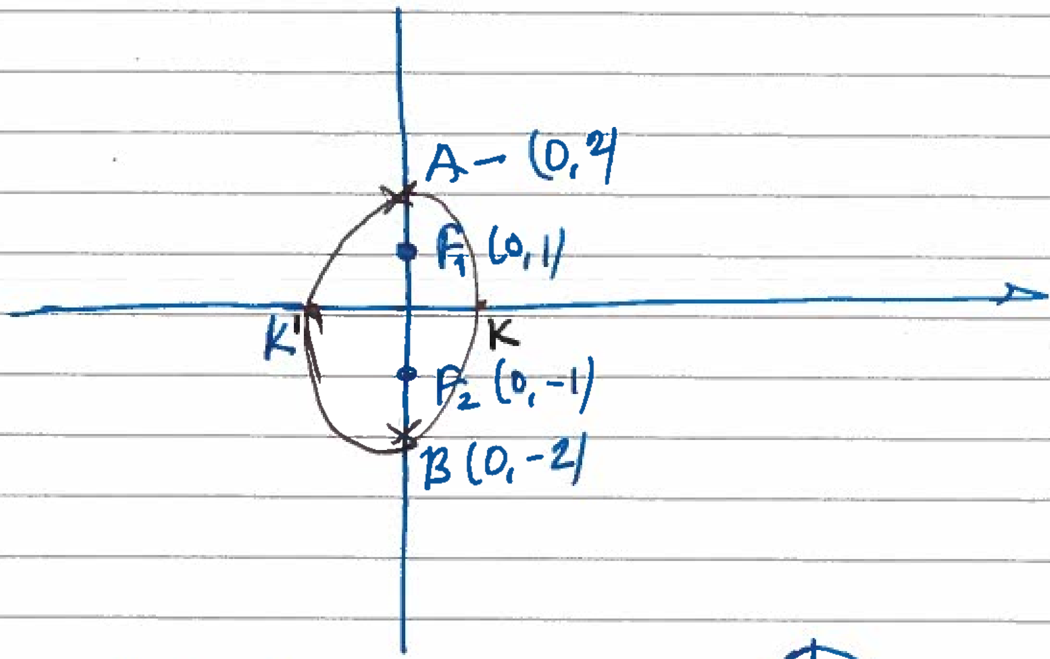
Q4

H hyperbola: $F_1 = (0,2)$, $F_2 = (0,-2)$

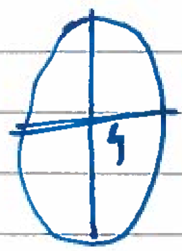
C - ellipse such that it passes through points F_1, F_2 and has foci at points $(0,1)$, $(0,-1)$

(see solution of exercise 2)

Write down equation of this ellipse.



$$|AF_1| + |AF_2| = 1 + 3 = 4.$$



Let K has coordinates $(x,0)$

$$|KF_1| + |KF_2| = 4$$

$$|KF_1| = \sqrt{1+x^2} = |KF_2|$$

$$2\sqrt{1+x^2} = 4$$

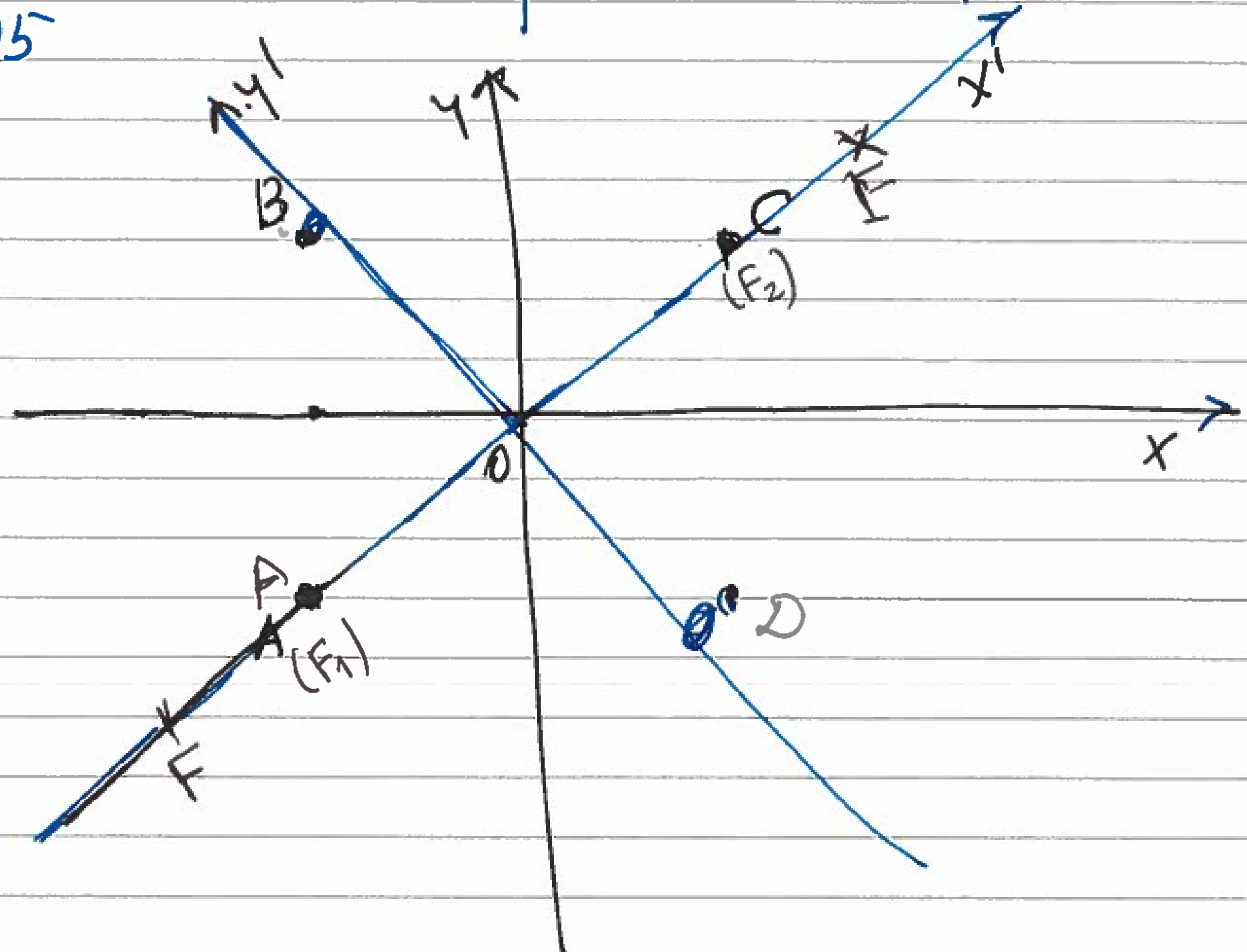
$$x = \pm\sqrt{3}.$$

Hence equation of ellipse

$$\boxed{\frac{x^2}{3} + \frac{y^2}{4} = 0}$$

Solutions of Homework C1.

Q5



Ellipse has foci at points A, C and passes via points B, D.

$|BO| = \sqrt{2}$

$|BA| + |BC| = 2+2=4 \quad |FA| + |FC| = 4$ Hence $|FO| = 2$.

In the new Cartesian Coordinates (with axis OX', OY') equation of ellipse

$$\frac{X'^2}{4}+\frac{Y'^2}{2}=1,$$

It is rotation on angle $\frac{\pi}{4}$:
$$\begin{cases} x' = \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y \\ y' = -\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y \end{cases}$$

$$\frac{1}{4}\left[\frac{x+y}{\sqrt{2}}\right]^2 + \frac{1}{2}\left[\frac{y-x}{\sqrt{2}}\right]^2 = 1$$

$$\boxed{3x^2+3y^2-2xy=8}$$

$$\text{Area} = \pi\cdot\frac{a}{2}\cdot\frac{b}{2} = \pi\cdot|OB|\cdot|OF| = \pi\sqrt{2}\cdot 2 = \boxed{2\pi\sqrt{2}}$$

Solutions of Homework C1

Q6

$$C: \quad px^2 + py^2 + 2xy + \sqrt{2}(x+y) = 0$$

Rotate coordinates on angle $\frac{\pi}{4}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} \qquad \begin{cases} x = \frac{u-v}{\sqrt{2}} \\ y = \frac{u+v}{\sqrt{2}} \end{cases}$$

$$C: \quad p\left(\frac{u-v}{\sqrt{2}}\right)^2 + p\left(\frac{u+v}{\sqrt{2}}\right)^2 + 2\left(\frac{u-v}{\sqrt{2}}\right)\left(\frac{u+v}{\sqrt{2}}\right) + 2u = 0$$

$$C: \quad (p+1)u^2 + (p-1)v^2 + 2u = 0.$$

1) $p > 1$ $\quad C: \quad (p+1)\left(u + \frac{1}{p+1}\right)^2 + (p-1)v^2 = \frac{1}{p+1}$. Ellipse.

2) $p = 1$. $\quad C: \quad 2u^2 + 2u = 0$. $u(u+1) = 0$. two parallel lines

$u =$

$u = -1$.

3) $-1 < p < 1$ $\quad (p+1)\left(u + \frac{1}{p+1}\right)^2 - (1-p)v^2 = \frac{1}{p+1}$ Hyperbola

4) $p = -1$ $\quad -2v^2 + 2u = 0$. $\quad u = v^2$ parabola

5) $p < -1$ $\quad (p+1)\left(u + \frac{1}{p+1}\right)^2 + (p-1)v^2 = \frac{1}{p+1}$

$$\cancel{\left(u + \frac{1}{p+1}\right)^2 + \frac{p-1}{p+1}v^2}$$

$$(p+1)^2\left(u + \frac{1}{p+1}\right)^2 + (p^2-1)v^2 = 1.$$

Ellipse.