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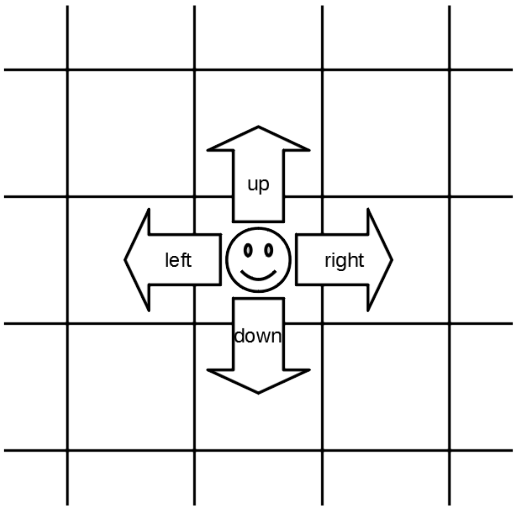
Theoretical Exercise Sheet 3

Deadline Friday, May 6, 23:59

About the submission of this sheet.

- You might submit the solutions to exercises in groups of up to 3 students.
- All students of a group need to be in the same tutorial.
- Write the names of **all** students of your group on your solution.
- Hand in the solution **in CMS**. **New: Starting from this sheet, we have activated "Team Groupings" in CMS**.
  - Go to your personal page in CMS. Here you find the entry "Teams".
  - When you click "Create team", you get an invite code.
  - Please share this with your team mates, who need to click on "Join team" and enter the code.

1. (5 points) Consider an unbounded regular 2D grid. The start state is at the origin, $(0, 0)$, and the goal state is at (x, y). You can move left, right, up and down. For example, if you are at the start and move to the right, you get to position $(1, 0)$. Moving down gets you to position $(0, -1)$. Each move has unit cost.



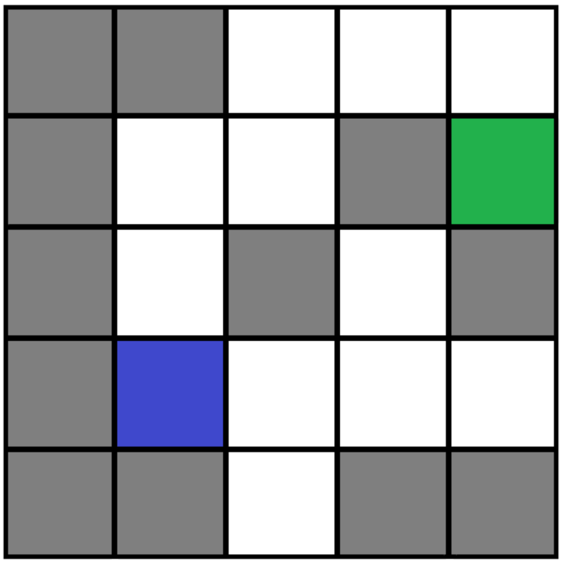
   1. Is $h((u, v)) = |u - x| + |v - y|$ a consistent heuristic for a state at (u, v)? Justify your answer (1-2 sentences).
   2. Is $h((u, v)) = |u - x| + |v - y|$ an admissible heuristic for a state at (u, v)? Justify your answer (1-2 sentences).

3. At most how many nodes are expanded by A* graph search using the Manhattan distance h_{Man} as your heuristic function? *You may assume that the Manhattan distance is admissible for this state space.*
4. Does h_{Man} remain admissible if some links are removed?
5. Does h_{Man} remain admissible if some links are added between nonadjacent states?

2. (6 points) Consider the following Labyrinth, in which the agent starts at the blue square and tries to get to the green one. The actor can only move up, down, left and right (**not** diagonally).The agent can walk on the white, green and blue squares, but not on the grey ones. Each square can be written down as a combination of its X and Y coordinates as $(X|Y)$, which begin at the bottom left at 0. For example, the bottom left ist (0|0) and the goal is (4|3).

   Perform Greedy Best-First Search with the manhattan distance as heuristic for six iterations of the main loop. For each iteration, write down the node that gets expanded, the states that get considered for adding to the frontier, the frontier after the expansion of the state, and the explored set. Make sure to keep the frontier ordered and break ties by lower Y value and if that is the same, lower X value. For example, if the following squares have the same heuristic value, they get ordered as $[(3|0), (2|1), (4|1)]$, where (3|0) gets picked first.

   You can use the table below, which already has the first steps.



expanded node	nodes that might get added	frontier	explored set
		[(1|1)]	{}
(1|1)	{(2|1), (1|2)}	[(2|1), (1|2)]	{(1|1)}

3. (8 points) Consider the state space from Figure 1.

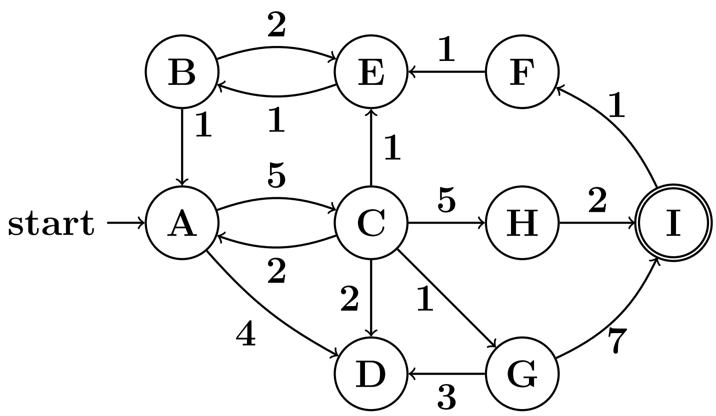


Figure 1: State space used in Exercise 3.

(i) Run A^* search on this problem. As a heuristic estimate for a state s, use the minimal number of edges that are needed to reach a goal state from s (or ∞ if s is not solvable, e.g., h(C)=2). Draw the search graph and **annotate each node with the g and h value as well as the order of expansion**. Draw duplicate nodes as well, and mark them accordingly by crossing them out. If the choice of the next state to be expanded is not unique, expand the lexicographically smallest state first. Give the solution found by A^* search. Is this solution optimal? Justify your answer (1-2 sentences).

(ii) Run the hill climbing algorithm, as stated in the lecture, on this problem. **Use the heuristic function from part (i)**. For each state, provide all applicable actions and the states reachable using these actions. **Annotate states with their heuristic value. Specify which node is expanded in each iteration of the algorithm.** If the choice of the next state to be expanded is not unique, expand the lexicographically smallest state first. Does the algorithm find a solution? If yes, what is it and is it optimal?

(iii) Could hill-climbing stop in a local minimum without finding a solution? If yes, give an example heuristic $h : \{A, B, ..., I\} \rightarrow \mathbb{N}_0^+ \cup \{\infty\}$ for the state space depicted in Figure 1 that leads hill-climbing into a local minimum, and explain what happens. If no, please explain why.