SAARLAND UNIVERSITY ARTIFICIAL INTELLIGENCE
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Theoretical Exercise Sheet 6

Deadline Friday, June 10, 23:59

About the submission of this sheet.

- You might submit the solutions to exercises in groups of up to 3 students.
- All students of a group need to be in the same tutorial.
- Write the names of **all** students of your group on your solution.
- Hand in the solution **in CMS** and use "Team Groupings".
  - Go to your personal page in CMS. Here you find the entry "Teams".
  - When you click "Create team", you get an invite code.
  - Please share this with your team mates, who need to click on "Join team" and enter the code.

1. (28 points) (7+7+7+7) Transform the following formulae into **CNF and DNF**. To do so follow the steps mentioned in **lecture 08 slide 21**. Specify each rule that is applied and corresponding intermediate result. For each formula also specify if they are (i) satisfiable, (ii) unsatisfiable, (iii) falsifiable, (iv) valid.

   a) $((P \implies Q) \wedge (Q \implies R)) \implies (R \wedge P)$

   b) $\neg(P \iff Q) \implies (Q \iff R)$

   c) $\big[(P \wedge Q) \implies R\big] \iff \big[P \implies (Q \implies R)\big]$

   d) $\Big[\big[(\neg P \implies Q) \wedge (P \implies Q)\big] \vee R\Big] \wedge \neg(\neg R \implies Q)$

2. (25 points) (5+(5+5)+10)

   a. Below is an example of a complex formula ψ.

   $$\psi = A \wedge [(\neg P \iff Q) \vee (Q \implies R) \vee S]$$

   Which of the following interpretations entail formula ψ? Mark all the statements that are True:

   ◯ a. $I = \{A = 1, P = 1, Q = 1, R = 0, S = 1\}$
   ◯ b. $I = \{A = 1, P = 1, Q = 1, R = 0, S = 0\}$
   ◯ c. $I = \{A = 0, P = 1, Q = 1, R = 0, S = 1\}$
   ◯ d. $I = \{A = 1, P = 1, Q = 0, R = 1, S = 0\}$
   ◯ e. $I = \{A = 1, P = 0, Q = 1, R = 0, S = 0\}$

b. For the formulae below, **use resolution to prove that they are unsatisfiable**. To do so, first give a set of clauses Δ that is equivalent to the formula and second, use resolution to prove that it is unsatisfiable. **Write the resolution process in the form of a tree for easier readability**.

i) $(A \vee B \vee C) \wedge (\neg A \vee B \vee C) \wedge (A \vee \neg B \vee C) \wedge (A \vee B \vee \neg C) \wedge (\neg A \vee \neg B \vee C) \wedge (A \vee \neg B \vee \neg C) \wedge (\neg A \vee B \vee \neg C) \wedge (\neg A \vee \neg B \vee \neg C)$

ii) $(\neg B \vee D) \wedge (A \vee C \vee \neg D) \wedge (B \vee C) \wedge (\neg A \vee E) \wedge (\neg C \vee E) \wedge (\neg D \vee \neg E) \wedge (B \vee \neg E)$

c.) Mr.X has invited Archie, Beto, Chelsea, Darth and Emma to a party. Now look at the following propositions.

- If Chelsea goes to the party, then Darth will too.
- If both Archie and Beto go to the party, then Emma wouldn't go.
- At least one of Chelsea and Darth would go to the party.
- If both Beto and Darth are going then so would Emma.
- If Archie goes then Emma would go too.
- If Beto wouldn't go then even Darth wouldn't.
- If Emma goes then Archie would join her.
- It can never be that both Beto and Charles wouldn't go to the party.

(i) Give a formula for each of the above propositions.

(ii) First write them in a CNF form and the list them as clauses.

(iii) Using resolution prove that **Archie wouldn't go to the party**. **Write the resolution process in the form of a tree for easier readability.**

3. (13 points) (1 + 10 + 2)
Given below is the formula ϕ. Use the **DPLL** procedure to determine whether ϕ is satisfiable or unsatisfiable. Assume that DPLL selects variables in the **splitting rule in alphabetical order** (i.e. A before B before C ...), that the **splitting rule first attempts the value True (1)** and then the value False (0) and **unit clauses with atoms are chosen before its negation** (i.e. A before $\neg A$).

$$\phi = (A \vee \neg B \vee E) \wedge (\neg A \vee \neg B) \wedge (\neg A \vee B) \wedge (C \vee B) \wedge (C \vee \neg D) \wedge (\neg C \vee \neg E) \wedge (C \vee D)$$

a) Start by writing the above formula ϕ in clause notation.

b) Run the DPLL procedure and write down each step in a numbered list as shown in **lecture 09 slide 15**. For each step, state the updated clause set.

c) Is ϕ **satisfiable**? If yes, what is the **satisfying (partial) assignment** and **how many** complete interpretation(s) satisfy ϕ?

4. (23 points) (8 + 15)

a) Perform **DPLL + Clause Learning** for Δ_1 up to the first conflict. If you have to choose something, go **lexicographically** (if UP rule: $P \mapsto 0$ and UP rule: $P \mapsto 1$ are possible start with 1). Write down each step including the updated clause set in a numbered list as in task 3.

$$\Delta_1 = \{\{A, B, \neg D\}, \{\neg A, \neg D\}, \{\neg B\}, \{B, C\}, \{\neg C, D\}\}$$

Draw the implication graph and mark the conflict graph in this implication graph. State what clause is learned.

b) Perform DPLL with clause learning on Δ_2. Write down each step including the updated clause set in a numbered list as in task 3. If you have to choose something, go **lexicographically**. If you encounter a case where two or more different unit propagation rules are applicable, choose the one which gets assigned to 1. Whenever you encounter a conflict, state which clause is learned with the clause learning method and backtrack before the last choice. Do this until the clause set is proven to be satisfiable or unsatisfiable. For the first conflict draw the **implication graph** and mark the **conflict** graph.

$$\Delta_2 = \{\{A, E\}, \{\neg A, \neg B, C\}, \{\neg A, \neg D, \neg E\}, \{\neg B, \neg C\}, \{B, D\}, \{\neg D, E\}\}$$