

## Readings:

- 
- Winter, Y. (2016). Elements of formal semantics: An introduction to the mathematical theory of meaning in natural language. Edinburgh University Press. (Ch. 1)
- Coppock, E., and Champollion, L. (2021). Invitation to formal semantics. Manuscript, Boston University and New York University (Ch.1)

## *Credits to:*

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## Recap: What notion of sentence meaning?

The standard answer in semantics is to follow logic: what we want to obtain are the sentence's **truth conditions** – More explicitly: semantics should determine what things have to be like in order for the sentence to be true.

For some purposes, this view is too narrow:

- First, language contains not only declarative sentences, but also interrogative sentences and imperatives, to which the notion of truth does not naturally apply.
  - (1) a. Alice went to Paris.  
b. Where did Alice go?  
c. Go to Paris!

We thus need a separate view about the semantics of interrogatives and imperatives.

- Second, even for declaratives, we seem to need something more fine-grained for some purposes. For instance, consider:
  - (2) a. Every dog is a dog.  
b. Either it rains or it doesn't rain.

They have the same truth-conditions (always true) but intuitively different meanings (the former is about dogs, the latter about rain).

Nevertheless, truth-conditions capture important aspects of the meaning of a (declarative) sentence.

- They determine the sentence's **informative content**: by asserting a sentence, a speaker conveys the information that the state of affairs is one of those where the sentence is true.
- They allow us to characterize the relation of **entailment**:  $\alpha$  entails  $\beta$  if in all circumstances where  $\alpha$  is true,  $\beta$  is true as well

The truth-conditional perspective on meaning naturally suggests two ways to assess the predictions of a theory.

- **Truth-value judgments**: check whether speakers of the language judge the sentence as true/false in the circumstances predicted by the theory.

(3) Most cats are sleeping.

3 is intuitively false in a situation where one cat out of ten is sleeping. We want the theory to predict this.

- **Entailment judgments**: check whether speakers judge a sentence as following from another in accordance with the theory's entailment predictions.

(4) Many cats are sleeping.

4 intuitively entails that *some animals are sleeping*. We want the theory to predict this.

## Entailment

One of the most important usages of natural language is for everyday reasoning.

(5) Tina is tall and thin.

From this sentence, any English speaker is able to draw the conclusion:

(6) Tina is tall.

Thus, any speaker who considers sentence (5) to be true, will consider sentence (6) to be true as well. We say that sentence (5) entails (6), and denote it  $(5) \implies (6)$ . Sentence (5) is called the premise, or antecedent, of the **entailment**. Sentence (6) is called the conclusion, or **consequent**.

However, the converse does not hold: (6) may be true while (5) is not – this is the case if Tina happens to be thin but not tall. Because of such situations, we conclude that sentence (6) does not entail (5). This is denoted  $(5) \not\Rightarrow (6)$ .

- (7) a. Sue only drank half a glass of wine  $\implies$  Sue drank less than one glass of wine.  
b. A dog entered the room  $\implies$  An animal entered the room.  
c. John picked a blue card from the pack  $\implies$  John picked a card from the pack.

These kinds of entailment are very common in natural language, but they were not systematically treated in classical logic.

- (8) a. Tina is a bird. b. Tina can fly.

The inferential relation between sentences like (8a) and (8b) is *defeasible*, or cancelable, reasoning, because:

- (9) Tina is a bird, but she cannot fly, because ... (she is too young to fly, a penguin, an ostrich, etc.).

Entailments are classified as *indefeasible* reasoning: all of the assumptions that are needed in order to reach the conclusion of an entailment are explicitly stated in the premise.

Sentence  $\phi$  **semantically entails** a sentence  $\psi$  iff:  
every situation that makes  $\phi$  true, makes  $\psi$  true  
(or: in all worlds in which  $\phi$  is true,  $\psi$  is also true)

Entailments between sentences allow us to define the related notion of **equivalence**.

- (10) (5) = Tina is tall and thin and the sentence S = Tina is tall and Tina is thin

are equivalent, they entail each other.

- (11) #Tina is tall and thin, but she is not thin.

is **contradiction**.

## Model

A model is an abstract mathematical structure that we construct for describing hypothetical situations. Models are used for analyzing natural language expressions (words, phrases and sentences) by associating them with abstract objects.

Let  $exp$  be a language expression, and let  $M$  be a model. We write  $\llbracket exp \rrbracket^M$  when referring to the denotation of  $exp$  in the model  $M$ .

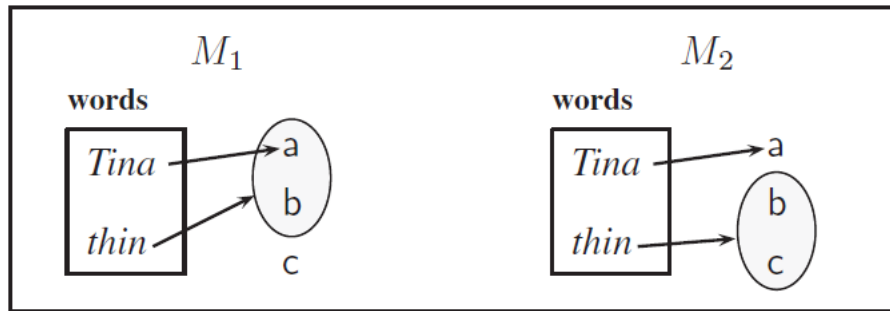


Figure 1: Models map words and other expressions to abstract mathematical objects.  $M_1$  and  $M_2$  are models with an entity denotation of *Tina* and a set denotation of *thin*. The arrows designate the mappings from the words to their denotations, which are part of the model definition.

We formally write it as follows:

$$\begin{aligned} \llbracket Tina \rrbracket^{M_1} &= a & \llbracket thin \rrbracket^{M_1} &= \{a, b\} \\ \llbracket Tina \rrbracket^{M_2} &= a & \llbracket thin \rrbracket^{M_2} &= \{b, c\} \end{aligned}$$

A semantic theory  $T$  satisfies the **truth-conditional criterion** (TCC) for sentences  $S_1$  and  $S_2$  if the following two conditions are equivalent:

1. Sentence  $S_1$  intuitively entails  $S_2$ .
2. For all models  $M$  in  $T$ :  $\llbracket S_1 \rrbracket^M \leq \llbracket S_2 \rrbracket^M$

We explicitly state the assumptions that we have so far made about our models:

1. In every model  $M$ , in addition to the two truth-values 0 and 1, we have an arbitrary non-empty set  $E^M$  of the entities in  $M$ . We refer to this set as the domain of entities in  $M$ . For instance, in models  $M^1$  and  $M^2$  of Figure 1 the entity domains  $E^{M_1}$  and  $E^{M_2}$  are the same: in both cases they are the set  $\{a, b, c\}$ .

Expression	Cat.	Type	Abstract denotation	Denotations in example models with $E = \{a, b, c, d\}$		
				$M_1$	$M_2$	$M_3$
Tina	PN	entity	<b>tina</b>	a	b	b
tall	A	set of entities	<b>tall</b>	{b,c}	{b,d}	{a,b,d}
thin	A	set of entities	<b>thin</b>	{a,b,c}	{b,c}	{a,c,d}
tall and thin	AP	set of entities	AND( <b>tall</b> , <b>thin</b> )	{b,c}	{b}	{a,d}
Tina is thin	S	truth-value	IS( <b>tina</b> , <b>thin</b> )	1	1	0
Tina is tall and thin	S	truth-value	IS( <b>tina</b> , AND( <b>tall</b> , <b>thin</b> ))	0	1	0

2. In any model  $M$ , the proper name *Tina* denotes an arbitrary entity in the domain  $E^M$
3. In any model  $M$ , the adjectives *tall* and *thin* denote arbitrary sets of entities in  $E^M$

The word ‘is’ denotes a *membership* function. Formally, we define *IS* as the function that satisfies the following, for every entity  $x$  in  $E$  and every subset  $A$  of  $E$ :

$$IS(x, A) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

We define the denotation of the word *and* to be the *intersection* function over  $E$ . This is the function *AND* that satisfies the following, for all subsets  $A$  and  $B$  of  $E$ :

$$AND(A, B) = A \cap B = \text{the set of all members of } E \text{ that are both in } A \text{ and in } B$$

Thus,

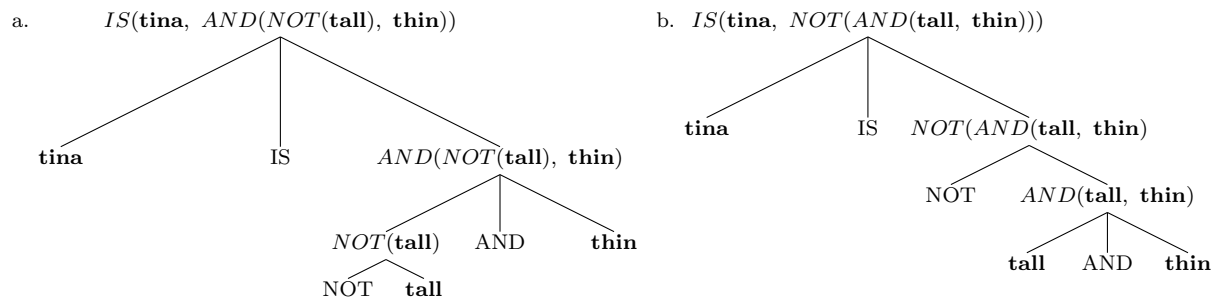
$$\begin{aligned} \llbracket Tina is thin \rrbracket^M &= IS(\mathbf{tina}, \mathbf{thin}) \\ \llbracket Tina is thin and tall \rrbracket^M &= IS(\mathbf{tina}, AND(\mathbf{thin}, \mathbf{tall})) \end{aligned}$$

The denotation of the negation word ‘not’ is the *complement* function, i.e. the function NOT that maps any subset  $A$  of  $E$  to its complement set:

$$NOT(A) = \bar{A} = E - A \text{ the set of all members of } E \text{ that are not in } A$$

**Compositionality:** the denotation of a complex expressions is determined by the denotations of its immediate parts and the ways they combine with each other.

Structural ambiguity:



$IS(tina, AND(NOT(tall), thin)) = 1$  iff  $tina \in \overline{tall} \cap thin$

$IS(tina, NOT(AND(tall, thin))) = 1$  iff  $tina \in \overline{tall} \cap \overline{thin}$ .

Other types of ambiguity (see Exercises):

- Type 1 :  $VP + NP + PP$  (attachment ambiguity): Guna ate an ice cream with fruits from Chennai.
- Type 2 :  $Gerund + VP$  Visiting relatives can be boring.
- Type 3 :  $VP + NP + more...than + NP$  Jerry loves the fans more than Sally.
- Type 4 :  $VP + NP + PP1 + PP2$  Put the bottle on the table in the kitchen.
- Type 5 :  $NP + Adj. Clause$  Tom got into the car which was parked behind the house.

The picture is complicated by two important linguistic phenomena which need to be factored in: (i) implicatures and ii) presuppositions -- > next Lecture