We have :

$$\frac{3}{2} = 2^x = 2^{\frac{1}{a_0+\frac{1}{a_1+\cdots}}} \Rightarrow \left(\frac{3}{2}\right)^{a_0+\frac{1}{a_1+\frac{1}{a_2+\cdots}}} = 2\,.$$

We have that $\frac{3}{2} < 2 < \frac{9}{4}$, hence $a_0 = 1$.

Thus we have that

$$\frac{3}{2} = 2^{\frac{1}{1+\frac{1}{a_1+\cdots}}} \Rightarrow \left(\frac{3}{2}\right)^{1+\frac{1}{a_1+\frac{1}{a_2+\cdots}}} = 2 \Rightarrow \left(\frac{4}{3}\right)^{a_1+\frac{1}{a_2+\frac{1}{a_3+\cdots}}} = \frac{3}{2}\,.$$

We have that $\frac{4}{3} < \frac{3}{2} < \left(\frac{4}{3}\right)^2$, hence $a_1 = 1$.