Software Engineering

WS 2022/23, Sheet 06

Prof. Dr. Sven Apel
Yannick Lehmen, Maurice Vincon

Handout: 05.12.2022

Task 1

a) Explain the concept of *parametric polymorphism*.

b) **Discussion:** Prepare arguments for and against implementing variability by using parametric polymorphism. You can present your arguments during the discussion in the tutorial sessions.

Solution

a) *Parametric Polymorphism* is the concept of writing functions or data types of a program generically. This means that the type of the values of data structures or functions can be changed at compile time.

Task 2

Prove the cases 5, 7, 10, and 12 from the *Coarsening and Refinement* slide in the lecture slides. Use the notation that is used to prove the cases 1–4 in the lecture.

Solution

Proof. We have to proof that for any $e, e' \in \mathit{VExpr}$ it holds that $e \equiv e'$ if and only if, for any given configuration $\psi \in \mathbb{B}_F$ with e and e' closed under ψ, $[\![e]\!]_\psi = [\![e']\!]_\psi$ holds. The proof is by structural induction over the terms $e, e' \in \mathit{VExpr}$.

Induction hypothesis: If e and e' both contain no choice expressions, then $e \equiv e'$ if and only if $e = e'$ and therefore also $\forall\, \psi \in \mathbb{B}_F.\ [\![e]\!]_\psi = [\![e']\!]_\psi$, as none of the transformation rules are applicable.

Induction step: $e_1, e_2, e_3, e_4 \in \mathit{VExpr}$, $\mathrm{closed}(e_1, \psi)$, $\mathrm{closed}(e_2, \psi)$, $\mathrm{closed}(e_3, \psi)$, $\mathrm{closed}(e_4, \psi)$

Case 5: $e = \mathsf{choice}\ \langle \phi, e_1, e_2 \rangle < e_3 \qquad e' = \mathsf{choice}\ \langle \phi, e_1 < e_3, e_2 < e_3 \rangle$

$$\begin{aligned} [\![e]\!]_\psi &= [\![\, \mathsf{choice}\ \langle \phi, e_1, e_2 \rangle < e_3 \,]\!]_\psi = [\![\mathsf{choice}\ \langle \phi, e_1, e_2 \rangle]\!]_\psi < [\![e_3]\!]_\psi \\ [\![e']\!]_\psi &= [\![\, \mathsf{choice}\ \langle \phi, e_1 < e_3, e_2 < e_3 \rangle \,]\!]_\psi \end{aligned}$$

if $\psi \Rightarrow \Phi$:

$$[\![\mathsf{choice}\ \langle \phi, e_1, e_2 \rangle]\!]_\psi < [\![e_3]\!]_\psi = [\![e_1]\!]_\psi < [\![e_3]\!]_\psi = [\![e_1 < e_3]\!]_\psi = [\![\, \mathsf{choice}\ \langle \phi, e_1 < e_3, e_2 < e_3 \rangle \,]\!]_\psi$$

if $\psi \Rightarrow \neg\Phi$:

$$[\![\mathsf{choice}\ \langle \phi, e_1, e_2 \rangle]\!]_\psi < [\![e_3]\!]_\psi = [\![e_2]\!]_\psi < [\![e_3]\!]_\psi = [\![e_2 < e_3]\!]_\psi = [\![\, \mathsf{choice}\ \langle \phi, e_1 < e_3, e_2 < e_3 \rangle \,]\!]_\psi$$

$\Rightarrow [\![e]\!]_\psi = [\![e']\!]_\psi \overset{\text{(I.H.)}}{\Longrightarrow} e \equiv e'$

Case 7: $e = \mathsf{if}\ \mathsf{choice}\ \langle \phi, e_1, e_2 \rangle\ \mathsf{then}\ e_3\ \mathsf{else}\ e_4 \qquad e' = \mathsf{choice}\ \langle \phi,\ \mathsf{if}\ e_1\ \mathsf{then}\ e_3\ \mathsf{else}\ e_4, \mathsf{if}\ e_2\ \mathsf{then}\ e_3\ \mathsf{else}\ e_4 \rangle$

$$\begin{aligned} [\![e]\!]_\psi &= [\![\mathsf{if}\ \mathsf{choice}\ \langle \phi, e_1, e_2 \rangle\ \mathsf{then}\ e_3\ \mathsf{else}\ e_4]\!]_\psi = \mathsf{if}\ [\![\mathsf{choice}\ \langle \phi, e_1, e_2 \rangle]\!]_\psi\ \mathsf{then}\ [\![e_3]\!]_\psi\ \mathsf{else}\ [\![e_4]\!]_\psi \\ [\![e']\!]_\psi &= [\![\mathsf{choice}\ \langle \phi, \mathsf{if}\ e_1\ \mathsf{then}\ e_3\ \mathsf{else}\ e_4, \mathsf{if}\ e_2\ \mathsf{then}\ e_3\ \mathsf{else}\ e_4 \rangle]\!]_\psi \end{aligned}$$

if $\psi \Rightarrow \Phi$:

$$\begin{aligned} \mathsf{if}\ [\![\mathsf{choice}\ \langle \phi, e_1, e_2 \rangle]\!]_\psi\ \mathsf{then}\ [\![e_3]\!]_\psi\ \mathsf{else}\ [\![e_4]\!]_\psi &= \mathsf{if}\ [\![e_1]\!]_\psi\ \mathsf{then}\ [\![e_3]\!]_\psi\ \mathsf{else}\ [\![e_4]\!]_\psi \\ &= [\![\mathsf{if}\ e_1\ \mathsf{then}\ e_3\ \mathsf{else}\ e_4]\!]_\psi \\ &= [\![\mathsf{choice}\ \langle \phi, \mathsf{if}\ e_1\ \mathsf{then}\ e_3\ \mathsf{else}\ e_4, \mathsf{if}\ e_2\ \mathsf{then}\ e_3\ \mathsf{else}\ e_4 \rangle]\!]_\psi \end{aligned}$$

if $\psi \Rightarrow \neg\Phi$:

$$\begin{aligned} \mathsf{if}\ [\![\mathsf{choice}\ \langle \phi, e_1, e_2 \rangle]\!]_\psi\ \mathsf{then}\ [\![e_3]\!]_\psi\ \mathsf{else}\ [\![e_4]\!]_\psi &= \mathsf{if}\ [\![e_2]\!]_\psi\ \mathsf{then}\ [\![e_3]\!]_\psi\ \mathsf{else}\ [\![e_4]\!]_\psi \\ &= [\![\mathsf{if}\ e_2\ \mathsf{then}\ e_3\ \mathsf{else}\ e_4]\!]_\psi \\ &= [\![\mathsf{choice}\ \langle \phi, \mathsf{if}\ e_1\ \mathsf{then}\ e_3\ \mathsf{else}\ e_4, \mathsf{if}\ e_2\ \mathsf{then}\ e_3\ \mathsf{else}\ e_4 \rangle]\!]_\psi \end{aligned}$$

$\Rightarrow [\![e]\!]_\psi = [\![e']\!]_\psi \overset{\text{(I.H.)}}{\Longrightarrow} e \equiv e'$

Case 10: $e = (\mathsf{let}\ x = \mathsf{choice}\ \langle\phi, e_1, e_2\rangle \mathsf{in}\ e_3) \quad e' = \mathsf{choice}\ \langle\phi, \mathsf{let}\ x = e_1\ \mathsf{in}\ e_3, \mathsf{let}\ x = e_2\ \mathsf{in}\ e_3\rangle$

$$\begin{aligned}
[\![e]\!]_\psi &= [\![\mathsf{let}\ x = \mathsf{choice}\ \langle\phi, e_1, e_2\rangle\ \mathsf{in}\ e_3]\!]_\psi = (\mathsf{let}\ x = [\![\mathsf{choice}\ \langle\phi, e_1, e_2\rangle]\!]_\psi\ \mathsf{in}\ [\![e_3]\!]_\psi)\\
[\![e']\!]_\psi &= [\![\mathsf{choice}\ \langle\phi, \mathsf{let}\ x = e_1\ \mathsf{in}\ e_3, \mathsf{let}\ x = e_2\ \mathsf{in}\ e_3\rangle]\!]_\psi
\end{aligned}$$

if $\psi \Rightarrow \Phi$:

$$\begin{aligned}
(\mathsf{let}\ x = [\![\mathsf{choice}\ \langle\phi, e_1, e_2\rangle]\!]_\psi\ \mathsf{in}\ [\![e_3]\!]_\psi) &= (\mathsf{let}\ x = [\![e_1]\!]_\psi\ \mathsf{in}\ [\![e_3]\!]_\psi)\\
&= [\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_3]\!]_\psi\\
&= [\![\mathsf{choice}\ \langle\phi, \mathsf{let}\ x = e_1\ \mathsf{in}\ e_3, \mathsf{let}\ x = e_2\ \mathsf{in}\ e_3\rangle]\!]_\psi
\end{aligned}$$

if $\psi \Rightarrow \neg\Phi$:

$$\begin{aligned}
(\mathsf{let}\ x = [\![\mathsf{choice}\ \langle\phi, e_1, e_2\rangle]\!]_\psi\ \mathsf{in}\ [\![e_3]\!]_\psi) &= (\mathsf{let}\ x = [\![e_2]\!]_\psi\ \mathsf{in}\ [\![e_3]\!]_\psi)\\
&= [\![\mathsf{let}\ x = e_2\ \mathsf{in}\ e_3]\!]_\psi\\
&= [\![\mathsf{choice}\ \langle\phi, \mathsf{let}\ x = e_1\ \mathsf{in}\ e_3, \mathsf{let}\ x = e_2\ \mathsf{in}\ e_3\rangle]\!]_\psi
\end{aligned}$$

$\Rightarrow [\![e]\!]_\psi = [\![e']\!]_\psi \overset{\text{(I.H.)}}{\Longrightarrow} e \equiv e'$

Case 12: $e = \mathsf{choice}\ \langle\phi, \mathsf{choice}\ \langle\phi, e_1, e_2\rangle, e_2\rangle \quad e' = \mathsf{choice}\ \langle\phi, e_1, e_2\rangle$

if $\psi \Rightarrow \Phi$:

$$[\![e]\!]_\psi = [\![\mathsf{choice}\ \langle\phi, \mathsf{choice}\ \langle\phi, e_1, e_2\rangle, e_2\rangle]\!]_\psi = [\![\mathsf{choice}\ \langle\phi, e_1, e_2\rangle]\!]_\psi = [\![e']\!]_\psi$$

if $\psi \Rightarrow \neg\Phi$:

$$[\![e]\!]_\psi = [\![\mathsf{choice}\ \langle\phi, \mathsf{choice}\ \langle\phi, e_1, e_2\rangle, e_2\rangle]\!]_\psi = [\![e_2]\!]_\psi = [\![\mathsf{choice}\ \langle\phi, e_1, e_2\rangle]\!]_\psi = [\![e']\!]_\psi$$

$\Rightarrow [\![e]\!]_\psi = [\![e']\!]_\psi \overset{\text{(I.H.)}}{\Longrightarrow} e \equiv e'$

$\square$