Let $u = u(x, y)$ be harmonic function in the disc, and let z_0 be its singulr point.

Consider conjugaate function v:

$$v_x = -u_y \, , v_y = -u_x$$

it is antiderivative of 1-form $dv = u_y dx - u_x dy$. It defines multivalued function $V = \int (u_y dx - u_x dy)$ whic is defined up to a period

$$\Pi = \int_C (u_y dx - u_x dy) \, .$$

We come to multivalued holomorphic function $u + iv$ with period Π. On the other hand the function

$$\frac{\Pi}{2\pi} Log(z - z_0)$$

is multivalued, with the same period. Hence the function:

$$F(z) = u(z) + iv(z) - \frac{\Pi}{2\pi} \log(z - z_0)$$

has disconnected leaves. This is multivalued function $f(z) \to f(z) + i\Pi$. Consider the function

$$G(z) = \exp\left(-\frac{2\pi F(z)}{\Pi}\right) = \exp\left(u(z) + iv(z) - \frac{\Pi}{2\pi} \log(z - z_0)\right) = (z - z_0) \exp\left(-\frac{2\pi F(z)}{\Pi}\right)$$

This is ONE-VALUED! holomorphic function:

$$f(z) \to f(z) + i\Pi \, , G(z) \to \exp \frac{2\pi}{\Pi} \left(F(z) + i\Pi\right) = G(z) \, .$$

(here ther are slight inconveniences.....)

This is holomorphic function in the disc. Consider function

$$G$$

, , !