On Z_2 grading

In fact Z_2 grading has sence only in free Clifford algebra?? We consider two examples, when it failes

Example 1 Consider Clifford algebra over $\mathbf{E}^3$ (We suppose $W(\mathbf{x}) = -(\mathbf{x}, \mathbf{x})$)

We know that

1. Odd elements $\mathbf{x}$ project to rotation of $\mathbf{E}^3$ on the angle π with respect to axis directed along $\mathbf{x}$.

2. The even element $\mathbf{x} \cdot \mathbf{y}$ projects to the rotation of $\mathbf{E}^3$ on the angle $2\angle(\mathbf{x}, \mathbf{y})$ with respect to axis which is orthogonal to both vectors. (It is identical transformation if $\mathbf{x}, \mathbf{y}$ are linear dependent)

We see that

$$L_{\mathbf{e_x}} L_{\mathbf{e_y}} = L_{\mathbf{e_z}} \,,$$

i.e. even and odd elements have the same projection.

Example 2 Consider Clifford $\mathtt{Cliff}_k$ algebra over $\mathbf{E}^k$

Consider map

$$\mathbf{E}^3 \to \mathtt{Cliff}_2\colon\ \beta(\mathbf{e}_x) = \mathbf{e}_x, \beta(\mathbf{e}_y) = \mathbf{e}_y, \beta(\mathbf{e}_z) = \mathbf{e}_x \cdot \mathbf{e}_y,$$

Yes, this map destroys parity, but it still obeys condition

$$\beta_{\mathbf{x}} \cdot \beta_{\mathbf{x}} = -(\mathbf{x}, \mathbf{x}) \cdot 1$$

hence by universality condition

$$\begin{array}{c} \mathbf{E}^3 \\ \swarrow\downarrow \\ \mathtt{Cliff}_3 \Rightarrow \mathtt{Cliff}_2 \end{array}$$

we come to a map

$$\mathtt{Cliff}_3 \to \mathtt{Cliff}_2$$

ne can see that $\mathtt{Cliff}_2$ is associative algebra over $\mathbf{E}^3$ with