Linear algebra of thick morphjisms

let X, Y be finite-dimensional vector spaces, and let $S\colon X \to Y$ be a linear map. We come to linear map $S^*\colon Y^* \to X^*$ and the bilinear form on the space $X \otimes Y^*$:

$$\mathbf{y} = S(\mathbf{x}), \quad y^\mu = S_i^\mu x^i \,, \tag{1}$$

$$\mathbf{p} = S^*(\mathbf{q}), \quad p_i = S_i^\mu q_\mu \,, \tag{2}$$

$$S(\mathbf{x}, \mathbf{q}) = S_i^\mu x^i q_\mu \,. \tag{3}$$

The object S defines the Lagrangian surface Λ_S in the space $T^*X \times (-T^*Y)$

$$\Lambda_S = \{(\mathbf{x}, \mathbf{p}, \mathbf{y}, \mathbf{q}\colon\ \mathbf{y} = S(\mathbf{x})\,, \quad \mathbf{p} = S^*(\mathbf{q})\}\,.$$

if we define symplectic structure in the space $T^*X \times (-T^*Y)$ by canonical 2-form

$$\omega = dp_i \wedge dx^i - dq_\mu dy^\mu \,.$$