We calculated in October 2018 the action of oscillator. Now I will try to calculate it in another way, and understand deeper the relation of action with canonical transformations....

Recall the blog on action:

`26 October 2018`

`We considered Lagrangian of harmonic osillator:`

$$L=\frac{m\dot{x}^2}{2}-\frac{mw^2x^2}{2}\,,$$

`and calculated` $S(x_1,t_1;x_0,t_0)$`:`

`First we wrote down the path` $x(t)$ `which obeys the differential equation` $\frac{d^2x}{dt^2}+\omega^2x=0$ `and the boundary condition` $x(t_0)=x_0$ `and` $x(t_1)=x_1$`. This is:`

$$x(t)=\frac{x_1\sin\omega(t-t_0)-x_0\sin\omega(t-t_1)}{\sin\omega(t_1-t_0)}\,.$$

`Respectively for velocity we have`

$$v(t)=\omega\frac{x_1\cos\omega(t-t_0)-x_0\cos\omega(t-t_1)}{\sin\omega(t_1-t_0)}\,.$$

`Thus we have for Lagrangian`

$$L(t)=\left(\frac{mv^2}{2}-\frac{mw^2x^2}{2}\right)_{x=x(t),v=v(t)}=\frac{m\omega^2}{2\sin^2\omega(t_1-t_0)}\times$$

$$\left[\left(x_1\cos\omega(t-t_0)-x_0\cos\omega(t-t_1)\right)^2-\left(x_1\sin\omega(t-t_0)-x_0\sin\omega(t-t_1)\right)^2\right]=$$

$$\frac{m\omega^2}{2\sin^2\omega(t_1-t_0)}\left[x_1^2\cos 2\omega(t-t_0)+x_0^2\cos 2\omega(t-t_1)-2x_1x_0\cos 2\omega\left(t-\frac{t_0+t_1}{2}\right)\right]\,.$$

`Finally we have that`

$$S(x_1,t_1;x_0,t_0)=\int_{t_0}^{t_1}\left(\frac{mv^2}{2}-\frac{mw^2x^2}{2}\right)_{x=x(t),v=v(t)}=\int_{t_0}^{t_1}L(t)dt=$$

$$\frac{m\omega^2}{2\sin^2\omega(t_1-t_0)}\int_{t_0}^{t_1}dt\left[x_1^2\cos 2\omega(t-t_0)+x_0^2\cos 2\omega(t-t_1)-2x_1x_0\cos 2\omega\left(t-\frac{t_0+t_1}{2}\right)\right]=$$

$$\frac{m\omega^2}{2\sin^2\omega(t_1-t_0)}\left[\left(x_1^2+x_0^2\right)\frac{\sin 2\omega(t_1-t_0)}{2\omega}-2x_1x_0\frac{\sin\omega(t_1-t_0)}{\omega}\right]=$$

$$\frac{m\omega}{2\sin\omega(t_1-t_0)}\left[\left(x_1^2+x_0^2\right)\cos\omega(t_1-t_0)-2x_1x_0\right]\,.$$

Change now the angle that we look at this problem.
Instead Lagrnagian $L = \frac{1}{2}\left(\dot{q}^2 - q^2\right)$ we consider Hamiltonian

$$H = \frac{1}{2}\left(p^2 + x^2\right)$$

Previously we calculated action as integral of Lagrangian over time:

$$S(x_0, x_1, t) = \frac{1}{2\sin t}\left[\left(x_1^2 + x_0^2\right)\cos t - 2x_1 x_0\right] .$$

It is useful to denote (x, p) initial coordinates and momenta and by (y, q) after time T