Homework 7

1 A point moves in $\mathbf{E}^2$ along a parabola with the law of motion $x = t$, $y = t - t^2$, $-\infty < t < \infty$. Find the velocity and acceleration vectors. Find the points of the parabola where the angle between velocity and acceleration vectors is acute. Find the points where speed attains its minimum value.

2 A point moves in $\mathbf{E}^2$ along an ellipse with the law of motion $x = a\cos t$, $y = b\sin t$, $0 \le t < 2\pi$, $(0 < b < a)$. Find the velocity and acceleration vectors. Find the points of the ellipse where the angle between velocity and acceleration vectors is acute. Find the points where speed attains its maximum value.

3 Find a natural parameter for the following interval of the straight line:

$$C\colon \begin{cases} x = t \\ y = 2t + 1 \end{cases}, \quad 0 < t < \infty\,.$$

4 Consider the following curve (a helix):

$$\mathbf{r}(t)\colon \quad \begin{cases} x(t) = R\cos t \\ y(t) = R\sin t \\ z(t) = ct \end{cases}.$$

Find a natural parameter of this curve.
What can you say about the acceleration of this curve? In particular show that the tangential acceleration is equal to zero.

5 Calculate the curvature of the parabola $x = t, y = at^2$ $(a > 0)$ at an arbitrary point.

† * Let s be a natural parameter on this parabola. Show that the integral $\int_{-\infty}^{\infty} k(s)ds$ of the curvature $k(s)$ over the parabola is equal to π.

6 Consider the parabola

$$\mathbf{r}(t)\colon \begin{cases} x = v_x t \\ y = v_y t - \frac{gt^2}{2} \end{cases}.$$

(It is path of the point moving under the gravity force with initial velocity $\mathbf{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$.)
Calculate the curvature at the vertex of this parabola.

7 Consider the ellipse $x = a\cos t, y = b\sin t$ $(a, b > 0,\ 0 \le t < 2\pi)$ in $\mathbf{E}^2$. Calculate the curvature $k(t)$ at an arbitrary point of this ellipse.

† Find the radius of a circle which has second order touching with the ellipse at the point $(0, b)$.

8 Find a curvature at an arbitrary point of the helix considered in Exercise 4.