Recall that generating function $S(x,q) = S(x^i, q_a)$ defines a map

$$\Phi_S^*\colon \quad C(N) \to C(M)$$

such that

$$\forall g(y) \in CM(N)C(M) \ni f(x) = \Phi_S^*(g) = g(y) + S(x,q) - y^a q_a\,,$$

where functions y^a, q_a are such that

$$y^a = \frac{\partial S(x,q)}{\partial q_a}\Big|_{q_a = \frac{\partial g}{\partial y^a}}\,. \qquad \Rightarrow \frac{\partial f}{x^i} = p_i = \frac{\partial S(x,q)}{\partial x^i}$$

NOw consider the case when $M = N$.
Let $H = H(x,p)$ be Hamiltonian on T^*M.
s Hamiltonian generates infinitesimal thick morphism

$$\Phi_{\varepsilon S}^*\colon g(x) \mapsto g(x) + \varepsilon H(x,p)\big|_{p = \frac{\partial g}{\partial y}}\,,$$

This infinitesimal map is nothing but the Hamilton-Jacobi vector field on $C(M)$

$$\mathbf{X}_S = \int dx H\left(x, \frac{\partial g}{\partial x}\right)\frac{\delta}{\delta x}\,.$$

This we see that for finite interval of the time

$$\Phi_S^{(t)*}(g) = A(g|x,t)\colon \quad \begin{cases} \frac{\partial A}{\partial t} = H\left(x, \frac{\partial A}{\partial x}\right) \\ A\big|_{t=0} = g \end{cases}$$

This is up to a sign Hamiton Jacobi equation.

Thus we see that the action of inverse image of thick diffeomorphism generated by the Hamiltonian H on the arbitrary function $g(y)$ is the action $A(g|x,t)$, which is equal to $g(x)$ at the moment $t = 0$.

It is difficult to avoid a temptation to claim that the quantum version of this equation has to be Shrodinger equation.