

Homework 9

1 Let F be a projective transformation of \mathbf{RP} such that

$$[x' : y'] = F([x : y]) = [2x + 3y : 3x + 2y].$$

Let $P = [6 : 2]$ be a point on \mathbf{RP} .

Find the affine coordinate u_P of this point, and find the affine coordinate u'_P of the point $F(P)$.

Find a point A such that $A = F(\infty)$.

Find a point B such that $F(B) = \infty$.

2 Consider the projective transformation

$$F: \quad F([x : y]) = [y : x].$$

Write down this transformation in affine coordinate.

Show by straightforward calculation, that the cross-ratio (A, B, C, D) of four points on the projective line is the invariant of this projective transformation.

3 Four points $A, B, C, D \in \mathbf{RP}^2$ are given in homogeneous coordinates by

$$A = [2 : -1 : 1], \quad B = [15 : -10 : 5], \quad C = \left[1 : -\frac{4}{5} : \frac{1}{5}\right], \quad D = [2 : 0 : 2].$$

Show that these points are collinear.

Calculate their cross-ratio.

4 Three points $A, B, C \in \mathbf{RP}^2$ are given in homogeneous coordinates by

$$A = [6 : 2 : 2], \quad B = [15 : 5 : 1], \quad C = [18 : 6 : 3].$$

Show that these points are collinear.

Find a point D on projective plane \mathbf{RP}^2 such that the cross-ratio $(A, B, C, D) = -1$.

5 Let A, B, C, D be four collinear points on projective plane \mathbf{RP}^2 .

Let $(A, B, C, D) = \lambda$. Calculate (B, A, C, D) , (A, B, D, C) and (B, A, D, C) .

6 On the projective line are given two points $A = [3 : 3]$ and $B = [7 : 1]$.

Find a point P on the projective line such that the ratio

$$(A, B, P) = \frac{u_A - u_P}{u_B - u_P} = -2.$$

Let F be a projective transformation such that $F([x : y]) = [x + y : x]$.

Consider also a point $Q = \infty$ and find images A', B', P', Q' of the points A, B, P, Q under the projective transformation F :

$$A' = F(A), \quad B' = F(B), \quad P' = F(P), \quad Q' = F(Q).$$

Calculate the cross ratio (A', B', P', Q') ,

Explain why the ratio (A, B, P) is equal to the cross-ratio (A', B', P', Q') .