Homework 5

1 Consider the following curves:

$C_1\colon \mathbf{r}(t)\ \begin{cases} x=t \\ y=2t^2-1 \end{cases},\ 0<t<1,\qquad C_2\colon \mathbf{r}(t)\ \begin{cases} x=t \\ y=2t^2-1 \end{cases},\ -1<t<1,$

$C_3\colon \mathbf{r}(t)\ \begin{cases} x=2t \\ y=8t^2-1 \end{cases},\ 0<t<\frac{1}{2},\qquad C_4\colon \mathbf{r}(t)\ \begin{cases} x=\cos t \\ y=\cos 2t \end{cases},\ 0<t<\frac{\pi}{2},$

$C_5\colon \mathbf{r}(t)\ \begin{cases} x=t \\ y=2t-1 \end{cases},\ 0<t<1,\qquad C_6\colon \mathbf{r}(t)\ \begin{cases} x=1-t \\ y=1-2t \end{cases},\ 0<t<1,$

$C_7\colon \mathbf{r}(t)\ \begin{cases} x=\sin^2 t \\ y=-\cos 2t \end{cases},\ 0<t<\frac{\pi}{2},\qquad C_8\colon \mathbf{r}(t)\ \begin{cases} x=t \\ y=\sqrt{1-t^2} \end{cases},\ -1<t<1,$

$C_9\colon \mathbf{r}(t)\ \begin{cases} x=\cos t \\ y=\sin t \end{cases},\ 0<t<\pi,\qquad C_{10}\colon \mathbf{r}(t)\ \begin{cases} x=\cos 2t \\ y=\sin 2t \end{cases},\ 0<t<\frac{\pi}{2},$

$C_{11}\colon \mathbf{r}(t)\ \begin{cases} x=\cos t \\ y=\sin t \end{cases},\ 0<t<2\pi,\qquad C_{12}\colon \mathbf{r}(t)\ \begin{cases} x=a\cos t \\ y=b\sin t \end{cases},\ 0<t<2\pi$ (ellipse),

Draw the images of these curves.

Write down their velocity vectors.

Indicate parameterised curves which have the same image (equivalent curves).

In each equivalence class of parameterised curves indicate curves with same and opposite orientations.

2 Consider the curves C_1, C_2 given by the parametric equations

$$C_1\colon \mathbf{r}(\tau)\ \begin{cases} r(\tau)=\frac{1}{2-\cos\tau} \\ \varphi(\tau)=\tau \end{cases},\ 0\le\tau<2\pi,\quad C_2\colon \mathbf{r}(t)\ \begin{cases} x(t)=\frac{2}{3}\cos t+\frac{1}{3} \\ y(t)=\frac{1}{\sqrt{3}}\sin t \end{cases},\ 0\le t<2\pi\,.$$

Here the curve C_1 is defined in polar coordinates r,φ, the curve C_2 is defined in usual Cartesian coordinates ($x=r\cos\varphi, y=r\sin\varphi$).

Show that the images of both curves are ellipses.

Check that these ellipses coincide.

† Find foci of this ellipse *.

3 Consider the following curve (helix): $\mathbf{r}(t)\colon\ \begin{cases} x(t)=R\cos\Omega t \\ y(t)=R\sin\Omega t \\ z(t)=ct \end{cases},\quad 0\le t\le t_0$. Show that the image of this curve belongs to the surface of cylinder $x^2+y^2=R^2$.

Find the velocity vector of this curve.

Find the length of this curve.

Finish the following sentence:

After developing the surface of cylinder to the plane the curve will develop to the...

4 Consider differential forms $\omega=xdy-ydx$, $\sigma=xdx+ydy$ and vector fields $\mathbf{A}=x\partial_x+y\partial_y$, $\mathbf{B}=x\partial_y-y\partial_x$.

a) Calculate $\omega(\mathbf{A}),\omega(\mathbf{B})$, $\sigma(\mathbf{A}),\sigma(\mathbf{B})$.

b) Calculate differential forms ω and σ in polar coordinates $x=r\cos\varphi$, $y=r\sin\varphi$.

5 Consider differential forms $\omega=xdy-ydx$ and $\sigma=xdx+ydy+zdz$ in $\mathbf{E}^3$. Calculate $\omega(\mathbf{v})$ and $\sigma(\mathbf{v})$ on the velocity vectors of helix considered in question 3).

---

* Ellipse can be defined as a locus of points in a plane such that the sum of the distances to two fixed points is a constant. These two fixed points are called foci of the ellipse.