Let $\mathbf{a}, \mathbf{b}$ be two arbitrary non-proportional unit vectors in $\mathbf{E}^2$, and let $\{\mathbf{e}, \mathbf{f}\}$ be an arbitrary orthonormal basis in $\mathbf{E}^2$.

Consider bilinear form:

$$B_P(\mathbf{x}, \mathbf{y}) = \langle P^{-1}\mathbf{x}, P^{-1}\mathbf{y}\rangle_{\mathbf{E}^2}$$

and consider an ellipse

$$B_P(\mathbf{x}, \mathbf{x}) = 1\,.$$

This is the image of circle under affine transformation. Its main directions will be eigenvectors of P.

Under rotation and affine transformation $x \mapsto ax, y \mapsto by$ it becomes the circle.