Normal gauge and combinatorial problem

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$$g_{ij}(x) = \delta_{IJ} + \frac{1}{3} R_{ipjq} x^p x^q$$

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Look in the different way an this formula: Let C_{ipjq} be a tensor such that

$$C_{ipjq} = C_{jpiq}$$

and conisder metric

$$g_{ij} = \delta_{ij} + kC_{ipjq}$$

then calculate in terms of this metric the Riemannian curvature tensor at $x = 0$. Calculations are boring but we come to the tensor

$$R^i_{jmn} = R^i_{jmn}(\Gamma)\,,\,,\,,\Gamma^i_{nj} = \text{Levi-Civita connection of } g_{ik}$$

This tensor is expressed via tensor C_{ipjq} and it obeys the properties:

$$R_{ijmn=-R_{ijnm}}$$

$$R_{ijmn} = R_{mnij}$$

$$R_{ijmn} + R_{imnj} + R_{injm} = 0\,.$$

Good combinatorial exersise, perform it!

$$g_{mn} = \delta_{mn} + kC_{mpnq} x^p x^q\,.$$

Calculate Levi-Civita connection at $x = 0$ We do not care about upper and lower indices since $g = \delta$ at $x = 0$

$$\Gamma^i_{nj} = \frac{1}{-}\left(\partial_n g_{ij} + \ldots\right) =$$

$$\frac{k}{2}\left(C_{ipjn} + C_{injp} + C_{ipnj} + C_{ijnp} - C_{npji} - C_{nijp}\right) x^p\,,$$

$$R_{ijmn}\big|_{x=0} = \partial_m \Gamma^i_{nj} - \partial_n \Gamma^i_{mj} =$$

$$\frac{k}{2}\left(C_{imjn} + C_{injm} + C_{imnj} + C_{ijnm} - C_{nmji} - C_{nijm} - C_{injm} - C_{imjn} - C_{inmj} - C_{ijmn} + C_{mnji} + \right.$$

doing some simple calculations we come to

$$\frac{k}{2}\left(C_{\overbrace{\underbrace{imnj}_{[1\leftrightarrow 4]}}^{[2\leftrightarrow 3]}} - C_{\underbrace{ij}_{[1\leftrightarrow 2]}} \right)$$