# Artificial Intelligence 12. Planning, Part I: Framework

How to Describe Arbitrary Search Problems

Jörg Hoffmann, Daniel Fiser, Daniel Höller, Sophia Saller

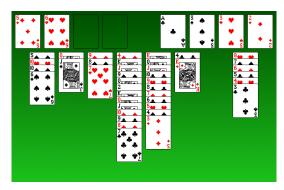


Summer Term 2022

### Agenda

- Introduction
- 2 The History of Planning
- The STRIPS Planning Formalism
- The PDDL Language
- 5 Why Complexity Analysis?
- **6** Planning Complexity
- Conclusion

#### Reminder: Discrete Search Problems



- States: Card positions (position\_Jspades=Qhearts).
- Actions: Card moves (move\_Jspades\_Qhearts\_freecell4).
- Initial state: Start configuration.
- Goal states: All cards "home".
- Solution: Card moves solving this game.

### **Planning**

#### **Ambition:**

Write one program that can solve all discrete search problems.

#### **Problem Descriptions**

- The **blackbox description** of a problem  $\Pi$  is an API (a programming interface) providing functionality allowing to construct the state space: InitialState(), GoalTest(s), . . .
  - $\rightarrow$  "Specifying the problem" = programming the API.
- The declarative description of  $\Pi$  comes in a problem description language. This allows to implement the API, and much more.
  - $\rightarrow$  "Specifying the problem" = writing a problem description.
  - $\rightarrow$  Here, "problem description language" = planning language.

# "Planning Language"?

Introduction

#### How does a planning language describe a problem?

- A *logical description* of the possible states (vs. Blackbox: data structures). E.g.: predicate Eq(.,.).
- A logical description of the initial state I (vs. data structures). E.g.: Eq(x,1).
- A logical description of the goal condition G (vs. a goal-test function). E.g.: Eq(x,2).
- A logical description of the set A of actions in terms of preconditions and effects (vs. functions returning applicable actions and successor states).
  - E.g.: "increment x: pre Eq(x,1), eff  $Eq(x,2) \wedge \neg Eq(x,1)$ ".
- $\rightarrow$  Solution (plan) = sequence of actions from A, transforming I into a state that satisfies G. E.g.: "increment x".

# "Planning Language"?

#### Disclaimer:

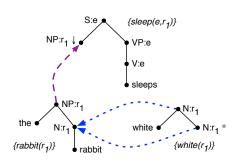
- ightarrow Planning languages go way beyond discrete search problems. There are variants for partially observable, stochastic, dynamic, continuous, and multi-agent settings.
  - We focus on discrete search problems for simplicity (combined with practical relevance).
  - For a comprehensive overview, see [Ghallab et al. (2004)].

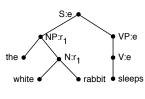
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### Planning: Language Generation

# (Project w/ CoLi Dept.)





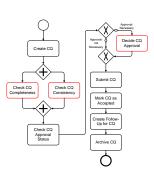
• Input: Tree-adjoining grammar, intended meaning.

Artificial Intelligence

• Output: Sentence expressing that meaning.

# Planning: Business Process Templates at SAP

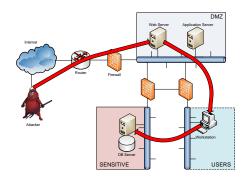
Action name	precondition	effect
Check CQ Completeness	CQ.archiving:notArchived	CQ.completeness:complete OR
		CQ.completeness:notComplete
Check CQ Consistency	CQ.archiving:notArchived	CQ.consistency:consistent OR
		CQ.consistency:notConsistent
Check CQ Approval Status	CQ.archiving:notArchived AND	CQ.approval:necessary OR
	CQ.approval:notChecked AND	CQ.approval:notNecessary
	CQ.completeness:complete AND	
	CQ.consistency:consistent	
Decide CQ Approval	CQ.archiving:notArchived AND	CQ.approval:granted OR
	CQ.approval:necessary	CQ.approval:notGranted
Submit CQ	CQ.archiving:notArchived AND	CQ.submission:submitted
	(CQ.approval:notNecessary OR	
	CQ.approval:granted)	
Mark CQ as Accepted	CQ.archiving:notArchived AND	CQ.acceptance:accepted
	CQ.submission:submitted	
Create Follow-Up for CQ	CQ.archiving:notArchived AND	CQ.followUp:documentCreated
	CQ.acceptance:accepted	
Archive CQ	CQ.archiving:notArchived	CQ.archiving:archived



- **Input:** SAP-scale model of behavior of activities on Business Objects, process endpoint.
- Output: Process template leading to this point.

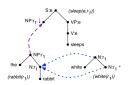
# Planning: Security Testing

# (Project w/ CISPA)

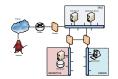


- Input: Network configuration, location of sensible data.
- Output: Sequence of exploits giving access to that data.

### Planning!

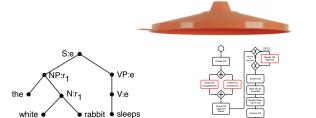


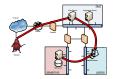
Action name	precondition	effect
Check CQ Completeness	CQ.archiving:notArchived	CQ.completeness:complete OF CQ.completeness:notComplete
Check CQ Consistency	CQ.archiving:notArchived	CQ consistency:consistent OR CQ consistency:notConsistent
Check CQ Approval Status	CQ.archiving:notArchived AND CQ.approval:notChecked AND CQ.completeness.complete AND CQ.comsistency:consistent	CQ.approval:notNecessary CQ.approval:notNecessary
Decide CQ Approval	CQ:archiving:notArchived AND CQ:approval:necessary	CQ.approval:granted OR CQ.approval:notGranted
Submit CQ	CQ.archiving:notArchived AND (CQ.approval:notNecessary OR CQ.approval:granted)	CQ submission: submitted
Mark CQ as Accepted	CQ:archiving:notArchived AND CQ:submission:submitted	CQ.acceptance:accepted
Create Follow-Up for CQ	CQ-archiving:notArchived AND CQ-acceptance:accepted	CQ.followUp:documentCreate
Archive CQ	CO.archiving:notArchived	CO.archiving:archived





#### Planning Domain Definition Language (PDDL) → Planning System





# Planning: Pros and Cons

- Powerful: In some applications, generality is absolutely necessary.
   (E.g. SAP)
- Quick: Rapid prototyping: 10s lines of problem description vs. 1000s lines of C++ code. (E.g. language generation)
- Flexible: Adapt/maintain the description. (E.g. network security)
- Intelligent: Determines automatically how to solve a complex problem effectively! (The ultimate goal, no?!)
- Efficiency loss: When humans develop domain-specific solvers, they
  invent tricks for efficiency (e.g. search bias, pruning, state
  representations suitable for neural network learning) . . . → Trade-off
  between "automatic and general" vs. "manual work but effective".

How to make fully automatic algorithms effective?

#### The Problem



- $\bullet$  n blocks, 1 hand.
- A single action either takes a block with the hand or puts a block we're holding onto some other block/the table.

blocks	states	blocks	states
1	1	9	4596553
2	3	10	58941091
3	13	11	824073141
4	73	12	12470162233
5	501	13	202976401213
6	4051	14	3535017524403
7	37633	15	65573803186921
8	394353	16	1290434218669921

 $\rightarrow$  State spaces typically are huge (not only for Go and Chess).

# Algorithmic Problems in Planning

#### Satisficing Planning

**Input:** A planning task  $\Pi$ .

**Output:** A plan for  $\Pi$ , or "unsolvable" if no plan for  $\Pi$  exists.

#### **Optimal Planning**

**Input:** A planning task  $\Pi$ .

**Output:** An *optimal* plan for  $\Pi$ , or "unsolvable" if no plan for  $\Pi$  exists.

ightarrow The techniques successful for either one of these are almost disjoint. And satisficing planning is *much* more effective in practice.

→ Programs solving these problems are called (optimal) planners, planning systems, or planning tools.

# Our Agenda for This Topic

- $\rightarrow$  Our treatment of the topic "Planning" consists of Chapters 12 and 13.
  - This Chapter: Background, planning languages, complexity.
    - $\rightarrow$  Sets up the framework. Computational complexity is essential to distinguish different algorithmic problems, and for the design of heuristic functions (see next).
  - Chapter 13: How to automatically generate a heuristic function, given planning language input?
    - $\rightarrow$  Focussing on heuristic search as the solution method, this is the main question that needs to be answered.
- → We focus on model-based techniques. The use of neural networks is an active research topic (in my research group among others). It's difficult due to the extremely general nature of planning languages.

# Our Agenda for This Chapter

- The History of Planning: How did this come about?
  - $\rightarrow$  Gives you some background, and motivates heuristic search.
- The STRIPS Planning Formalism: Which concrete planning formalism will we be using?
  - → Lays the framework we'll be looking at.
- The PDDL Language: What do the input files for off-the-shelf planning software look like?
  - $\rightarrow$  So you can actually play around with such software. (Exercises!)
- Why Complexity Analysis? Why do we bother?
  - $\rightarrow$  I'll try to convince you that this is USEFUL.
- Planning Complexity: How complex is planning?
  - ightarrow The price of generality is complexity. Here's what that "price" is, exactly.

### In the Beginning . . .

#### ... Man invented Robots:

"Planning" as in "the making of plans by an autonomous robot".

#### In a little more detail:

- Newell and Simon (1963) introduced general problem solving.
- ... not much happened (well not much we still speak of today) ...
- Early 70s Stanford Research Institute developed a robot.
- They needed a "planning" component taking decisions.
- They took inspiration from general problem solving and theorem proving, and called their algorithm "STRIPS" (see slide 25).

#### And then:

### History of Planning Algorithms

#### Compilation into Logics/Theorem Proving:

- **Popular when:** Stone Age 1990.
- Approach: From planning task description, generate PL1 formula  $\varphi$  that is satisfiable iff there exists a plan; use a theorem prover on  $\varphi$ .
- Keywords/cites: Situation calculus, frame problem, . . .

#### Partial-Order Planning:

- **Popular when:** 1990 1995.
- Approach: Starting at goal, extend partially ordered set of actions by inserting achievers for open sub-goals, or by adding ordering constraints to avoid conflicts.
- Keywords/cites: UCPOP [Penberthy and Weld (1992)], causal links, flaw-selection strategies, . . .

# History of Planning Algorithms, ctd.

#### GraphPlan:

- **Popular when:** 1995 2000.
- Approach: In a forward phase, build a layered "planning graph"
  whose "time steps" capture which pairs of actions can achieve
  which pairs of facts; in a backward phase, search this graph starting
  at goals and excluding options proved to not be feasible.
- **Keywords/cites:** [Blum and Furst (1995, 1997); Koehler *et al.* (1997)], action/fact mutexes, step-optimal plans, . . .

#### Planning as SAT:

- **Popular when:** 1996 today.
- **Approach:** From planning task description, generate propositional CNF formula  $\varphi_k$  that is satisfiable iff there exists a plan with k steps; use a SAT solver on  $\varphi_k$ , for different values of k.
- **Keywords/cites:** [Kautz and Selman (1992, 1996); Rintanen *et al.* (2006); Rintanen (2010)], SAT encoding schemes, BlackBox, ...

### History of Planning Algorithms, ctd.

#### Planning as Heuristic Search:

Introduction

- Popular when: 1999 today.
- Approach: Devise a method  $\mathcal{R}$  to simplify ("relax") any planning task  $\Pi$ ; given  $\Pi$ , solve  $\mathcal{R}(\Pi)$  to generate a heuristic function h for informed search.
- **Keywords/cites:** [Bonet and Geffner (1999); Haslum and Geffner (2000); Bonet and Geffner (2001); Hoffmann and Nebel (2001); Edelkamp (2001); Gerevini *et al.* (2003); Helmert (2006); Helmert *et al.* (2007); Helmert and Geffner (2008); Karpas and Domshlak (2009); Helmert and Domshlak (2009); Richter and Westphal (2010); Nissim *et al.* (2011); Katz *et al.* (2012); Keyder *et al.* (2012); Katz *et al.* (2013); Domshlak *et al.* (2015)], critical path heuristics, ignoring delete lists, relaxed plans, landmark heuristics, abstractions, partial delete relaxation, LP heuristics, . . .

References

# The International Planning Competition (IPC)

#### Competition?

"Run competing planners on a set of benchmarks devised by the IPC organizers. Give awards to the most effective planners."

- 1998, 2000, 2002, 2004, 2006, 2008, 2011, 2014, 2018
- PDDL [McDermott et al. (1998); Fox and Long (2003); Hoffmann and Edelkamp (2005); Gerevini et al. (2009)]
- $\bullet \approx 70$  domains, > 1500 instances, 74 planning systems in 2011
- Optimal track vs. satisficing track
- Various others: uncertainty, learning, ...

http://ipc.icaps-conference.org/

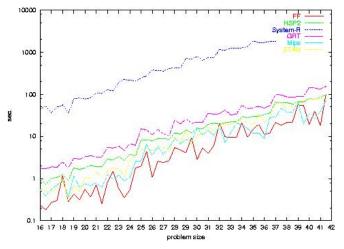
# IPC 2000: Competitors

Introduction

- BlackBox: Compilation to SAT [Kautz and Selman (1999)].
- HSP: Heuristic search [Bonet and Geffner (2001)].
- IPP: GraphPlan variant [Koehler et al. (1997)].
- STAN: Heuristic search.
- GRT: Heuristic search.
- Mips: Heuristic search.
- FF: Heuristic search [Hoffmann and Nebel (2001)].
- ... (13 altogether)

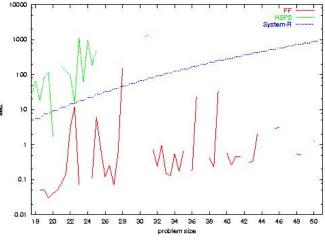
### IPC 2000: Benchmark Domains

- Blocksworld: Move around blocks on a table (yeah, I know).
- Freecell: The card game.
- Logistics: Transport packages using trucks and airplanes.
- Miconic-ADL: A complex elevator-control problem (see slide 61).
- Schedule: A simple scheduling problem where objects must be processed with various machines.

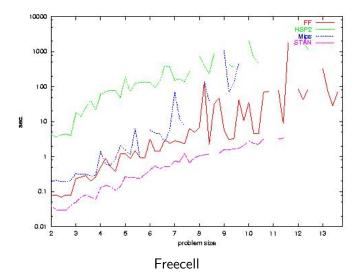


Logistics

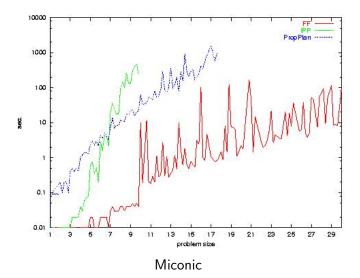
# IPC'00 Results, Fully Automatic Track

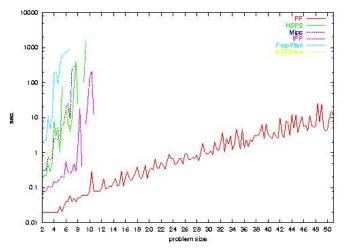


Blocksworld









Schedule

#### And Since Then?

- IPC 2000: Winner heuristic search.
- IPC 2002: Winner heuristic search.
- IPC 2004: Winner satisficing heuristic search; optimal compilation to SAT.
- IPC 2006: Winner satisficing heuristic search; optimal compilation to SAT.
- **IPC 2008:** Winner satisficing heuristic search; optimal symbolic search.
- IPC 2011: Winner satisficing heuristic search; optimal heuristic search.
- **IPC 2014:** Winner satisficing heuristic search; optimal symbolic search.
- IPC 2018: Winner satisficing heuristic search; optimal portfolio/symbolic search/heuristic search.
- ightarrow For the rest of this topic, we focus on planning as heuristic search.
- $\rightarrow$  This is a VERY short summary of the history of the IPC! There are many different categories, and many different awards.

### Questionnaire

#### Question!

If planners x,y both compete in IPC'YY, and x wins, is x "better than" y?

(A): Yes. (B): No.

- $\rightarrow$  Yes, but only on the IPC'YY benchmarks, and only according to the criteria used for determining a "winner"! On other domains and/or according to other criteria, you may well be better off with the "loser".
- $\rightarrow$  It's complicated. Over-simplification is dangerous. (But, of course, nevertheless is being done all the time).

# "STRIPS" Planning

- STRIPS = Stanford Research Institute Problem Solver.
   STRIPS is the simplest possible (reasonably expressive) logics-based planning language.
- STRIPS has only Boolean variables: propositional logic atoms.
- Its preconditions/effects/goals are as canonical as imaginable:
  - Preconditions, goals: conjunctions of positive atoms.
  - Effects: conjunctions of literals (positive or negated atoms).
- We use the common set-based notation for this simple formalism.
- I'll outline some extensions beyond STRIPS later on, when we discuss PDDL.
- $\rightarrow$  Historical note: STRIPS [Fikes and Nilsson (1971)] was originally a planner (cf. slide 14), whose language actually wasn't quite that simple.

# STRIPS Planning: Syntax

**Definition (STRIPS Planning Task).** A STRIPS planning task, short planning task, is a 4-tuple  $\Pi = (P, A, I, G)$  where:

- P is a finite set of facts (aka propositions).
- A is a finite set of actions; each  $a \in A$  is a triple  $a = (pre_a, add_a, del_a)$  of subsets of P referred to as the action's precondition, add list, and delete list respectively; we require that  $add_a \cap del_a = \emptyset$ .
- $I \subseteq P$  is the initial state.
- $G \subseteq P$  is the goal.

We will often give each action  $a \in A$  a name (a string), and identify a with that name.

**Note:** We assume unit costs for simplicity: every action has cost 1.

### "TSP" in Australia



# STRIPS Encoding of "TSP"



- Facts P:  $\{at(x), visited(x) \mid x \in \{Sydney, Adelaide, Brisbane, Perth, Darwin\}\}$ .
- Initial state I: {at(Sydney), visited(Sydney)}.
- Goal G:  $\{at(Sydney)\} \cup \{visited(x) \mid x \in \{Sydney, Adelaide, Brisbane, Perth, Darwin\}\}.$
- Actions  $a \in A$ : drive(x,y) where x,y have a road. Precondition  $pre_a$ :  $\{at(x)\}$ . Add list  $add_a$ :  $\{at(y), visited(y)\}$ . Delete list  $del_a$ :  $\{at(x)\}$ .
- Plan:  $\langle drive(Sydney, Brisbane), drive(Brisbane, Sydney), drive(Sydney, Adelaide), drive(Adelaide, Perth), drive(Perth, Adelaide), drive(Adelaide, Darwin), drive(Darwin, Adelaide), drive(Adelaide, Sydney).$

### STRIPS Planning: Semantics

**Definition (STRIPS State Space).** Let  $\Pi = (P, A, c, I, G)$  be a STRIPS planning task. The state space of  $\Pi$  is  $\Theta_{\Pi} = (S, A, T, I, S^G)$  where:

- The states (also world states)  $S = 2^P$  are the subsets of P.
- A is Π's action set.

Planning History

Introduction

- The transitions are  $T = \{s \xrightarrow{a} s' \mid pre_a \subseteq s, s' = appl(s, a)\}.$ If  $pre_a \subseteq s$ , then a is applicable in s and  $appl(s,a) := (s \cup add_a) \setminus del_a$ . If  $pre_a \not\subseteq s$ , then appl(s,a) is undefined.
- I is Π's initial state.
- The goal states  $S^G = \{s \in S \mid G \subseteq s\}$  are those that satisfy  $\Pi$ 's goal.

An (optimal) plan for  $s \in S$  is an (optimal) solution for s in  $\Theta_{\Pi}$ , i.e., a path from s to some  $s' \in S^G$ . A solution for I is called a plan for  $\Pi$ .  $\Pi$  is solvable if a plan for  $\Pi$  exists.

For  $\vec{a} = \langle a_1, \dots, a_n \rangle$ ,  $appl(s, \vec{a}) := appl(\dots appl(appl(s, a_1), a_2) \dots, a_n)$  if each  $a_i$  is applicable in the respective state; else,  $appl(s, \vec{a})$  is undefined.

**Note:** This is exactly like the state spaces of **Chapter 3** (with unit costs). Solutions are defined as before (paths from I to a state in  $S^G$ ).

# STRIPS Encoding of Simplified "TSP"

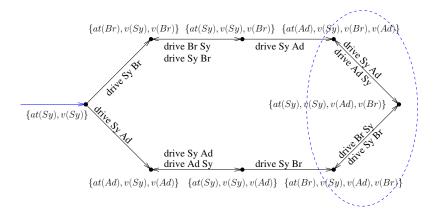


- Facts P:  $\{at(x), visited(x) \mid x \in \{Sydney, Adelaide, Brisbane\}\}$ .
- $\bullet \ \ \mathsf{Initial\ state}\ I\colon \{at(Sydney), visited(Sydney)\}.$
- Goal G:  $\{visited(x) \mid x \in \{Sydney, Adelaide, Brisbane\}\}$ . (Note: no "at(Sydney)".)

Add list  $add_a$ :  $\{at(y), visited(y)\}$ .

Delete list  $del_a$ :  $\{at(x)\}$ .

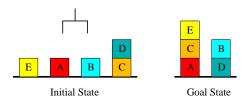
# STRIPS Encoding of Simplified "TSP": State Space



 $\rightarrow$  Is this actually the state space? No, only the reachable part. E.g.,  $\Theta_{\Pi}$  also includes the states  $\{v(Sy)\}$  and  $\{at(Sy), at(Br)\}$ .

Hoffmann, Fiser, Höller, Saller

## (Oh no it's) The Blocksworld



- Facts: on(x, y), onTable(x), clear(x), holding(x), armEmpty().
- Initial state:  $\{onTable(E), clear(E), ..., onTable(C), on(D, C), clear(D), armEmpty()\}.$
- Goal:  $\{on(E,C), on(C,A), on(B,D)\}.$
- Actions: stack(x, y), unstack(x, y), putdown(x), pickup(x).
- stack(x, y)?  $pre: \{holding(x), clear(y)\}$   $add: \{on(x, y), armEmpty()\}$  $del: \{holding(x), clear(y)\}.$

## Questionnaire

#### Question!

Introduction

```
Which are correct encodings (part of some correct overall encoding) of the STRIPS Blocksworld pickup(x) action schema? pre_a = \{onTable(x), clear(x), armEmpty()\}, add_a = \{holding(x)\}, \\ del_a = \\ \text{(A): } \{onTable(x)\}. \\ \text{(B): } \{armEmpty()\}. \\ \text{(C): } \{onTable(x), armEmpty(), \\ clear(x)\}. \\ \text{(D): } \{onTable(x), \\ armEmpty()\}. \\ \text{(D): } \{onTable(x), \\ armEmp
```

 $\rightarrow$  (A): No, must delete armEmpty(). (B): No, must delete onTable(x). (C), (D): Both yes: We can, but don't have to, encode the single-arm Blocksworld so that the block currently in the hand is not clear. (For (C), stack(x,y) and putdown(x) need to add clear(x), so the encoding on the previous slide does not work.)

## PDDL History

Introduction

### Planning Domain Description Language:

- A description language for planning in the STRIPS formalism and various extensions.
- Used in the International Planning Competition (IPC).
- 1998: PDDL [McDermott et al. (1998)].
- 2000: "PDDL subset for the 2000 competition" [Bacchus (2000)].
- 2002: PDDL2.1, Levels 1-3 [Fox and Long (2003)].
- 2004: PDDL2.2 [Hoffmann and Edelkamp (2005)].
- 2006: PDDL3 [Gerevini et al. (2009)].

## PDDL Quick Facts

### PDDL is not a propositional language:

- Representation is "lifted", using variables ranging over a universe of objects like in predicate logic. The universe is finite however.
- Predicates as in predicate logic.
- Action schemas parameterized by objects.

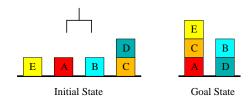
### A PDDL planning task comes in two pieces:

- The domain file and the problem file.
- The problem file gives the objects, the initial state, and the goal state.
- The domain file gives the predicates and the operators; each benchmark domain has one domain file.

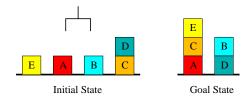
#### Examples & planner:

http://fai.cs.uni-saarland.de/hoffmann/PlanningForDummies.zip http://editor.planning.domains

### The Blocksworld in PDDL: Domain File



### The Blocksworld in PDDL: Problem File



## **PDDL** in 1998

### **STRIPS** + **ADL** (Action Description Language):

- Arbitrary first-order logic formulas in action preconditions and the goal.
- Conditional effects, i.e., effects that occur only if their separate effect condition holds.

#### ADL is a real headache to implement:

- The systems that do handle ADL compile it down to simpler formats [Gazen and Knoblock (1997)]. (Typically, STRIPS with conditional effects.)
- Example FF: 7000 C lines for compilation, 2000 lines core planner.

## Miconic-ADL "Stop" Action Schema in PDDL

Artificial Intelligence

```
(:action stop
:parameters (?f - floor)
:precondition (and (lift-at ?f)
                    (imply
                    (exists
                    (?p - conflict-A)
                    (or (and (not (served ?p))
                             (origin ?p ?f))
                    (and (boarded ?p)
                             (not (destin ?p ?f)))))
                    (forall
                    (?q - conflict-B)
                    (and (or (destin ?q ?f)
                             (not (boarded ?q)))
                    (or (served ?q)
                             (not (origin ?q ?f))))))
                    (imply (exists
                    (?p - conflict-B)
                    (or (and (not (served ?p))
                             (origin ?p ?f))
                    (and (boarded ?p)
                             (not (destin ?p ?f)))))
                    (forall
                    (?q - conflict-A)
                    (and (or (destin ?q ?f)
                             (not (boarded ?q)))
                    (or (served ?q)
                             (not (origin ?q ?f))))))
                    (imply
                    (exists
                    (?p - never-alone)
```

Hoffmann, Fiser, Hondanga(Prigin ?p ?f)

### PDDL in 2000

Fahiem Bacchus selected a subset of the ADL subset of McDermott's PDDL for the 2000 competition.

(Actually, he first designed a whole new language all of his own, but the IPC'00 organizing committee didn't like it.)

## PDDL in 2002

Introduction

Maria Fox and Derek Long promoted numeric and temporal planning:

- PDDL 2.1 level 1 Bacchus's PDDL
- PDDL 2.1 level 2: Level 1 extended with numeric state variables. Comparisons between numeric expressions are allowed as logical atoms (" $fuel(v) \ge dist(x, y) * consumption(v)$ "). Effects can assign the value of an expression to a numeric variable ("fuel(v) := fuel(v) - dist(x, y) \* consumption(v)").
- PDDL 2.1 level 3: Level 2 extended with action durations. Actions take an amount of time given by the value of a numeric expression ("dist(x,y)/speed(v)"). Conditions and effects are evaluated at either the start or the end of the action, and several actions can be executed in parallel.

### PDDL After 2002

For IPC'04, Stefan Edelkamp and I deemed PDDL2.1 to be challenging enough, so made only two small language extensions for **PDDL 2.2**: Derived Predicates (e.g., flow of current in an electricity network) and Timed Initial Literals (e.g., sunrise and sunset, shop closing times).

Gerevini & Long thought that PDDL2.2 is still not enough, and extended it with various complex notions of soft goals and preferences to obtain PDDL 3.

- → The good news (from my perspective): Since 2008, PDDL has remained largely stable.
- $\rightarrow$  Having said that: There's variants for partial observability, stochastic effects, uncertain initial states, multi-agents, . . .

## Questionnaire

### Question!

### What is PDDL good for?

(A): Nothing. (B): Free beer.

(C): Those AI planning guys. (D): Being lazy at work.

- $\rightarrow$  (A): Nah, it's definitely good for *something* (see remaining answers).
- $\rightarrow$  (B): Generally, no. Sometimes, yes: PDDL is needed for the IPC, and if you win the IPC you get price money (= free beer).
- $\rightarrow$  (C): Yep. (When I started in this area, every system had its own language, so running experiments felt a lot like "Lost in Translation".)
- $\rightarrow$  (D): Yep. You can be a busy bee, programming a solver yourself. Or you can be lazy and just write the PDDL. (I think I said that before . . . )

## Why Complexity Analysis?

### Why? Why?

#### Two very good reasons:

- It saves you from spending lots of time trying to invent algorithms that do not exist.
- Willer app in planning: tractable fragments for heuristic functions.
  - $\rightarrow$  Identify special cases that can be solved in polynomial time.
  - $\rightarrow$  Relax the input into the special case to obtain a heuristic function! ( $\rightarrow$  Chapter 3)
- $\rightarrow$  I'll next briefly remind you of the basic concepts in complexity theory, then I'll illustrate both 1 and 2 with an example. Afterwards we'll have a look at the complexity of the main decision problems in STRIPS planning.

## Reminder (?): NP and PSPACE

**Def Turing machine:** Works on a tape consisting of tape cells, across which its R/W head moves. The machine has internal states. There are transition rules specifying, given the current cell content and internal state, what the subsequent internal state will be, how what the R/W head does (write a symbol and/or move). Some internal states are accepting.

**Def NP**: Decision problems for which there exists a *non-deterministic* Turing machine that runs in *time* polynomial in the size of its input. Accepts if *at least one* of the possible runs accepts.

**Def PSPACE**: Decision problems for which there exists a *deterministic* Turing machine that runs in *space* polynomial in the size of its input.

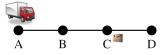
**Relation:** Non-deterministic polynomial space can be simulated in deterministic polynomial space. Thus **PSPACE** = **NPSPACE**, and hence (trivially) **NP**  $\subseteq$  **PSPACE**. It is commonly believed that **NP**  $\not\supseteq$  **PSPACE** (similar to **P**  $\subseteq$  **NP**).

→ For comprehensive details, please see a text book. My personal favorite is [Garey and Johnson (1979)]. (On the first 3 pages, they explain why knowing about **NP**-hardness will help you talk to your future boss.)

## The "Only-Adds" Relaxation

#### Example: "Logistics"

Introduction



- Facts P:  $\{truck(x) \mid x \in \{A, B, C, D\}\} \cup \{pack(x) \mid x \in \{A, B, C, D, T\}\}.$
- Initial state I:  $\{truck(A), pack(C)\}$ .
- Goal G:  $\{truck(A), pack(D)\}$ .
- Actions A: (Notated as "precondition ⇒ adds, ¬ deletes")
  - drive(x, y), where x, y have a road: " $truck(x) \Rightarrow truck(y), \neg truck(x)$ ".
  - load(x): " $truck(x), pack(x) \Rightarrow pack(T), \neg pack(x)$ ".
  - unload(x): " $truck(x), pack(T) \Rightarrow pack(x), \neg pack(T)$ ".

### Only-Adds Relaxation: Drop the preconditions and deletes.

"drive(x,y):  $\Rightarrow truck(y)$ "; "load(x):  $\Rightarrow pack(T)$ "; "unload(x):  $\Rightarrow pack(x)$ ".

 $\rightarrow$  Say we want to use this for generating a heuristic function: We solve the relaxed problem on state s to obtain h(s) (details see next chapter).

## Solving Only-Adds STRIPS Tasks

### Our problem:

Introduction

- Given STRIPS task  $\Pi = (P, A, I, G)$ .
- Find action sequence  $\vec{a}$  leading from I to a state that contains G, when pretending that preconditions and deletes are empty.

### **Solution 1:** (simplest possible approach)

```
\begin{split} \vec{a} &:= \langle \rangle \\ \text{while } G \neq \emptyset \text{ do} \\ &\text{select } a \in A \\ &G := G \setminus add_a \\ &\vec{a} := \vec{a} \circ \langle a \rangle; \ A := A \setminus \{a\} \\ \text{endwhile} \\ &\text{return } h := |\vec{a}| \end{split}
```

 $\rightarrow$  Is this h admissible? No. Admissibility is only guaranteed if we find a shortest possible  $\vec{a}$ ; else,  $\vec{a}$  might be longer than a plan for  $\Pi$  itself. Selecting an arbitrary action each time,  $\vec{a}$  may be longer than needed.

## Solving Only-Adds STRIPS Tasks, ctd.

```
So, what about this?  \vec{a} := \langle \rangle \\  \text{while } G \neq \emptyset \text{ do} \\  \text{select } a \in A \text{ s.t. } |add_a| \text{ is maximal} \\  G := G \setminus add_a \\  \vec{a} := \vec{a} \circ \langle a \rangle; \ A := A \setminus \{a\} \\  \text{endwhile} \\  \text{return } h := |\vec{a}|
```

ightarrow h admissible? No, large  $add_a$  doesn't help if the intersection with G is small.

```
And this?
```

```
\begin{array}{l} \vec{a} := \langle \rangle \\ \text{while } G \neq \emptyset \text{ do} \\ \qquad \qquad \text{select } a \in A \text{ s.t. } |add_a \cap G| \text{ is maximal} \\ \qquad G := G \setminus add_a \\ \qquad \vec{a} := \vec{a} \circ \langle a \rangle; \ A := A \setminus \{a\} \\ \text{endwhile} \end{array}
```

 $\text{return } h := |\vec{a}|$ 

```
→ h admissible? Still no. Example: G = \{A, B, C, D, E, F\}; add_{a_1} = \{A, C, E\}; add_{a_2} = \{A, B\}; add_{a_3} = \{C, D\}; add_{a_4} = \{E, F\}.
```

→ Maybe a post-process removing redundant actions could help?

## Solving Only-Adds STRIPS Tasks, ctd.

## From [Garey and Johnson (1979)]:

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NP-COMPLETE PROBLEMS

#### [SP5] MINIMUM COVER

INSTANCE: Collection C of subsets of a finite set S, positive integer  $K \le |C|$ . QUESTION: Does C contain a cover for S of size K or less, i.e., a subset  $C' \subseteq C$  with  $|C'| \le K$  such that every element of S belongs to at least one member of C'?

Reference: [Karp, 1972]. Transformation from X3C.

Comment: Remains NP-complete even if all  $c \in C$  have  $|c| \le 3$ . Solvable in polynomial time by matching techniques if all  $c \in C$  have  $|c| \le 2$ .

#### So what?

- Given STRIPS task  $\Pi = (P, A, I, G)$ .
- Find  $\vec{a}$  of length  $\leq K$  leading from I to a state that contains G, when pretending that preconditions and deletes are empty.
- $ightarrow \vec{a}$  leads to  $G \Leftrightarrow \bigcup_{a \in \vec{a}} add_a \supseteq G \Leftrightarrow$  the add lists in  $\vec{a}$  cover G. QED.

## Questionnaire

# (From [Garey and Johnson (1979)])

**Assume:** In 2 years from now, you have finished your studies and are working in an industry job. Your boss Mr. X gives you a problem and says "Solve It!". By which he means, "write a program that solves it efficiently".

#### Question!

## Could knowing about NP-hardness help?

(A): Yes. (B): No.

→ Yes! Say that, after trying in vain for 4 weeks, you got the next meeting with Mr. X. Do you want to say "Um, sorry, but I couldn't find an efficient solution, please don't fire me"?

Or would you rather say "Look, I didn't find an efficient solution. But neither could all the Turing-award winners out there put together, because the problem is **NP**-hard"?

## Reminder: Algorithmic Problems in Planning

### Satisficing Planning

**Input:** A planning task  $\Pi$ .

**Output:** A plan for  $\Pi$ , or "unsolvable" if no plan for  $\Pi$  exists.

### Optimal Planning

**Input:** A planning task  $\Pi$ .

**Output:** An *optimal* plan for  $\Pi$ , or "unsolvable" if no plan for  $\Pi$  exists.

# Decision Problems in (STRIPS) Planning

**Definition (PlanEx).** By PlanEx, we denote the problem of deciding, given a STRIPS planning task  $\Pi$ , whether or not there exists a plan for  $\Pi$ .

 $\rightarrow$  Corresponds to satisficing planning.

**Definition (PlanLen).** By PlanLen, we denote the problem of deciding, given a STRIPS planning task  $\Pi$  and an integer K, whether or not there exists a plan for  $\Pi$  of length at most K.

 $\rightarrow$  Corresponds to optimal planning.

**Definition (PolyPlanLen).** By PolyPlanLen, we denote the problem of deciding, given a STRIPS planning task  $\Pi$  and an integer K bounded by a polynomial in the size of  $\Pi$ , whether or not there exists a plan for  $\Pi$  of length at most K.

→ Corresponds to optimal planning with "small" plans. Example of a planning domain with exponentially long plans? Towers of Hanoi.

Introduction

Why C.?

## Complexity of PlanEx [Bylander (1994)]

**Lemma.** PlanEx is **PSPACE**-hard.

→ "At least as hard as any other problem contained in **PSPACE**."

**Proof Sketch.** Given a Turing machine with space bounded by polynomial p(|w|), we can in polynomial time (in the size of the machine and its input w) generate an equivalent STRIPS planning task. Say the possible symbols in tape cells are  $x_1, \ldots, x_m$  and the internal states are  $s_1, \ldots, s_n$ , accepting state  $s_{acc}$ .

- The contents of the tape cells:
- $in(1,x_1),\ldots,in(p(|w|),x_1),\ldots,in(1,x_m),\ldots,in(p(|w|),x_m).$
- The position of the R/W head:  $at(1), \ldots, at(p(|w|))$ .
- The internal state of the machine:  $state(s_1), \ldots, state(s_n)$ .
- Transitions rules  $\mapsto$  STRIPS actions; accepting state  $\mapsto$  STRIPS goal  $\{state(s_{acc})\}$ ; initial state obvious.
- This reduction to STRIPS is polynomial-time because we need only
  polynomially many facts for content of tape cells and position of R/W
  head.

## Complexity of PlanEx, ctd. [Bylander (1994)]

**Lemma.** PlanEx is a member of **PSPACE**.

→ "At most as hard as any other problem contained in **PSPACE**."

**Proof.** Because **PSPACE** = **NPSPACE**, it suffices to show that PlanEx is a member of **NPSPACE**:

- s := I; l := 1;
- Guess an applicable action a, compute the outcome state s', set l := l + 1;
- $\bullet$  If s' contains the goal then succeed;
- If  $l > 2^{|P|}$  then fail else goto 2;
- $\rightarrow$  Remembering the actual action *sequence* would take exponential space in case of exponentially long plans (cf. slide 55). But, to decide PlanEx, we only need to remember its length.

**Theorem (Complexity of PlanEx).** PlanEx is **PSPACE**-complete. (Immediate from previous two lemmas)

# Complexity of PlanLen [Bylander (1994)]

PlanLen isn't any easier than PlanEx:

**Corollary.** PlanLen is **PSPACE**-complete.

**Proof.** Membership: Same as PlanEx but failing at l > K. Hardness? Setting  $K := 2^{|P|}$ , PlanLen answers PlanEx.

### PolyPlanLen is easier than PlanEx:

**Theorem.** PolyPlanLen is **NP**-complete.

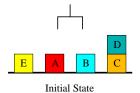
**Proof.** Membership? Same as PlanLen. This now runs in polynomial time because K is polynomially bounded. Hardness: E.g., by reduction from SAT. (Exercises, perhaps)

 $\rightarrow$  Bounding plan length does not help in the general case as we can set the bound to a trivial (exponential) upper bound on plan length. If we restrict plan length to be "short" (polynomial), planning becomes easier.

## Domain-Specific PlanEx vs. PlanLen . . .

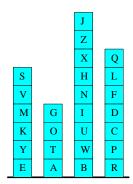
- ... is more interesting than the general case.
  - In general, both have the same complexity.
  - Within particular applications, bounded length plan existence (optimal planning) is often harder than plan existence (satisficing planning).
  - This happens in many planning competition benchmark domains:
     PlanLen is NP-complete while PlanEx is in P.
  - For example: Blocksworld and Logistics.
- → In practice, optimal planning is (almost) never easy.

### The Blocksworld is Hard?

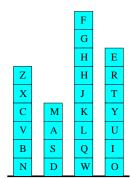




### The Blocksworld is Hard!



Initial State

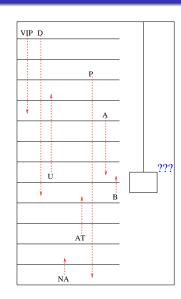


Goal State

### Miconic-ADL: PlanEx is Hard



- VIP: Served first.
- D: Lift may only go down when inside; similar for U.
- NA: Never-alone; AT: Attendant.
- A, B: Never together in the same elevator (!)
- P: Normal passenger :-)



## Summary

- General problem solving attempts to develop solvers that perform well across a large class of problems.
- Planning, as considered here, is a form of general problem solving dedicated to the class of classical search problems. (Actually, we also address inaccessible, stochastic, dynamic, continuous, and multi-agent settings.)
- Heuristic search planning has dominated the International Planning Competition (IPC). We focus on it here.
- STRIPS is the simplest possible, while reasonably expressive, language for our purposes. It uses Boolean variables (facts), and defines actions in terms of precondition, add list, and delete list.
- PDDL is the de-facto standard language for describing planning problems.
- Plan existence (bounded or not) is PSPACE-complete to decide for STRIPS. If we bound plans polynomially, we get down to NP-completeness.

## Reading

• Chapters 10: Classical Planning and 11: Planning and Acting in the Real World [Russell and Norvig (2010)].

Content: Ok as a background read, but not a good introduction to modern planning techniques.

Chapter 10 gives some background. Some issues are, imho, misrepresented, and it's far from being an up-to-date account. But it's Ok to get some additional intuitions in words different from my own.

Chapter 11 is useful in our context here because I don't cover any of it. If you're interested in extended/alternative planning paradigms, do read it.

## Reading, ctd.

 Everything You Always Wanted to Know About Planning (But Were Afraid to Ask) [Hoffmann (2011)].

#### Available at:

http://fai.cs.uni-saarland.de/hoffmann/papers/ki11.pdf

Content: My personal perspective on planning. Excerpt from the abstract:

The area has long had an affinity towards playful illustrative examples, imprinting it on the mind of many a student as an area concerned with the rearrangement of blocks, and with the order in which to put on socks and shoes (not to mention the disposal of bombs in toilets). Working on the assumption that this "student" is you – the readers in earlier stages of their careers – I herein aim to answer three questions that you surely desired to ask back then already:

What is it good for? Does it work? Is it interesting to do research in?

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