Homework 4

1 Calculate the derivatives of the functions $f = x^2 + y^2$, $g = e^{-(x^2+y^2)}$ and $h = q\log|r| = q\log\left(\sqrt{x^2+y^2}\right)$ (q is a constant) along vector fields $\mathbf{A} = x\partial_x + y\partial_y$ and $\mathbf{B} = x\partial_y - y\partial_x$, i.e. calculate $\partial_{\mathbf{A}} f, \partial_{\mathbf{A}} g, \partial_{\mathbf{A}} h$, $\partial_{\mathbf{B}} f, \partial_{\mathbf{B}} g, \partial_{\mathbf{B}} h$.

2 Perform the calculations of the previous exercise using polar coordinates.

3 Consider in $\mathbf{E}^2$ vector fields $\mathbf{A} = x\partial_x + y\partial_y$, $\mathbf{B} = x\partial_y - y\partial_x$, $\mathbf{C} = \partial_x$, $\mathbf{D} = \partial_y$. Calculate the values of 1-forms df, dg on these vector fields if $f = (x^2+y^2)^n$ and $g = \frac{y}{x}$. For vector fields $\mathbf{A}, \mathbf{B}$ perform calculations also in polar coordinates.

4 Calculate the integrals of the form $\omega = \sin y\, dx$ over the following three curves. Compare answers.

C_1: $\mathbf{r}(t)\ \begin{cases} x = 2t^2 - 1 \\ y = t \end{cases}$, $0 < t < 1$, $\qquad C_2$: $\mathbf{r}(t)\ \begin{cases} x = 8t^2 - 1 \\ y = 2t \end{cases}$, $0 < t < 1/2$,

C_3: $\mathbf{r}(t)\ \begin{cases} x = \cos 2t \\ y = \cos t \end{cases}$, $0 < t < \frac{\pi}{2}$

5 Calculate the integral of the form $\omega = e^{-y}dx + \sin x dy$ over the segment of straight line which connects the points $A = (1,1)$, $B = (2,3)$. At what extent an answer depends on a chosen parameterisation?

6 Calculate the integral of the form $\omega = xdy$ over the upper arc of the unit circle starting at the point $A = (1,0)$ and ending at the point $(0,1)$.

7 Solve the previous problem for the arc of the ellipse $x^2 + y^2/9 = 1$ defined by the condition $y \geq 0$.

8 Calculate the integral $\int_C \omega$ where $\omega = xdx + ydy$ and C is

a) the straight line segment $x = t, y = 1 - t$, $0 \leq t \leq 1$

b) the segment of parabola $x = t$, $y = 1 - t^n$, $0 \leq t \leq 1$, $n = 2, 3, 4, \ldots$

c) the segment of the sinusoid $x = t$, $y = \cos\frac{\pi}{2}t$, $0 \leq t \leq 1$

d) **an arbitrary** curve starting at the point $(0,1)$ and ending at the point $((1,0)$.

9 Calculate the integral of the form $\omega = \frac{xdy - ydx}{x^2+y^2}$ over the curves $a), b), c)$ from the previous exercise.

10* What values can take the integral $\int_C \omega$ if C is an arbitrary curve starting at the point $(0,1)$ and ending at the point $((1,0)$ and $\omega = \frac{xdy-ydx}{x^2+y^2}$.