# Verstärkung

Freitag, 5. Februar 2021 17:33

$$\forall xs \in L(X): \forall n \in \mathbb{N}: |xs| + n = \text{foldl}(\underbrace{(\lambda (x,s) \in X \times \mathbb{N}.\ s+1)}_{f}, \underbrace{\overset{n}{\cancel{0}}}, xs)$$

Beweis durch strukturelle Induktion über $xs \in L(X)$.

1. Fall: $xs = nil$:

Sei $n \in \mathbb{N}$ beliebig

$$\begin{aligned}
\text{foldl}(f, \overset{n}{\cancel{0}}, xs) &= \text{foldl}((f, \overset{n}{\cancel{0}}, nil) && \mid xs = nil \\
&= n && \mid \text{Def foldl} \\
&= 0 + n && \mid \text{Arithmetik} \\
&= |nil| + n && \mid \text{Def } |_| \\
&= |xs| + n && \mid xs = nil
\end{aligned}$$

2. Fall $xs = x :: xr$:

Induktionsannahme: $\forall n \in \mathbb{N}: \text{foldl}((f, \overset{n}{\cancel{0}}, xr) = |xr| + n$

Sei $n \in \mathbb{N}$ beliebig.

$$\begin{aligned}
\text{foldl}((f, \overset{n}{\cancel{0}}, xs) &= \text{foldl}((f, \overset{n}{\cancel{0}}, x :: xr) && \mid xs = x :: xr \\
&= \text{foldl}((f, \underbrace{f(x, \overset{n}{\cancel{0}})}_{\text{Ein mögl. } n}, xr) && \mid \text{Def foldl} \\
&= |xr| + f(x, n) && \mid \text{Induktion für } xr \\
&= |xr| + (n + 1) && \mid \text{Def } f \\
&= \underbrace{|xr| + 1} + n && \mid \text{Mathe} \\
&= |x :: xr| + n && \mid \text{Def } |_| \\
&= |xs| + n && \mid xs = x :: xr
\end{aligned}$$

Def $|\_|$

$|nil| = 0$

$|x :: xr| = |xr| + 1$

$\square$

Somit gilt mit $n = 0$ die geforderte Aussage.