

Homework 3.

1 Let $\{\mathbf{e}_x, \mathbf{e}_y\}$ be an orthonormal basis in \mathbf{E}^2 . Consider the following ordered pairs:

a) $\{\mathbf{e}_y, \mathbf{e}_x\}$

b) $\{\mathbf{e}_y, -\mathbf{e}_x\}$

c) $\{\frac{\sqrt{2}}{2}\mathbf{e}_x + \frac{\sqrt{2}}{2}\mathbf{e}_y, -\frac{\sqrt{2}}{2}\mathbf{e}_x + \frac{\sqrt{2}}{2}\mathbf{e}_y\}$

d) $\{\frac{\sqrt{3}}{2}\mathbf{e}_x + \frac{1}{2}\mathbf{e}_y, \frac{1}{2}\mathbf{e}_x - \frac{\sqrt{3}}{2}\mathbf{e}_y\}$

Show that all these ordered pairs are orthonormal bases in \mathbf{E}^2 .

Find amongst them the bases which have the same orientation as the orientation of the basis $\{\mathbf{e}_x, \mathbf{e}_y\}$.

Find amongst them the bases which have the orientation opposite to the orientation of the basis $\{\mathbf{e}_x, \mathbf{e}_y\}$.

2 Let $\{\mathbf{e}, \mathbf{f}\}$ be a basis in two-dimensional linear space V . Consider an ordered pair $\{\mathbf{a}, \mathbf{b}\}$ such that

$$\mathbf{a} = \mathbf{f}, \quad \mathbf{b} = \gamma\mathbf{e} + \mu\mathbf{f},$$

where γ, μ are arbitrary real numbers.

Find values γ, μ such that an ordered pair $\{\mathbf{a}, \mathbf{b}\}$ is a basis and this basis has the same orientation as the basis $\{\mathbf{e}, \mathbf{f}\}$.

3 Let $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ be an arbitrary basis in \mathbf{E}^3 . Show that the basis $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ either has the same orientation as the basis $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\}$, or the same orientation as the basis $\{\mathbf{e}_y, \mathbf{e}_x, \mathbf{e}_z\}$.

4 Let $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\}$ be an orthonormal basis in \mathbf{E}^3 . Consider the following ordered triples:

a) $\{\mathbf{e}_x, \mathbf{e}_x + 2\mathbf{e}_y, 5\mathbf{e}_z\}$,

b) $\{\mathbf{e}_y, \mathbf{e}_x, 5\mathbf{e}_z\}$,

c) $\{\mathbf{e}_y, \mathbf{e}_x, -5\mathbf{e}_z\}$,

d) $\{\frac{\sqrt{3}}{2}\mathbf{e}_x + \frac{1}{2}\mathbf{e}_y, -\frac{1}{2}\mathbf{e}_x + \frac{\sqrt{3}}{2}\mathbf{e}_y, \mathbf{e}_z\}$,

e) $\{\mathbf{e}_y, \mathbf{e}_x, \mathbf{e}_z\}$,

f) $\{\mathbf{e}_y, \mathbf{e}_x, -\mathbf{e}_z\}$.

Show that all ordered triples a), b), c), d), e), f) are bases.

Show that the bases a), c), d) and f) have the same orientation as the basis $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\}$, and the bases b) and e) have the orientation opposite to the orientation of the basis $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\}$.

Show that bases d), e) and f) are orthonormal bases and bases a), b) and c) are not orthonormal bases.

[†] Show that a linear operator P which transforms the basis $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\}$ to the basis $\{\mathbf{e}_x, \mathbf{e}_y, -\mathbf{e}_z\}$ is a rotation. Find an axis and an angle of this rotation.

5 Let $\{\mathbf{e}, \mathbf{f}, \mathbf{g}\}$ be a basis in linear three-dimensional space V .

Consider the following ordered triples:

$$\{\mathbf{f}, \mathbf{e} + 2\mathbf{f}, 3\mathbf{g}\}, \quad \{\mathbf{e}, \mathbf{f}, 2\mathbf{f} + 3\mathbf{g}\}$$

Show that these ordered triples are bases and these bases have opposite orientations.