Recalling on fibre bundles

Let $P \to B$ be principal fibre bundle, with sturcture group G. Let V be a space where group G acts. We consider associated bundle $P \times_G V$ of pairs

$$P \times_G V = \{(p, \mathbf{v}): \quad (pg, \mathbf{v}) = (p, \rho(g)\mathbf{v})\}$$

Many years ago I learned the fact, that sections of the associated fibre bundle $P \times_G V$ are in one-one corresondence with equivariant maps from P to V:

Theorem

$$Hom_G(P, V) \approx \quad \Gamma\,(P \times_G V)$$

Indeed let $f = f(p)$ be an equivariant map from P to V: $f(pg) = \rho(g)f(p)$.

One can assign to this map the section $\sigma(b) = (p, f(p))$, where p is an arbitrary element which projects to b.

On the other hand let $\sigma(b) = (p(b), \mathbf{v}(b))$ be an arbitrary section of the fibre bundle $P \times_G V$. It defines the equivariant map such that the value of this map on arbitrary $p' = pg$ $(\pi(p' = \pi(p)) = b)$ is equal to

$$f(p') = f(pg) = \rho(g)\mathbf{v}(b)\,.$$

I spent the considerable time to recover this textbook result...