How look linear factional maps which send disc $x^2 + y^2 = 1$ onto itself: this is representation of the group $SL(2, R)/??$ in Poincare disc.

(Linear fractional bijections of upper-half plane onto itself are $SL(2, R)/Z_2?$)

If

$$w = \frac{Az + B}{Cz + D}$$

and $|w| = 1$ if $|z| = 1$, then

$$z\bar{z} = 1 \to w\bar{w} = 1, i.e.(Az + B)(\bar{A}\bar{z} + \bar{B}) = (Cz + D)(\bar{C}\bar{z} + \bar{D})$$

we come to

$$|A|^2 + |B|^2 + (A\bar{B}z + c.c.) = |C|^2 + |D|^2 + (C\bar{D}z + c.c.)\,.$$

We see that for $g = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$,

$$|A|^2 + |B|^2 = |C|^2 + |D|^2\,, \quad \text{and}\,, \quad A\bar{B} - C\bar{D} = 0 \tag{$*$}$$

This means that g up to dilation preserves hermitian scalar product with signature (1.1), we come to the group $SU(1, 1)/\{1. -1\}$.

Indeed Let $g = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ Then conditions (*) means that

$$g^+ \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot g = \begin{pmatrix} \bar{A} & \bar{C} \\ \bar{B} & \bar{D} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \lambda \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

i.e. removing scaling factor l and putting $\det g = 1$ we come to

$$g^+ = \begin{pmatrix} \bar{A} & \bar{C} \\ \bar{B} & \bar{D} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} D & -B \\ -C & A \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} D & B \\ C & A \end{pmatrix}\,,$$

i.e. $D = \bar{A}, C = \bar{B}$ Thus symmetries of Poincare disc is $SU(1, 1)/\{1, -1\}$, where

$$SU(1.1) = \left\{ g = \begin{pmatrix} A & B \\ \bar{B} & \bar{A} \end{pmatrix} : |A|^2 - |B|^2 = 1 \right\}\,.$$