

Upper half plane is not equivalent to plane

Consider upper half-plane: $\mathbf{H}: \{(x, y), y > 0\}$ and all the plane \mathbf{C} . They are not holomorphically equivalent, i.e. it does not exist analytical bijection $\mathbf{H} \leftrightarrow \mathbf{C}$. Indeed \mathbf{H} is holomorphically equivalent to disc $D_2: \{(x, y), x^2 + y^2 < 1\}$ (Klein map). This implies that disc is equivalent to \mathbf{C} . Let $F: \mathbf{C} \leftrightarrow \mathbf{H}$. Then F is analytical bounded function on \mathbf{C} , and due to Liouville Theorem F maps \mathbf{C} to the point.

Really crazy effective application of Liouville theorem!!!