Let $\rho(dx)$ be left-invariant measure on the group G:

$$\int f(g^{-1}x)\rho(dx) = \int f(x)\rho(dx)\,.$$

For an arbitrary $g \in G$ consider the new measure $\rho_g(dx)$ such that

$$\int f(x)\rho_g(dx) = \int f(g^{-1}xg)\rho(dx)\,.$$

This measure is also left-invariant:

$$\int f(hx)\rho_g(dx) = \int f(g^{-1}hxg)\rho(dx) =$$

and due to left invariance of the measure $\rho(dx)$

$$\int f(xg)\rho(dx) = \int f(g^{-1}xg)\rho(dx) = \int f(x)\rho_g(dx)\,.$$

The uniquencess of left-invariant Haar meausure implies that this new measure is proportional to the initial one:

$$\int f(x)\rho_g(dx) = \Delta(g)\int f(x)\rho$$

One can see that modular form is multiplicative: