Theoretical Exercise Sheet 10
Deadline Friday, July 8, 23:59

About the submission of this sheet.
- You might submit the solutions to exercises in groups of up to 3 students.
- All students of a group need to be in the same tutorial.
- Write the names of **all** students of your group on your solution.
- Hand in the solution **in CMS** and use "Team Groupings".
    - Go to your personal page in CMS. Here you find the entry "Teams".
    - When you click "Create team", you get an invite code.
    - Please share this with your team mates, who need to click on "Join team" and enter the code.

1. (5 points) Given two random variables X with the domain $\{x^0, x^1\}$, and Y with the domain $\{y^0, y^1, y^2\}$. Which of the following are valid (joint) probability distributions?

   ○
	x^0	x^1
y^0	0	0
y^1	0	1
y^2	0	0

   ○
	x^0	x^1
y^0	-0.1	0.2
y^1	0.3	0.3
y^2	0.1	0.2

   ○
	x^0	x^1
y^0	0	1
y^1	0	1
y^2	0	1

   ○
	x^0	x^1
y^0	0.1	0.15
y^1	0.15	0.2
y^2	0.1	0.3

   ○
	x^0	x^1
y^0	0.2	0.3
y^1	0.1	0.2
y^2	0.3	0.2

2. (3 points) Given the probability distribution P defined by the following table:

	x^0	x^1
y^0	0.15	0.35
y^1	0.41	0.09

Calculate the marginal probability of $P(x^1)$. Show how you arrived at the answer.

3. (3 points) Given a probability distribution P defined by the following table:

	x^0	x^1
y^0	0.02	0.28
y^1	0.08	0.62

Calculate the conditional probability $P(y^1|x^0)$. Show how you arrived at the answer.

4. (6 points) Given a probability distribution P defined by the following table:

	x^0	x^1
y^0	0.02	0.18
y^1	0.08	0.72

Show that the random variables X and Y are stochastically independent in P.

5. (24 points) (4 + 4 + 8 + 8) Consider the following domain including three variables: $\underline{toothache}$, $\underline{cavity}$ and $\underline{catch}$, denoted T, C and A. The variables are Boolean. t^1 means that T is true, t^0 means that T is false, and so on. The full probability distribution is given in the following table:

T	C	A	P
t^0	c^0	a^0	0.576
t^0	c^0	a^1	0.144
t^0	c^1	a^0	0.008
t^0	c^1	a^1	0.072
t^1	c^0	a^0	0.064
t^1	c^0	a^1	0.016
t^1	c^1	a^0	0.012
t^1	c^1	a^1	0.108

Calculate the following probabilities. Show how you arrived at the answer.

1. $P(t^1)$
2. $P(c^1)$
3. $P(c^1|a^1)$
4. $P(c^1|t^0)$