Homework 4

1 Calculate the area of parallelograms formed by the vectors $\mathbf{a}, \mathbf{b}$ if

a) $\mathbf{a} = (1, 2, 3), \mathbf{b} = (1, 0, 1)$;

b) $\mathbf{a} = (2, 2, 3), \mathbf{b} = (1, 1, 1)$;

c) $\mathbf{a} = (5, 8, 4), \mathbf{b} = (10, 16, 8)$.

2 Prove the inequality $(ad - bc)^2 \leq (a^2 + b^2)(c^2 + d^2)$

a) by a direct calculation

b) considering vector product of vectors $\mathbf{x} = a\mathbf{e}_x + b\mathbf{e}_y$ and vectors $\mathbf{y} = c\mathbf{e}_x + d\mathbf{e}_y$

3 Show that for any two vectors $\mathbf{a}, \mathbf{b} \in \mathbf{E}^3$ the following identity is satisfied

$$(\mathbf{a}, \mathbf{a})(\mathbf{b}, \mathbf{b}) = (\mathbf{a}, \mathbf{b})^2 + (\mathbf{a} \times \mathbf{b}, \mathbf{a} \times \mathbf{b}) \,.$$

Write down this identity in components.
Compare this identity with CBS inequality See the problem 10 in the Homework 1).

4 Find a vector $\mathbf{n}$ such that the following conditions hold:
1) It has a unit length
2) it is orthogonal to the vectors $\mathbf{a} = (1, 2, 3)$ and $\mathbf{b} = (1, 3, 2)$.
3) An ordered triple $\{\mathbf{a}, \mathbf{b}, \mathbf{n}\}$ has an orientation opposite to the orientation of the basis of Euclidean space.

5 Consider system of simultaneous equations $\begin{cases} ax + by + cz = d \\ x + 2y + 3z = 1 \end{cases}$
Find conditions on parameters a, b, c, d such that this system has no solutions.
Could this system have exactly one solution?

6 Write down an equation of the plane α such that α is orthogonal to the vector $\mathbf{N} = (1, 2, 3)$ and the point $A = (2, 3, 5)$ belongs to this plane.
Find the distance between this plane and the point $B = (1, 0, 0)$.

7 Write down an equation of the plane passing through the points $A = (x_1, y_1, z_1) = (1, 1, 1)$, $B = (x_2, y_2, z_2) = (1, 2, 3)$, $C = (x_3, y_3, z_3) = (2, 2, 0)$.

8† Find a line passing through the point $(1, 0, 0)$ such that all points of this line belong to the one-sheeted hyperboloid $x^2 + y^2 - z^2 = 1$.