Homework 9(C3)

1 Four points A, B, C, D are given on the projective line $\mathbf{RP}^1$. Their homogeneous coordinates are

$$A = [2:2]\,, B = [1:5]\,, C = [3:7]\,, D = [2:1]\,.$$

Calculate the affine coordinate u of these points, ($u = \frac{x}{y}$) and calculate cross-ratio of these points.

2 As usual denote by (A, B, C, D) the cross-ratio of the four points A, B, C, D on the projective line.

a) Does the cross-ratio change if we change the order of these points?

b) Let $(A, B, C, D) = \lambda$.

Calculate the cross-ratios (B, A, C, D), (A, B, D, C) and (B, A, D, C).

* Calculate cross- ratio (A, C, B, D).

* Calculate cross-ratio of arbitrary permutation of the points A, B, C, D.

3 Four points A, B, C, D sre given on the projective line. Show that the cross-ratio

$$(A, B, C, D) = \frac{u_A - u_C}{u_B - u_C}\,,$$

in the case if point D is at infinity.

4 Four points $A, B, C, D \in \mathbf{RP}^2$ are given in homogeneous coordinates by

$$A = [1:-1:1]\,, \quad B = [10:-15:5]\,, \quad C = \left[1:-\frac{9}{5}:\frac{1}{5}\right]\,, \quad D = [1:0:2]\,.$$

Show that these points are collinear.

Calculate their cross-ratio.

5 Two points $A, B \in \mathbf{RP}^2$ are given in homogeneous coordinates, $A = [2:2:4]$, $B = [3:7:2]$. Consider the projective line AB passing through the points A and B.

Show that the point $C = [1:2:1]$ belongs to the line AB, i.e. the points A, B, C are collinear.

Show that a point $E_{\lambda,\mu} = [2\lambda + 3\mu : 2\lambda + 7\mu : 4\lambda + 2\mu]$ where λ, μ are arbitrary real numbers belongs to the line AB

Show that the point $K = [2:0:1]$ does not belong to the line AB, i.e. the points A, B, K are not collinear.

Consider a point $D = [1:3:0]$ which is at infinity. Show that this point is collinear with the points A and B, i.e. it belongs to the projective line AB.

* Calculate the cross-ratio (A, B, C, D).

***6** Let $\triangle ABC$ be a triangle in the Euclidean plane $\mathbf{E}^2$.

Let P, Q be two points on the line AB such that the segment CP is the bisectrix of the angle ACB, and the segment CQ is the bisectrix of the external angle ACB. (We suppose that $|AC| \neq |BC|$. In the case if $|AC| = |BC|$ then the bisectrix of the external angle is parallel to the line l_{AB}.)

Calculate the cross-ratio (A, B, P, Q).

One can include $\mathbf{E}^2$ in projective plane $\mathbf{RP}^2$. What happens with $\triangle ABC$ if we will perform a projective transformation of $\mathbf{RP}^2$ which sends the point Q to infinity?