Homework 8

1 Consider in $\mathbf{E}^2$ the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

Find the foci of this ellipse.

Define focal polar coordinates for this ellipse and write down the equation of this ellipse in polar coordinates.

2 Consider a curve in $\mathbf{E}^2$ defined in polar coordinates (r, φ) by the equation

$$r = \frac{p}{1 - e\cos\varphi}, \quad p > 0\,. \tag{1}$$

a) Write down the equation of this curve in Cartesian coordinates $\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$ in the case if $p = 2, e = \frac{1}{3}$, show that this curve is an ellipse, and find the foci and the centre of this ellipse. Calculate the area of this ellipse.

b) Justify by straighforward calculations that in the case $0 \leq e < 1$ the curve (1) is indeed an ellipse with foci at the origin and at the point $(2c, 0)$, where $c = \frac{pe}{1-e^2}$, and with semi-major axis $a = \frac{p}{1-e^2}$.

c) How does the curve defined by equation (1) look in the case if $e = 1$?

3 Let C be the curve defined by the intersection of the plane $\alpha\colon 2z - x = 2$ with the conic surface M: $x^2 + y^2 = z^2$.

Let C_{proj} be the orthogonal projection of this curve onto the plane $z = 0$.

Show that the curve C_{proj} is an ellipse.

Explain why the curve C is also an ellipse.

Find the foci of the curve C_{proj}. In particular show that the vertex of the conic surface M is a focus of the ellipse C_{proj}.

Find the areas of the ellipses C and C_{proj}.

4 Let C be the curve defined by the intersection of the plane $\alpha\colon kx + z = 1$ (where k is real parameter) with the conic surface M: $2x^2 + 2y^2 = 9z^2$.

Let C_{proj} be the orthogonal projection of this curve onto the plane $z = 0$.

Find the values of parameter k such that the curve C and the curve C_{proj} are ellipses.

Find the values of parameter k such that the curve C and the curve C_{proj} are parabolas.

In the case if a curve C (and a curve C_{proj}) are parabolas,show that the vertex of the conic surface M, the origin, is the focus of this parabola C_{proj}

Find the directrix of this parabola.

5 Find the foci and directrix of the parabola $y = ax^2$, $(a > 0)$.

Choose focal polar coordinates and write down the equation of this parabola in these polar coordinates.