The Lecture of Why

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Questions of the day

What is **causation**,
how can we **measure** it,
and how can **discover** it?

Dependence vs. Causation

Machine learning is amazing!

It's a cow!

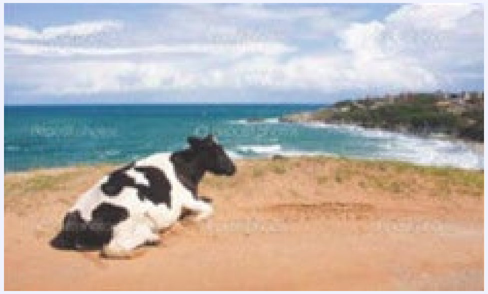
cow · milk · agriculture · farm · cattle · livestock · dairy · beef · hayfield · field · grass · mammal · pasture · calf · farmland · rural · animal · pastoral · bull · grassland

cow · beef · agriculture · cattle · milk · pasture · mammal · livestock · farmland · grass · farm · hayfield · rural · herd · dairy · pastoral · grassland · field · calf · bull

cow · mammal · pasture · grass · animal · no person · nature · agriculture · livestock · hayfield · cattle · farm · rural · field · milk · grassland · beef · pastoral · countryside

It’s… not a cow!

but not always
in the right way…



beach · sand · travel · no person · water · sea · seashore
summer · sky · outdoors · ocean · nature

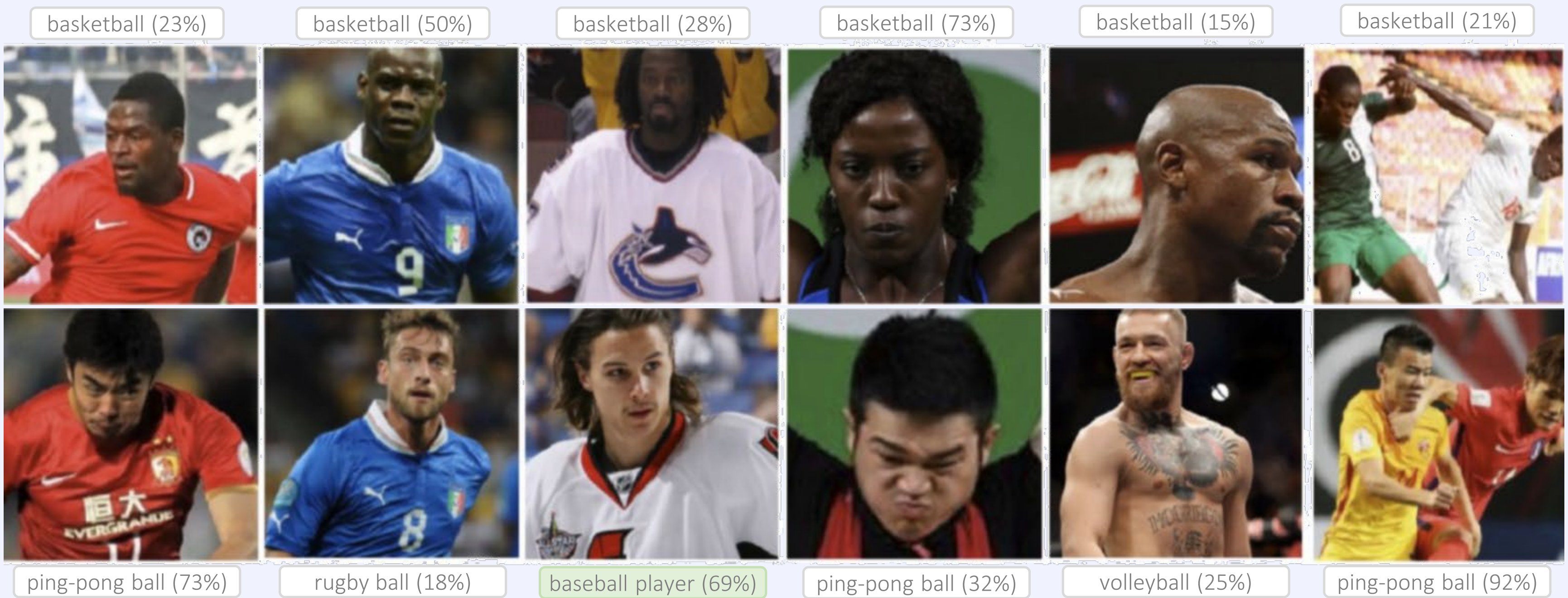
water · no person · beach · seashore · sea · sand · mammal
outdoors · travel · ocean · surf · sky

no person · water · mammal · cattle · outdoors · cow
landscape · travel · sky · livestock

(Lopez-Paz et al 2016, Perona 2017)

It’s a ball!

sometimes in a very bad way



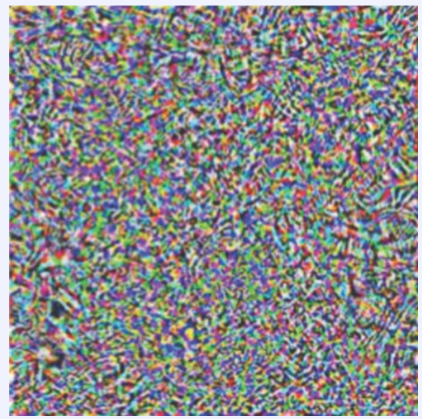
(Stock & Cisse 2017)

It’s an airplane!

sometimes in an exploitable way

pig

$+0.005 \times$



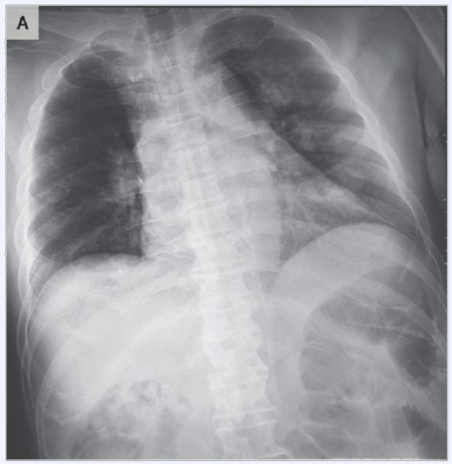
$=$

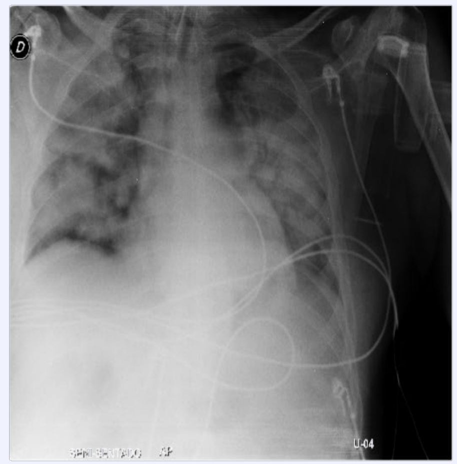
airliner

(Szegedy et al 2013)

COVID-19

True Label	COVID-19 (Training Data)	COVID-19 (Unseen Data)





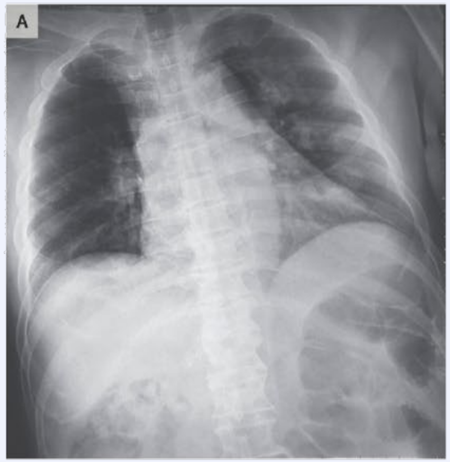
Model	Prediction	Confidence	Prediction	Confidence
DNN	COVID-19	99.7%	Non-COVID	75.1%
BNN	COVID-19	95.5%	COVID-19	67.1%

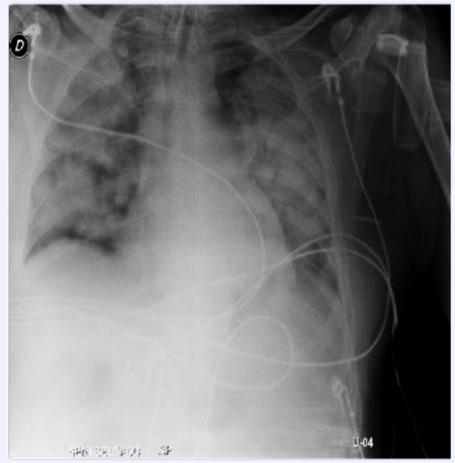
(Mallick et al 2020)

CATVID-19

and often in a useless way

True Label	COVID-19 (Training Data)	COVID-19 (Unseen Data)	Cat (Unrelated Data)

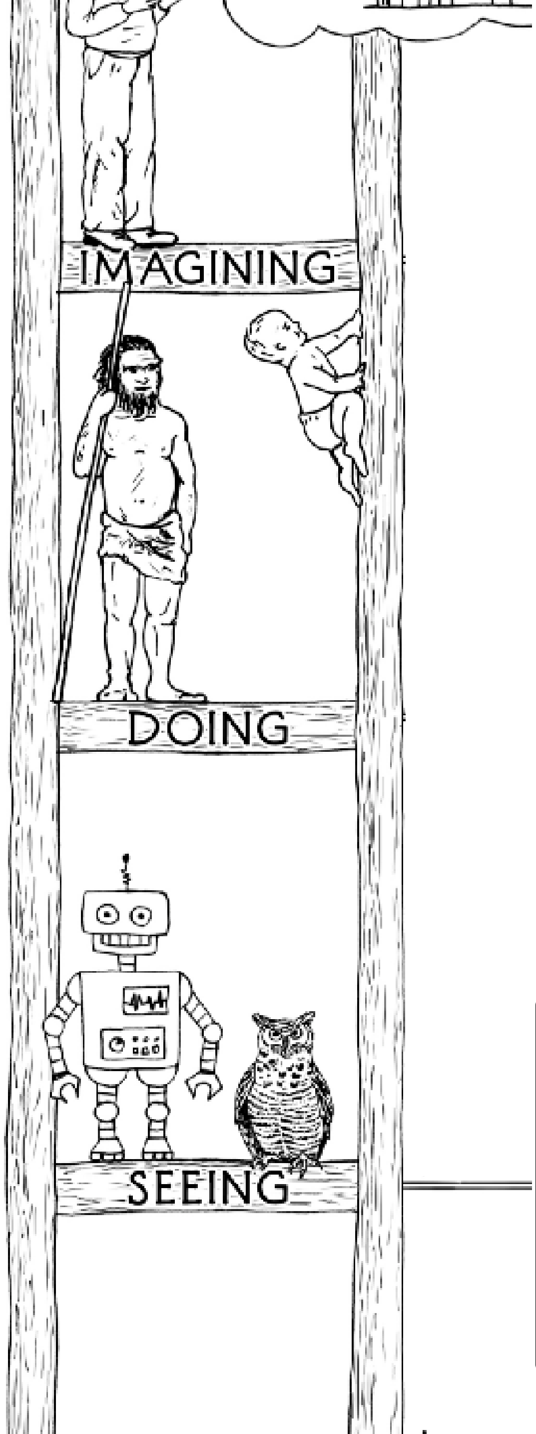




Model	Prediction	Confidence	Prediction	Confidence	Prediction	Confidence
DNN	COVID-19	99.7%	Non-COVID	75.1%	COVID-19	100%
BNN	COVID-19	95.5%	COVID-19	67.1%	COVID-19	99.8%

(Mallick et al 2020)


BIG
DATA



1. ASSOCIATION

ACTIVITY: Seeing, Observing

QUESTIONS: *What if I see …?*
(How are the variables related?
How would seeing X change my belief in Y?)

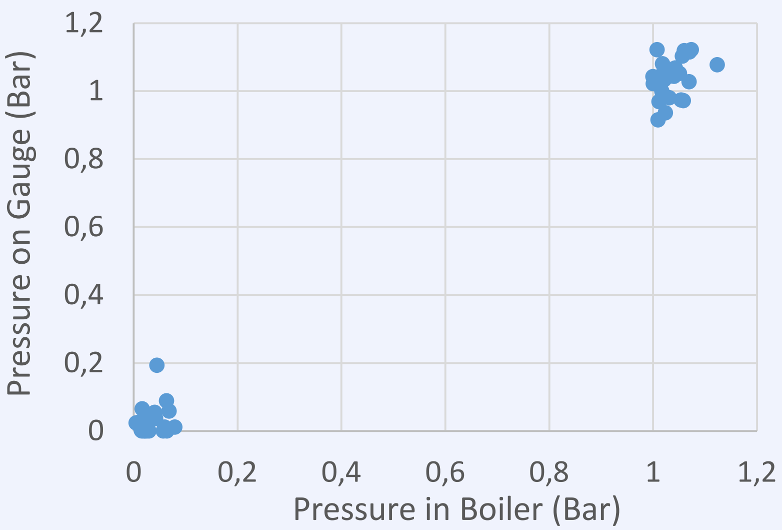
EXAMPLES: What does a symptom tell me about a disease?
What does a survey tell us about
the election results?

(Pearl 2018)

Time for a coffee

Let's consider my espresso machine

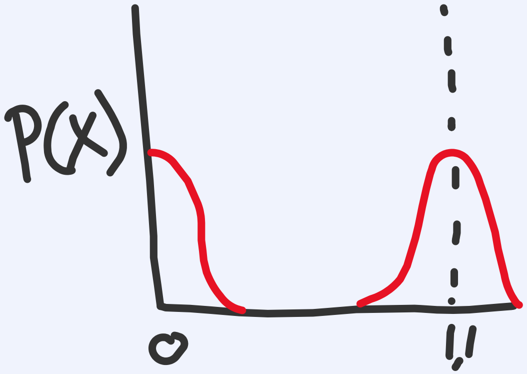
- X is the actual pressure in the boiler
- Y is the pressure measured by front gauge



Time for a coffee

Can we decide cause from effect based on data?

- we can compute **marginal probabilities**
- $P(X)$ is the probability of measuring a certain pressure X in the boiler
- $P(Y)$ is the probability of measuring a certain pressure Y on the gauge

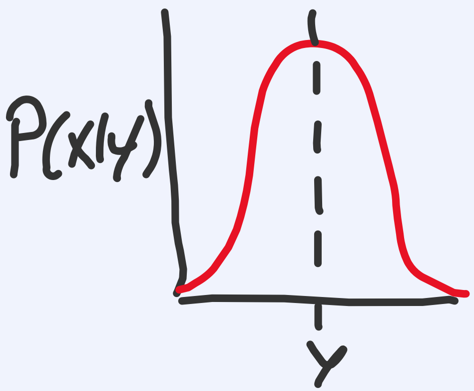


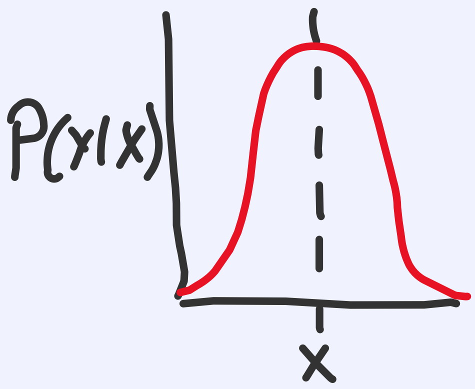
- **Marginals** are **insufficient** to tell cause from effect

Time for a coffee

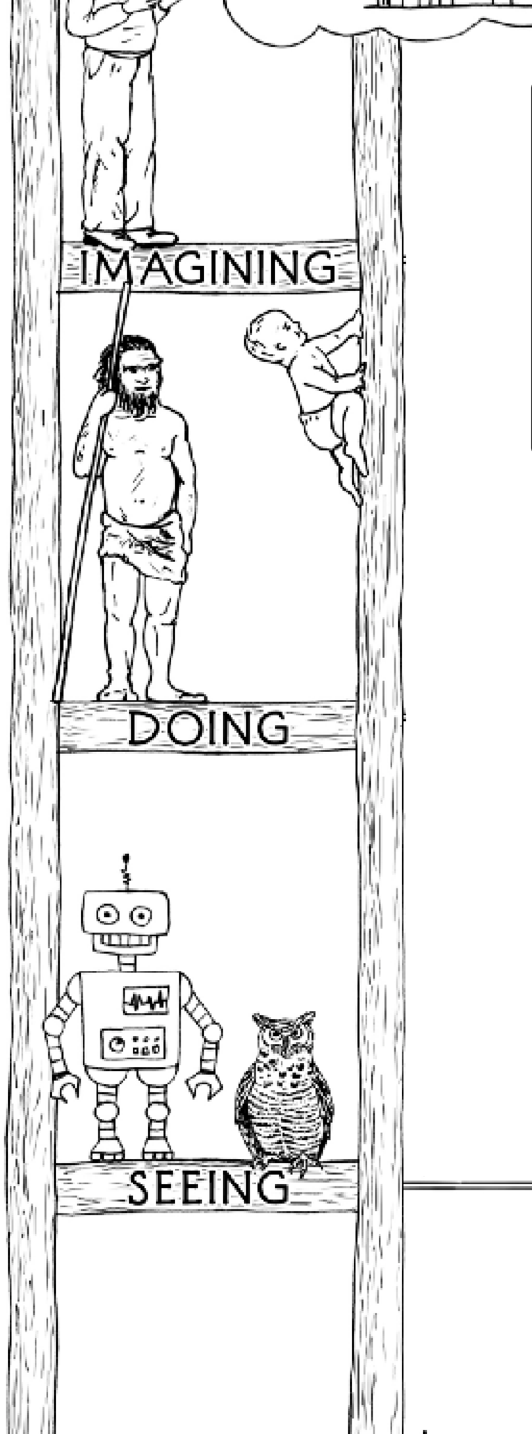
Can we decide cause from effect based on data?

- we can compute **conditional probabilities** from the data
- $P(X \mid Y)$ is the probability of a pressure X in the boiler, knowing the gauge says Y
- $P(Y \mid X)$ vice versa





- **Conditionals** are **insufficient** to tell cause from effect



3. COUNTERFACTUALS

ACTIVITY: Imagining, Retrospection, Understanding

QUESTIONS: *What if I had done ...? Why?*
(Was it X that caused Y? What if X had not occurred? What if I had acted differently?)

EXAMPLES: Was it the asprin that stopped my headache?
Would I have bought a laptop, if I would not have bought a backpack?

$$P(y \mid x * 2)$$

2. INTERVENTION

ACTIVITY: Doing, Intervening

QUESTIONS: *What if I do...? How?*
(What would Y be if I do X?
How can I make Y happen?)

EXAMPLES: If I take asprine, will my headache be cured?
If make someone buy a backpack, will they also buy a laptop?

$$P(y \mid do(x))$$

1. ASSOCIATION

ACTIVITY: Seeing, Observing

QUESTIONS: *What if I see ...?*
(How are the variables related?
How would seeing X change my belief in Y?)

EXAMPLES: What does a symptom tell me about a disease?
How often do people who buy a backpack also buy a laptop?

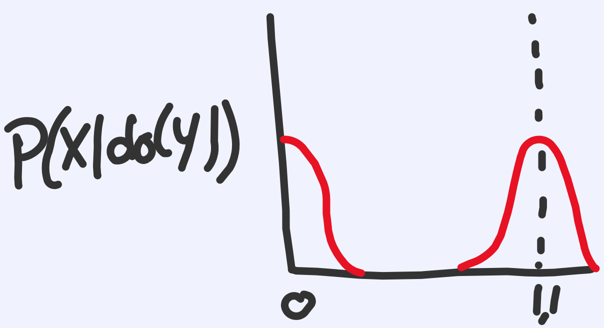
$$P(y \mid x)$$

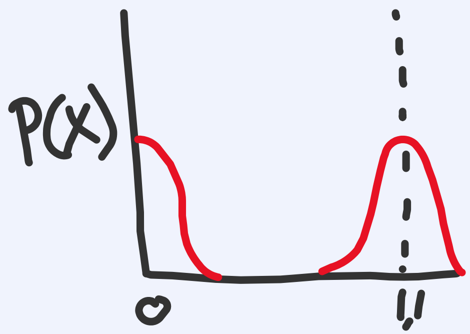
(Maayan Harel)

Time for a coffee

How can we decide on causality?

- **intervening** on the barometer, e.g. moving its needle up or down, has **no effect** on the actual pressure



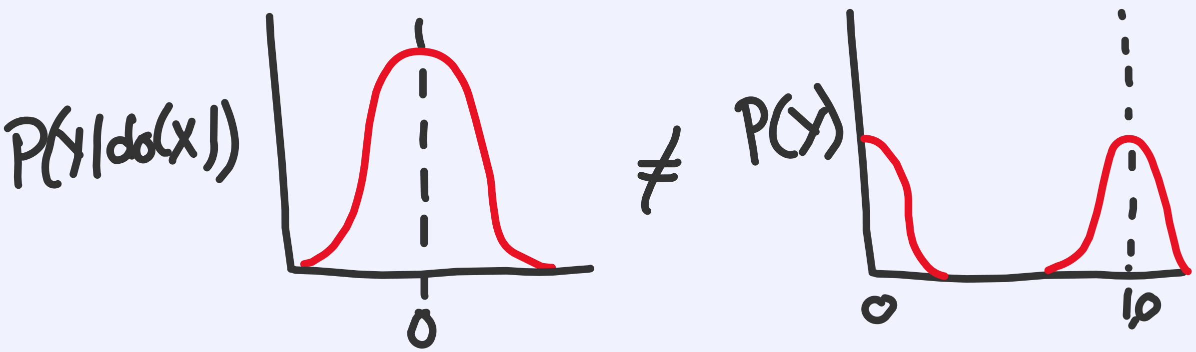


- $P(X \mid do(Y)) = P(X)$
- Clearly, Y **does not cause** X

Time for a coffee

How can we decide on causality?

- **intervening** on the boiler, e.g. adding extra pressure, **does** move the needle of the barometer



- $P(Y \mid do(X)) \neq P(Y)$
- Clearly, X has a causal effect on Y

Randomized Controlled Trials

Randomized controlled trials are the de-facto standard for determining whether X causes Y

- treatment $X \in \{0,1,\ldots\}$, potential effect Y and co-variates Z

Simply put, we

1. gather a **large population** of test subjects
2. **randomly split** the population into two equally sized groups A and B, making sure that Z is **equally distributed** between A and B
3. **apply treatment** $X = 0$ to group A, and treatment $X = 1$ to group B
4. **determine** whether Y and X are dependent

If $Y \not\perp\!\!\!\perp X$, we conclude that X causes Y

Randomized Controlled Trials

Randomized controlled trials are the de-facto
standard f

- treatmen Z

Ultimate, but not ideal

- Often impossible or unethical
- Large populations needed
- Difficult to control for Z

Simply pu

1. gather a
2. **random** ups A and B, making B
3. **apply tre** to group B
4. **determine** whether Y and X are dependent

If $Y \not\perp\!\!\!\perp X$, we conclude that X causes Y

Observational Data

If we cannot measure $P(Y \mid do(X))$ directly in a randomized trial, can we estimate it based on data we observed outside of a controlled experiment?

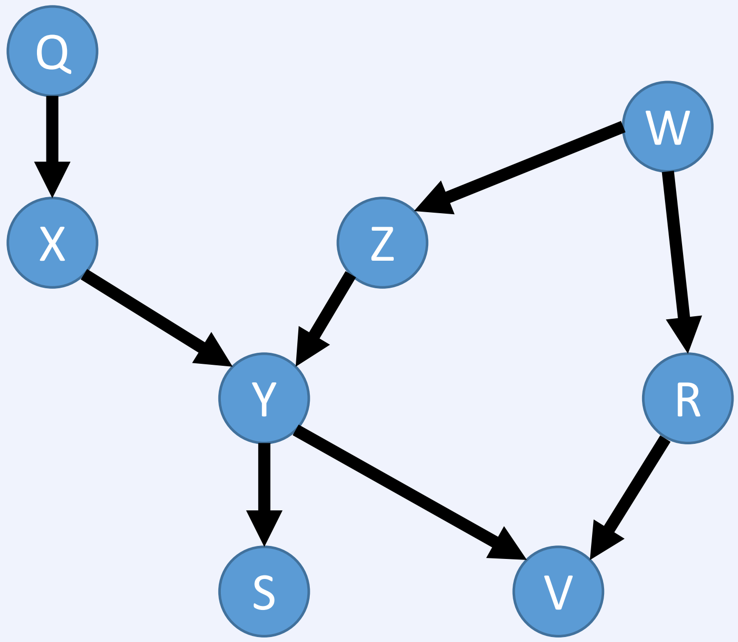
Structural Causal Model

What happens if we **intervene**?
What happens if we $do(X)$?

$Q \; := U_Q$
$W \; := U_W$

$X := 2Q + U_X$
$Z := 5W + U_Z$
$R := 2W + U_R$

$Y := f_Y(X, Z, U_Y)$
$S := f_S(Y, U_S)$
$V := f_V(Y, R, U_V)$



Doo-doo-doo baby shark

What happens if we **intervene**?
What happens if we $do(X)$?

$Q \coloneqq U_Q$
$W \coloneqq U_W$

$\boldsymbol{X \coloneqq 42}$
$Z \coloneqq 5W + U_Z$
$R \coloneqq 2W + U_R$

$Y \coloneqq f_Y(X, Z, U_Y)$
$S \coloneqq f_S(Y, U_S)$
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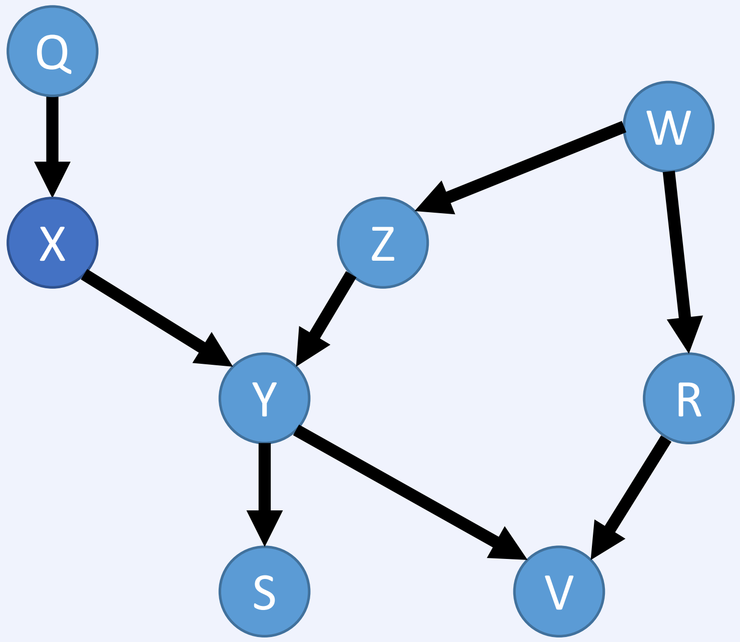
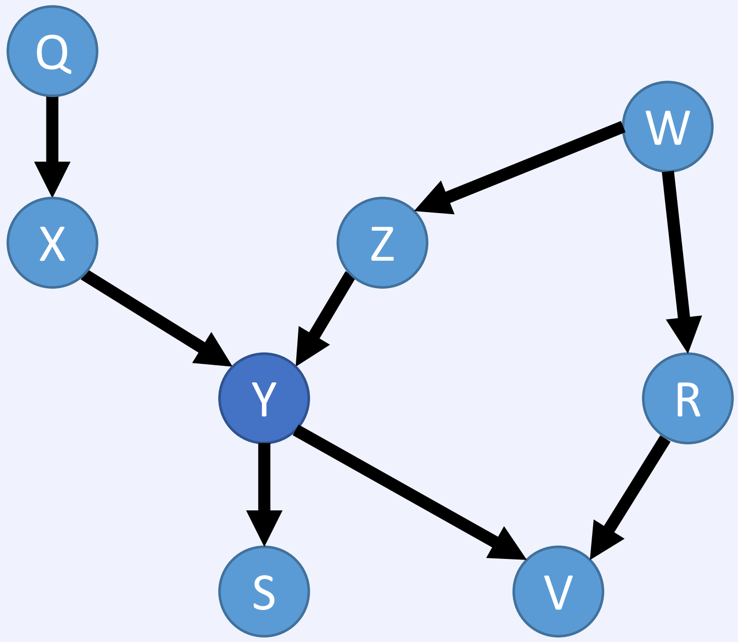


Doo-doo-doo baby shark

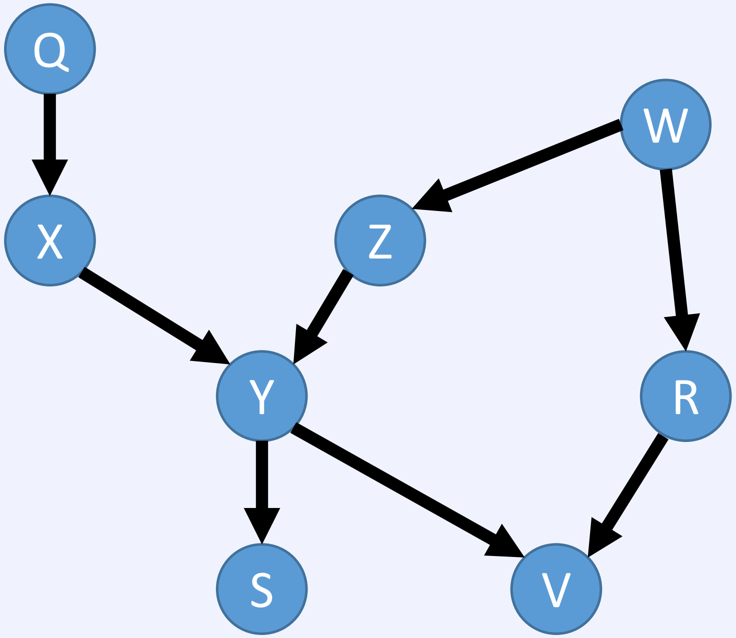
What happens if we **intervene**?
What happens if we $do(Y)$?

A do-intervention on Y means removing **all** external influence on Y, i.e. removing all incoming edges



Do-calculus

Do-calculus rules allow us to transform between **interventional** conditional probabilities, and **observational** conditional probabilities

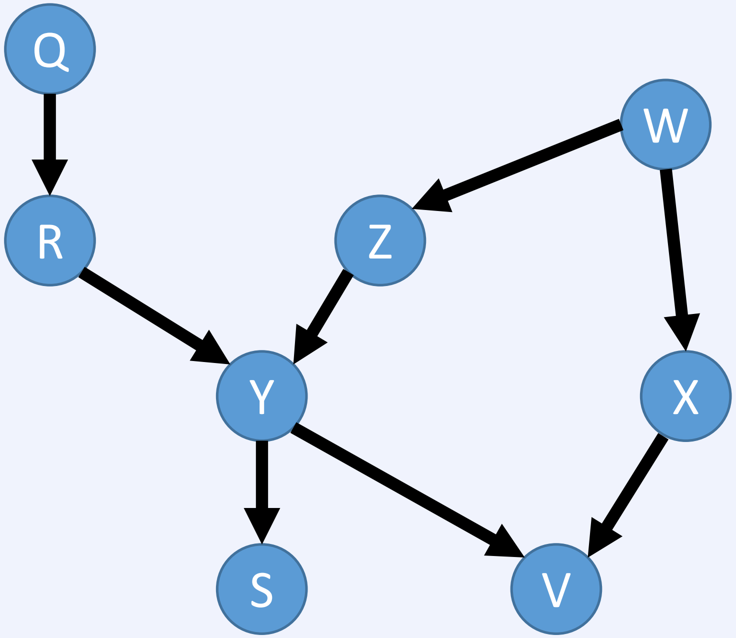


Do-calculus

Rule 1 (Interventions)

$$P(Y \mid do(X)) \stackrel{?}{=} P(Y)$$

iff no causal path connects X and Y

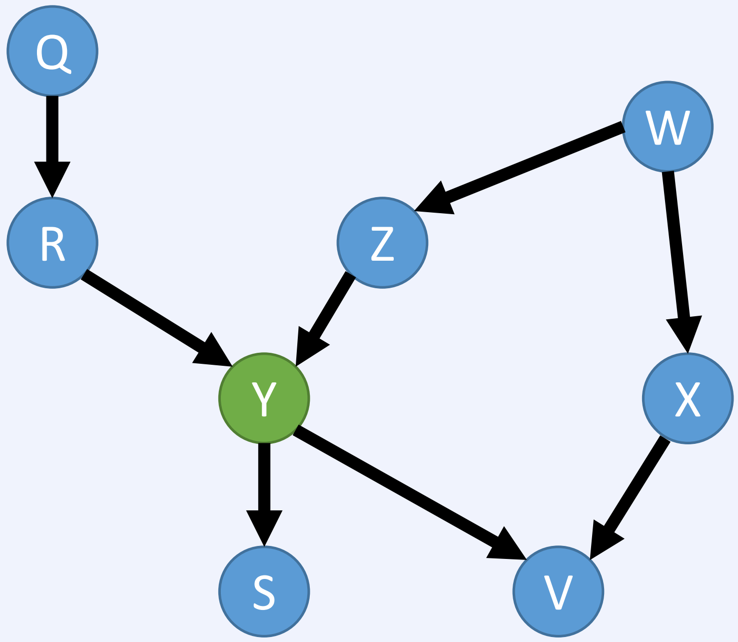


Do-calculus

Rule 1 (Interventions)

$$P(\boldsymbol{Y} \mid do(X)) = P(\boldsymbol{Y})$$

iff no causal path connects X and $\boldsymbol{Y}$



Do-calculus

Rule 1 (Interventions)

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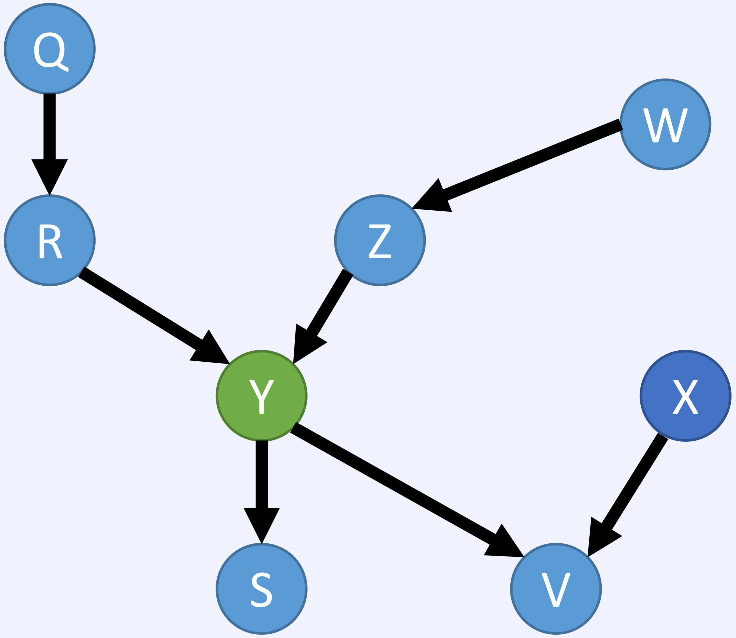

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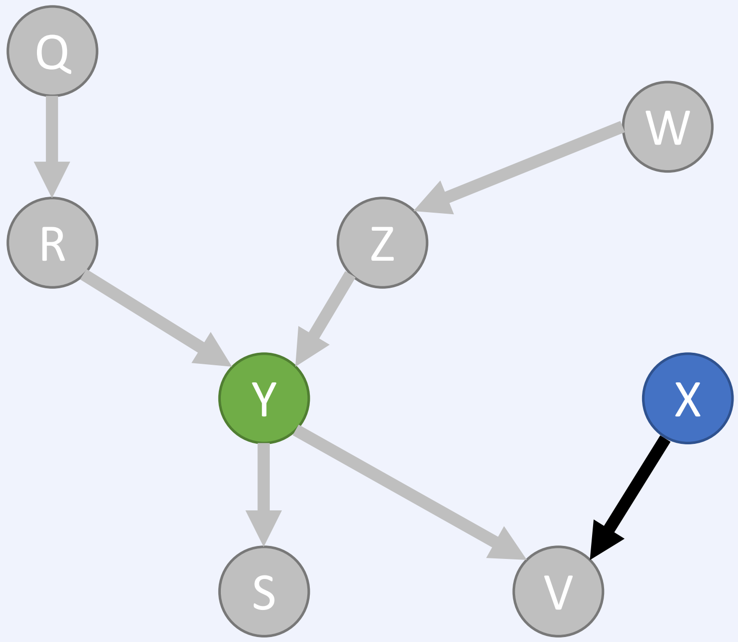
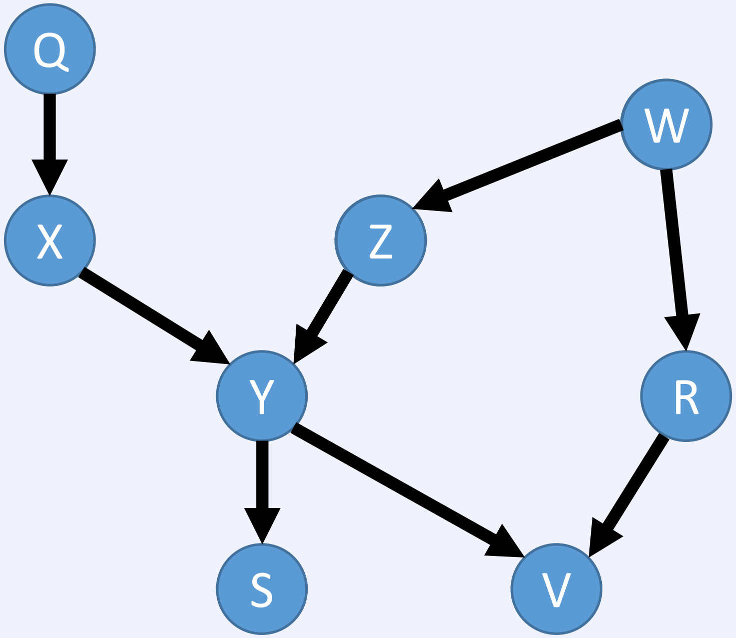


Do-calculus

Rule 2 (Observations)

$$P(Y \mid do(X), Z, W) \stackrel{?}{=} P(Y \mid do(X), Z)$$

iff Z blocks all paths from W to Y, and all arrows leading into X have been deleted



Do-calculus

Rule 2 (Observations)

$$P(\boldsymbol{Y} \mid do(X), Z, W) = P(\boldsymbol{Y} \mid do(X), Z)$$

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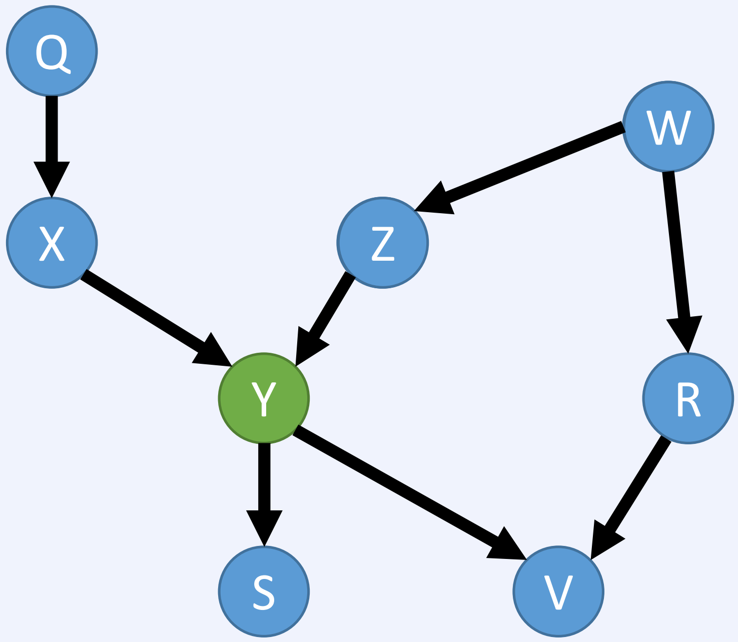

Do-calculus

Rule 2 (Observations)

$$P(\boldsymbol{Y} \mid do(\boldsymbol{X}), Z, W) = P(\boldsymbol{Y} \mid do(\boldsymbol{X}), Z)$$

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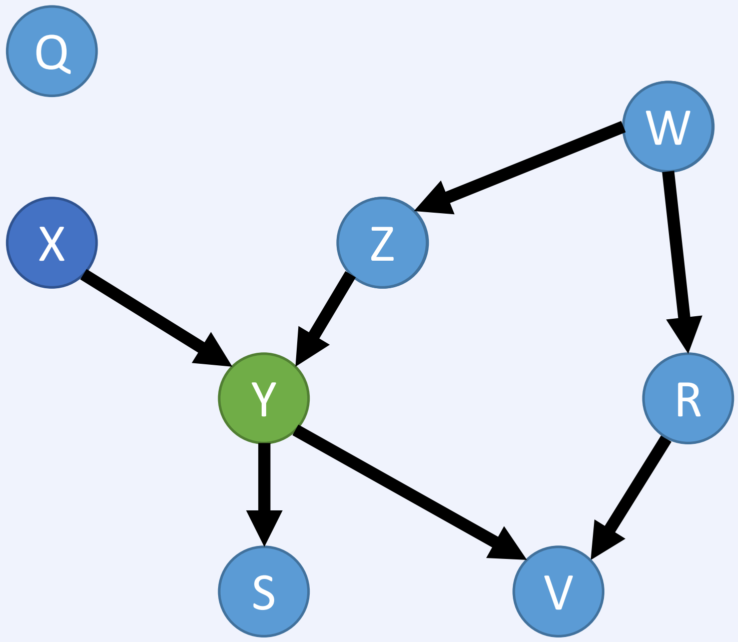

Do-calculus

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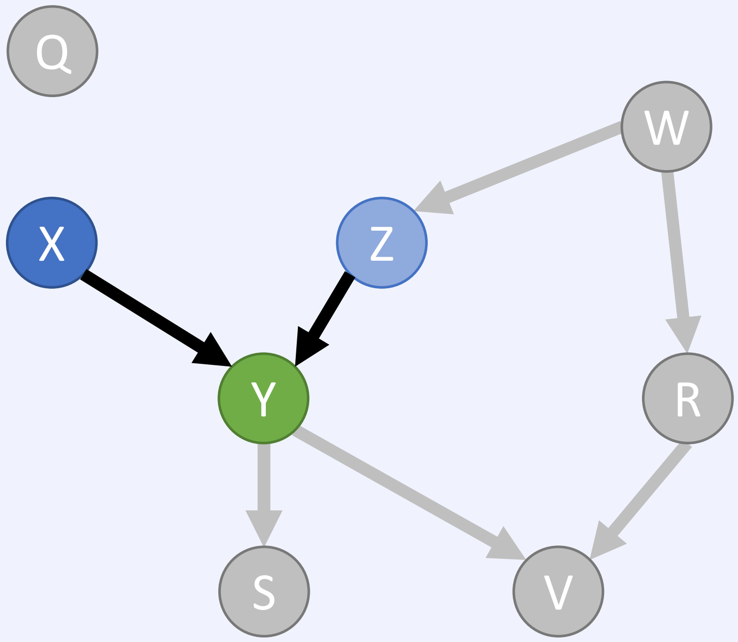
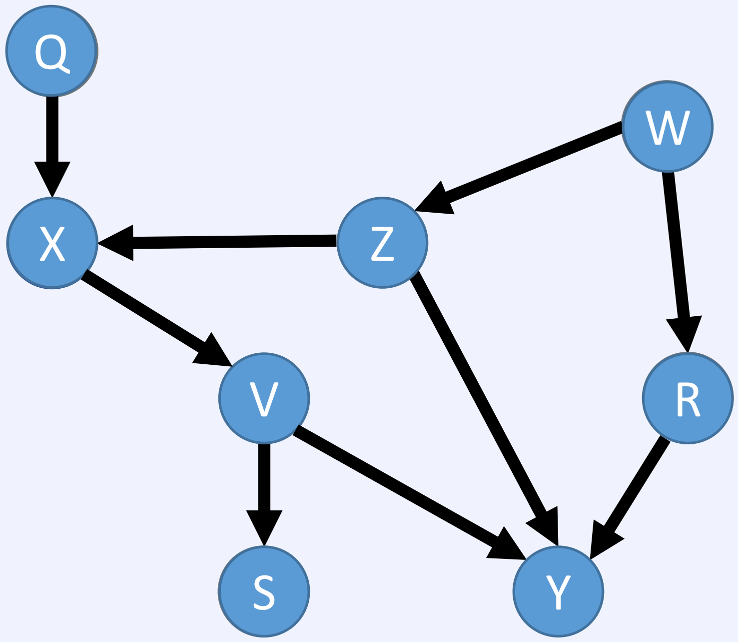


Do-calculus

Rule 3 (Exchange)

$$P(Y \mid do(X), Z) \stackrel{?}{=} \sum_z P(Y|X, Z = z)P(Z = z) \stackrel{?}{=} P(Y \mid X, Z)$$

iff Z satisfies the back-door criterion



Do-calculus

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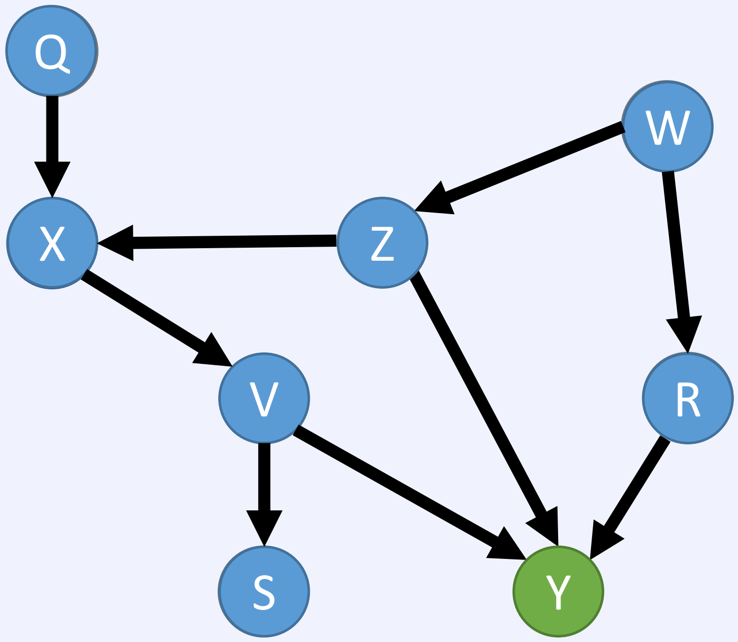

Do-calculus

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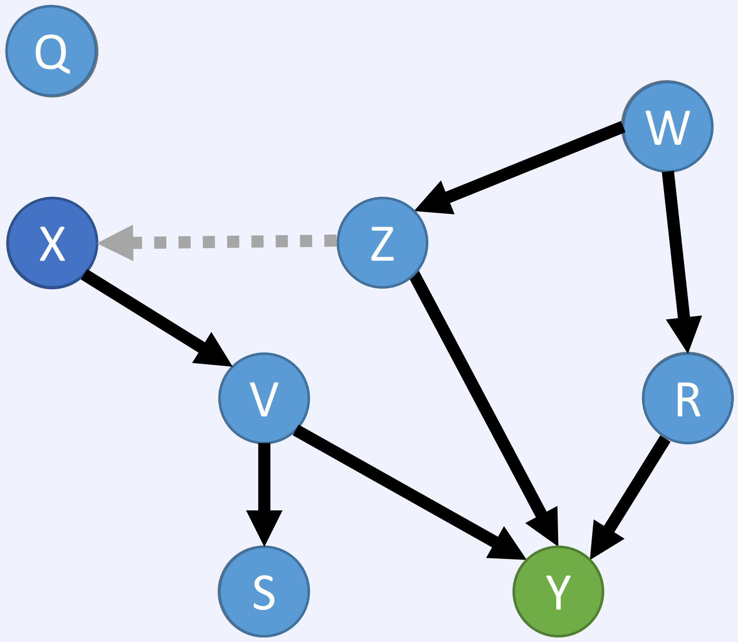
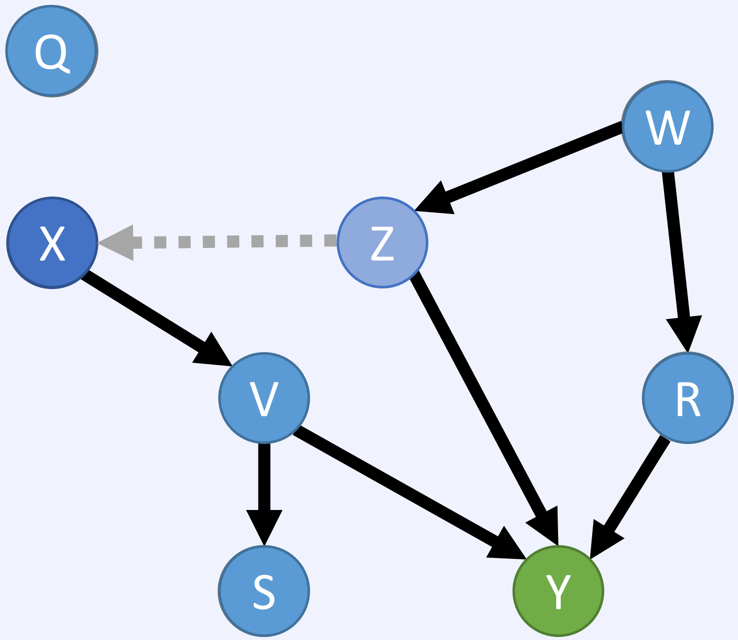


Do-calculus

Rule 3 (Exchange)

$$P(\boldsymbol{Y} \mid do(\boldsymbol{X}), \boldsymbol{Z}) =$$
$$\sum_z P(Y|X, Z=z)P(Z=z)$$
$$= P(\boldsymbol{Y} \mid \boldsymbol{X}, \boldsymbol{Z})$$

iff $\boldsymbol{Z}$ *satisfies the back-door criterion*



Do-calculus

Rule 3 (Exchange)

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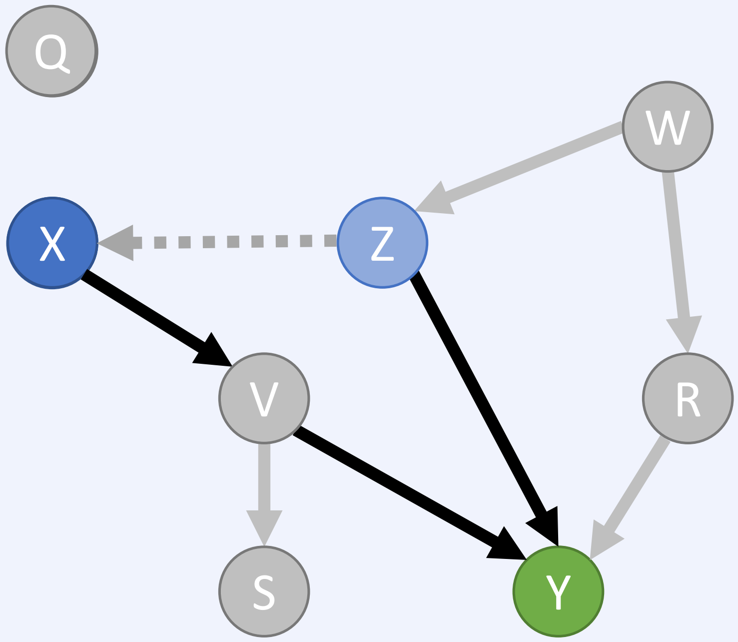
iff ***Z*** *satisfies the back-door criterion*

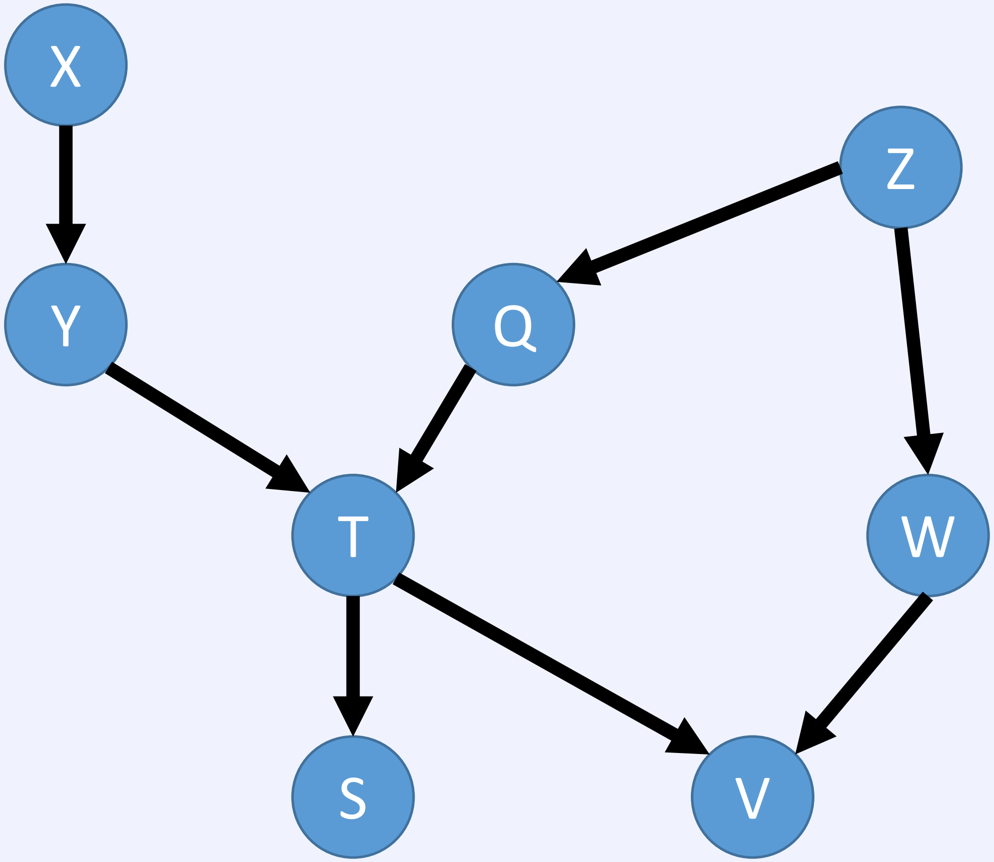


Observational Data

If we cannot measure $P(Y \mid do(X))$ directly in a randomized trial, can we estimate it based on data we observed outside of a controlled experiment?

Sometimes, yes!

Causal Discovery



Causal Markov Condition

The world is a DAG

Causal Markov Condition

Any distribution generated by a Markovian model M can be factorized as

$$P(X_1, X_2, \ldots, X_n) = \prod_i P(X_i \mid pa_i)$$

where $X_1, X_2, \ldots, X_n$ are the **endogenous** variables in M, and pa_i are (values of) the **endogenous** "parents" of X_i in the causal diagram associated with M

(Spirtes, Glymour, Scheines 1982; Pearl 2009)

Causal Markov Condition

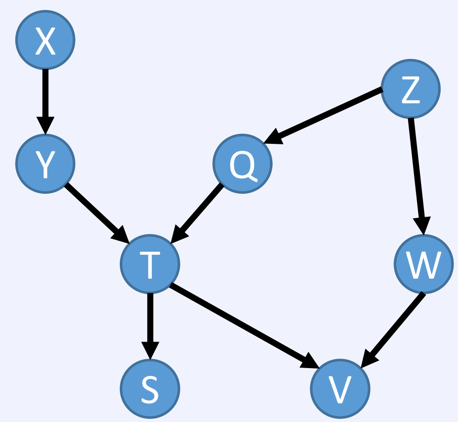
Data Table
(drawn iid ~ P)

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
[1,]	-1.111280231	-2.0140969	0.38660442	0.05846330	0.5205943	2.40911019
[2,]	1.417817353	-0.3615978	-0.19247032	0.66140629	-0.1432120	-0.11834670
[3,]	-0.570567540	-1.6432378	-0.01828731	0.63376433	1.0635629	1.38168120
[4,]	-0.266227679	0.3599688	-0.24996129	-0.71842864	1.3101086	-0.12842456
[5,]	-0.427288260	-0.2755770	0.18415136	-0.38490679	0.7879288	0.17226519
[6,]	1.071823011	-2.2669731	-0.12903350	1.20311317	-0.9858127	-0.80158209
[7,]	0.837535622	1.1515241	1.59051510	0.38925330	0.1345126	-0.67640590
[8,]	-0.390993411	-1.1961786	-0.39611883	-0.03885206	0.6040686	-1.48233781
[9,]	0.362079425	-0.1536282	-0.07836638	0.35483976	-0.7917826	1.03274031
[10,]	0.458338530	-0.0165398	-2.03619702	-0.52135067	-0.4390771	1.20154780
[11,]	0.501343446	0.2389414	0.29264235	2.22713490	-1.0410120	-0.89328211
[12,]	-1.415642964	-0.1702699	2.38358494	-0.81265492	-0.6158825	1.26850073
[13,]	-0.046928402	-0.3022692	1.13007307	0.42498056	-0.1353464	-0.32156204
[14,]	-0.102232153	1.2782075	0.04981187	-0.20025751	-0.3551035	0.96481313
[15,]	1.341928249	0.1602453	-2.00424050	0.73607678	-0.7738258	-1.23018988
[16,]	0.379343237	0.8455179	0.38334824	-1.10415371	1.3109047	0.51595299
[17,]	0.992962014	-0.1822972	-0.62581816	-0.24508326	-1.0401618	-0.40046472
[18,]	0.148449812	1.8961460	-1.80999444	1.15871379	-0.4712393	-0.11946830
[19,]	0.343098853	-0.8892800	-0.99248067	1.25076084	-1.3800977	-0.49034137
[20,]	-0.694376265	1.0474346	-1.18596211	0.58955030	-0.1164544	-0.60899072
[21,]	-0.228495189	-0.2954567	-0.71869073	-0.45818747	-0.1463725	0.10861868
[22,]	0.452582822	1.2291624	1.93100711	1.28179874	0.5874635	-1.11419976
[23,]	0.935567535	-0.2807363	-2.28854793	-0.80001996	0.2223043	0.34980701
[24,]	0.894893812	1.6273959	0.49487719	0.83645987	1.2652432	-0.56321515
[25,]	0.007212357	-1.5697742	1.94262455	-1.32507779	0.5770311	-0.27249976
[26,]	-1.662708965	0.1443786	1.40188962	0.86200639	0.6357342	0.55804169
[27,]	-1.198212728	0.1428584	-0.67462761	0.48976698	0.6945944	-0.94838562

Assumptions

Markov Condition ($P \Leftarrow G$)

Causal Graph G



Faithfulness

Independence in the data,

means independence in the generating graph

Faithfulness

$$\mathit{If}\ X \perp\!\!\!\perp Y\ \mathit{in\ the\ data,}$$

$$X \perp\!\!\!\perp Y\ \mathit{in\ the\ generating\ graph}$$

Faithfulness

Data Table
(drawn iid ~ P)

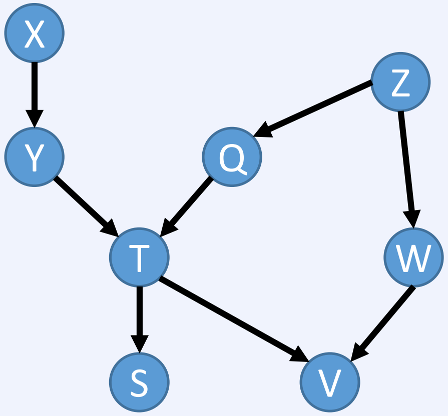
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
[1,]	-1.111280231	-2.0140969	0.38660442	0.05846330	0.5205943	2.40911019
[2,]	1.417817353	-0.3615978	-0.19247032	0.66140629	-0.1432120	-0.11834670
[3,]	-0.570567540	-1.6432378	-0.01828731	0.63376433	1.0635629	1.38168120
[4,]	-0.266227679	0.3599688	-0.24996129	-0.71842864	1.3101086	-0.12842456
[5,]	-0.427288260	-0.2755770	0.18415136	-0.38490679	0.7879288	0.17226519
[6,]	1.071823011	-2.2669731	-0.12903350	1.20311317	-0.9858127	-0.80158209
[7,]	0.837535622	1.1515241	1.59051510	0.38925330	0.1345126	-0.67640590
[8,]	-0.390993411	-1.1961786	-0.39611883	-0.03885206	0.6040686	-1.48233781
[9,]	0.362079425	-0.1536282	-0.07836638	0.35483976	-0.7917826	1.03274031
[10,]	0.458338530	-0.0165398	-2.03619702	-0.52135067	-0.4390771	1.20154780
[11,]	0.501343446	0.2389414	0.29264235	2.22713490	-1.0410120	-0.89328211
[12,]	-1.415642964	-0.1702699	2.38358494	-0.81265492	-0.6158825	1.26850073
[13,]	-0.046928402	-0.3022692	1.13007307	0.42498056	-0.1353464	-0.32156204
[14,]	-0.102232153	1.2782075	0.04981187	-0.20025751	-0.3551035	0.96481313
[15,]	1.341928249	0.1602453	-2.00424050	0.73607678	-0.7738258	-1.23018988
[16,]	0.379343237	0.8455179	0.38334824	-1.10415371	1.3109047	0.51595299
[17,]	0.992962014	-0.1822972	-0.62581816	-0.24508326	-1.0401618	-0.40046472
[18,]	0.148449812	1.8961460	-1.80999444	1.15871379	-0.4712393	-0.11946830
[19,]	0.343098853	-0.8892800	-0.99248067	1.25076084	-1.3800977	-0.49034137
[20,]	-0.694376265	1.0474346	-1.18596211	0.58955030	-0.1164544	-0.60899072
[21,]	-0.228495189	-0.2954567	-0.71869073	-0.45818747	-0.1463725	0.10861868
[22,]	0.452582822	1.2291624	1.93100711	1.28179874	0.5874635	-1.11419976
[23,]	0.935567535	-0.2807363	-2.28854793	-0.80001996	0.2223043	0.34980701
[24,]	0.894893812	1.6273959	0.49487719	0.83645987	1.2652432	-0.56321515
[25,]	0.007212357	-1.5697742	1.94262455	-1.32507779	0.5770311	-0.27249976
[26,]	-1.662708965	0.1443786	1.40188962	0.86200639	0.6357342	0.55804169
[27,]	-1.198212728	0.1428584	-0.67462761	0.48976698	0.6945944	-0.94838562

Assumptions

Markov Condition ($P \Leftarrow G$)

Faithfulness ($P \Rightarrow G$)

Causal Graph G



Causal Sufficiency

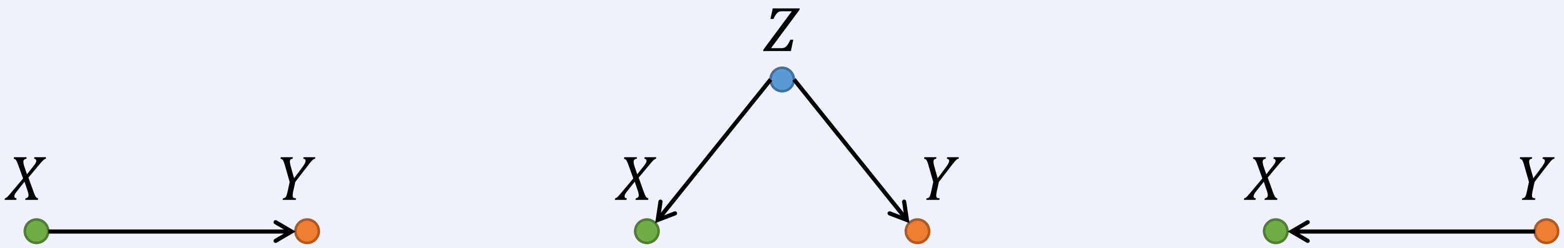
We have measured
all common causes of
all measured variables

There are no hidden confounders

Statistical Causality

Reichenbach's
common cause principle
links causality and probability

if X and Y are statistically dependent then either



When Z **screens** X and Y from each other,
given Z, X and Y become **independent**.

In other words…

For all variables X and Y, if Y does not cause X,
then $P(X \mid Y, pa_X) = P(X \mid pa_X)$

In other words, we can weed out non-causal edges: if the data shows independence, **no edge**, and if a dependence can be explained away, also **no edge**!

All together, these assumptions allow us to **identify** causal DAGs **up to** Markov equivalence

Constraint-Based Causal Discovery

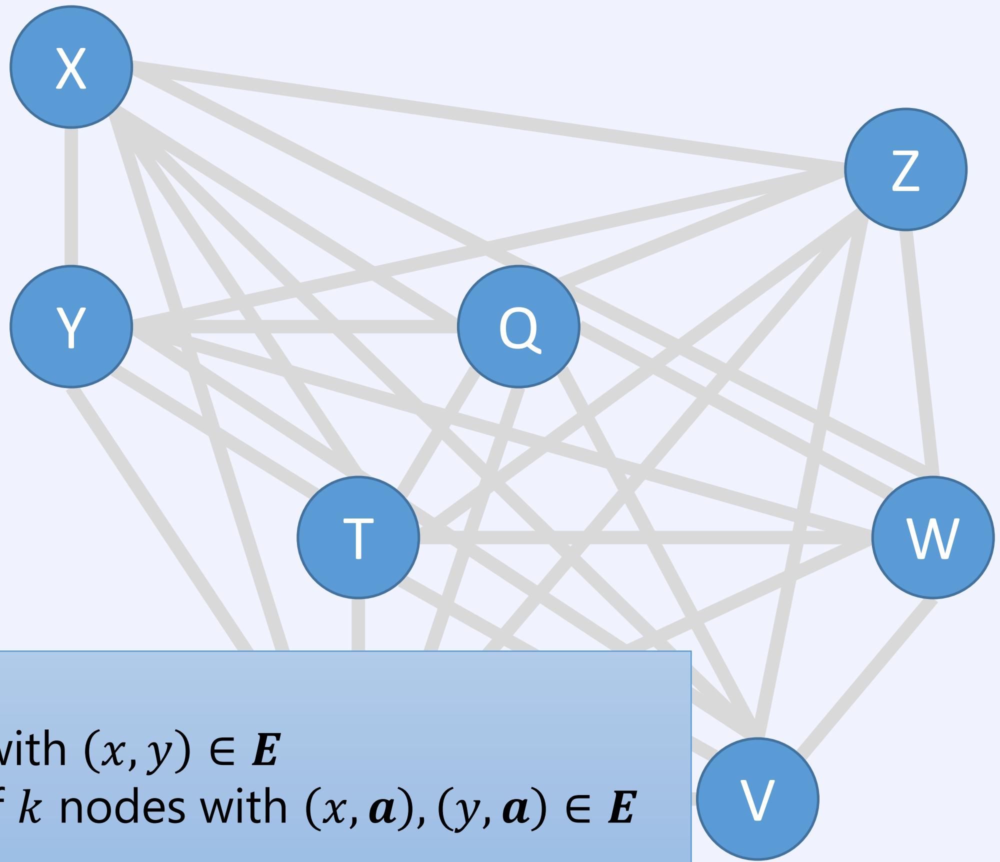
The PC algorithm is one of the most well-known, and most relied upon causal discovery algorithms

- proposed by Peter Spirtes and Clark Glymour

Two main steps

1) use conditional independence tests to determine the undirected causal graph (aka the skeleton)
2) apply constraint-based rules to direct (some) edges

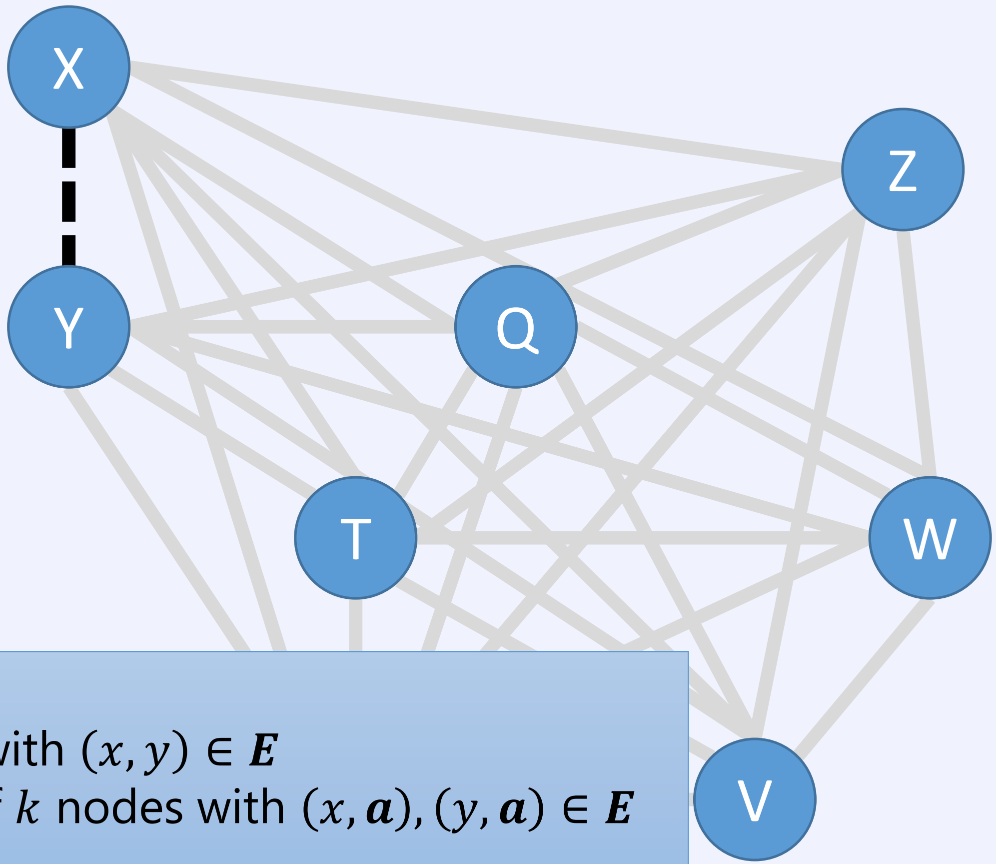
Step 1: Discover the Skeleton



for $k = 0\ to\ n$
  for all $X, Y \in \boldsymbol{V}$ with $(x, y) \in \boldsymbol{E}$
    for all $\boldsymbol{A} \subseteq \boldsymbol{V}$ of k nodes with $(x, \boldsymbol{a}), (y, \boldsymbol{a}) \in \boldsymbol{E}$
      if $X \perp\!\!\!\perp Y \mid \boldsymbol{A}$
        remove (x, y) from $\boldsymbol{E}$

Step 1: Discover the Skeleton

$X \not\perp\!\!\!\perp Y$ → **cannot prune edge**



for $k = 0\ to\ n$
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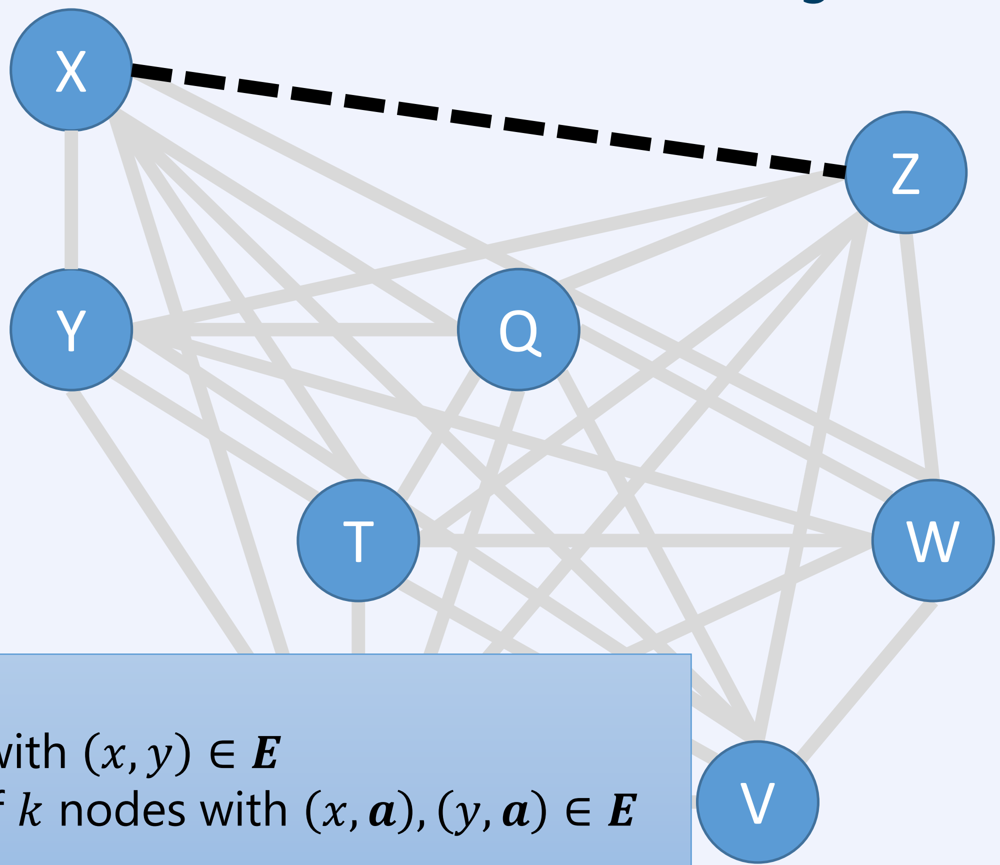
        remove (x, y) from $\boldsymbol{E}$

Step 1: Discover the Skeleton

$X \perp\!\!\!\perp Z \rightarrow$ no causal edge



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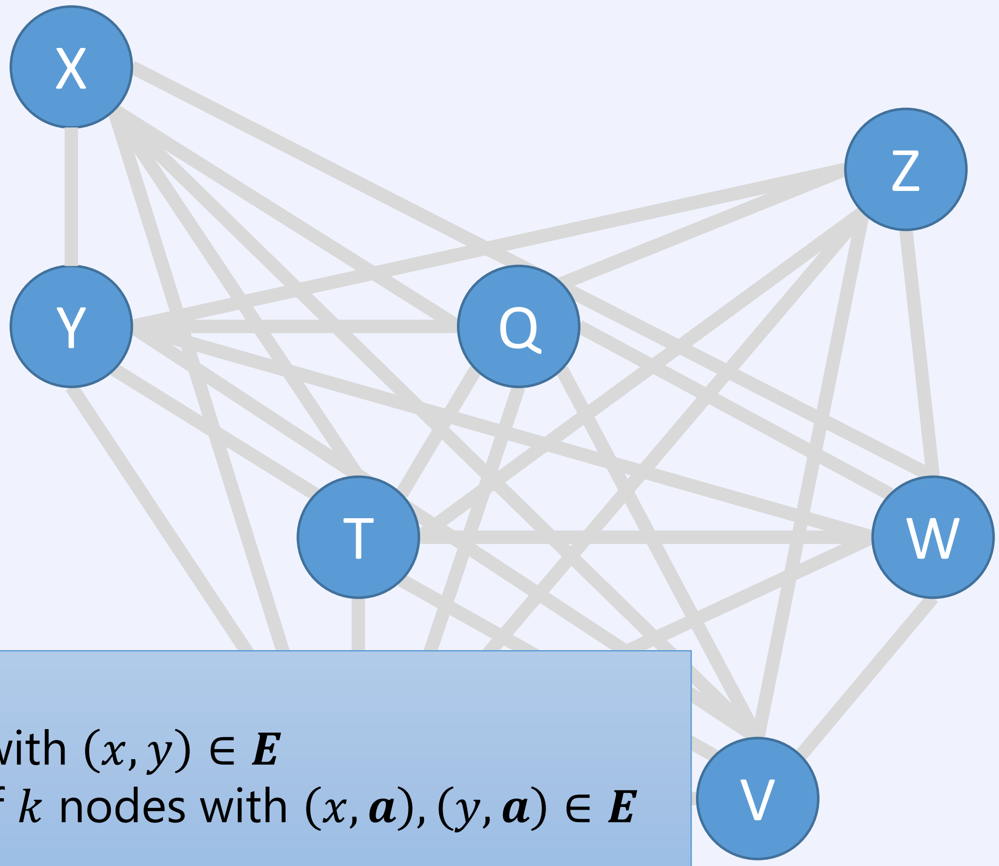

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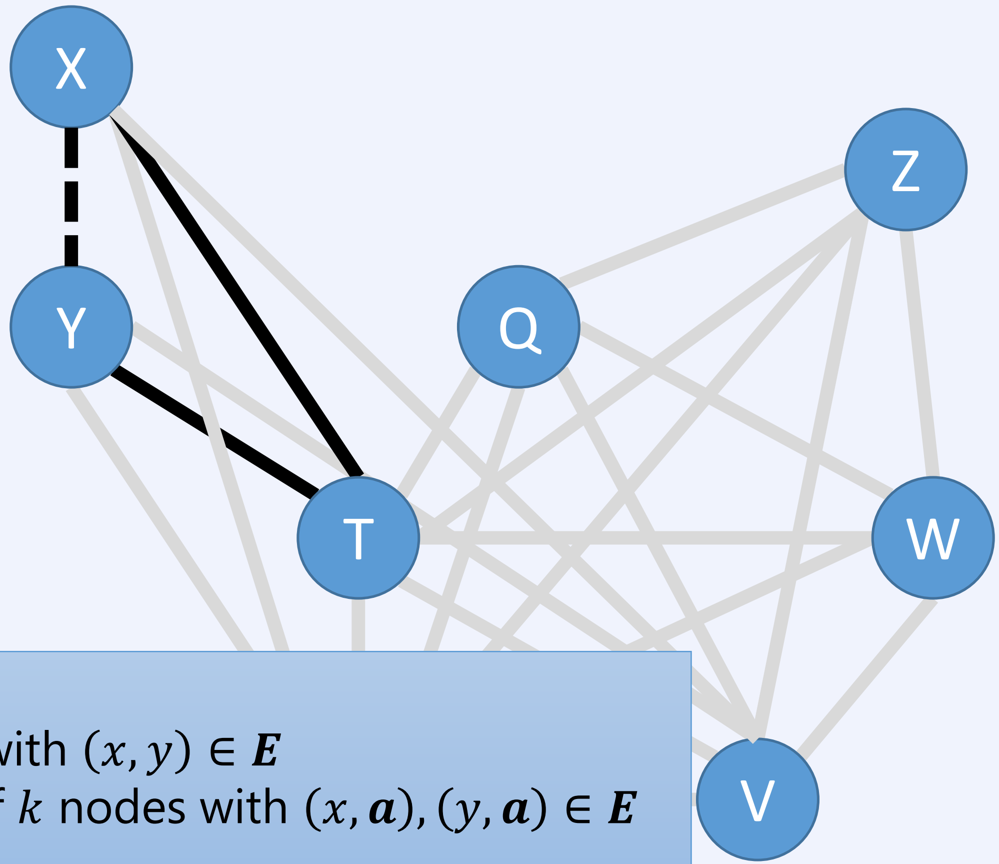
Step 1: Discover the Skeleton

$X \not\perp\!\!\!\perp Y \mid T \rightarrow$ **cannot prune edge**



for $k = 0\ to\ n$
  for all $X, Y \in \boldsymbol{V}$ with $(x, y) \in \boldsymbol{E}$
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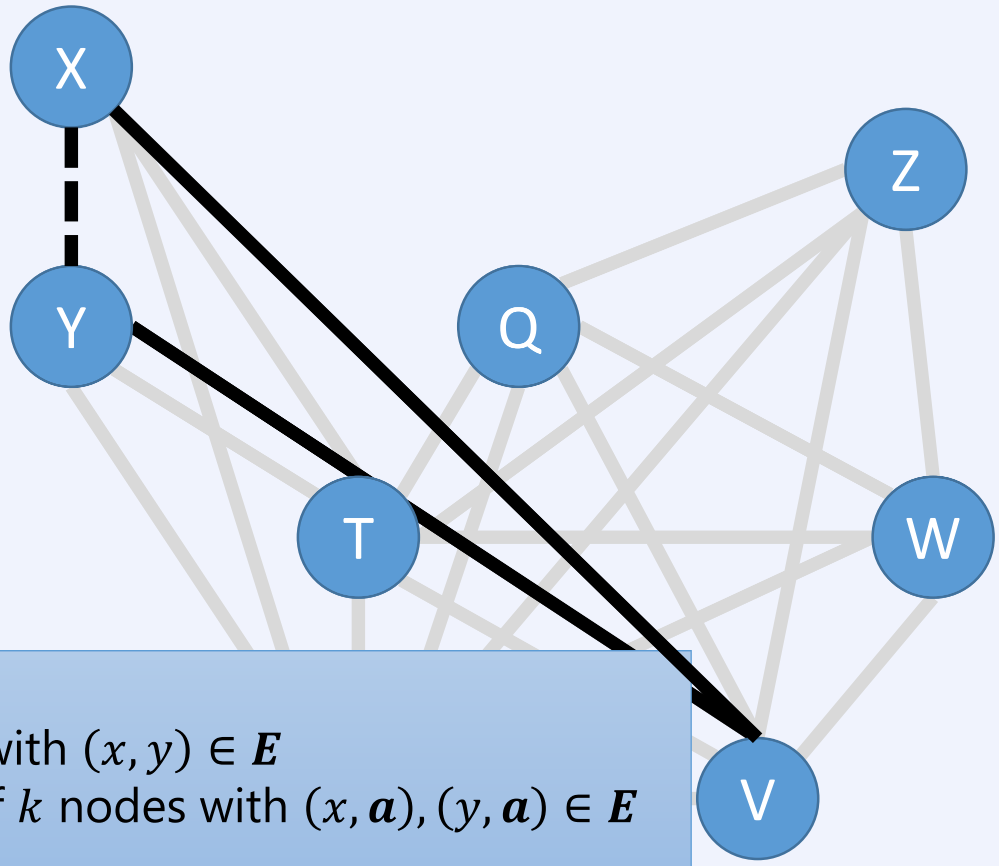
Step 1: Discover the Skeleton

$X \not\perp\!\!\!\perp Y \mid V$ → **cannot prune edge**



for $k = 0\ to\ n$
  for all $X, Y \in \boldsymbol{V}$ with $(x, y) \in \boldsymbol{E}$
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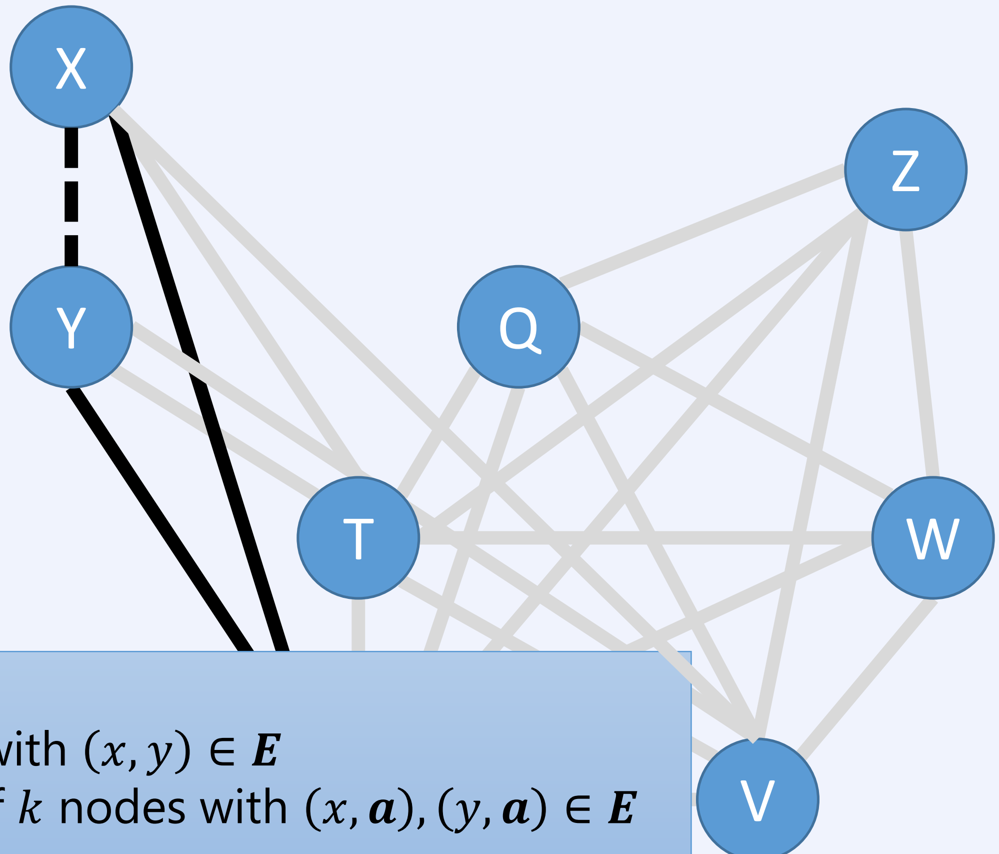
Step 1: Discover the Skeleton

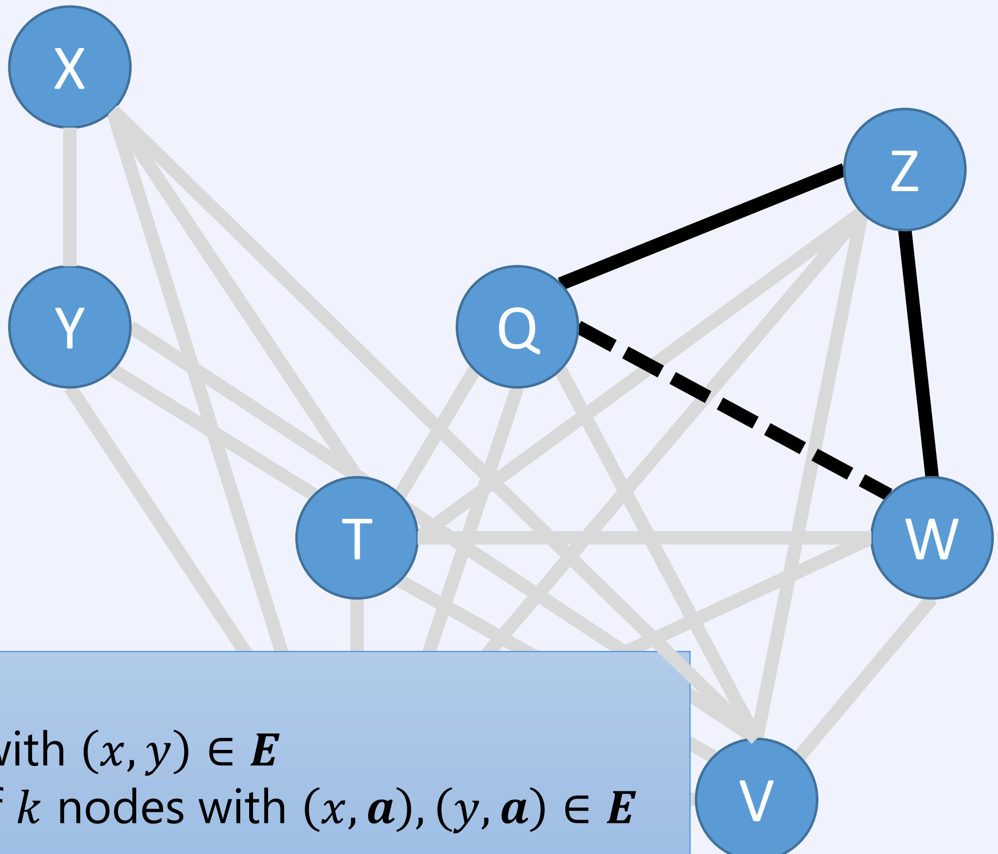
$X \not\perp\!\!\!\perp Y \mid S$ → **cannot prune edge**



for $k = 0\ to\ n$
  for all $X, Y \in \boldsymbol{V}$ with $(x, y) \in \boldsymbol{E}$
    for all $\boldsymbol{A} \subseteq \boldsymbol{V}$ of k nodes with $(x, \boldsymbol{a}), (y, \boldsymbol{a}) \in \boldsymbol{E}$
      if $X \perp\!\!\!\perp Y \mid \boldsymbol{A}$
        remove (x, y) from $\boldsymbol{E}$

Step 1: Discover the Skeleton

$Q \perp\!\!\!\perp W \mid Z$ → no causal edge



for $k = 0\ to\ n$
 for all $X, Y \in \boldsymbol{V}$ with $(x, y) \in \boldsymbol{E}$
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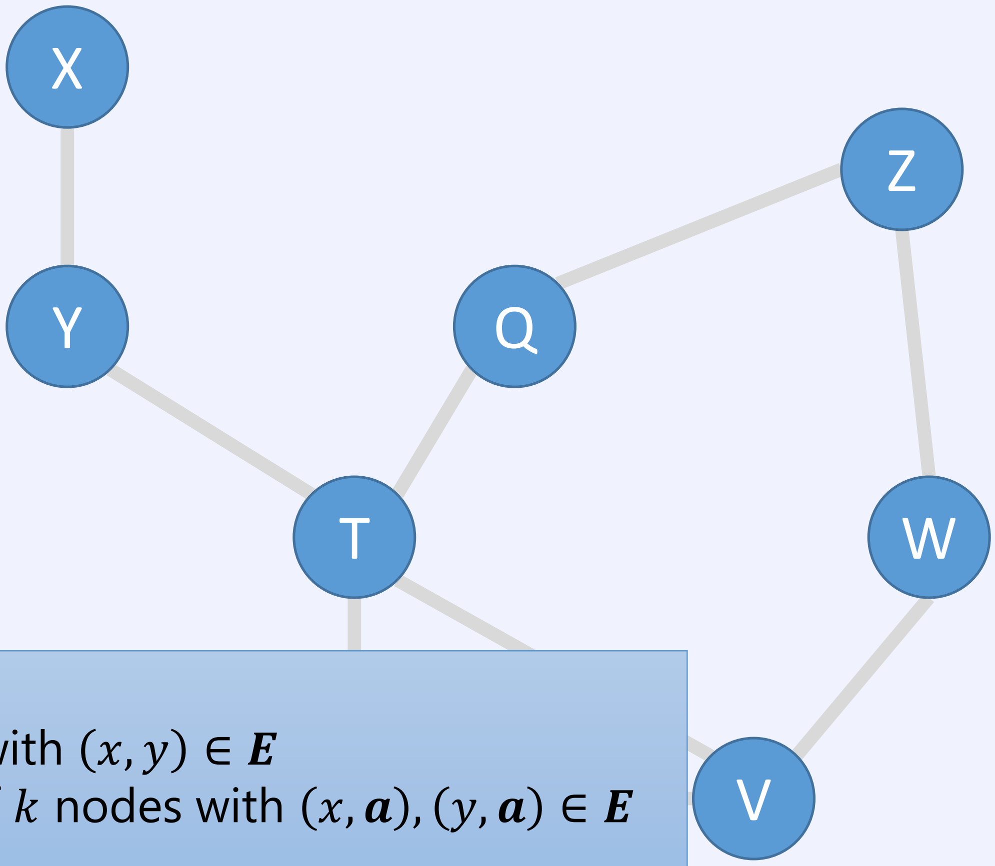
Step 1: Discover the Skeleton



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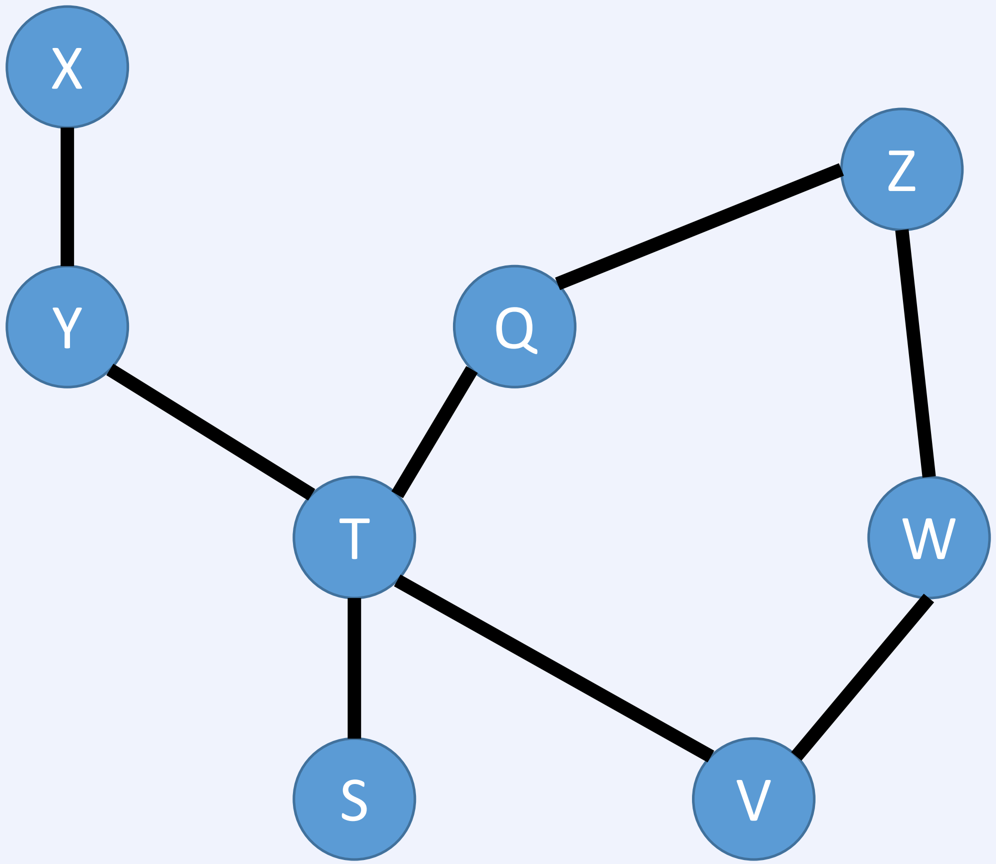
Step 1: Discover the Skeleton

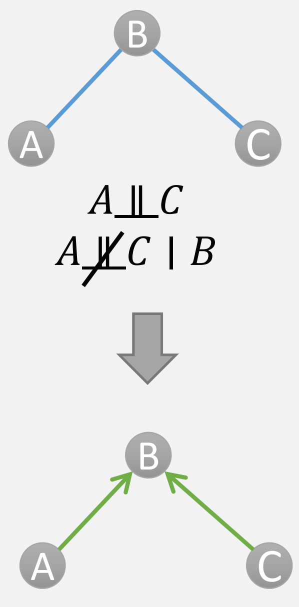
We now have the **causal skeleton**

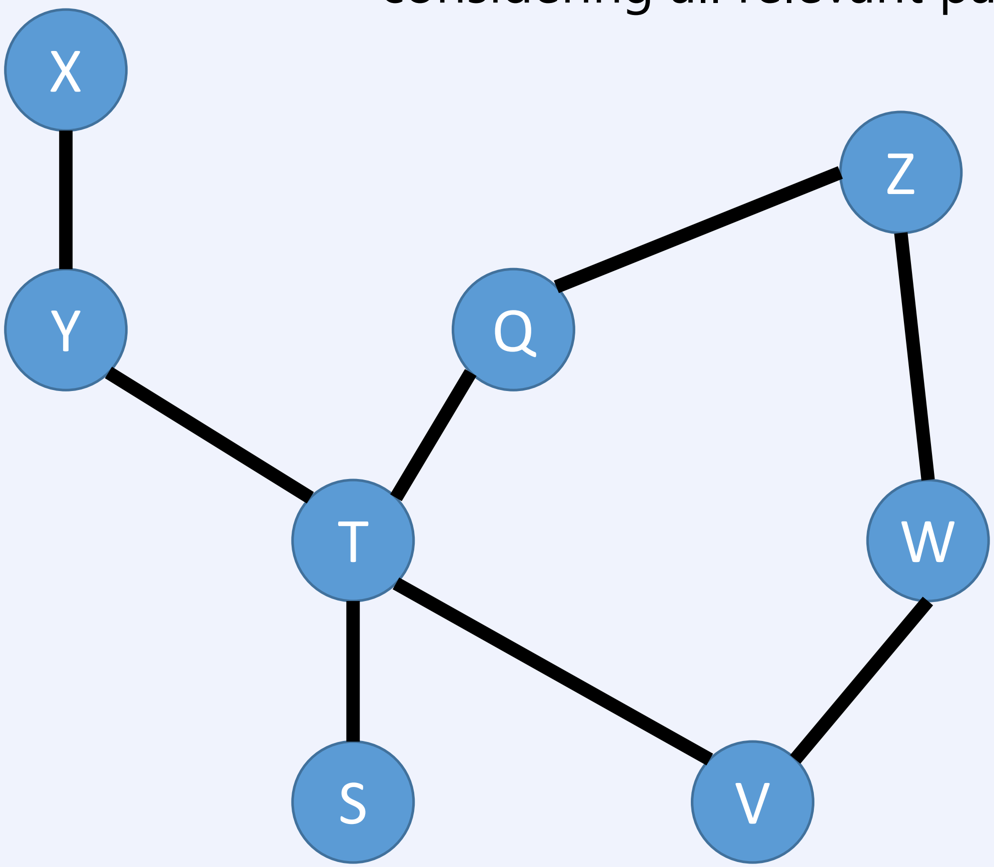


Step 2: Orientation

We now identify all **colliders** $A \rightarrow B \leftarrow C$ considering all relevant pairs **once**

Rule 1 (collider)





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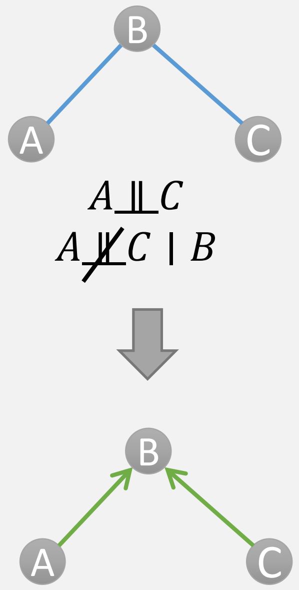
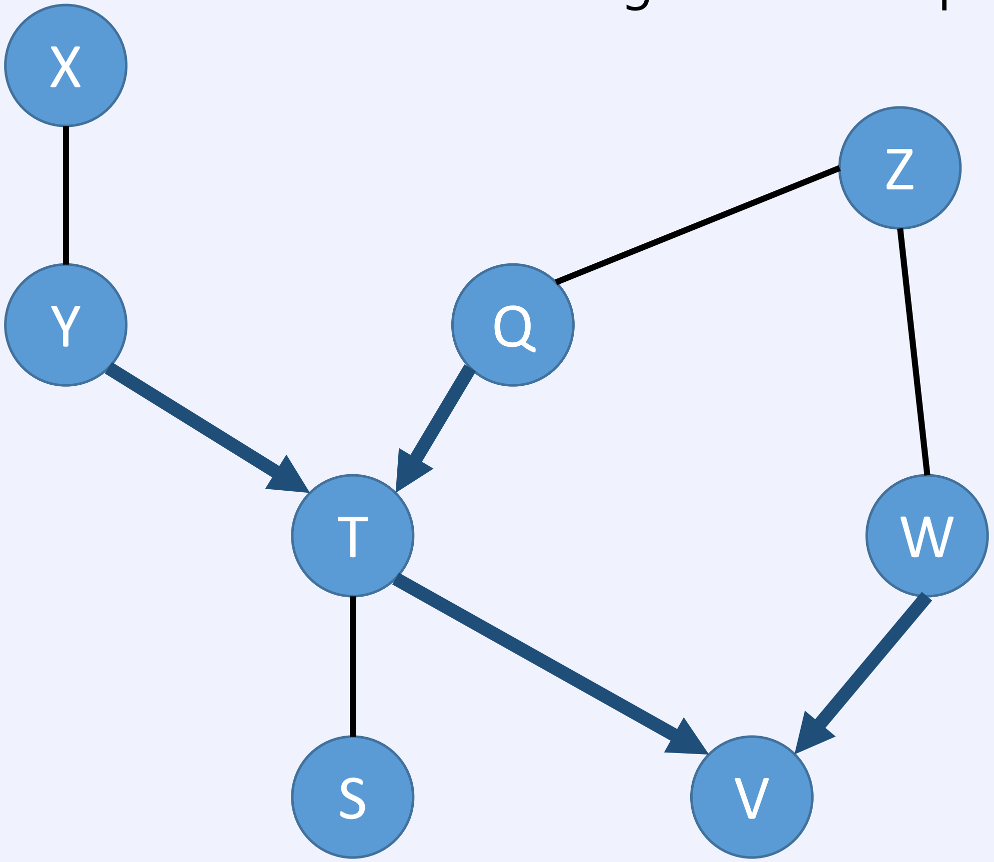
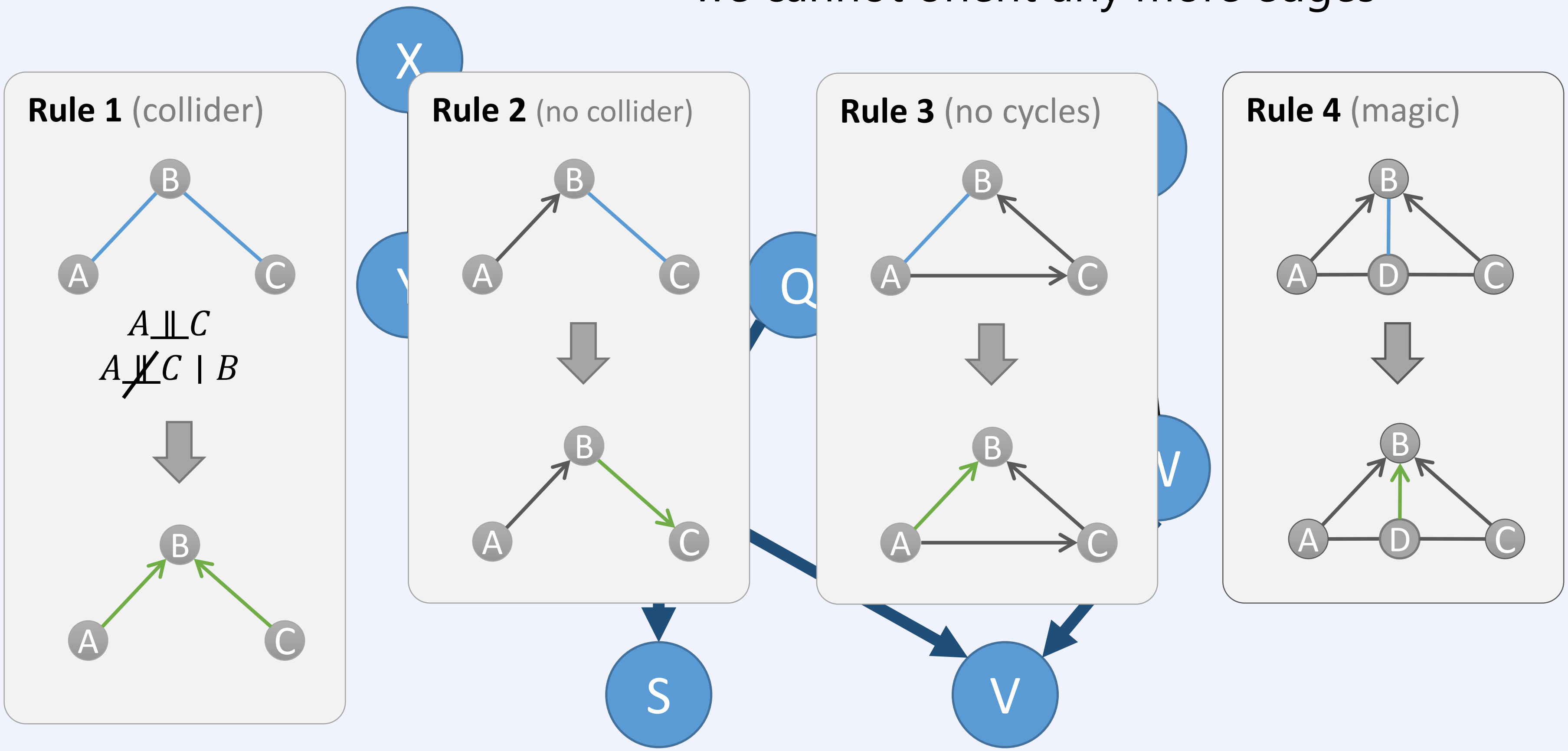


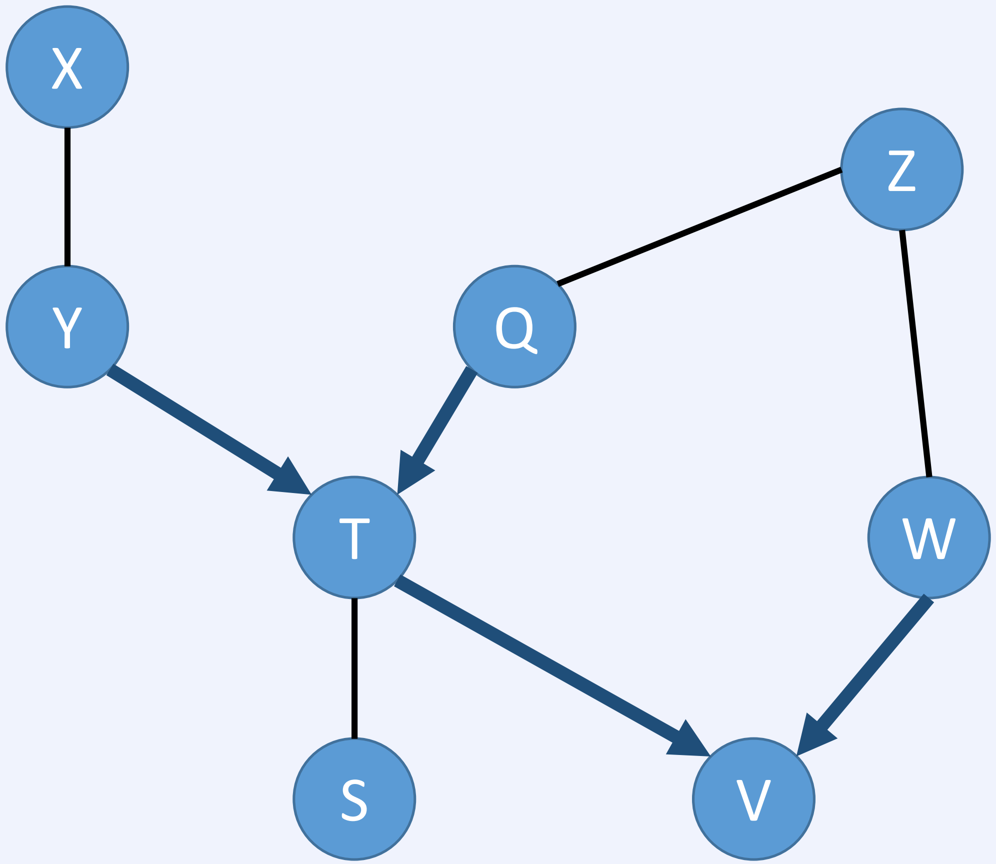


Step 2: Orientation

We then iteratively apply Rules 2—4 until
we cannot orient any more edges



Done! We discovered causality!



Summary

We learned about the **ladder of causation**

- causal conclusions are **impossible** without causal assumptions
- *no causation in, no causation out*

We learned about **do-calculus**

- allows us to determine if under our current assumptions, observational data suffices to estimate causal effects

We learned about **causal discovery**

- how and when we can discover a causal graph from data

Welcome to the causal revolution!

Thank you!

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Welcome to the causal revolution!