Statement

Let L be functional from the space of fuctions on linear space V to the space of functions on the linear space U:

$$L\colon \quad C(V) \ni g(\mathbf{y}) \mapsto L(g)(x) \in C(U)$$

(y^a are coordinates on V and x^μ coordinates on U)

Suppose that for every function $g \in C(V)$, there exist functions 'coordinates' $y^a_g(x)$ on U such that

$$L(g + tH) - L(G) = tH(y^a_g(x))\,, \quad \text{where } t \text{ is nilpotent such that } t^2 = 0 \tag{1}$$

Consdider functions $g_{\mathbf{k}} = \mathbf{k}\mathbf{y} = k_a y^a$, and coordinate functions $y^a_{\mathbf{k}}(x) = y^a_{\mathbf{k}\mathbf{y}}(x)$ (see the previous blog 030118.tex)

Lemma

$$\frac{\partial y^a_{\mathbf{k}}}{\partial k_b} - \frac{\partial y^b_{\mathbf{k}}}{\partial k_a} = 0 \tag{2}$$

Proof. Calculate $L(\mathbf{k}\mathbf{y} + t_1\mathbf{k}_1\mathbf{y} + t_2\mathbf{k}_2\mathbf{y})$ where for nilpotents t_1, t_2

$$t_1^2 = t_2^2 = 0$$

Then

$$L(\mathbf{k}\mathbf{y} + t_1\mathbf{k}_1\mathbf{y} + t_2\mathbf{k}_2\mathbf{y}) = L(\mathbf{k}\mathbf{y} + t_1\mathbf{k}_1\mathbf{y}) + t_2\mathbf{k}_2\mathbf{y}\left(y^a_{\mathbf{k}+t_1\mathbf{k}_1}\right) =$$

$$L(\mathbf{k}\mathbf{y}) + t_1\mathbf{k}_1\mathbf{y}\left(y^a_{\mathbf{k}}\right) + t_2\mathbf{k}_2\mathbf{y}\left(y^a_{\mathbf{k}} + \left(\frac{\partial y^a_{\mathbf{k}}}{\partial \mathbf{k}} t_1\mathbf{k}_1\right) = \right)$$

$$L(\mathbf{k}\mathbf{y}) + t_1 k_{1_a} y^a_{\mathbf{k}} + t_2 k_{2_a}\left(y^a_{\mathbf{k}} + \left(\frac{\partial y^a_{\mathbf{k}}}{\partial \mathbf{k}_1} t_1\mathbf{k}_1\right)\right) =$$

$$L(\mathbf{k}\mathbf{y}) + t_1 k_{1_a} y^a_{\mathbf{k}} + t_2 k_{2_a} y^a_{\mathbf{k}} + t_2 t_1 k_{2_a} \frac{\partial y^a_{\mathbf{k}}}{\partial k_{1_b}} k_{1_b}$$

The second variaton of functional is symmetric: in temrs of these nilpotents it means that

$$L(\mathbf{k}\mathbf{y} + t_1\mathbf{k}_1\mathbf{y} + t_2\mathbf{k}_2\mathbf{y}) = L(\mathbf{k}\mathbf{y} + t_2\mathbf{k}_2\mathbf{y} + t_1\mathbf{k}_1\mathbf{y})\,.$$

Hence

$$t_2 t_1 k_{2_a} \frac{\partial y^a_{\mathbf{k}}}{\partial k_{_b}} k_{1_b} = t_2 t_1 k_{1_a} \frac{\partial y^a_{\mathbf{k}}}{\partial k_{_b}} k_{2_b}$$

THus we come to the satement of lemma.

Now based on the lemma consider the function $S(\mathbf{x}, \mathbf{k})$ such that

$$\frac{\partial S}{\partial k_a} = y^a_{\mathbf{k}}$$

and consider the thick morphism Φ^*_S

This thick morphism coincides with functional L on linear functions.