

Homework 7

1) Let M be a surface embedded in Euclidean space \mathbf{E}^3 . We say that the triple of vector fields $\{\mathbf{e}, \mathbf{f}, \mathbf{n}\}$ is adjusted to the surface M if $\mathbf{e}, \mathbf{f}, \mathbf{n}$ be three vector fields defined on the points of this surface such that they form an orthonormal basis at any point, so that the vectors \mathbf{e}, \mathbf{f} are tangent to the surface and the vector \mathbf{n} is orthogonal to the surface.

Consider the derivation formulae for adjusted vector fields $\{\mathbf{e}, \mathbf{f}, \mathbf{n}\}$:

$$d \begin{pmatrix} \mathbf{e} \\ \mathbf{f} \\ \mathbf{n} \end{pmatrix} = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix} \begin{pmatrix} \mathbf{e} \\ \mathbf{f} \\ \mathbf{n} \end{pmatrix}, \quad (1)$$

where a, b, c are 1-forms on the surface M .

Write down the explicit expression for connection, Weingarten operator, the mean curvature and the Gaussian curvature of M in terms of 1-forms a, b, c and vector fields $\{\mathbf{e}, \mathbf{f}, \mathbf{n}\}$.

2) Show that in derivation formulae $da + b \wedge c = 0$.

3) Find explicitly a triple of vector fields $\{\mathbf{e}, \mathbf{f}, \mathbf{n}\}$ adjusted to the surface M if M is

- a) cylinder
- b) cone
- c) sphere

4) Using results of the previous exercise find explicit expression for derivation formulae (1) in the case if the surface M is a) cylinder, b) cone, c) sphere

Deduce from these results the formulae for Gaussian and mean curvature for cylinder, cone and sphere

5)* Let $\{\mathbf{e}, \mathbf{f}, \mathbf{n}\}$ and $\{\tilde{\mathbf{e}}, \tilde{\mathbf{f}}, \tilde{\mathbf{n}}\}$ be two triples of vector fields adjusted to the surface M .

What is the relation between these triples?

How 1-forms a, b, c in derivation formulae will change if we will change $\{\mathbf{e}, \mathbf{f}, \mathbf{n}\}$ to $\{\tilde{\mathbf{e}}, \tilde{\mathbf{f}}, \tilde{\mathbf{n}}\}$?

Show that 2-form da and 2-form $b \wedge c$ are independent on the choice of the adjusted triple.

6) What will be the result of the parallel transport of an arbitrary tangent vector along the closed curve C on the cone?

How this result correlates with the fact that the Gaussian connection of the cone equals to zero.