Another way to calculate way thick morphisms

Let $M = \mathbf{R}^m$, $N = \mathbf{R}^n$ and $S(x,q) = \varphi(x)q + \frac{1}{2}q^2$, then we know that

$$\Phi_S^{\mathrm{q}\cdot}(w(y)) = \left(\exp\left[\frac{i}{\hbar}a\left(\frac{\hbar}{i}\right)^2\frac{d^2}{dy^2}\right]w(y)\right)_{y=\varphi(x)} . \tag{1}$$

(see mathblog on 26-th September 2019.)

Choose $w(y) = \exp(\frac{i}{\hbar}g(y))$ We have that

$$f_\hbar(x) = L_\hbar(g) = \frac{\hbar}{i}\log\left(\exp\left[\frac{i}{\hbar}a\left(\frac{\hbar}{i}\right)^2\frac{d^2}{dy^2}\right]e^{\frac{i}{\hbar}g(y)}\right)_{y=\varphi(x)} . \tag{2}$$

In the previous mathblog we came to the answer that

$$f_\hbar(x) = g(y) + \frac{\hbar}{i}\log\left(\exp\left[\frac{i}{\hbar}a\left(\frac{dg}{dy}\right)^2\right] + 0(1)\right)_{y=\varphi(x)} \tag{3}$$

Now perform calculation of (2) in another way

$$\frac{\hbar}{i}\log\left[\sum_{n=0}^{\infty}\left(\frac{\hbar}{i}\right)^n a^n\frac{d^{2n}}{dy^{2n}}\left(e^{\frac{i}{\hbar}g(y)}\right)\right] =$$

$$\frac{\hbar}{i}\log\left[e^{\frac{i}{\hbar}g(y)} + \left(\frac{\hbar}{i}\right)a\frac{d^2}{dy^2}\left(e^{\frac{i}{\hbar}g(y)}\right) + \left(\frac{\hbar}{i}\right)^2a^2\frac{d^4}{dy^4}\left(e^{\frac{i}{\hbar}g(y)}\right) + \left(\frac{\hbar}{i}\right)^3a^3\frac{d^6}{dy^6}\left(e^{\frac{i}{\hbar}g(y)}\right) + \dots\right] =$$

$$g(y) + \frac{\hbar}{i}\log\left[1 + e^{-\frac{i}{\hbar}g(y)}\left(\frac{\hbar}{i}\right)a\frac{d^2}{dy^2}\left(e^{\frac{i}{\hbar}g(y)}\right) + e^{-\frac{i}{\hbar}g(y)}\left(\frac{\hbar}{i}\right)^2a^2\frac{d^4}{dy^4}\left(e^{\frac{i}{\hbar}g(y)}\right) + \dots\right] =$$

$$g(y) + \frac{\hbar}{i}\log\left[1 + e^{-\frac{i}{\hbar}g(y)}a\frac{d}{dy}\left(g'(y)e^{\frac{i}{\hbar}g(y)}\right) + e^{-\frac{i}{\hbar}g(y)}\left(\frac{\hbar}{i}\right)a^2\frac{d^3}{dy^3}\left(g'(y)e^{\frac{i}{\hbar}g(y)}\right) + \dots\right] =$$

$$g(y) + \frac{\hbar}{i}\log\left[1 + ag''(y) + \frac{i}{\hbar}\left(g'(y)\right)^2 + e^{-\frac{i}{\hbar}g(y)}\left(\frac{\hbar}{i}\right)a^2\frac{d^2}{dy^2}\left(g''(y)e^{\frac{i}{\hbar}g(y)} + \frac{i}{\hbar}(g'(y))^2e^{\frac{i}{\hbar}g(y)}\right) + \dots\right] =$$

$$g(y) + \frac{\hbar}{i}\times$$

$$\log\left[1 + ag''(y) + \frac{i}{\hbar}\left(g'(y)\right)^2 + e^{-\frac{i}{\hbar}g(y)}\left(\frac{\hbar}{i}\right)a^2\frac{d}{dy}\left(g'''(y)e^{\frac{i}{\hbar}g(y)} + 3\frac{i}{\hbar}g'g^{ii}e^{\frac{i}{\hbar}g(y)} + \left(\frac{i}{\hbar}\right)^2(g')^3e^{\frac{i}{\hbar}g(y)}\right)\right.$$

$$g(y) + \frac{\hbar}{i}\times$$

$$\log\left[1 + ag''(y) + \frac{i}{\hbar}\left(g'(y)\right)^2 + e^{-\frac{i}{\hbar}g(y)}\left(\frac{\hbar}{i}\right)a^2\frac{d}{dy}\left(g'''(y)e^{\frac{i}{\hbar}g(y)} + 3\frac{i}{\hbar}g'g^{ii}e^{\frac{i}{\hbar}g(y)} + \left(\frac{i}{\hbar}\right)^2(g')^3e^{\frac{i}{\hbar}g(y)}\right)\right.$$

$$g(y) + \frac{\hbar}{i}\log\left[1 + ag'' + \frac{i}{\hbar}g'^2 + \frac{\hbar}{i}a^2g'''' + 4a^2g'''g' + 3a^2\left(g''\right)^2 + 6\frac{i}{\hbar}g'^2g'' + \left(\frac{i}{\hbar}\right)^2g'^4\right]$$