Few days ago Yurij Bazlov told me about one very beautiful problem were Fermat (small) Theorem appears.

The problem is following.

Calculate the number of subsets of the set $\{1, 2, 3, \ldots, p\}$ such that the number p divides the sum of elements in this subset. Suppose that p is a prime number.

This problem was given to schoolkids on something like International Olympiad.

In this problem the Fermat (small) theorem is unexpectably appears.

Yurij solved this problem in a very beautiful way.

I spent the Saturday aolving this problem; as a result I failed to produce the coursework in a time, however, I solved it `Paris il vaut bien une messe!`.

My solution is the following:

Consider polynomial

$$P_p(x) = (1 + x)(1 + x^2)(1 + x^3) \ldots (1 + x^p)$$

where x is undeterminate, and show that

$$P_p(x) = 1 + C_p \frac{x^p - 1}{x - 1} .$$