i

$$[SU(2), CP] = [SO(3), E^3]$$

I know this hundred year but still I enjoy this

Let $\Psi = \begin{pmatrix} c_+ \\ c_- \end{pmatrix}$ be spinor in C^2. It defines the vector

$$\mathbf{s} = \begin{pmatrix} \overline{s_x} \\ \\ \overline{s_y} \\ \\ \overline{s_z} \end{pmatrix} = \frac{1}{\langle \Psi, \Psi \rangle} \begin{pmatrix} \langle \Psi, \hat{s_x} \Psi \rangle \\ \\ \langle \Psi, \hat{s_y} \Psi \rangle \\ \\ \langle \Psi, \hat{s_z} \Psi \rangle \end{pmatrix} = \frac{1}{|c_+|^2 + |c_-|^2} \begin{pmatrix} \overline{c_+} c_- + c_+ \overline{c_-} \\ \\ \frac{\overline{c_+} c_- - c_+ \overline{c_-}}{i} \\ \\ |c_+|^2 - |c_-|^2 \end{pmatrix},$$

where

$$s_x = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad s_y = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad s_z = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

are Pauli matrices.

This map is nothing but stereographic projection: If

$$\Psi = \begin{pmatrix} c_+ \\ c_- \end{pmatrix} = \begin{pmatrix} u + iv \\ 1 \end{pmatrix}$$

is represented by the point at $C \cup \{\infty\}$ then

$$\mathbf{s} = \begin{pmatrix} \overline{s_x} \\ \\ \overline{s_y} \\ \\ \overline{s_z} \end{pmatrix} = \frac{1}{\langle \Psi, \Psi \rangle} \begin{pmatrix} \langle \Psi, \hat{s_x} \Psi \rangle \\ \\ \langle \Psi, \hat{s_y} \Psi \rangle \\ \\ \langle \Psi, \hat{s_z} \Psi \rangle \end{pmatrix} = \frac{1}{|c_+|^2 + |c_-|^2} \begin{pmatrix} \overline{c_+} c_- + c_+ \overline{c_-} \\ \\ \frac{\overline{c_+} c_- - c_+ \overline{c_-}}{i} \\ \\ |c_+|^2 - |c_-|^2 \end{pmatrix} =$$

$$\frac{1}{u^2 + v^2 + 1} \begin{pmatrix} 2u \\ \\ 2v \\ \\ u^2 + v^2 - 1 \end{pmatrix}$$

This is just standard formula for stereographic projection.