Introduction to Formal Semantics

Tutorial Lecture 8: Intensional and Modal Logic

Nicolaie Dominik Dascalu, Dr. Volha Petukhova
Spoken Language System Group
Saarland University

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Overview for today

1. **Coextensionality**
2. **Informative/Uninformative**
3. **Towards Intensionality**
4. **Contingent and Necessary Truth**
5. **Propositional Attitudes**
6. **Intension and composition**

Reading:

- Coppock, E., and Champollion, L. (2021). Invitation to formal semantics. Manuscript, Boston University and New York University (Ch.13).
- Gutzmann, D. (2020). Semantik. Semantik. Einführungen in die Sprachwissenschaft. J.B. Metzler, Stuttgart (Ch.11). NM
- Von Fintel, K., & Heim, I. (2011). Intensional semantics. Unpublished Lecture Notes. NM

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Substitutability of coextensionals

If two expressions[1] have the same extension, then if one is substituted for the other in any given sentence, the truth value of the sentence remains the same. **(Coppock & Champollion 2022, p. 489)**

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$[\![\text{Peter Parker}]\!] \longleftrightarrow [\![\text{Spiderman}]\!]$ 

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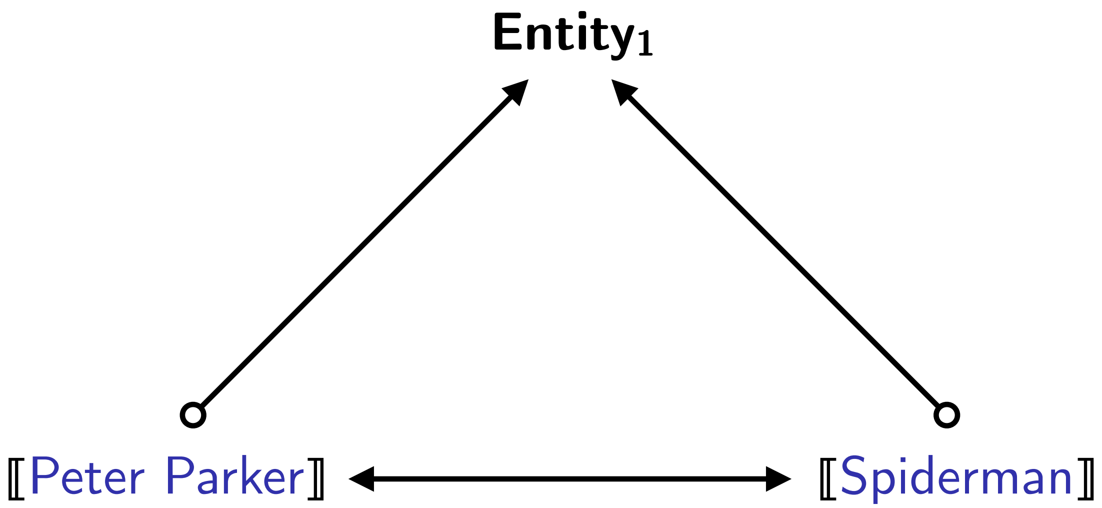

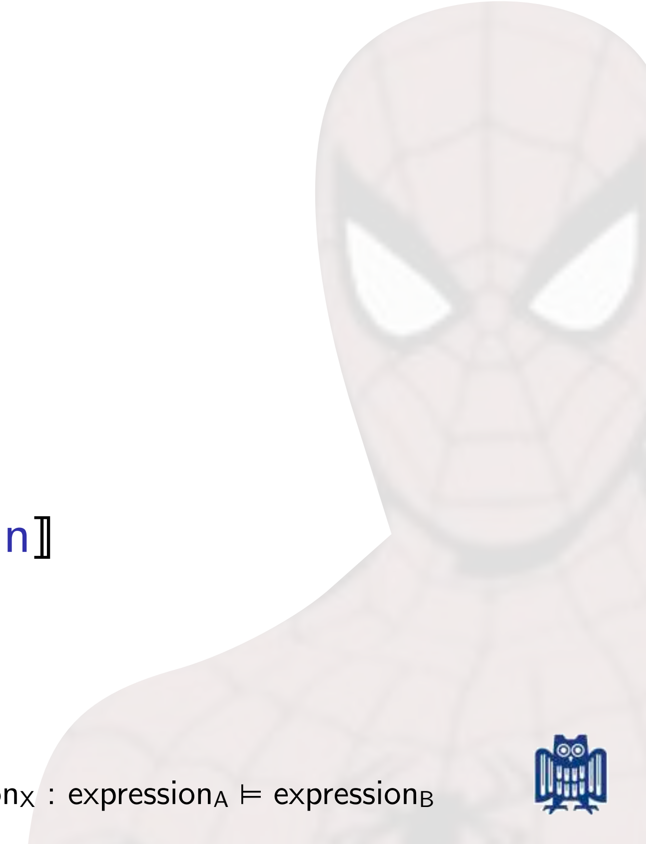

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$M_{De} = \{e_1, e_2, e_3\}$

$I_M(\text{peter parker}) = I_M(\text{spiderman}) = e_1$

$I_M(\text{mary jane}) = e_2$

$I_M(\text{harry osborne}) = e_3$

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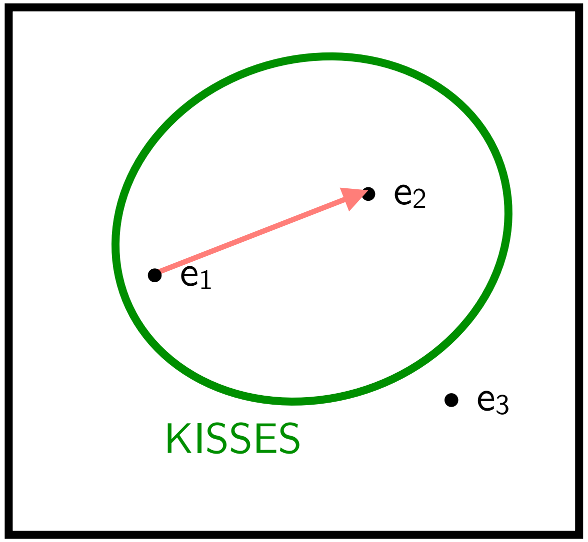

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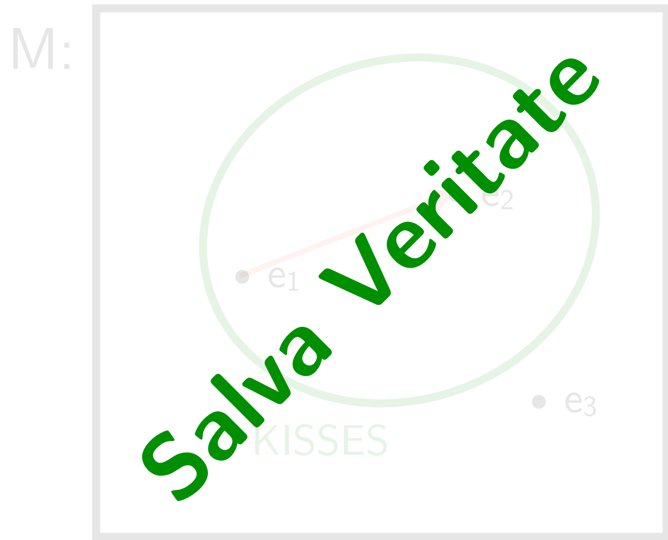

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1. Coextensionality

- If "a" and "b" are **co-referential**[2] **expressions** then by substituting one with the other "$\phi^{[a/b]}$" they are **co-extensional**: $[\![\phi^{[a]}]\!] = T \longleftrightarrow [\![\phi^{[b]}]\!] = T$

2. Here co-referentiality does not subsume deictic terms/anaphors as they first need to be anchored within the context.

2. Informative/Uninformative

Informative Vs Uninformative

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(1a) J.K.Rowling is the author of Harry Potter.

(2a) The empire state building is the 4th tallest building in New York City.

- Uninformative (a = a)

(3a) Susan is Susan

(4a) Boys are boys

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- **Informative** ($a = b$), **under negation** ($a = \neg b$)

Note: under negation $\mathbf{a = \neg b}$ is still informative! This renders it **contingent** in nature and will allow us to consider modal logic and possible worlds.

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(1b) J.K.Rowling is NOT the author of Harry Potter[4].

4. We might learn J.K. Rowling had a ghostwriter.

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- Is the principle of co-extensionality always effective?

3. Towards Intensionality

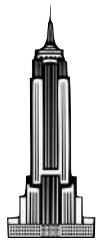
The problem of substitutivity of co-referential terms

(5) The empire state building is the 4th tallest building in NYC.

Intension $\downarrow$⟦esb⟧

- a = the tallest building in NYC.
- b = the 2nd tallest building in NYC.
- c = the 3rd tallest building in NYC.
- d = the 4th tallest building in NYC.
- e = the 424 ft building located in e*

Extension ⟦esb⟧



*e ...the 424 ft building located in 20 W 34th St, NT, 1001, U.S.

The problem of substitutivity of co-referential terms

(5) The empire state building is the 4th tallest building in NYC **(Contingent truth)**

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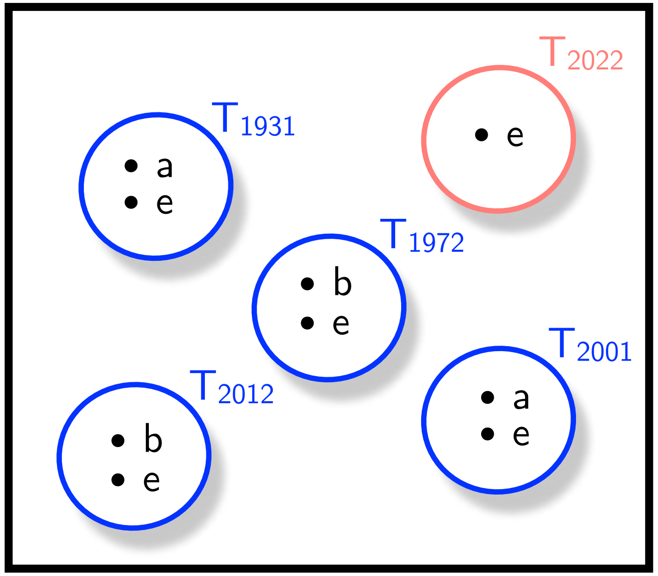

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Necessary and Contingent truths

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- What is the causes this?

4. Contingent and Necessary Truth

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"Whether or not a proposition **necessarily** holds depends on its truth value in **every world**, not just the world under consideration. In other words, **necessarily** depends on the **INTENSION** of the sentence it combines with, and not just its **EXTENSION**. The extension of an expression is its semantic value at a particular world (so, for formulas, the extension is a truth value), while the intension is a function from possible worlds to the extensions they have at those worlds." **(p. 491)**

Necessary truth

A **necessary truth** is one that could not have been otherwise. In **all circumstances**, a necessary truth expresses a true proposition.

(6) Human beings are mortal.

...is the sentence true or false?

Necessary truth

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(6) Human beings are mortal.

(i) $\underbrace{\textbf{Necessarily},}_{\textbf{Alethic}}$ $\underbrace{\text{human beings are mortal.}}_{\textbf{proposition}}$

$\Box$ ϕ

...is true in every-possible situation

4. Contingent and Necessary Truth

Contingent truth

A **contingent truth** is one that is true, but could have been false. In **some circumstances**, a contingent truth expresses a true proposition.

(7) The empire state building is the 4th tallest building in NYC.

...is the sentence true or false?

Contingent truth

A **contingent truth** is one that is true, but could have been false. In **some circumstances**, a contingent truth expresses a true proposition.

(7) The empire state building is the 4th tallest building in NYC.

(ii) $\underbrace{\textbf{Possibly}}_{\textbf{Alethic}}$, $\underbrace{\text{the empire state building is ...}}_{\textbf{proposition}}$

Alethic $\Diamond$ **proposition** ϕ

...is true in some-possible situations

Intensional operators

... we call these "circumstances" worlds **w** $\in$ **W** — **Leibniz** first defined possible worlds, **Kripke** later formalized this idea — and are now able to extend our traditional model into an intensional $\langle$**D,W,I**$\rangle$. The following are the two basic new intensional operators: "**Box operator** $\Box$", "**Diamond operator** $\Diamond$".

- *If ϕ is a formula, then $\Box\phi$ is a formula.*
- *If ϕ is a formula, then $\Diamond\phi$ is a formula.*

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Truth conditions

- $[\![\Box\phi]\!]^{M,g,w} = \top$ iff $[\![\phi]\!]^{M,g,w'} = \top$ for all w'
- $[\![\Diamond\phi]\!]^{M,g,w} = \top$ iff $[\![\phi]\!]^{M,g,w'} = \top$ for some w'

Reflexitivity of the intensional operators

Reflexivity of $\Box$:

If $\Box\phi$ is true in all worlds ($w \in W$), then ϕ is also true in the actual world ($w_@$):

- $\Box\phi \rightarrow \phi$

Non Reflexivity of $\Diamond$:

If $\Diamond\phi$ is true in some worlds, then ϕ is not necessarily true in the actual world:

- $\Diamond\phi \not\rightarrow \phi$, however, the following holds: $\phi \rightarrow \Diamond\phi$

$\Box\phi \longleftrightarrow \neg\Diamond\neg\phi$ are equivalent

Intensional interpretation

Expressions based on their intentionality are interpreted as follow:

- **Proper name:**

  $[\![\mathbf{a}_{e}]\!]^{M,g,w} = [\ [w_@ \longmapsto a], \ldots, [w_n \longmapsto a]\ ]$ **(rigid designators)**

- **Predicates:**

  $[\![P_{et}]\!]^{M,g,w} = [\ [w_@ \longmapsto \{a,b\}], \ldots, [w_n \longmapsto \{a,b,d\}]\ ]$

- **Sentences:**

  $[\![\phi_{t}]\!]^{M,g,w} = [\ [w_@ \longmapsto 1], \ldots, [w_n \longmapsto 0]\ ]$

Intensional interpretation

Expressions based on their intentionality are interpreted as follow:

- **Proper name:**
  $I(w_3, \text{john}) = \text{john}$
- **Predicates:**
  $I(w_3, \text{Happy}) = \{\text{john, sabrina, lela}\}$
- **Sentences:**
  $[\![\text{Happy(john)}]\!]^{M,g,w_3} = 1$

- Now we can move from the extensional reading...

(8) … is Spiderman.

a. $([\![$  $]\!]^{M,g} = [\![$  $]\!]^{M,g}) = \,?$

b. $([\![$  $]\!]^{M,g} = [\![$  $]\!]^{M,g}) = \,?$

c. $([\![$  $]\!]^{M,g} = [\![$  $]\!]^{M,g}) = \,?$

d. $([\![$  $]\!]^{M,g} = [\![$  $]\!]^{M,g}) = \,?$

…and which wouldn't be that helpful.

- To an intensional reading...

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...where, given our possible worlds, one interpretation holds rather than the other.

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b. $([\![$  $]\!]^{M,g} = [\![$  $]\!]^{M,g})(w_2) = ?$

c. $([\![$  $]\!]^{M,g} = [\![$  $]\!]^{M,g})(w_3) = ?$

d. $([\![$  $]\!]^{M,g} = [\![$  $]\!]^{M,g})(w_4) = ?$

w1 = marvel comics, w2 = marvel Sony universe, w3 = marvel sony universe, w4 marvel cinematic universe

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(8) $\downarrow[\![a]\!]$ $\downarrow[\![b]\!]$ $\downarrow[\![c]\!]$ $\downarrow[\![d]\!]$

$$\begin{bmatrix} w_1 & \mapsto & 1 \\ w_2 & \mapsto & 1 \\ w_3 & \mapsto & 1 \\ w_@ & \mapsto & 1 \end{bmatrix} \begin{bmatrix} w_1 & \mapsto & 0 \\ w_2 & \mapsto & 1 \\ w_3 & \mapsto & 0 \\ w_@ & \mapsto & 1 \end{bmatrix} \begin{bmatrix} w_1 & \mapsto & 0 \\ w_2 & \mapsto & 1 \\ w_3 & \mapsto & 0 \\ w_@ & \mapsto & 1 \end{bmatrix} \begin{bmatrix} w_1 & \mapsto & 0 \\ w_2 & \mapsto & 1 \\ w_3 & \mapsto & 1 \\ w_@ & \mapsto & 1 \end{bmatrix}$$

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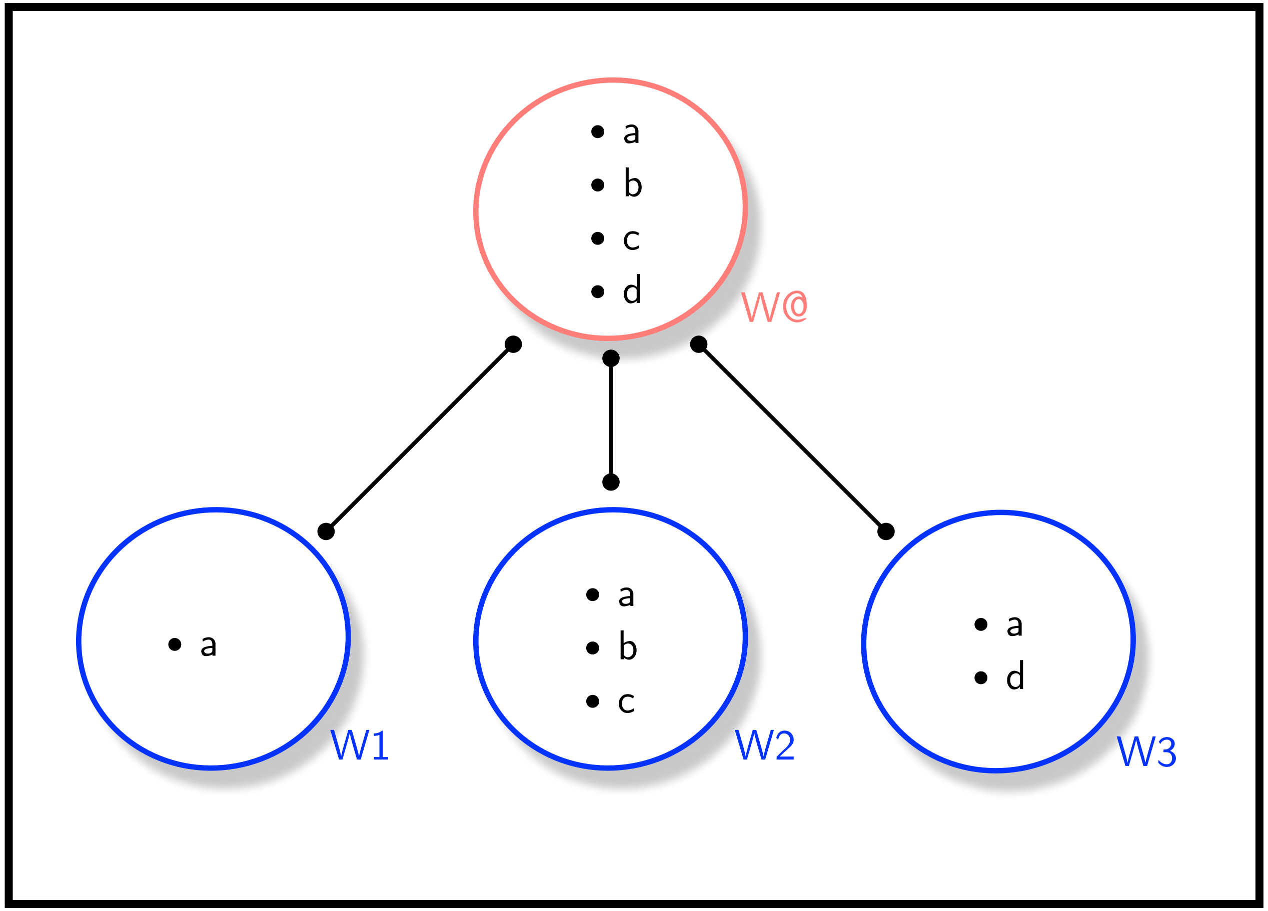
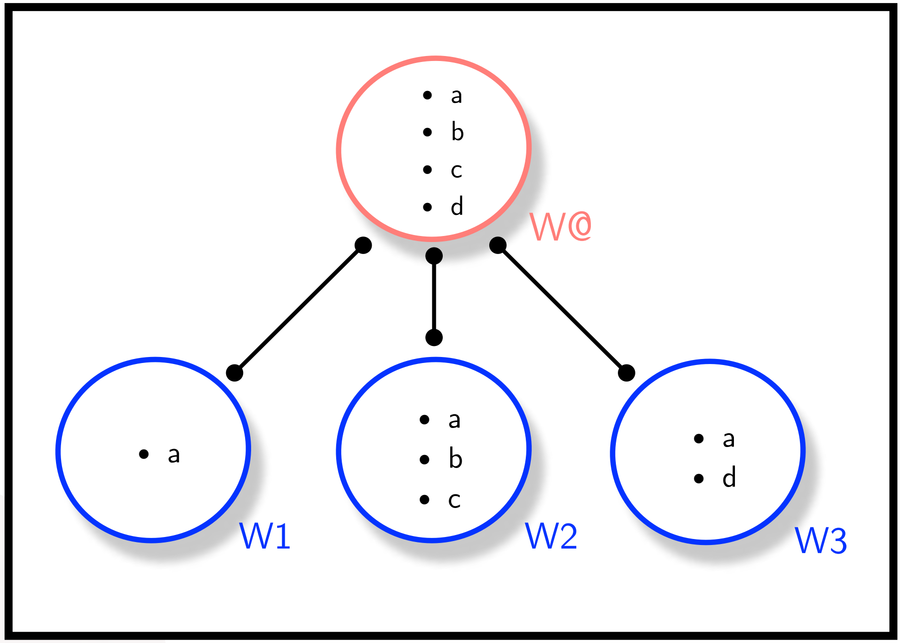
4. Contingent and Necessary Truth





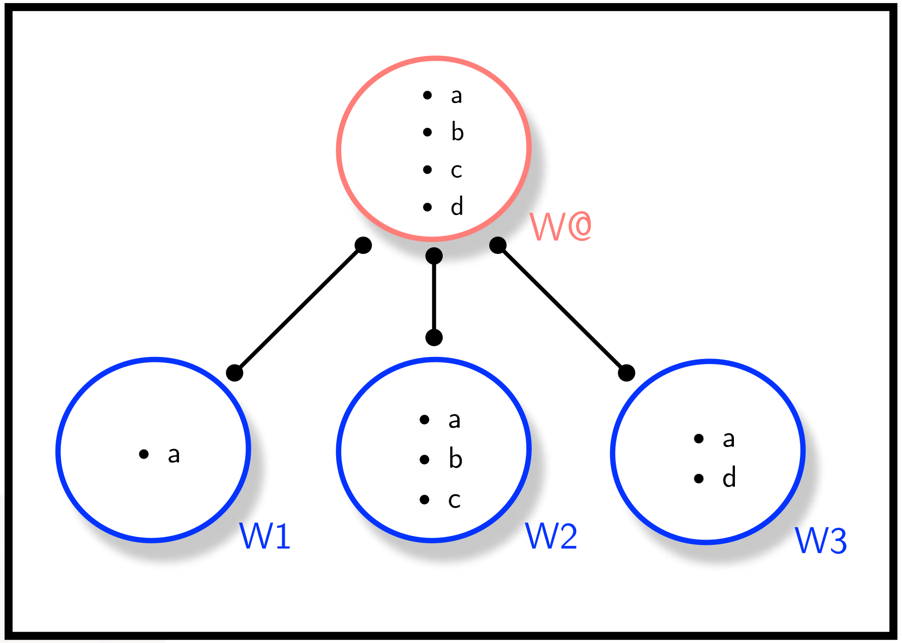
(8)

a. $M_{\{w@,w1,w2,w3\}} \models$ Peter Parker is Spiderman

b. $M_{\{w@,w2\}} \models$ Toby Maguire is Spiderman

c. $M_{\{w@,w2\}} \models$ Andrew Garfield is Spiderman

d. $M_{\{w@,w3\}} \models$ Tom Holland is Spiderman



(8)

a. $M_{\{w@,w1,w2,w3\}} \models \Box$ Peter Parker is Spiderman

b. $M_{\{w@,w2\}} \models \Diamond$ Toby Maguire is Spiderman

c. $M_{\{w@,w2\}} \models \Diamond$ Andrew Garfield is Spiderman

d. $M_{\{w@,w3\}} \models \Diamond$ Tom Holland is Spiderman

5. Propositional attitudes

Propositional attitude verbs

These express the speaker's attitude towards a certain proposition. e.g. *believe*, *know*, *want*. In particular, **belief states** are crucial to reason about the **common ground** of two or more speakers.

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(9) Susan believes Peter Parker is Iron Man.

Propositional attitude verbs

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(9) Susan $\underbrace{\textbf{believes}}_{\textbf{Dox}}$ $\underbrace{\text{Peter Parker is Iron Man.}}_{\phi}$

(9) Susan **believes** Peter Parker is Iron Man. $\not\models \phi$

5. Propositional Attitudes

Belief states and the common ground

These express the speaker's attitude towards a certain proposition. e.g. *believe*, *know*, *want*. In particular, **belief states** are crucial to reason about the **common ground** of two or more speakers.

$$(10)\ [\![\text{Peter Parker is Spider Man}]\!]^{M,g} = \begin{bmatrix} w_{mj} & \mapsto & 1 \\ w_{ned} & \mapsto & 1 \\ w_{oct} & \mapsto & 1 \\ w_{sus} & \mapsto & 0 \end{bmatrix}$$

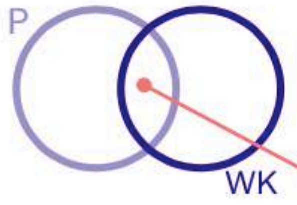
5. Propositional Attitudes

2.2 Pre-tests and Item conditions

Pre-Test 2

Access availability of relevant beliefs of the participants, testing for **common ground**. e.g. if the participants failed to guess the right country the sentence got **discarded** and substituted by a sentence that passed the pre-test.

Common Ground



Cloze Test

1. The fall of the Berlin Wall reunited ________
2. The .... ________
n. The .... ________

λx.[The fall of the Berlin Wall reunited (x)] (Germany)$^{\text{COUNTRY}}$

5. Propositional Attitudes

Veridicals

However, not all propositional attitude verbs give an intensional reading of their complement.

- **Note**: remember **factive verbs**?

  (11) Tidus **knows** that ϕ.

  $>>$ ϕ is the case; $\vDash \phi$

  (12) Auron **noticed** that ϕ.

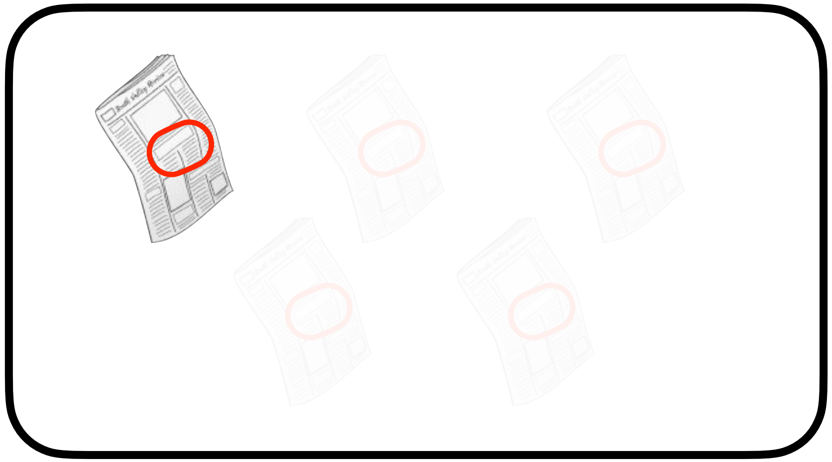
  $>>$ ϕ is the case; $\vDash \phi$

De Re vs. De Dicto Reading

These two readings are called **DE RE** ('of the object') and **DE DICTO** ('of the word'). In the **de re reading**, Andrew noticed a specific job offer. According to the **de dicto reading**, Andrew desires a job offer in general.

(13) Andrew **saw** *a job offer.* — De Re

(14) Andrew wants *a job offer*.



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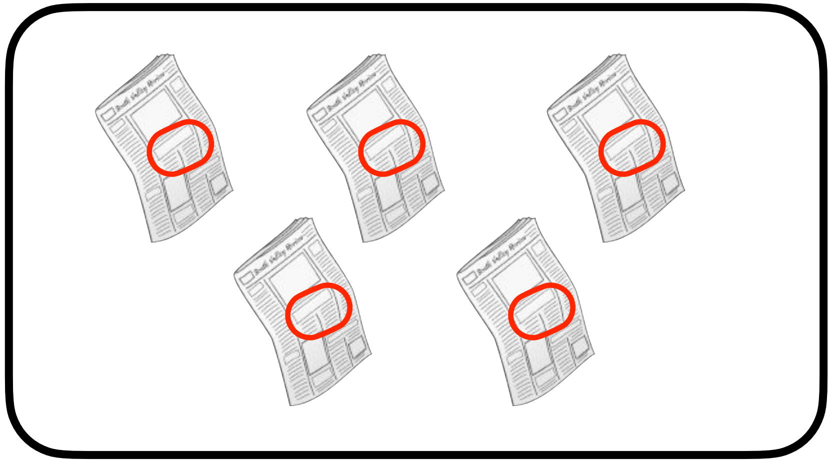
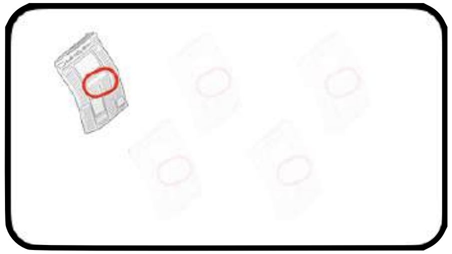
De Dicto



De Re (specific object) vs. De Dicto (nonspecific object)

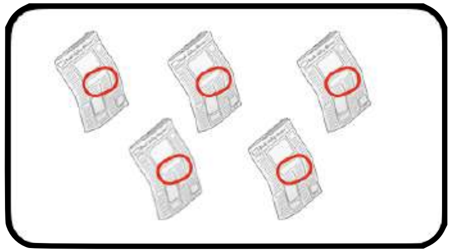
The *de re/de dicto* distinction is related to the distinction between specific and nonspecific objects. In many languages, indefinites can be marked for specificity using what is known as **Differential Object Marking (DOM)**. **(p. 493)**

(13) Andrew **saw** *a job offer.*



Specific object

(14) Andrew **wants** *a job offer.*



Category object

De Re/De Dictio as Language universal

The *de re/de dicto* distinction is related to the distinction between specific and nonspecific objects. In many languages, indefinites can be marked for specificity using what is known as **Differential Object Marking (DOM)**. **(p. 493)**

(15) a. Juan busca **a un profesor**.

b. Juan busca **un profesor**.

Specific object

Category object

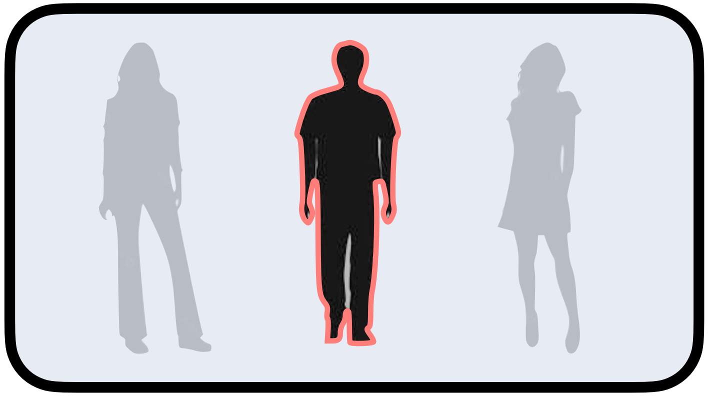
- How to treat belief states?

De Re vs. De Dicto Reading with epistemic markers

On the ***de re*** reading, Ralph has a belief about a particular object/ individual: There is someone about whom Ralph believes that they are a spy. **(see Quine 1956)**

(16) a. Ralph believes that **someone is a spy** — **De Re**

b. $\exists x\underbrace{[Bel(ralph,[Spy(x)])]}_{Dox_{Scope}}$

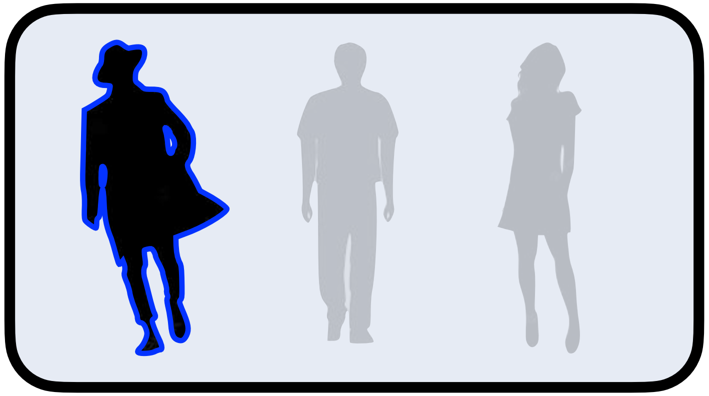


De Re vs. De Dicto Reading with epistemic markers

On the ***de dicto*** reading, Ralph has no particular individual in mind; he just believes that there are spies. The belief is not about a particular individual, rather it's about the category, spies[6].

(16) a. Ralph believes that **someone is a spy** — **De Dicto**

c. $\mathrm{Bel}(\mathrm{ralph}, \underbrace{\exists x[\mathrm{Spy}(x)]}_{\mathbf{Dox_{Scope}}})$



6. Among all $e \in D_e$ some are spies.

Opacity vs. Transparency

If the context of speech is clear we can substitute co-referential terms and implement them into reasoning patterns. However, **propositional attitude verbs** — in particular the verbs *believe* and *know* — give rise to environments where the principle of co-extensionality might fail. These are called **opaque**.

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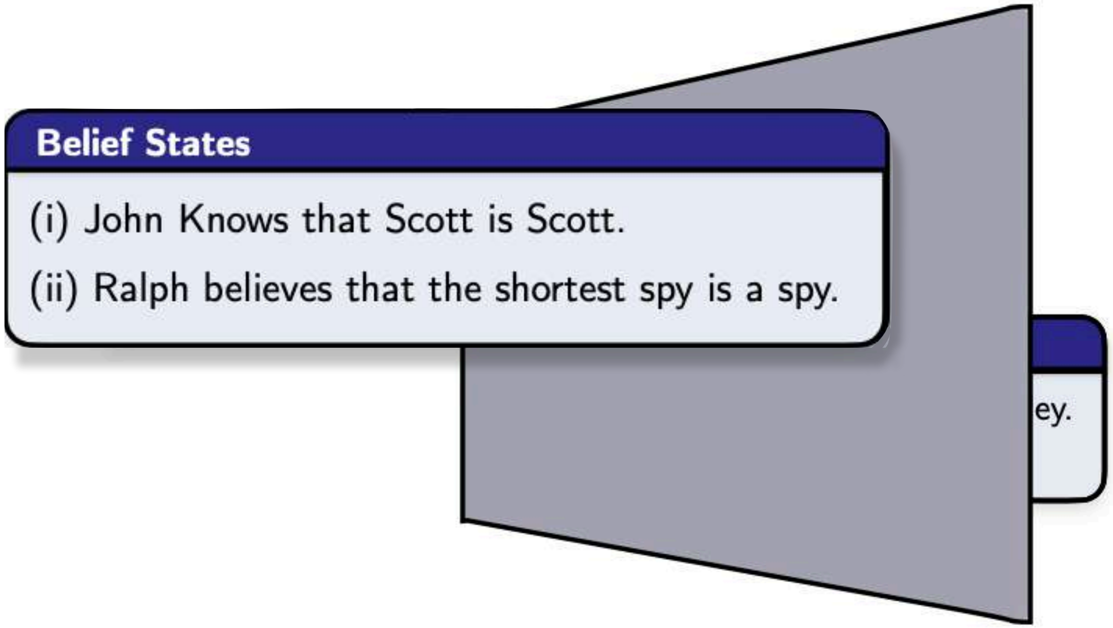
If the context of speech is clear we can substitute co-referential terms and implement them into reasoning patterns. However, **propositional attitude verbs** — in particular the verbs *believe* and *know* — give rise to environments where the principle of co-extensionality might fail. These are called **opaque**.



Belief States

(i) John Knows that Sc

(ii) Ralph believes that

States of affairs

(iii) Scott is the author of Waverley.

(iv) The shortest spy is Ortcutt.

Opaque

The speakers holds only certain mental states.

Belief States

(i) John Knows that Scott is Scott.

(ii) Ralph believes that the shortest spy is a spy.

ey.

Belief Integration

(i) $\not\models$ Scott is the author of Waverley.

(ii) $\not\models$ The shortest spy is Ortcutt.

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Belief States

(i) John Knows that Scott is Scott.

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Transparent

Belief States

(i) John Knows that Scott is Scott.

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Belief Integration

(i) $\models$ Scott is the author of Waverley.

(ii) $\models$ The shortest spy is Ortcutt.

Transparent

- Champollion's solution...

De Re vs. De Dicto disambiguation

Given that **believe** combines first with its clausal complement and then with its subject, its type should then be $\langle\langle s,t\rangle,\langle e,t\rangle\rangle$. **(Champollion 2021, p. 506)**

(16) John believes a republican will win.

a. $[\mathrm{Bel}(\mathrm{john}, {}^\wedge\exists x[\mathrm{Repub}(x)\wedge\mathrm{Win}(x)]])$ **De Dicto**

b. $\exists x[\mathrm{Repub}(x)\wedge\mathrm{Bel}(\mathrm{john}, {}^\wedge[\mathrm{Win}(x)])$ **De Re**

De Re vs. De Dicto - beta reduction

When **m** is in the scope of the **Bel operator**, its interpretation may vary from **world to world** (**De Dicto**) but when it is outside (**De Re**), it just denotes whoever Miss America is in the current world. **(p. 507)**

(17) John believes miss America is bald.

a. $[\lambda x.Bel(john, {}^{\wedge}Bald(x))](m)$ **De Re**

- John believes of the person who actually holds the title of Miss America that she is bald.

De Re vs. De Dicto - beta reduction

When **m** is in the scope of the **Bel operator**, its interpretation may vary from **world to world** (**De Dicto**) but when it is outside (**De Re**), it just denotes whoever Miss America is in the current world. **(p. 507)**

(17) John believes miss America is bald.

b. Bel(john, ^Bald(m)) **De Dicto**

- John would assent to the statement "Miss America is bald".

De Re vs. De Dicto - beta reduction

Crucially, the Ty_2 translation of c does not beta-reduce to that of d. However, the variable is bound in d such that it ranges over of the worlds where "Miss America" is bald.

(17) John believes miss America is bald.

c. [λx.Bel(w,john(w), λw.Bald(w,x))](m(w)) **De Re**

d. Bel[w,john(w), λw.Bald(w,m(w))] **De Dicto**

- w denotes w_0, John(w) denotes always John (w_0/w_1)
- m(w) denotes Camille in w_0 and Victoria in w_1
- **Problem:** (m(w)) is free in c. Instead d's reading works.

6. Intension and composition

6. Intension and composition

Intensional definition of semantic types

Letting s stand for the type of possible worlds, we now have, for every type τ, a new type $\langle s, \tau \rangle$. The complete type system is now as follows:

- t is a type
- e is a type
- If σ and τ are types, then so is $\langle \sigma, \tau \rangle$
- If τ is any type, then $\langle s, \tau \rangle$ is a type.

This rule says that for any extensional type you can define you can also add an intensional type which is a function from possible

Intensional definition of types

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Expression	Example	E-Type	I-Type
Proper name	Luke	e	⟨s,e⟩
Predicate*	Jedi	⟨e,t⟩	⟨s,⟨e,t⟩⟩
Sentence	Luke is a jedi.	t	⟨s,⟨t⟩⟩

This rule says that for any extensional type you can define you can also add an intensional type which is a function from possible

Intensional definition of types

α is an expression of type τ, then $\hat{}\alpha$ is an expression of type $\langle\ s,\ \tau\ \rangle$. Any expression of type $\langle\ s,\ \tau\ \rangle$ will denote a function from possible worlds to $D\tau$, where $D\tau$ is the domain of entities denoted by expressions of type τ. The official semantic rule for $\hat{}$ is as follows:

- If α is an expression of type τ, then $[\![\hat{}\alpha]\!]^{M,g,w}$ is that function f with domain W such that for all $w \in W : f(w)$ is $[\![\alpha]\!]^{M,g,w}$

6. Intension and composition

Example 1

The intension of floats "$^\wedge[\![\text{floats}]\!]^{M,g,w}$" is a function from possible worlds to a function of individuals in truth values (sets of individuals: $D_s \rightarrow D_{et}$).

$\textbf{floats}(w')(\textbf{yoda}) : t$

$\textbf{yoda} : e$ $\quad$ $\textbf{floats}(w') : \langle e,t \rangle$

$w' : s$ $\quad$ $\textbf{floats} : \langle s,\langle e,t\rangle\rangle$

(18) Yoda floats

If not further specified "w'" stands for the actual world (w@)

Example 2

However, Intensions should be used only when a syntactic-semantic context requires it, rather than always combining everything with intensions. Also, alternatively floats(w') = $\text{floats}_{w'}$

$$\textbf{floats}_{w'}(\textbf{yoda}) : t$$

$$\textbf{yoda} : e \qquad \textbf{floats}_{w'} : \langle e,t \rangle$$

(18) Yoda floats

6. Intension and composition

Example 3

Here we see clearly that the extension of “floats" depends on the respective evaluative world. If we abstract based on the **world variable** ($\lambda w'$) for the whole sentence $\langle s,t\rangle$, we get the intension of the sentence, which depends on the intension of **floats**.

$$\lambda w'.\mathbf{floats}_{w'}(\mathbf{yoda}) : \langle s,t\rangle$$

$$\lambda w' \qquad \mathrm{floats}_{w'}(\mathbf{yoda}) : t$$

$$\mathrm{yoda} : e \qquad \mathrm{floats}_{w'} : \langle e,t\rangle$$

(18) Yoda floats

Logical Space

the intension of a proposition refers to the **set of all possible worlds** (where this proposition holds). This is representable as the so called **logical spaced** which is divided into two subspaces: **(1)** the set of worlds which are part of the proposition (or mapped to it) and **(2)** the set of worlds which are not contained in the proposition.

w_{29}	w_6	w_{23}	w_{16}	w_{11}	w_8	w_{15}	w_{26}
w_{30}	w_{12}	w_4	w_{19}	w_9	w_5	w_{24}	w_{20}
w_{22}	w_{13}	w_{28}	w_{17}	w_3	w_{25}	$w_@$	w_{27}
w_1	w_{10}	w_{31}	w_{21}	w_{14}	w_2	w_{18}	w_7

$$^\wedge[\![\lambda w^*.\mathbf{every}(\mathbf{cat}_{w^*})(\mathbf{sleeps}_{w^*})]\!]$$

Composition and propositional attitudes

"**Believe**" is a relation between an individual (**attitude holder**) and a **proposition**. If we apply this directly to our analysis, then we can assume that an expression like belief takes a proposition (the object clause) and an individual (the subject) as arguments, resulting in a truth value.

- believe $\longrightarrow$ $\text{believe}_{w'}$: $\langle\langle s,t\rangle,\langle e,t\rangle\rangle$

6. Intension and composition

Example 4

In the embedded sentence (the complementizer), we first combine the extensional expression **unicorn**$_{w'}$ with its argument **frodo** and obtain an expression of type t.

unicorn$_{w'}$(**frodo**)) : ⟨t⟩

frodo : e   **unicorn**$_{w'}$: ⟨e,t⟩

(19) Gandalf believes Frodo is a unicorn.

6. Intension and composition

Example 4

Because we need the intension of this expression in order to use it as an argument for **believe**, we need the **world variable** $\lambda w'$ to abstract it. As a result we to obtain an expression of type ⟨s, t⟩ .

believes$_{w'}$: ⟨⟨s,t⟩,⟨e,t⟩⟩ $\lambda w'$.**unicorn**$_{w'}$(**frodo**) : ⟨s,t⟩

$\lambda w'$ **unicorn**$_{w'}$(**frodo**)) : ⟨t⟩

frodo : e **unicorn**$_{w'}$: ⟨e,t⟩

(19) Gandalf believes Frodo is a unicorn.

Example 4

Once the complementizer has been combined as argument with the Doxastic operator it needs competitions from the subject.

$\mathbf{believes}_{w'}(\lambda w'.\mathbf{unicorn}_{w'}(\mathbf{frodo}))(\mathbf{gandalf}) : \langle t \rangle$

gandalf : e $\quad$ $\mathbf{believes}_{w'}(\lambda w'.\mathbf{unicorn}_{w'}(\mathbf{frodo})) : \langle e,t \rangle$

$\mathbf{believes}_{w'} : \langle\langle s,t\rangle,\langle e,t\rangle\rangle$ $\quad$ $\lambda w'.\mathbf{unicorn}_{w'}(\mathbf{frodo}) : \langle s,t \rangle$

$\lambda w'$ $\quad$ $\mathbf{unicorn}_{w'}(\mathbf{frodo})) : \langle t \rangle$

$\mathbf{frodo} : e$ $\quad$ $\mathbf{unicorn}_{w'} : \langle e,t \rangle$

(19) Gandalf believes Frodo is a unicorn.

Example 4

The following tree can be paraphrased like this:

- $[\![\text{believes}_{w'}(\lambda w'.\text{unicorn}_{w'}(\text{frodo}))(\text{gandalf})]\!] = 1$, iff Gandalf believes in w' that Frodo is a unicorn.

$\textbf{believes}_{w'}(\lambda w'.\textbf{unicorn}_{w'}(\textbf{frodo}))(\textbf{gandalf}) : \langle t \rangle$

- gandalf : e
- $\textbf{believes}_{w'}(\lambda w'.\textbf{unicorn}_{w'}(\textbf{frodo})) : \langle e,t \rangle$
  - $\textbf{believes}_{w'} : \langle\langle s,t\rangle,\langle e,t\rangle\rangle$
  - $\lambda w'.\textbf{unicorn}_{w'}(\textbf{frodo})$:
    - $\lambda w'$
    - $\textbf{unicorn}_{w'}(\textbf{frodo})) : \langle t \rangle$
      - $\textbf{frodo} : e$
      - $\textbf{unicorn}_{w'} : \langle e,t \rangle$

(19) Gandalf believes Frodo is a unicorn.

Thank you all for the kind attention!

Conclusion

If you need further help or have additional questions, please contact us.