

Examples of Haar measure

Examples

Group of affine transformations of \mathbf{R} :

$$(a, b)|x \mapsto ax + b$$

We have

$$(a, \alpha) \circ (b, \beta)|x \mapsto (a, \alpha)|bx + \beta \mapsto |a(bx + \beta) + \alpha \mapsto (ab, a\beta + \alpha)|x \mapsto .$$

Thus

$$(a, \alpha)(b, \beta) = (ab, a\beta + \alpha)$$

Haar measure

$$\mu(db, d\beta) = \omega = \omega(b, \beta)db \wedge d\beta$$

Left-invariance means that for an arbitrary element (a, α)

$$\omega(b, \beta)db \wedge d\beta = \omega(ab, a\beta + \alpha)d(ab) \wedge d(a\beta + \alpha) = \omega(ab, a\beta + \alpha)a^2db \wedge d\beta$$