On two ways

Reading AKSZ paper I realised that the chidlish version of BRST—-BV relation is the following:

homological vector field $\mathbf{X}$ on M, $[\mathbf{X},\mathbf{X}]=0$		linear Hamiltonian $S=X^a x^*_a$ on ΠT^*M
if P is Poisson and $\mathbf{X}$ has Hamiltonian Q	$\Rightarrow$	for function f on M
$\{Q,f\}=\mathbf{X}f, Q$ BRST charge		$[S,f]=\{Q,f\}$
BRST equation $\{Q,Q\}=0$		BV equation $[S,S]=0$

(One can consider instead BV linear even action S on ΠT^*M the linear odd action $S^{\rm odd}=X^a(x)p_a$ in the space T^*M respectively with bracket $(\_,\_)$ on T^*M and master equation $(S,S)=0$)

Meaning of the equation above is the following: An action of homological vector field $\mathbf{X}$ on arbitrary function f on M is equal to canoncial Schouten bracket on ΠT^*M of even linear Hamiltonian $S^{\rm odd}=X^a(x)x^*_a$ (master-action) defined on ΠT^*M with this function f, or alternaticely it is equatl to canonical Poisson bracket in T^*M of odd linear Hamiltonian $S^{\rm odd}=X^a(x)p^*_a$ defined on T^*M with this function f:

$$\mathbf{X}f=[X^a(x)x^*_a,f(x)]=(X^a(x)p_a,f(x)) \tag{1}$$

On the other hand this can be represented as a derived bracket. Equation above means that the action of vector field $\mathbf{X}$ on function f can be considered as homotopy Schouten bracket (with Hamiltonian $X^a(x)x^*_a$) or it can be considered as homotopy Poisson bracket (with Hamiltonian $X^a(x)p_a$). What choice is better? Suppose that the field $\mathbf{X}$ is Hamitonian. Let $P=P(x,x^*)$ be an even function which generates Poisson bracket $\{\_,\_\}$: $(P=\{x^a,x^b\}x^*_a x^*_b)$:

$$\{f,g\}=\{f,g\}_P=[[P,f],g]\,,$$

and let $Q_\mathbf{X}$ be Hamiltonian of $\mathbf{X}$. Then

$$\mathbf{X}f=\{Q_\mathbf{X},f\}_P=[[P,Q_\mathbf{X}],f]$$

i.e. Hamiltonian which generates action of vector field $[P,Q_\mathbf{X}]$ and it is homotopy Schouten bracket.

This quesiont arises when we consider two little bit different scemes of construction higher Koszul bracket. In the first approach (paper 2008) we considered linear Hamiltonian $H=(d_P)^a\,p_a$ for Lichnerowicz differential d_P on ΠT^*M (Then applying MX symplectomorphsim to this Hamiltonian we came to the new Hamiltonian which produces higher Koszul bracket via derived.)

In the new approach we consider Lichnerowicz differential d_P on ΠT^*M as homotopy Schouten bracket generated by Hamiltonian H and this is the same linear Hamiltonian as before.