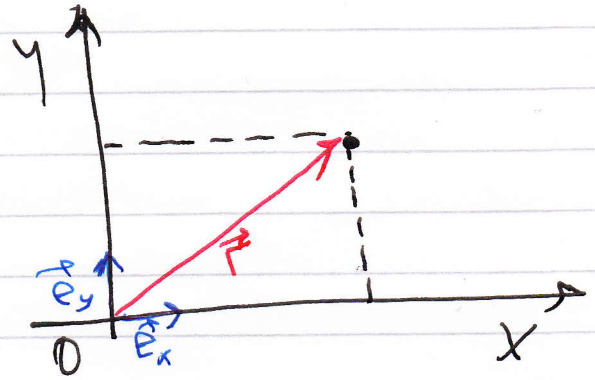
23 March

Lecture CII

Cartesian coordinates (orthogonal, rectangular)

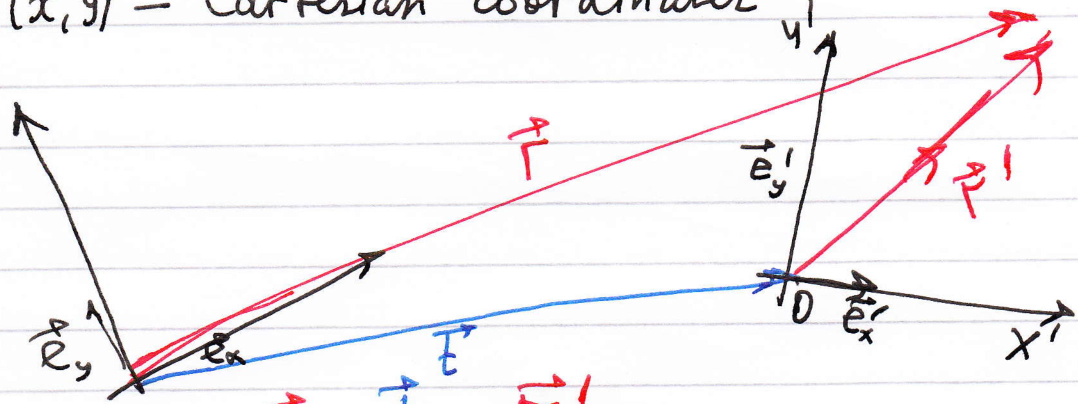
Transformation from Cartesian coordinates to Cartesian coordinates

Plane - $\mathbb{E}^2$ Space - $\mathbb{E}^3$



$\vec{r} = x\vec{e}_x + y\vec{e}_y$ $\{\vec{e}_x, \vec{e}_y\}$ - orthonormal basis

(x, y) - Cartesian coordinates



$\vec{r} = \vec{t} + \vec{r}'$

$$(\vec{e}_x, \vec{e}_y)\begin{pmatrix} x \\ y \end{pmatrix} = \vec{r} = \underbrace{(\vec{e}_x, \vec{e}_y)\begin{pmatrix} a \\ b \end{pmatrix}}_{\vec{t}} + \underbrace{(\vec{e}_x', \vec{e}_y')\begin{pmatrix} x' \\ y' \end{pmatrix}}_{\vec{r}'}$$

$$x\vec{e}_x + y\vec{e}_y = \vec{r} = \vec{t} + \vec{r}' = a\vec{e}_x + b\vec{e}_y + x'\vec{e}_x' + y'\vec{e}_y'$$

$$(\vec{e}_x', \vec{e}_y') = (\vec{e}_x, \vec{e}_y)\begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}$$

$$(\vec{e}_x, \vec{e}_y)\begin{pmatrix} x \\ y \end{pmatrix} = (\vec{e}_x, \vec{e}_y)\begin{pmatrix} a \\ b \end{pmatrix} + (\vec{e}_x, \vec{e}_y)\begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}\begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}\begin{pmatrix} x' \\ y' \end{pmatrix}$$

orthogonal matrix

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Transformation from Cartesian Coordinates to another Cartesian coordinates (in plane)

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \underbrace{\begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}}_{\text{orthogonal}} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\begin{cases} x = a + p_{11}x' + p_{12}y' \\ y = b + p_{21}x' + p_{22}y' \end{cases}$$

Example. $\begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}}_{\text{rotation}}$

$$\begin{cases} x = a + x'\cos\varphi - y'\sin\varphi \\ y = b + x'\sin\varphi + y'\cos\varphi \end{cases}$$

$$\begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\varphi & \sin\varphi \\ \sin\varphi & -\cos\varphi \end{pmatrix}}_{\text{rotation + reflection}}$$

$$\begin{cases} x = a + x'\cos\varphi + y'\sin\varphi \\ y = b + x'\sin\varphi - y'\cos\varphi \end{cases}$$

Lecture C II

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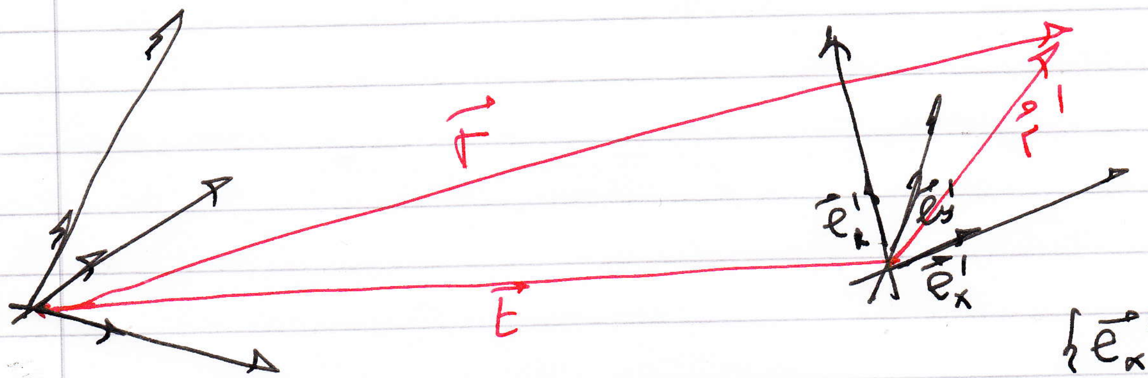
Transformation of coordinates in $\mathbb{E}^3$



$\{\vec e_x, \vec e_y, \vec e_z\}$ orthonormal basis

$\{\vec e\,'_x, \vec e\,'_y, \vec e\,'_z\}$ orthonormal basis

$$\vec r = \vec t + \vec r\,'$$

$$\underbrace{(\vec e_x, \vec e_y, \vec e_z)\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{\vec r} = \underbrace{(\vec e_x, \vec e_y, \vec e_z)\begin{pmatrix} a \\ b \\ c \end{pmatrix}}_{\vec t} + (\vec e\,'_x, \vec e\,'_y, \vec e\,'_z)\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

$$(\vec e\,'_x, \vec e\,'_y, \vec e\,'_z) = (\vec e_x, \vec e_y, \vec e_z)\underbrace{\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}}_{\text{orthogonal matrix}}$$

$P^T P = E$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} a \\ b \\ c \end{pmatrix}}_{\text{translation}} + \underbrace{\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}}_{\text{rotation + reflection}}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

Exemples.

Rotation around axis Oz on angle φ

$$\begin{pmatrix} x^* \\ y^* \\ z^* \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

Rotation around axis OX and translation

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

$$\begin{cases} x = a + x' \\ y = b + y'\cos\theta - z'\sin\theta \\ z = c + y'\sin\theta + z'\cos\theta \end{cases}$$