Homework 8(C2)

1 Let C be the curve defined by the intersection of the plane $\alpha\colon x + 2z = 2$ with the conic surface M: $x^2 + y^2 = z^2$.

Let C_{proj} be the orthogonal projection of this curve onto the plane $z = 0$.

Show that the curve C_{proj} is an ellipse.

Explain why the curve C is also an ellipse.

Find the foci of the curve C_{proj}. In particular show that the vertex of the conic surface M is a focus of the ellipse C_{proj}.

Find the areas of the ellipses C and C_{proj}.

Write down a parameterisation of the ellipse C and of the ellipse C_{proj} (you may choose any parameterisation)

2 Let C be the curve defined by the intersection of the plane $\alpha\colon kx + z = 1$ (where k is real parameter) with the conic surface M: $2x^2 + 2y^2 = z^2$.

Let C_{proj} be the orthogonal projection of this curve onto the plane $z = 0$.

Find the values of parameter k such that the curve C and the curve C_{proj} are ellipses.

Find the values of parameter k such that the curve C and the curve C_{proj} are hyperbolas.

Show that for $k = \pm\sqrt{2}$ the curves C and C_{proj} are parabolas.

Show that the vertex of the conic surface M, the origin, is the focus of the parabola C_{proj} and that the intersection of the plane α and the horizontal plane ($z = 0$) is the directrix of this parabola.

3 Let C be the ellipse in $\mathbf{E}^2$ with foci $F_1 = (0, 0)$, $F_2 = (6, 0)$ which passes through the point $B = (0, 8)$. Write down the equation of this ellipse.

Choose a parameterisation of this ellipse and calculate $\int_C xdy - ydx$.

To what extent does this integral depend on the choice of parameterisation?

4 Let C be the curve defined by the intersection of the plane $\alpha\colon 2x + z = 1$ with the conic surface M: $5x^2 + 5y^2 = z^2$

Choose a parameterisation of this conic section and calculate the integral of the 1-form $\omega = xdy - ydx + dz$ over this conic section.

To what extent does this integral depend on the choice of parameterisation?

5 Let C be the curve in $\mathbf{E}^3$, defined by the intersection of the conic surface $x^2+y^2 = z^2$ with the plane $kx + z = 1$, and let C_{proj} be the orthogonal projection of the curve C onto the plane $z = 0$.

Show that if $|k| < 1$ then the curve C is an ellipse.

Show that the curve C_{proj} is a parabola in the case if $k = 1$, and find focus and directrix of this parabola.