Homework 2a

1 Let $\mathbf{e}, \mathbf{f}$ be orthonormal basis in Euclidean space $\mathbf{E}^2$. Consider a vector

$$\mathbf{n}_\varphi = \mathbf{e}\cos\varphi + \mathbf{f}\sin\varphi\,.$$

Let A be a linear orthogonal operator acting on the space $\mathbf{E}^2$ such that $A(\mathbf{e}) = \mathbf{n}$.

We know that $\det A = \pm 1$ since A is orthogonal operator.

In the case if $\det A = 1$, find the image $A(\mathbf{f})$ of vector $\mathbf{f}$ and an image $A(\mathbf{x})$ of arbitrary vector $\mathbf{x} = a\mathbf{e} + b\mathbf{f}$, write down the matrix of operator A in the basis $\mathbf{e}, \mathbf{f}$ and explain geometrical meaning of the operator A.

† How the answer will change if $\det A = -1$?

2 Let $\mathbf{e}, \mathbf{f}$ be an orthonormal basis in Euclidean space $\mathbf{E}^2$. Consider a vector $\mathbf{N} = \mathbf{e}+\mathbf{f}$ in $\mathbf{E}^2$.

Let A be an orthogonal operator acting on the space $\mathbf{E}^2$ such that $A\mathbf{N} = \mathbf{N}$. ($\mathbf{N}$ is eigenvector of A with eigenvalue 1.) Suppose that A is not identity operator.

a) Find an action of operator A on the vector $\mathbf{R} = \mathbf{e} - \mathbf{f}$ in $\mathbf{E}^2$.

b) Write down the matrix of operator A in the basis $\mathbf{e}, \mathbf{f}$.

c) Explain geometrical meaning of the operator A.