Generalised functions and operators

We have that operator $a(f)$ acting on N particles state $\Psi = K_N(x_1, \ldots, x_N)$ sends it to $N-1$ particle state

$$K'_{N_1}(x_1, \ldots, x_{N-1}) = \sqrt{N} \int K_N(x_1, \ldots, x_{N-1}, y) f(y) dy \,.$$

On the other hand it can be written as

$$K'_{N-1}(x_1, \ldots, x_{N-1}) = \int L(x_1, \ldots, x_{N-1}; y_1, \ldots, y_N; \xi) f(\xi) K_N(y_1, \ldots, y_N) d\xi dy_1 \ldots dy_N \,.$$

Comparing these formulae we see that kernel of the annilihation operator can be written as

$$L_{\text{anilih.}}(x_1, \ldots, x_{N-1}; y_1, \ldots, y_N; \xi) = \sqrt{N} \delta(x_1 - y_1) \ldots \delta(x_{N-1} - y_{N-1}) \delta(y_N - \xi) \,.$$

Thus we see that

$$L_{\text{anilih.}}(x_\emptyset; y_1; \xi) = \delta(y_1 - \xi) \,, \quad L_{\text{anilih.}}(x_1; y_1, y_2; \xi) = \sqrt{2} \delta(x_1 - y_1) \delta(y_2 - \xi) \,,$$

$$L_{\text{anilih.}}(x_1, x_2; y_1, y_2, y_3; \xi) = \sqrt{3} \delta(x_1 - y_1) \delta(x_2 - y_2) \delta(y_3 - \xi) \,,$$

$$L_{\text{anilih.}}(x_1, x_2, x_3; y_1, y_2, y_3, y_4; \xi) = \sqrt{3} \delta(x_1 - y_1) \delta(x_2 - y_2) \delta(x_3 - y_3) \delta(y_4 - \xi) \,,$$

and so on.

Note that the kernel of the operator $a_B(f)$ and $a_F(f)$ looks the same.

`Now for creation operators:`

We have that operator $a^*(f)$ acting on N particles state $\Psi = K_N(x_1, \ldots, x_N)$ sends it to $N+1$ particle state

$$\tilde{K}_{N+1}(x_1, \ldots, x_N, x_{N+1}) = \sqrt{N+1} K_N(x_1, \ldots, x_N) f(x_{N+1}) \,.$$

It can be written as

$$\tilde{K}_{N+1}(x_1, \ldots, x_{N+1}) = \int L_{\text{creation}}(x_1, \ldots, x_{N+1}; y_1, \ldots, y_N; \xi) f(\xi) K_N(y_1, \ldots, y_N) d\xi dy_1 \ldots dy_N \,.$$

Comparing these formulae we see that

$$L_{\text{creation}}(x_1, \ldots, x_{N+1}; y_1, \ldots, y_N; \xi) = \sqrt{N+1} \delta(x_1 - y_1) \ldots \delta(x_N - y_N) \delta(x_{N+1} - \xi) \,.$$

Thus we see that

$$L_{\text{creation}}(x_1; y_\emptyset; \xi) = L(y_1, \xi) = \delta(x_1 - \xi) \,, \quad L_{\text{creation}}(x_1, x_2; y_1; \xi) = \sqrt{2} \delta(x_1 - y_1) \delta(x_2 - \xi) \,,$$

$$L_{\text{creation}}(x_1, x_2, x_3; y_1, y_2; \xi) = \sqrt{3} \delta(x_1 - y_1) \delta(x_2 - y_2) \delta(x_3 - \xi) \,,$$

$$L_{\text{creation}}(x_1, x_2, x_3, x_4; y_1, y_2, y_3; \xi) = 2 \delta(x_1 - y_1) \delta(x_2 - y_2) \delta(x_3 - y_3) \delta(x_4 - \xi) \,,$$

and so on.

Note that the kernel of the operator $a_B(f)$ and $a_F(f)$ looks the same.