

Integration of cocycles

Let $c(K)$ be cocycle on algebra of vector fields on \mathbf{R} . How to take its integral. I still did not calculate the role of being cocycle, e,t,x,, but if $z = z(t; x)$ be an arbitrary diffeomorphism, and Φ be one cocycle on group then for infinitesimal τ ($\tau^2 = 0$)

$$\Phi(z(t + \tau; x)) - \Phi(z(t; x)) = c(K = z_t) \text{ at the point } z,$$

i.e.

$$\frac{\partial \Phi(z)}{t} = c(K = z_t) \text{ at the point } z,$$

Example. Consider cocycle

$$c(K) = K_{xx}|dx|$$

Then we have

$$\begin{aligned} \frac{\partial \Phi(z(t, x))}{\partial t} &= z_{tzz}|dz| = \left(\left(\frac{z_{tx}}{z_x} \right)_x \cdot \frac{1}{z_x} \right) z_x |dx| = \\ \left(\frac{z_{tx}}{z_x} \right)_x |dx| &= \frac{\partial^2}{\partial x \partial t} (\log z_x) |dx| = \frac{\partial}{\partial t} \left(\frac{\partial \log z_x}{\partial x} \right) |dx| \\ \Phi(z(x)) &= \frac{z_{xx}}{z_x} |dx| \end{aligned}$$

sure here we suppose that a function $z(x)$ is ubcluded in the exponent $z(t, x)$. (il faut formuler plus exactement.....)

$$\text{Schwarzian: } c(K) = K_{xxx}|dx|^2$$