Partition of unity snd identities

Consider S^1 in stereographic coordinates:

$$2\pi = \int_{\infty}^{\infty} \frac{dt}{1+t^2} = \int_{\infty}^{\infty} \frac{(\rho_+(t) + \rho_- t)dt}{1+t^2}\,,$$

where $(\rho_+(t), \rho_(t))$ is partition of unity for S^1.

$$2\pi = \int_{\infty}^{\infty} \frac{(\rho_+(t) + \rho_- t)dt}{1+t^2} = \int_{\infty}^{\infty} \frac{\rho_+(t)dt}{1+t^2} + \int_{\infty}^{\infty} \frac{\rho_-(t))dt}{1+t^2}\,,$$

where $\rho_+(t) = f(t)$ is an arbitrary function which vanishes at infinity and $\rho_t = 1 - \rho_+(t)$. (We suppose also that $0 < \rho, 1$.) Consider new coordinate $s = \frac{1}{t}$, then

$$2\pi = \int_{\infty}^{\infty} \frac{\rho_+(t)dt}{1+t^2} + \int_{\infty}^{\infty} \frac{\rho_-(t)dt}{1+t^2} = \int_{\infty}^{\infty} \frac{\rho_+(t)dt}{1+t^2} + \int_{\infty}^{\infty} \frac{\rho_-\left(\frac{1}{s}\right)ds}{1+s^2} =$$

$$\int_{\infty}^{\infty} \frac{\left(\rho_+(t) + \rho_-\left(\frac{1}{t}\right)\right)dt}{1+t^2}\,.$$

We come to the identity:

$$\int_{\infty}^{\infty} \frac{f(t) - f\left(\frac{1}{t}\right)}{1+t^2} dt = 0$$

for an arbitrary function $f(t)$.