Affine transformation of coordinates

and area of ellipse.

Let $C: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be an ellipse

Consider affine transformation of coordinates

$$\begin{cases} x = ax' \\ y = by' \end{cases} \quad \left(\begin{array}{l} x', y' \text{ are not Cartesian coordinates!} \\ \text{if} \end{array}\right.$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$
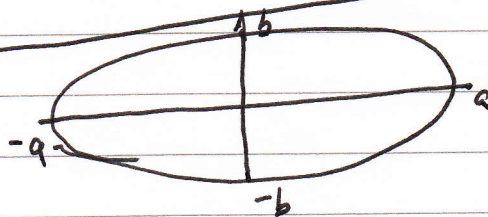
$$C: \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \iff \boxed{x'^2 + y'^2 = 1}$$

$$\underbrace{}_{\text{interior}} \text{of ellipse}$$

Calculate area of

$$\int\limits_{\frac{x^2}{a^2}+\frac{y^2}{b^2}\leq 1} dx\, dy = \int\limits_{x'^2+y'^2\leq 1} \det \frac{\partial(x,y)}{\partial(x',y')} dx'\, dy' =$$

$$= \int\limits_{x^2+y'\leq 1} ab\, dx'\, dy' = ab \cdot \text{area of interior of} = \pi ab.$$
$$\text{unit circle}$$

We see that

area of ellipse = π multiplied on lengths of half-axes

$$\boxed{S = \pi ab}$$