Geometry of first order equation

Let J^1M be a space of first jets of functions on manifold M. Cooridnates on J^1M are (p_i, q^j, u), where q^j are coordinates on M. Jet of every function $u = u(x)$ has coordinates $\left(p_i = \frac{\partial u}{\partial x q^i}, q^i, u\right)$.

Consider $\mathcal{C}$, the Cartan distribution of $2n$-dimensional planes in J^1M defined by the form $\omega = p_i dq^i - du$

$$\mathcal{C}_{\mathbf{p}} \subset T_{\mathbf{p}} J^1 M = \{T_{\mathbf{p}}(J^1 M) \ni \mathbf{X} \colon\ \omega(\mathbf{X}) = 0\}\,,$$

Vector field

$$M^i \frac{\partial}{\partial q^i} + N_i \frac{\partial}{\partial p_i} + A \frac{\partial}{\partial u} \text{ belongs to Cartan distribution } \mathcal{C} \text{ if } A = p_i M^i\,.$$

$\mathcal{C}$ is non-integrable distribution.

Consider differential equation,

$$\mathcal{E} \colon F(p, q, u) = 0\,.$$

Differential equation is sumbmanifold of codimension 1 in the space $J^1(M)$.

The Cartan distribution $\mathcal{C}$ of hyperplanes on J^1M defines distribution $\mathcal{C}(\mathcal{E})$ in $T\mathcal{E}$:

$$\mathcal{C}(E) = \mathcal{C} \cap T\mathcal{E}\,.$$

$$\mathbf{X} = M^i \frac{\partial}{\partial q^i} + N_i \frac{\partial}{\partial p_i} + A \frac{\partial}{\partial u} \in \mathcal{C}(\mathcal{E}) \text{ if } A = p_i M^i \,\&\, \left(M^i \frac{\partial}{\partial q^i} + N_i \frac{\partial}{\partial p_i} + A \frac{\partial}{\partial u}\right) F(p, q, u)\big|_{F=0} = 0\,.$$

The solution of differential equation (1) is the maximal integral of the distribution $\mathcal{C}(\mathcal{E})$, which is the n-dimensional submanifold in M. (We come to the n-parametric family of solutions?)

`Quesion: Why`

Let N be an arbitrary solution, N is the surface of dimension n and any tangent plane to N belongs to Cartan distribution and is tangent to $\mathcal{E}$. Consider an arbitrary point $\mathbf{p} \in N$. Let $\alpha = \alpha_{\mathbf{p}}$ be the tangent plane. The vectors in tangent plane belong to distribution $\mathcal{C}_{\mathcal{E}} = \mathcal{C} \cap \mathcal{E}$, i.e. they are orthogonal to the covector (1-form) in $2n+1$-dimensional space

$$\omega_C = (1, 0, \dots, 0, -p_1, \dots, -p_n)\,, \qquad \text{(the vector belongs to Cartan distribution } \mathcal{C})$$

and to the covector

$$dF = \left(F_u, \frac{\partial F}{\partial q^1}, \dots, \frac{\partial F}{\partial q^n}; \frac{\partial F}{\partial p_1}, \dots \frac{\partial F}{\partial p_n}\right)\,, \qquad \text{(the vector is tangent to the differential equation } \mathcal{E} =$$

$$\alpha_{\mathbf{p}} \ni \mathbf{X} \Leftrightarrow \omega_C(\mathbf{X}) = dF(\mathbf{X}) = 0\,.$$

Definition The point $\mathbf{p}$ is not singular point if tnhe rank of the matrix $\begin{pmatrix} \omega_C \\ dF \end{pmatrix}$ is equal to 2. IN this case the dimension of

`I continue this file the first day of the New Year.....`