Z4-Induktion

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Listen nil

Bäume

$\subset$

(=) Konstituente

$1 :: 2 :: 3 :: nil$

$2 :: 3 :: nil$

$\cancel{1 :: 2 :: nil}$

$[X]$

$$foldr(\underbrace{\lambda(x, s) \in X \times L(X).\ x :: s}_{f},\ nil,\ xs) = xs$$

Beweis durch Induktion über $xs \in L(X)$.

1. Fall $xs = nil$:

$$\begin{aligned} foldr(f, nil, xs) &= foldr(f, nil, nil) && \mid xs = nil \\ &= nil && \mid \text{Def. foldr} \\ &= xs && \mid xs = nil \end{aligned}$$

2. Fall $xs = x :: xr$:

Induktionsannahme: $foldr(f, nil, xr) = xr$

$$\begin{aligned} foldr(f, nil, xs) &= foldr(f, nil, x :: xr) && \mid xs = x :: xr \\ &= f(x, foldr(f, nil, xr)) && \mid \text{Def foldr} \\ &= f(x, xr) && \mid \text{Induktion für } xr \\ &= x :: xr && \mid \text{Def } f \\ &= xs && \mid xs = x :: xr \end{aligned}$$

$f(x,\ foldr(f, nil, xr))$

$\square$