CS271Project5

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October 2019

1. Proof: Hypothesis: A complete binary tree with height h contains $2^{h+1}-1$ total nodes.
   Base Case: When a complete binary tree with height 0, it is simply contain only one node, which is the root. And, $2^{0+1}-1=2-1=1$. Therefore, the base case holds true.
   Inductive Step: We suppose the hypothesis holds for the complete binary tree with height h, for $1 \leq h$ and prove that it also holds for $h+1$. We will consider node x with height $h+1$. Each of children of node x has height h. Then, we know the subtree rooted at the child of node x, with height h contains $2^{h+1}-1$ total nodes. (by Inductive Hypothesis) And we know the fact that the total number of a complete binary tree rooted at node x is equal to the total number of nodes in left subtree adds the total number of nodes in right subtree and plus one. Then, the number of a completer binary tree with height h
   $= 2^{h+1}-1+2^{h+1}-1+1$
   $= 2^{h+1}-1+2^{h+1}$
   $= 2 * 2^{h+1}-1$
   $= 2^{h+2}-1$
   $= 2^{h+1+1}-1$
   Therefore, the hypothesis holds for $h+1$. Therefore, a complete binary tree with height h contains $2^{h+1}-1$ total nodes.

2. Proof: Hypothesis:A complete binary tree with n nodes has $(n-1)/2$ internal nodes.
   Base Case: When a complete binary tree with 1 node, it will have zero internal nodes. Since the number of nodes in binary tree is 1, $(1-1)/2=0$. Therefore, the base case holds.
   Inductive Step: We assume the hypothesis holds for complete binary tree with k nodes, for $k \geq 1$. Then, we need to consider a complete binary tree rooted at node x with $k+2$ nodes. Suppose the subtree rooted at left child of node x with y nodes and the subtree rooted at right child of node x with z nodes. And we know that in that they are the children of node x,then $y, z \leq k$ and $y+z=k+2-1=k+1$ because the sum of the number of nodes of left subtree and right subtree is equal to the total nodes minus 1. The subtree rooted at left child of node x has $(y-1)/2$ internal nodes(by

Induction Hypothesis). The subtree rooted at right child of node x has $(z-1)/2$ internal nodes(by Induction Hypothesis). Since we have known that the number of internal nodes of a complete binary tree rooted at node x is equal to the number of internal nodes of subtree rooted at the left child of node x add the number of internal nodes of subtree rooted at the right child of node x and plus one. Then, we know that the number of internal nodes of a complete binary tree rooted at node x with $k+2$ nodes
$= (y-1)/2 + (z-1)/2 + 1$
$= (y-1+z-1+2)/2$
$= (k+1)/2$
$= (k+2-1)/2$
Therefore, the hypothesis holds true for $k+2$. Therefore, a complete binary tree with n nodes has $(n-1)/2$ internal nodes.