

On one definition of rank of matrix

Sasha Borovik told me about one extremely helpful definition of the rank of the matrix. Here I will reproduce it.

Let $M = M_{m \times n}$ be $m \times n$ matrix. Then *rank* of M is the minimal positive number r such that there exist $m \times r$ matrix $L = L_{m \times r}$ and $r \times n$ matrix $R = R_{r \times n}$ such that

$$M_{m \times n} = L_{m \times r} \circ R_{r \times n} .$$

We suppose that $\text{rank } M = 0$ if all the entries of this matrix vanish.