SAARLAND UNIVERSITY — ARTIFICIAL INTELLIGENCE
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Theoretical Exercise Sheet 7

Deadline Friday, June 17, 23:59

About the submission of this sheet.

- You might submit the solutions to exercises in groups of up to 3 students.
- All students of a group need to be in the same tutorial.
- Write the names of **all** students of your group on your solution.
- Hand in the solution **in CMS** and use "Team Groupings".
  - Go to your personal page in CMS. Here you find the entry "Teams".
  - When you click "Create team", you get an invite code.
  - Please share this with your team mates, who need to click on "Join team" and enter the code.

1. (16 points) $(4 + 4 + 4 + 4)$

   1. For each of the following sentences, write down a **predicate logic formula that has the same meaning**. You may only use the following predicates: $\text{Tutor}(x), \text{Student}(x), \text{Assistant}(x), \text{Likes}(x, y), \text{GivesGoodGradesTo}(x, y),$ $\text{GoodStudent}(x)$
      (i) If a tutor gives good grades to every student, every student likes them.
      (ii) Someone is a good student if and only if there is a tutor that gives them good grades but does not give good grades to every student.
   2. For each of the following predicate logic formulas, write down an **English sentence with the same meaning.**
      (i) $\exists x[\text{Student}(x) \wedge \neg\exists y[\text{Tutor}(y) \wedge \text{Likes}(x, y)]]$
      (ii) $\forall x \forall y[\text{Assistant}(x) \wedge \text{Tutor}(y) \wedge \neg\text{Likes}(x, y)] \rightarrow$ $\exists z[\text{GoodStudent}(z) \wedge \text{Student}(z) \wedge \neg\text{GivesGoodGradesTo}(y, z)]$

2. (22 points) $(5 + 8 + 9)$ Transform the following predicate logic formulas into **Clausal Normal Form**. Write down the results of all intermediate steps, specifying which steps you are applying and giving the intermediate results.

   Note: Simplify the formulas where possible.

   (a) $\varphi_1 = \forall x[\,(P(x) \wedge \exists y Q(y)) \rightarrow \exists y \neg R(y, x)]$
   (b) $\varphi_2 = \neg\exists x[\,\exists y[\,(\neg P(x) \wedge \neg R(y)) \rightarrow Q(x, y)] \rightarrow \exists y(P(x) \wedge Q(x, y) \wedge \forall z R(z))\,]$
   (c) $\varphi_3 = \neg\exists x \forall y \forall z[\,P(x, y) \leftrightarrow (\neg Q(x, z) \vee R(y))]$

3. (21 points) (5 + 6 + 2 + 8)
Run the **unification** algorithm from the lecture on each of the given sets. Assume w, x, y, z to be variables and a, b, c to be constants. For each step, write down the **set to unify**, the **disagreement set**, and the **updated set of substitutions**, i.e., T_0, D_0, s_1, T_1, D_1, ... until the algorithm terminates. In particular, if the algorithm does not output a failure, list the *entire* content of the substitutions $s_1, \ldots, s_n$. In any case, state the **output** of the algorithm. If multiple substitutions are possible, choose **lexicographically** (e.g., $\frac{x}{f(a)}$ before $\frac{x}{f(y)}$).

   (a) $\{P(a, b, c), P(x, y, f(x))\}$
   (b) $\{P(g(a), y, f(z), h(x, b)), P(g(x), b, f(b), h(a, z))\}$
   (c) $\{P(x, y), P(g(a, x), a)\}$
   (d) $\{P(a, f(b, g(x)), g(z)), P(z, f(x, g(b)), w), P(a, y, g(a))\}$

4. (23 points) (3 + 8 + 12) In this exercise you will see whether Rapunzel is saved by the prince, or cursed by the sorcerer. For this consider the following statements and the set $\theta^* = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5\}$ of corresponding predicate logic formulas in Skolem normal form:

   1. Rapunzel lives in a tower.
      $\varphi_1 = lives(Rapunzel, tower)$
   2. If the Prince does not save Rapunzel, then she can't escape the tower.
      $\varphi_2 = saves(Prince, Rapunzel) \vee \neg escapes(Rapunzel, tower)$
   3. Everyone who lives in the tower and does not escape will be cursed by the sorcerer.
      $\varphi_3 = \forall x[\neg lives(x, tower) \vee escapes(x, tower) \vee curse(sorcerer, x)]$
   4. The Prince does not save anyone that lives in a tower.
      $\varphi_4 = \forall x[\neg saves(Prince, x) \vee \neg lives(x, tower)]$
   5. The sorcerer does not curse Rapunzel.
      $\varphi_5 = \neg curse(sorcerer, Rapunzel)$

   (a) Write down $CF(\theta^*)$ and the resulting Herbrand universe $HU(\theta^*)$.
   (b) Write down the Herbrand expansion $HE(\theta^*)$. Bring each of the formulas in $HE(\theta^*)$ into CNF, resulting in a set of clauses Δ.
   (c) Use **propositional resolution** to prove that Δ is unsatisfiable. Assuming that statements 1.–4. are true, will Prince save Rapunzel from the sorcerer?

5. (10 points) Consider the following set of predicate logic formulas in Skolem Normal Form:

   1. Every university mascot is an animal.
      $\varphi_1 = \forall x[\neg Mascot(x) \vee Animal(x)]$
   2. Steffi the owl or Kasper the clown are university mascots.
      $\varphi_2 = Mascot(Steffi) \vee Mascot(Kasper)$
   3. Every animal is liked by some student.
      $\varphi_3 = \forall x[\neg Animal(x) \vee (Student(f(x)) \wedge Likes(f(x), x))]$
   4. No student likes Kasper the clown.
      $\varphi_4 = \forall x[\neg Student(x) \vee \neg Likes(x, Kasper)]$

   We want to prove by contradiction that Steffi is an animal. Therefore, we add to our set of clauses:

   5. Steffi the owl is not an animal.
      $\varphi_5 = \neg Animal(Steffi)$

   These clauses give us Δ:

$$
\begin{aligned}
\varphi_1 &= \{\neg Mascot(x), Animal(x)\} \\
\varphi_2 &= \{Mascot(Steffi), Mascot(Kasper)\} \\
\varphi_{3,1} &= \{\neg Animal(y), Student(f(y))\} \\
\varphi_{3,2} &= \{\neg Animal(z), Likes(f(z), z))\} \\
\varphi_4 &= \{\neg Student(w), \neg Likes(w, Kasper)\} \\
\varphi_5 &= \{\neg Animal(Steffi)\}
\end{aligned}
$$

   Use **binary PL1 resolution** to show that Δ is unsatisfiable. Also specify the unifiers when applying a resolution step.
   Note: If you need to use a clause more than once, make a copy of the clause and rename the variables to avoid name collisions.

   Depict the resolution process in form of a tree for easier readability. Label edges with the corresponding unifying substitution