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`Fact`

Cosnider differential equations

There is one-one correspondence between solutions $u = \varphi(x^1, \dots, x^n)$ of the differential equation

$$u(x^1, \dots x^n) \colon \; A^i(x^j) \frac{\partial u(x)}{\partial x^i} = F(x^1, \dots x^n, u)\,, \tag{1}$$

and the one-parametric families of solutions equation

$$\Phi = \Phi(x^1, \dots, x^n, u) \colon \; A^\mu(x) \frac{\partial \Phi(x,u)}{\partial x^i} + F(x^i, u) \frac{\partial \Phi(x,u)}{\partial u} = 0\,. \tag{2}$$

Namely, let $\Phi = W(x^1, \dots, x^n, u)$ be the solution of the equation (2). Then *one-parametric family* of functions

$$u = \varphi_c(x) \colon W(x,u)\big|_{u=\varphi_c(x)}$$

is the solution of equation* (1):

$$\forall c \in \mathbf{R}\,,\; A^i(x^j) \frac{\partial u(x)}{\partial x^i}\big|_{u(x=\varphi_c(x))} = F(x^1, \dots x^n, u)\,,$$

and vice versa: Let $u(c,x) = \varphi_c(x)$ be *an one parametric family* of functions ($c \in \mathbf{R}$) such that all these functions are solutions of the equation (1). Then a function $c = c(x,u)$ is the solution of the equation (2).

* we suppose that $\Phi_u \neq 0$