Homework 6

1 A point moves in $\mathbf{E}^2$ along an ellipse with the law of motion $x = a\cos t$, $y = b\sin t$, $0 \le t < 2\pi$, $(0 < b < a)$. Find the velocity and acceleration vectors. Find the points of the ellipse where the angle between velocity and acceleration vectors is acute. Find the points where speed attains its maximum value.

2 Find a natural parameter for the line $y = kx + b$.

3 Consider the following curve (a helix):

$$\mathbf{r}(t)\colon \quad \begin{cases} x(t) = R\cos t \\ y(t) = R\sin t \\ z(t) = ct \end{cases} .$$

Show that the tangential acceleration is equal to zero.
Find a natural parameter of this curve.

4 Find a natural parameter for the parabola $x = t, y = t^2$.

5 Calculate the curvature of the parabola $x = t, y = at^2$ $(a > 0)$ at an arbitrary point.

Find an equation of the circle which has a second order touching with this parabola at its vertex (the point $(0, 0)$).

* Let s be a natural parameter on this parabola. Show that the integral $\int_{-\infty}^{\infty} k(s)ds$ of the curvature $k(s)$ over the parabola is equal to π.

5a For any function $f = f(x)$ one can consider its graph as not-parameterised curve C_f. Calculate curvature of the curve C_f at any point $(x, f(x))$.

Find a radius of circle which have second order touching with the curve C_f at the point $(x, f(x))$.

6 Consider the parabola

$$\mathbf{r}(t)\colon \quad \begin{cases} x = v_x t \\ y = v_y t - \frac{gt^2}{2} \end{cases} .$$

(It is path of the point moving under the gravity force with initial velocity $\mathbf{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$.)
Calculate the curvature at the vertex of this parabola.

7 Find a curvature at an arbitrary point of the helix considered in Exercise 3.

8 The curve C in $\mathbf{E}^3$ is given by the parameterisation $x = t,\ y = t^2,\ z = t^3,\ 0 \le t \le 2$. Find the velocity and acceleration vectors for this curve.

Consider the plane α given by the equation $3x - 3y + z = 1$.

Prove that α is the plane spanned by the velocity and acceleration vectors at the point $(1, 1, 1)$ of the curve C.