Clairaut equation

Solving "naively" Clairaut equation

$$y = xy' + f(y')$$

differentiating we come to equation

$$y''(x + f'(y')) = 0$$

When solving equation x + f'(y') = 0 we come to solutions defined up to the constant.

$$y' = \varphi(-x) \Rightarrow y = C - \dots, \quad \varphi \circ f' = id$$

On the other hand we do not need to solve the equation x + f'(y') = 0. The equation defines the parameteric solution

$$\begin{cases} x = -f'(p) \\ y = p \end{cases}$$
, where $p = y'$

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