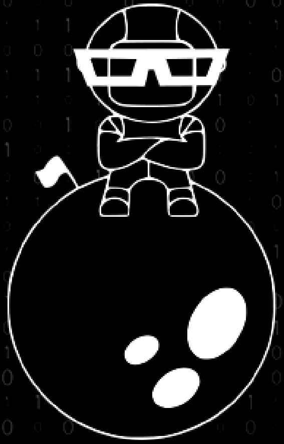


Physics and Orbits

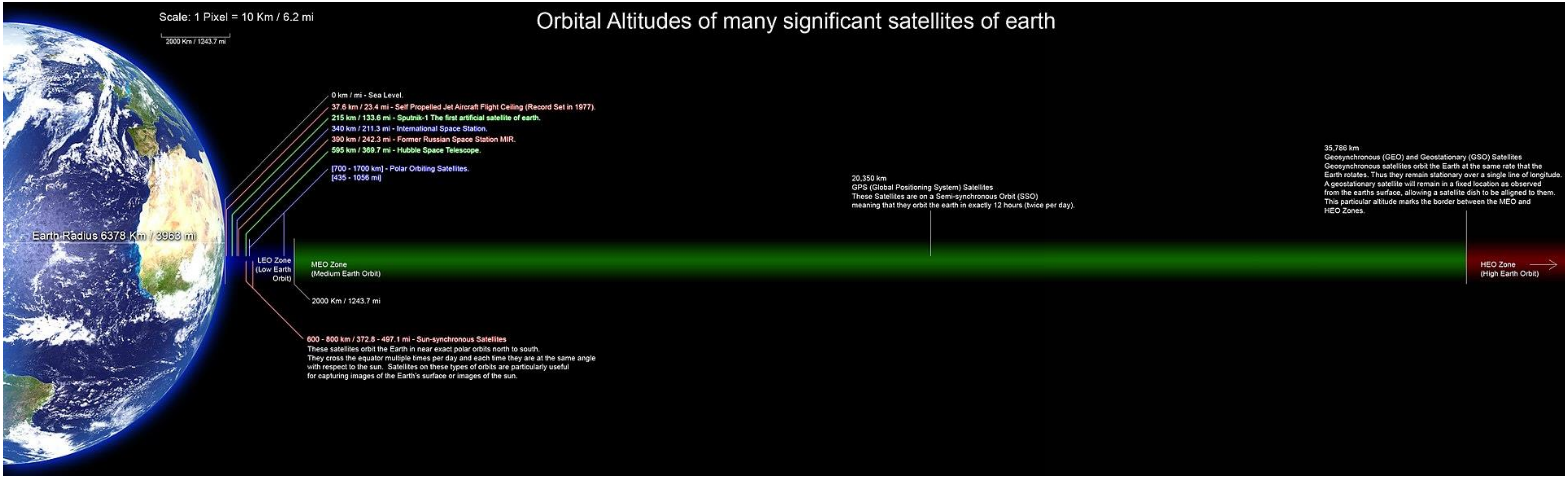
SPACE
INFORMATICS

SIC Saarland Informatics Campus

Trajectory and Orbit

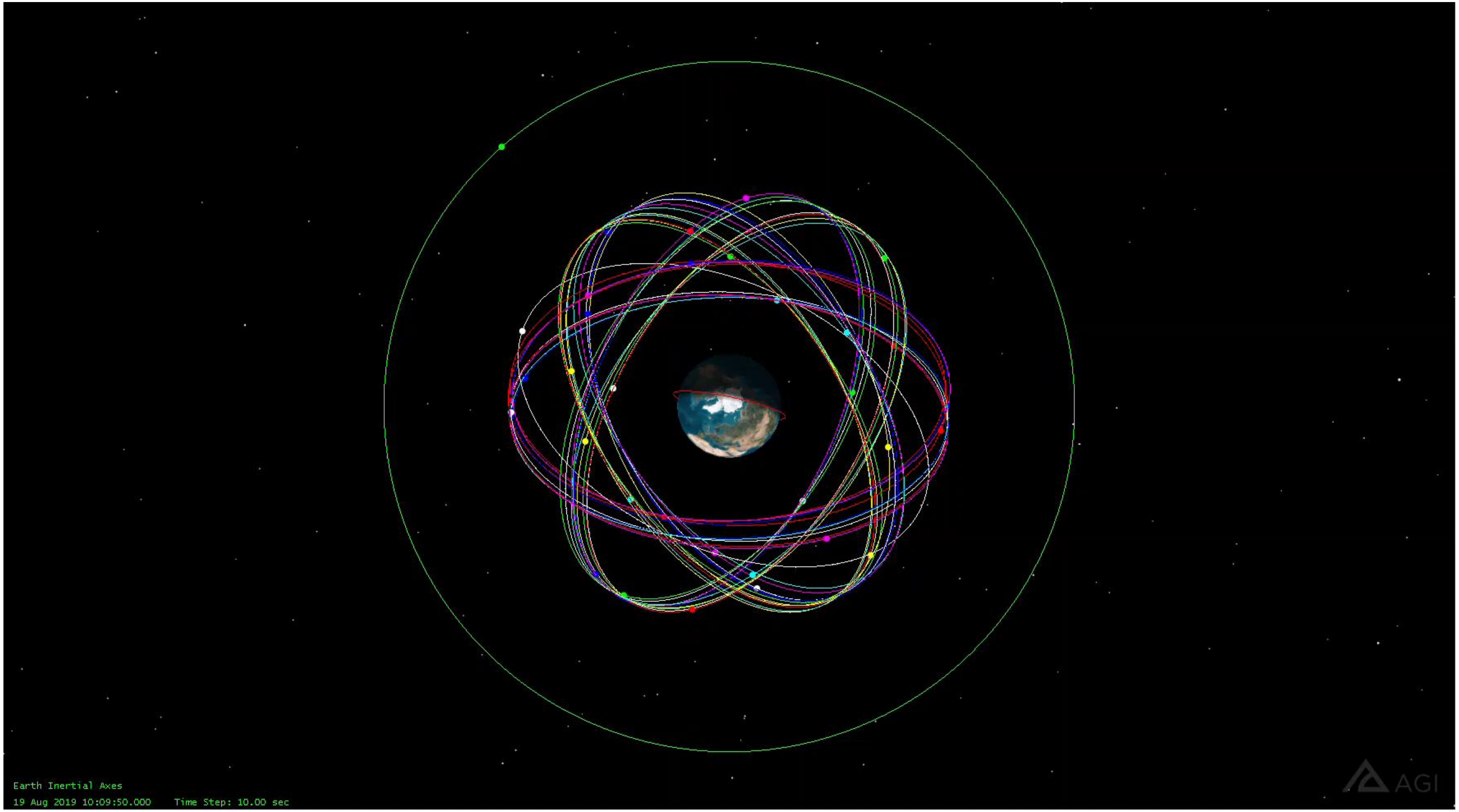
LEO – MEO - GEO

- Types of orbits
  - Low Earth Orbit (**LEO**) → (500;2000) km height
  - Medium Earth Orbit (**MEO**) → (2000;35786) km height
  - Geostationary Orbit (**GEO**) → 35786 km height



Trajectory and Orbit

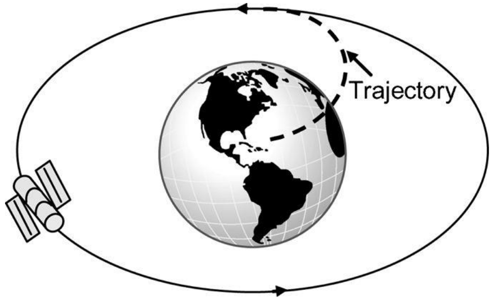
LEO – MEO - GEO



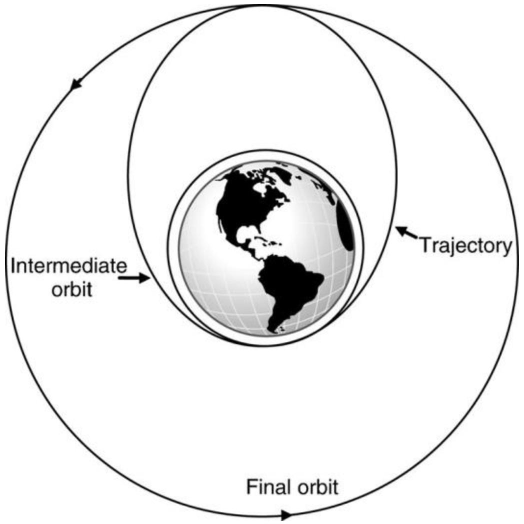
Trajectory and Orbit

Definitions

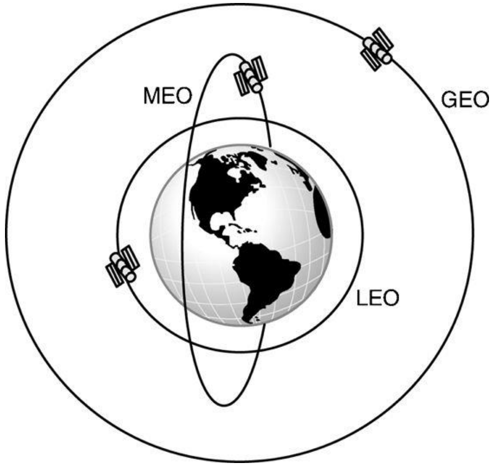
- A **trajectory** is a path traced by a moving body
  - Path followed by a **launch vehicle**
  - Path followed when satellites move from one orbit (**transfer**) to another
- An **orbit** is a trajectory that is **periodically** repeated
  - Artificial **satellite** around Earth



a) Launch trajectory



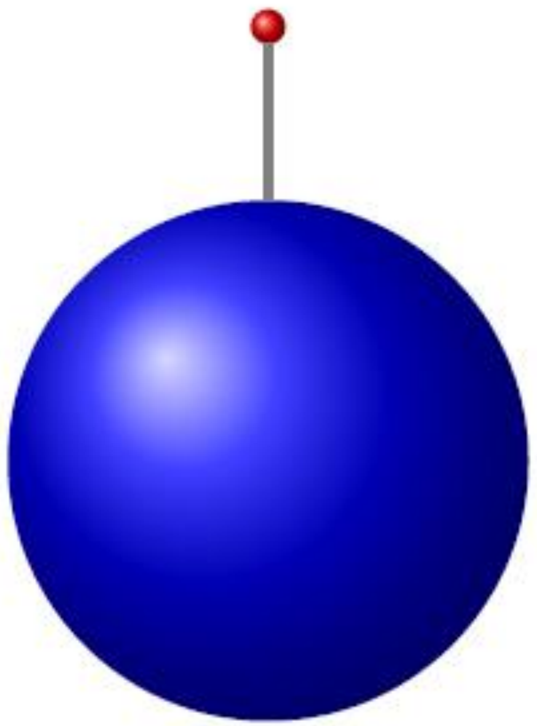
b) orbit transfer



c) Satellite orbit

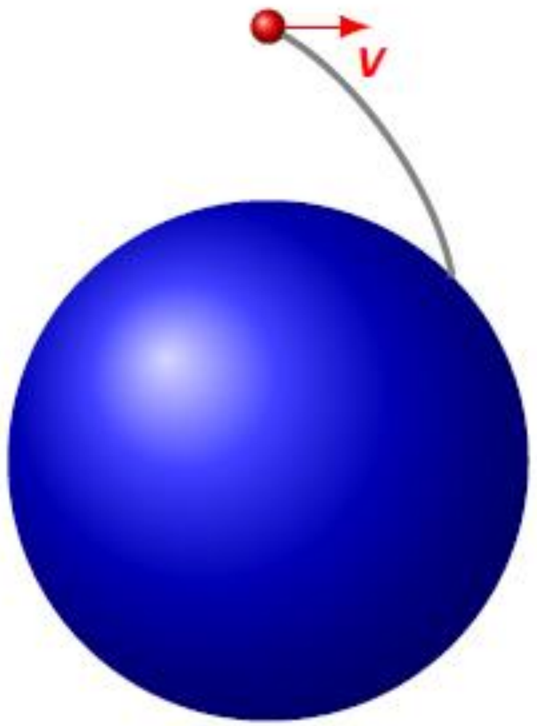
Trajectory and Orbit

From Trajectory to Orbit



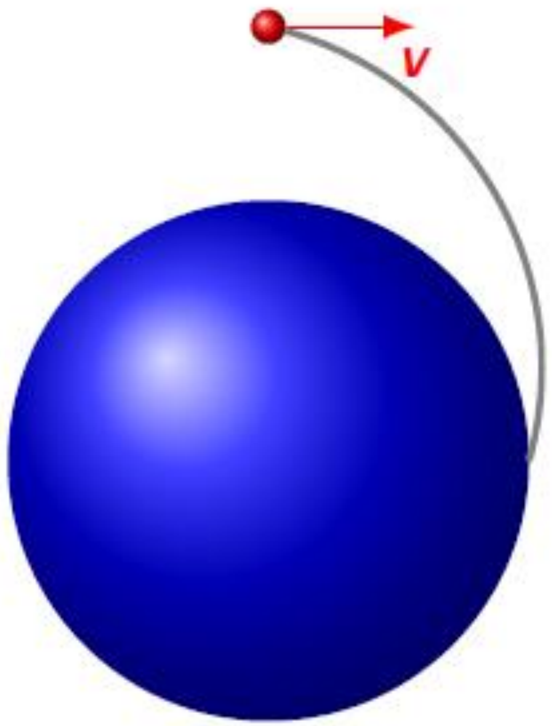
Trajectory and Orbit

From Trajectory to Orbit



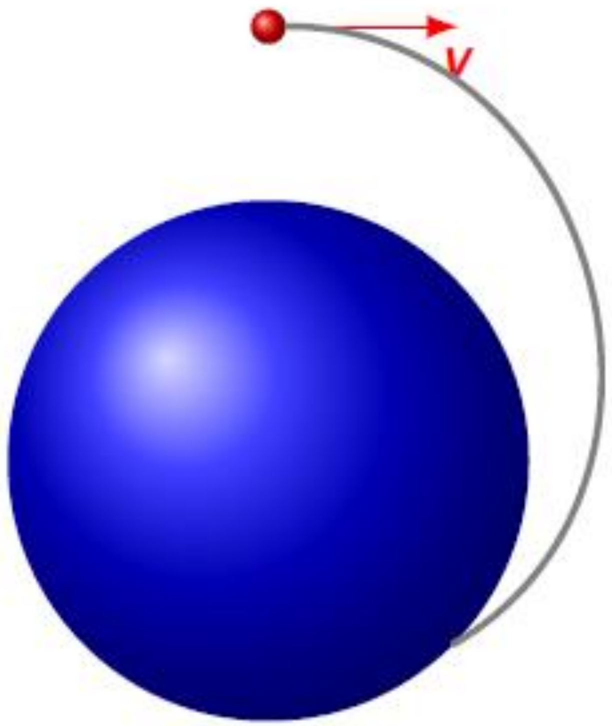
Trajectory and Orbit

From Trajectory to Orbit



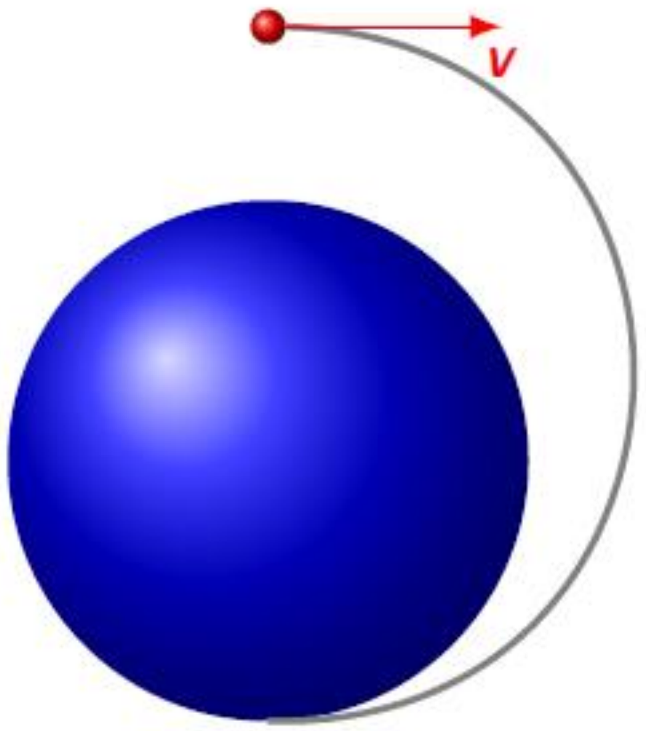
Trajectory and Orbit

From Trajectory to Orbit



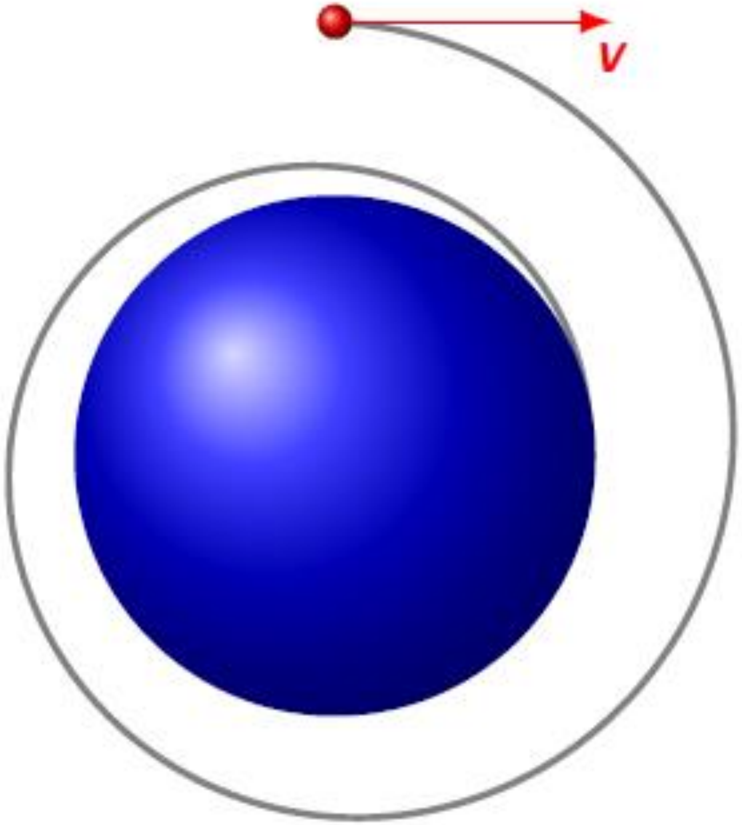
Trajectory and Orbit

From Trajectory to Orbit



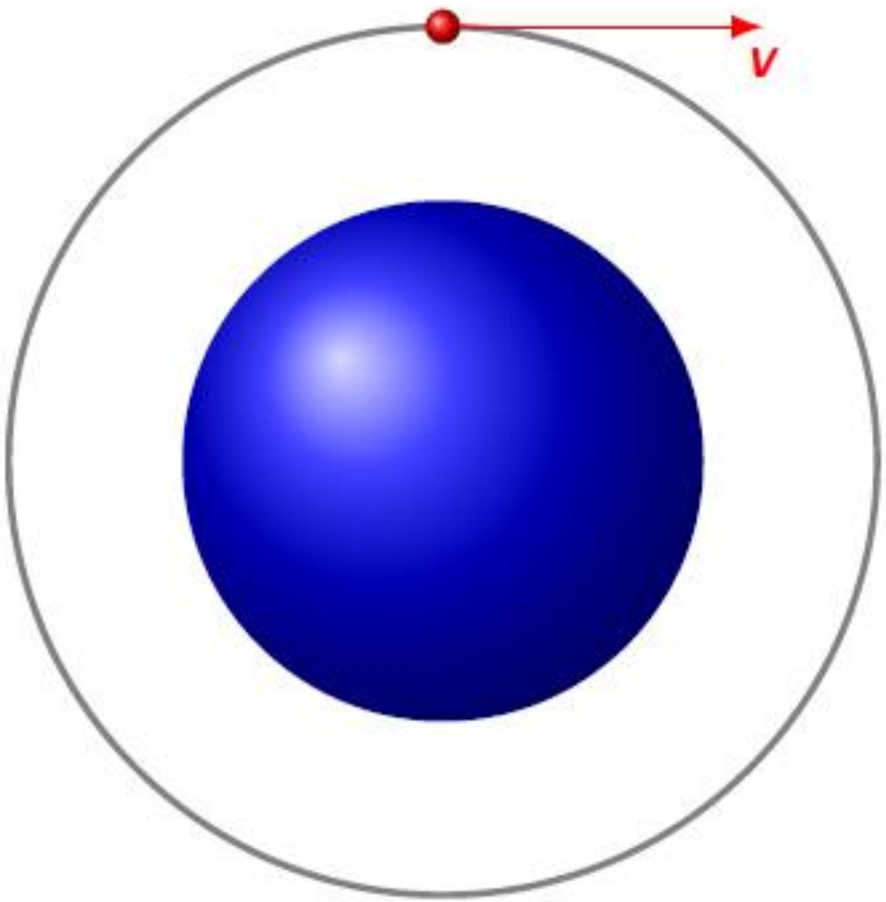
Trajectory and Orbit

From Trajectory to Orbit



Trajectory and Orbit

From Trajectory to Orbit



Basic Principles

Newton's laws of motion

- **First law:** an object either **remains at rest or continues to move at a constant velocity** unless acted upon by a force.
- **Second law:** the **vector sum of the forces** F on an object is equal to the mass m of that object multiplied by the **acceleration** $(F = ma)$
- **Third law:** when one body exerts a force on a second body, the second body exerts a force equal in magnitude and opposite direction on the first body.

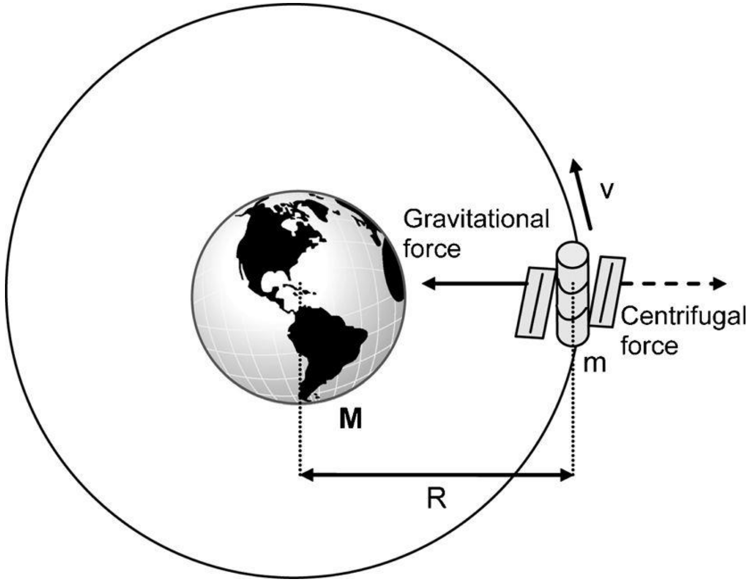
https://en.wikipedia.org/wiki/Newton%27s_laws_of_motion

Basic Principles

Newton's laws of motion - Centripetal and Centrifugal Force

- The motion of natural and artificial satellites is governed by two forces
  - **Centripetal force**: directed towards the center of the Earth (gravity force)
  - **Centrifugal force**: acts outwards from the center of the Earth

In the absence of centripetal force, the satellite would have **continued to move in a straight line** at a constant speed after injection



Centripetal force leads to an acceleration called **centripetal acceleration** as it causes a change in the direction of the satellite's velocity vector

Newton 3rd law of motion: there is a **centrifugal acceleration** acting outwards from the center of the Earth (but with no practical effect due to its mass) due to the **centripetal acceleration** acting towards the center of the Earth

Basic Principles

Geometrical Analysis

We have (with angles in radians)

$$\Delta s = \alpha r \Leftrightarrow \alpha = \frac{\Delta s}{r}$$

and

$$\Delta v = v_{t_0+\Delta t} - v_{t_0} = v_{t_0}\alpha$$

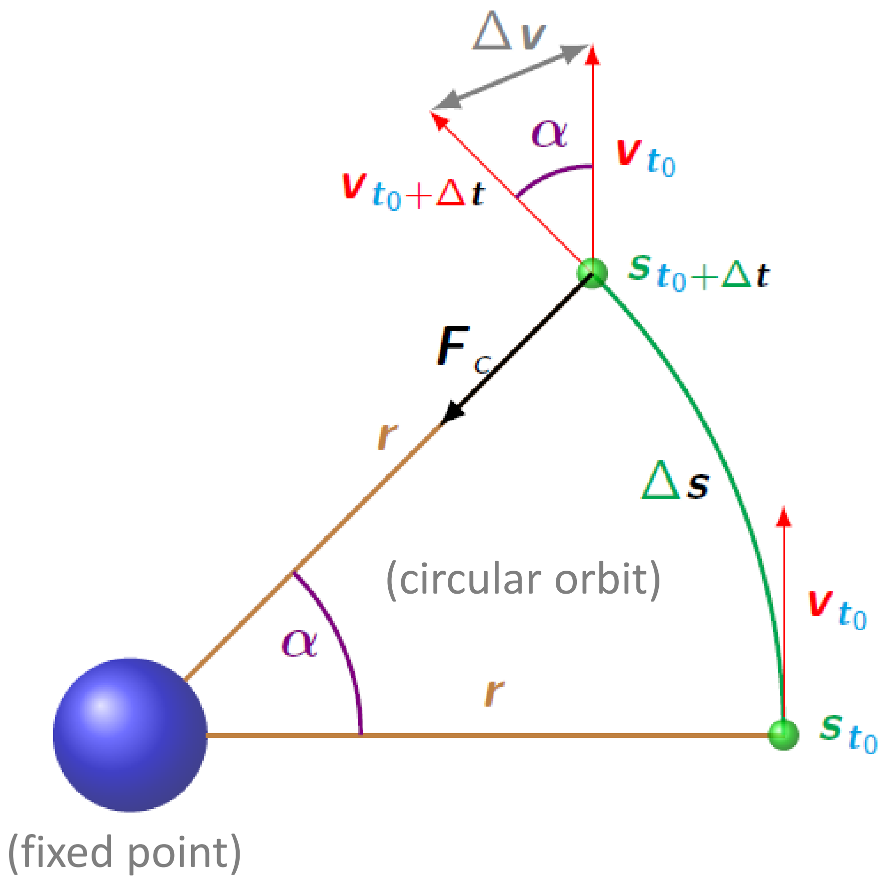
Then,

$$a_c = \frac{\Delta v}{\Delta t} = \frac{v\alpha}{\Delta t} = \frac{v\Delta s}{\Delta t\, r}.$$

Since $\frac{\Delta s}{\Delta t} = v$ we conclude that $a_c = \frac{v^2}{r}$.
Multiplying both sides by m, we end up with

Satellite mass

$$\boxed{m \cdot a_c = F_c = m\frac{v^2}{r}}$$



Fc = centripetal, which is equal to the **centrifugal force** directed outwards from the center of the Earth.

Basic Principles

Newton's Law of Gravitation

Geometrical analysis $F_c = m\frac{v^2}{r}$

- Every particle irrespective of its mass attracts every other particle as

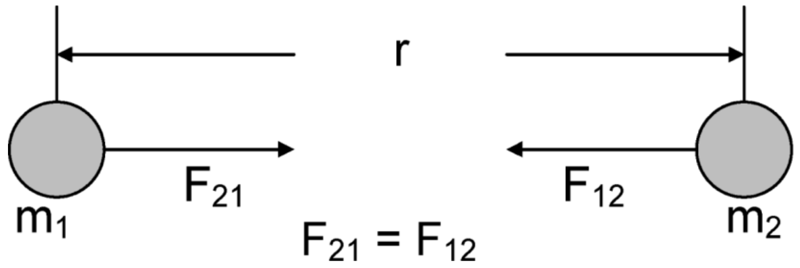
$$F = \frac{Gm_1m_2}{r^2}$$

where

- m_1m_2 = masses of two particles
- r = distance between particles
- G = gravitational constant (6.67×10^{-11} $m^3/kg\ s^2$) – not Earth gravity (g)

(empirical physical constant)

Resulting **forces** are equal in magnitude but opposite in direction



Resulting **accelerations** (force × mass) will depend on object mass

Basic Principles

Equating Centripetal and Centrifugal Force

$$F_c = m\frac{v^2}{r}$$

- Equating the **two** forces, satellite velocity v: (circular orbit)

Newton's law of gravitation

$$\boxed{F = \frac{Gm_1m_2}{r^2}} \rightarrow \frac{Gm_1m_2}{r^2} = \frac{m_2v^2}{r} \leftarrow \boxed{F_c = m\frac{v^2}{r}}$$

Geometrical analysis

$$v = \sqrt{\frac{Gm_1}{r}} = \sqrt{\frac{\mu}{r}}\ \left[\frac{dist}{time}\right] \qquad \omega = \frac{v}{r} = \sqrt{\frac{\mu}{r^3}}\left[\frac{rad}{time}\right]$$

Satellite speed in a circular orbit

where

- m_1 is the mass of Earth
- m_2 is the mass of the satellite
- $\mu = Gm_1 = 3.986013 \times 10^5$ km³/s²

Basic Principles

Orbital Period

- The **orbital period** T can be computed from the distance in the **circular** obit $2\pi r$ and the velocity v

$$T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{\mu}}$$

$$T = 2\pi \sqrt{\frac{r^3}{\mu}} \; [time]$$

Orbital period in a circular orbit

Basic Principles

Radius, Velocity and Periods

- Use Orbit-Wizard in STK to create:
  - **Circular LEO orbit**: 400 km height (+ avg. Earth radius: 6371 km)
  - **Circular MEO orbit**: 20000 km height (+ avg. Earth radius: 6371 km)
  - **Circular GEO orbit**: 42166.3 km radius (semi-major axis)
  - (Use orbits with **0° inclination**)
- Use the **3D and 2D windows** to visualize the orbits
- Use the **report tool** to explore numerical values
  - Create **custom report** with **velocity** and **period** to evaluate the equations

Alternative tool to STK (free)
GMAT Download: https://sourceforge.net/projects/gmat/
GMAT Tutorials: https://gmat.sourceforge.net/docs/R2022a/html/

Basic Principles

Circular Orbits

- So far for **perfectly circular** orbits…

Basic Principles

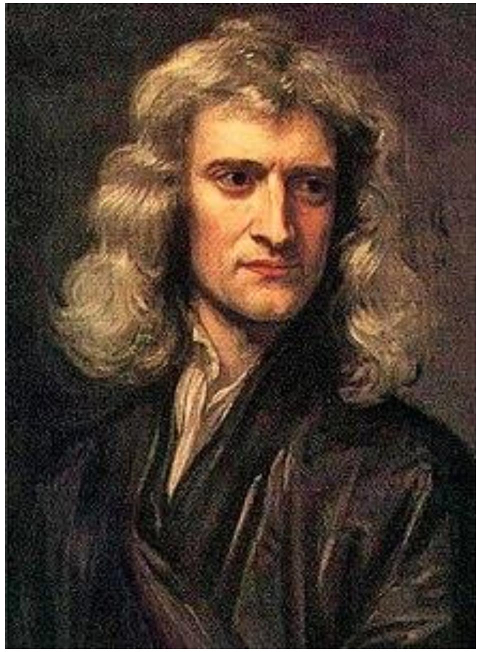
Kepler’s Laws

- **Johannes Kepler**, based on his lifetime study, gave a set of three empirical expressions that **described** planetary (orbital) motion
- These laws were later **explained** (Kepler did not explain the of *why* orbital motion) when **Newton** gave the law of motion and gravitation

Johannes Kepler
German
1571 -1630

Mars

Isaac Newton
English
1642 – 1726



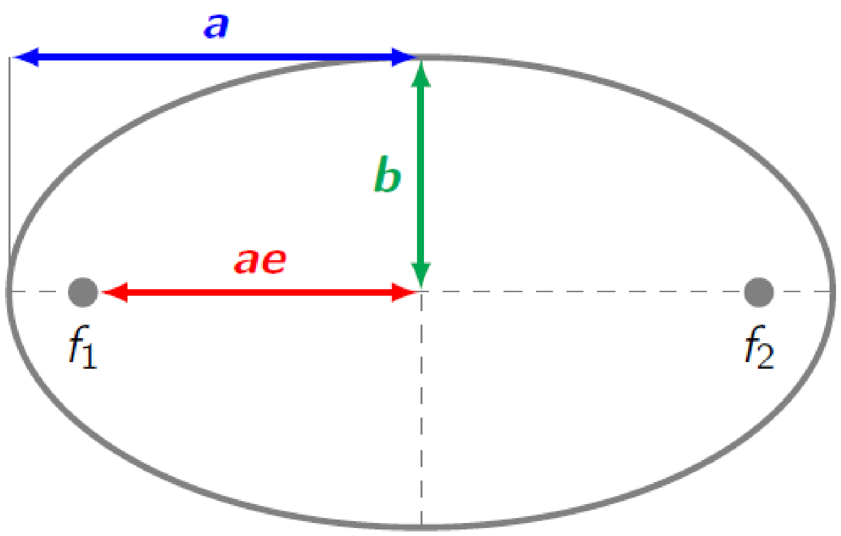
Basic Principles

Kepler's First Law

1
- "The orbit of a satellite (*planet*) around Earth is **elliptical** with the center of the Earth (*Sun*) lying at **one of the focus** of the ellipse"
- The elliptical orbit is characterized by its **semi-major axis** a**,** **semi-minor axis** b and **eccentricity** e

Eccentricity e is the ratio of the distance between the center of the ellipse and either of its focus

$$e = \frac{ae}{a}$$



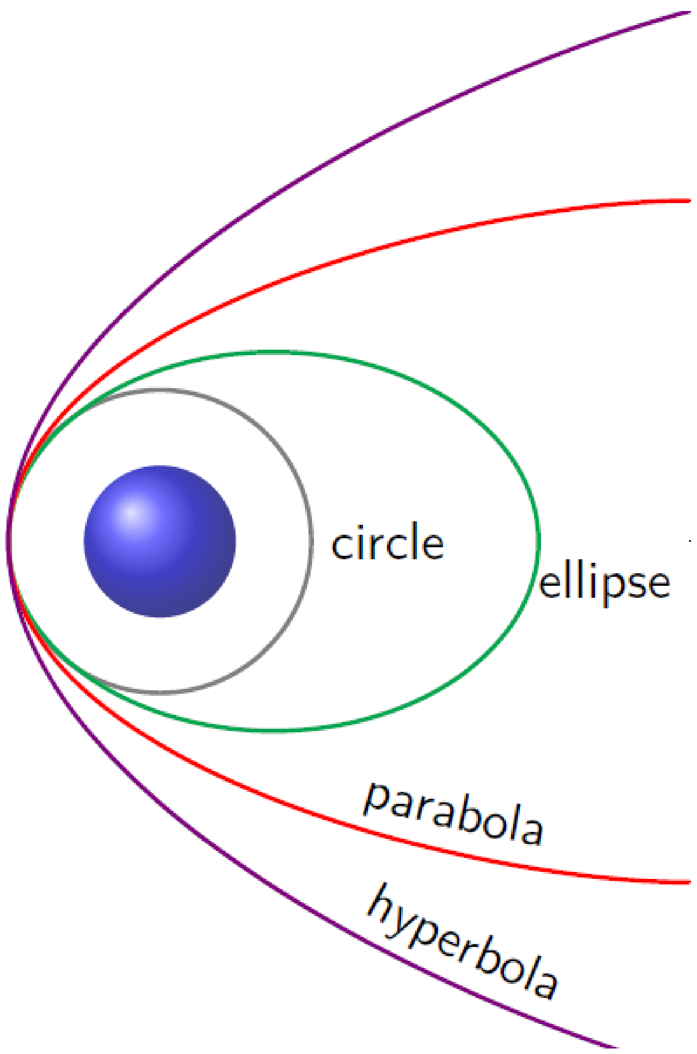
Eccentricity is a measure of how circular an orbit is.

In a circular orbit semi-major axis $a = b$ is the radius and $e = 0$

https://www.mathsisfun.com/geometry/eccentricity.html

Basic Principles

Kepler's First Law - Eccentricity



Mercury: 0.2056
Venus: 0.0068
Earth: 0.0167
Mars: 0.0934
Jupiter: 0.0484
Saturn: 0.0542
Uranus: 0.0472
Neptune: 0.0086

Orbits

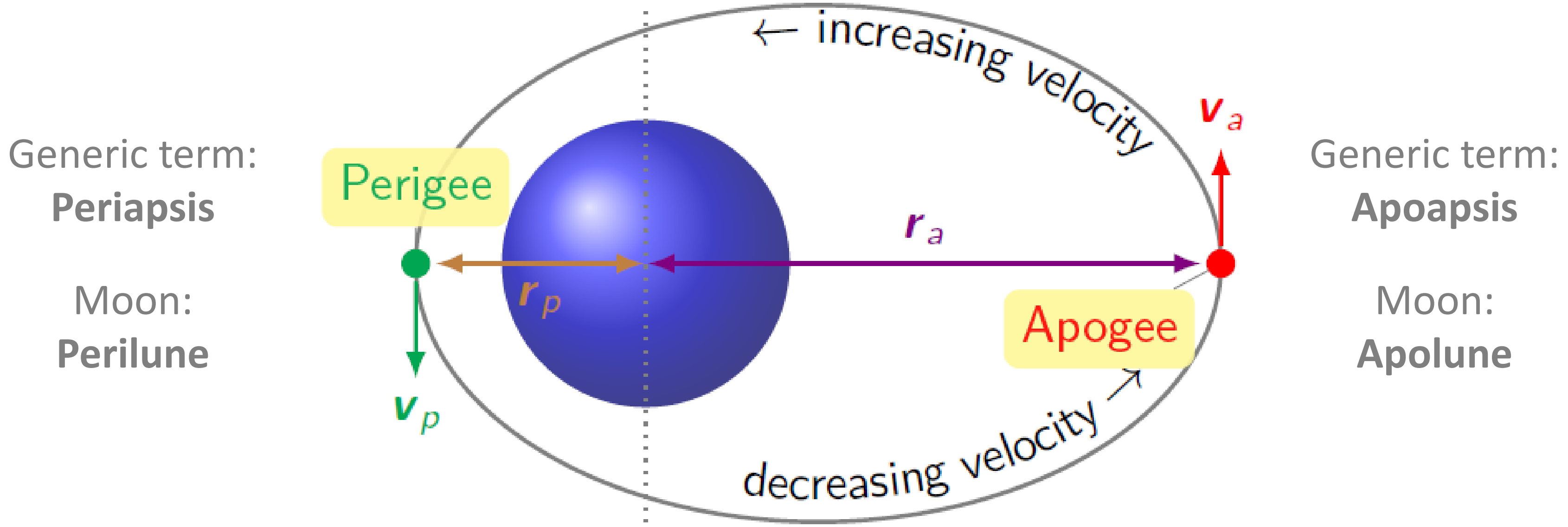
shape	e	Semi-major axis	Orbital Energy
circle	0	radius	< 0
ellipse	$\in]0, 1[$	> 0	< 0
parabola	1	∞	0
hyperbola	> 1	< 0	> 0

Trajectories

Basic Principles

Kepler's First Law: Perigee and Apogee

- Other important parameters of an elliptical satellite orbit are
  - **Apogee:** farthest point of the orbit from the Earth's centre
  - **Perigee:** nearest point of the orbit from the Earth's centre



Basic Principles

Kepler's First Law: Non-uniform velocity

With ellipsoidal orbits the velocity is not uniform.
Conservation of energy $\Rightarrow$ total energy constant

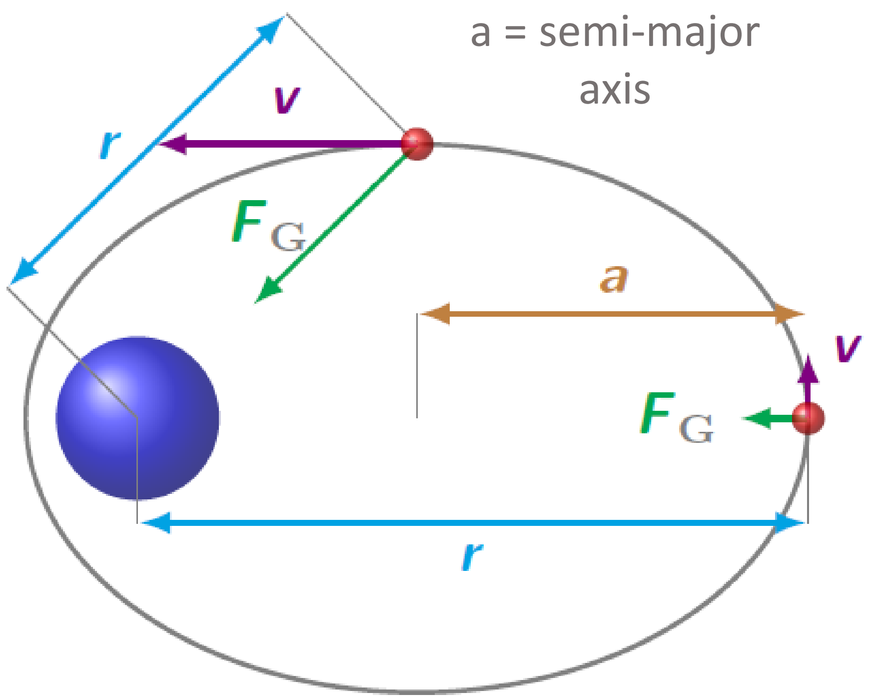
$$-\frac{G m_{\bullet} m_{\bullet}}{2a}$$

$\rightarrow$ difference of potential and kinetic energy is constant:

$$\frac{1}{2} m_{\bullet} v^2 - \frac{G m_{\bullet} m_{\bullet}}{r} = -\frac{G m_{\bullet} m_{\bullet}}{2a}$$

which leads to

$$\boxed{v = \sqrt{G m_{\bullet} \left(\frac{2}{r} - \frac{1}{a} \right)}}$$



When $a = r$ (circular orbit),
this reduces to $v = \sqrt{\frac{Gm_1}{r}}$

Basic Principles

Kepler's First Law: Radius, Velocity and Periods

- Use Orbit-Wizard in STK to create a **Molniya** orbit:
  - **Inclination**: 63.4°
  - **Apogee Longitude**: 0 °
  - **Perigee Altitude**: 500 km
  - **Argument of perigee**: 270 °
- Observe (report) the **velocity**

The name comes from a series of Soviet Russian Molniya communications satellites that have used this type of orbit since the mid-1960s.

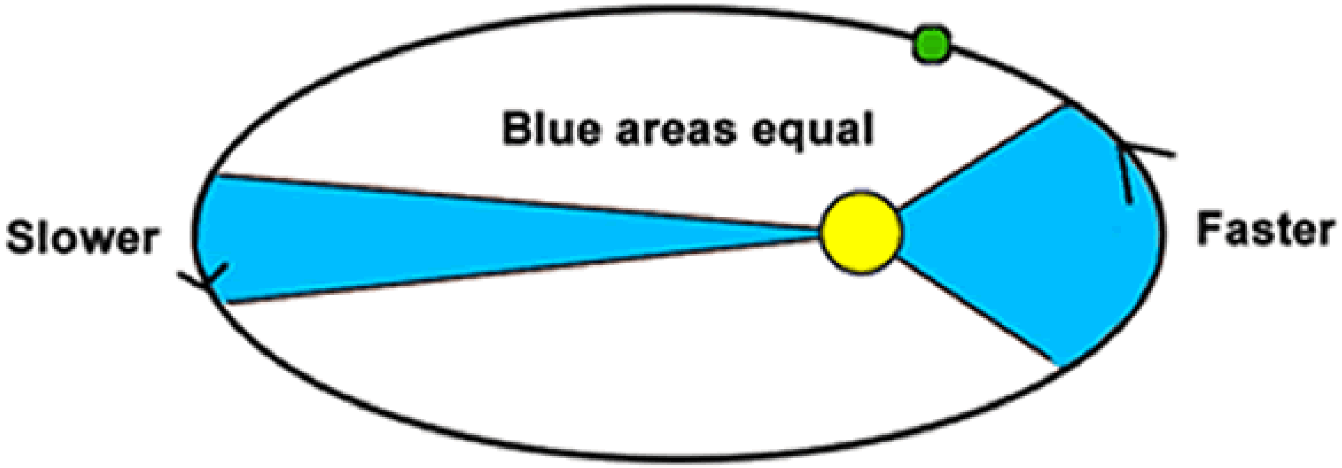
- **Advantages**:
  - A satellite in this orbit **spends most of its time in the northern hemisphere** and passes quickly over the southern hemisphere.
  - The elevation angle to geostationary orbits requires significant power in high latitudes.
- **Disadvantages**
  - The ground station needs a steerable antenna
  - Links must be switched between satellites
  - The range varies,
  - The spacecraft will pass the Van Allen radiation belt several times per day

GMAT Download: https://sourceforge.net/projects/gmat/
GMAT Tutorials: https://gmat.sourceforge.net/docs/R2022a/html/

Basic Principles

Kepler's Second Law

2 ▪ "A line joining a planet and the Sun **sweeps out equal areas** during **equal intervals** of time"



Basic Principles

Kepler's Third Law - Also known as the law of periods

3

- "The **square of the period** of any satellite is proportional to the **cube of the semi-major axis** of its elliptical orbit"
- We've said that (for circular orbits…)

$$T = \frac{2\pi r}{v} = \left(\frac{2\pi}{\sqrt{\mu}}\right) r^{3/2}$$

The same T we derived from Newton's equations.
Here, r can be replaced by semi-major axis a in the case of elliptical orbits

$$T^2 \propto r^3$$

Orbital Parameters

Towards defining an orbit

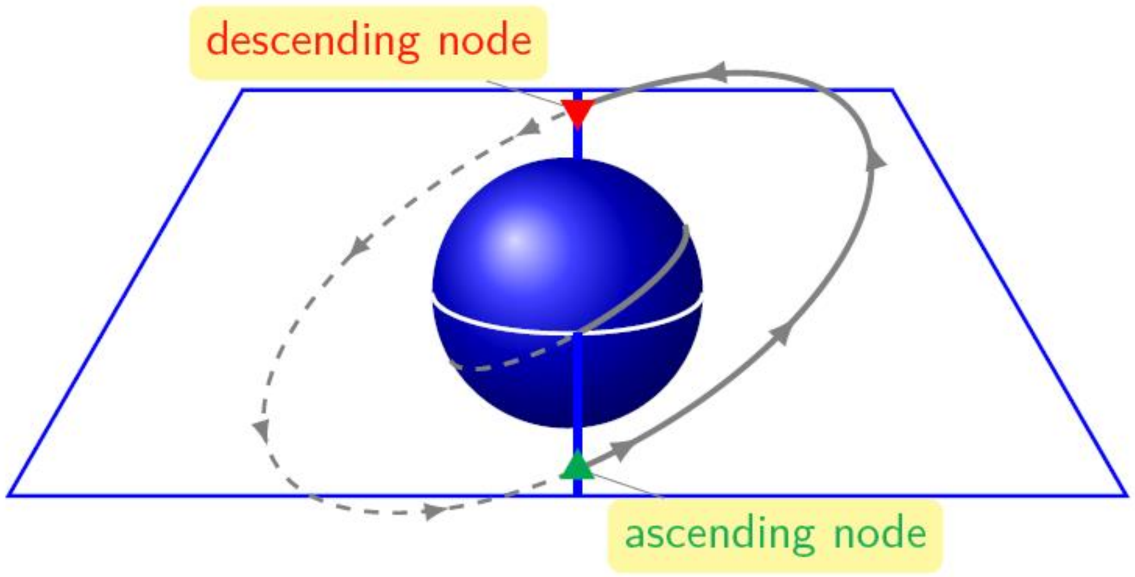
- We want to **define an orbit uniquely in space**
  - We saw that these define the **shape** of an orbit:
    - Semi-major axis, Semi-minor axis
    - Eccentricity
  - We now need to define
    - The orbit **orientation in space** and
    - The **satellite position** within the orbit path

Orbital Parameters

Orientation

Ascending and descending nodes

- The satellite orbit cuts the equatorial plane at two points:
  - The **descending node**, where the satellite passes from the northern hemisphere to the southern hemisphere
  - The **ascending node**, where the satellite passes from the southern hemisphere to the northern hemisphere

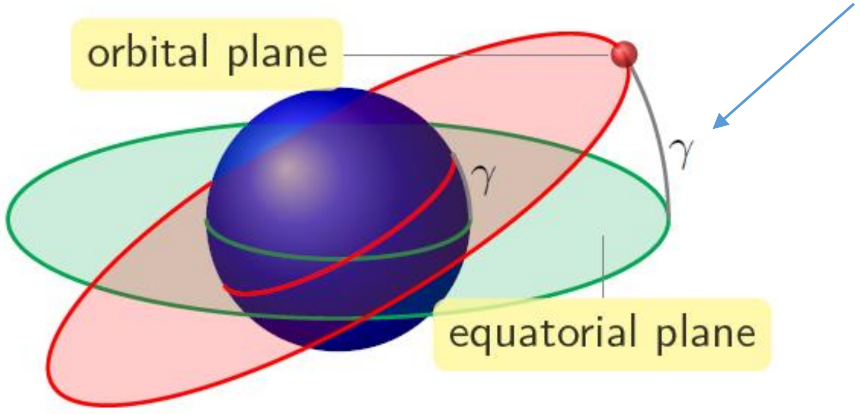


Orbital Parameters

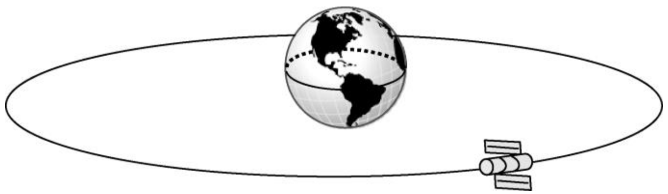
Orientation

Inclination

- Inclination is the angle that **the orbital plane of the satellite** makes with the **Earth's equatorial plane**



Equatorial Orbit (0° inc.)



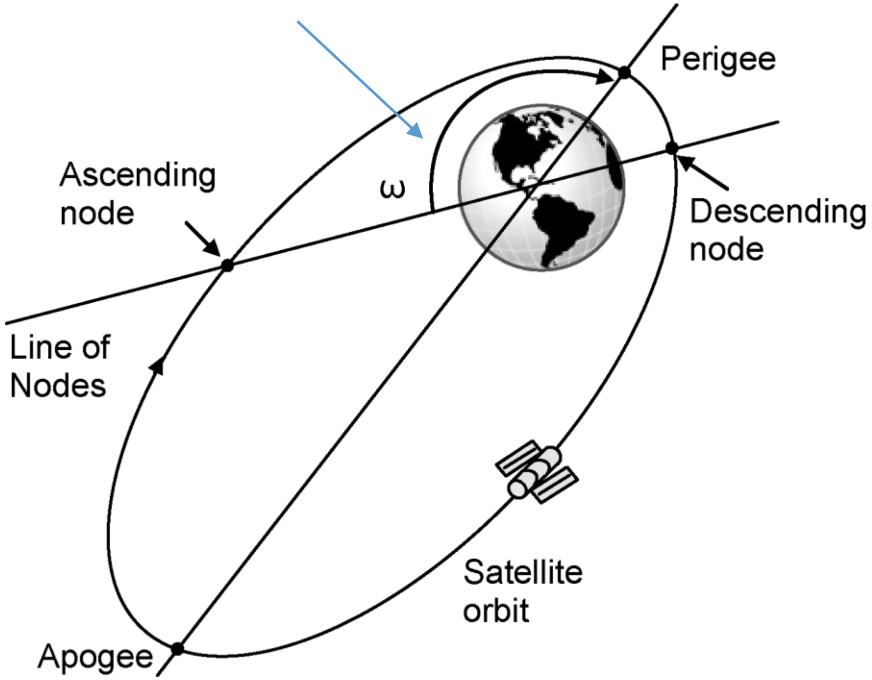
Polar Orbit (90 ° inc.)

Orbital Parameters

Orientation

Argument of the perigee

- It is measured as the angle ω between the **line joining the perigee and the centre** of the Earth and the **line of nodes** from the ascending node to the descending node **in the direction of the satellite orbit**



The argument of perigee defines the location of the major axis of the satellite orbit

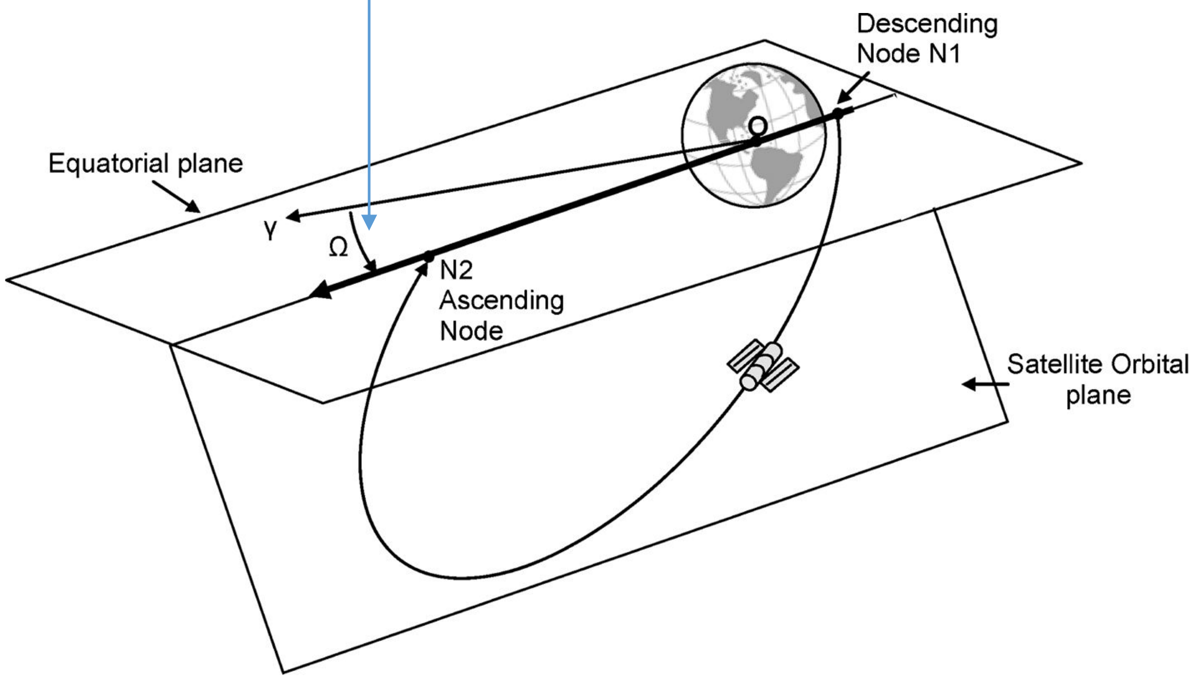
Orbital Parameters

Orientation

RAAN

- Is the angle Ω between the **line joining the ascending and descending nodes**, with respect to the **vernal equinox** (?) in the direction of rotation of Earth **at orbit epoch**

RAAN = Right Ascension of the Ascending Node

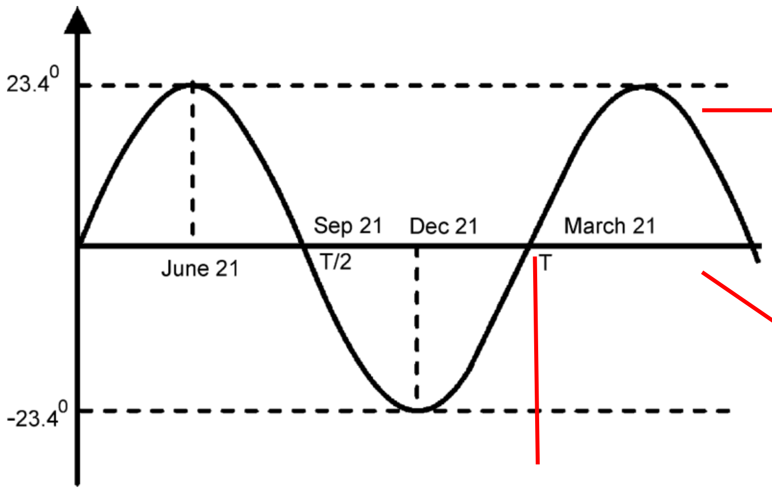


Orbital Parameters

Reference

Vernal Equinox

- The **inclination** of the **equatorial plane** of Earth with respect to the **direction of the sun**[1] follows a **sinusoidal variation** with period T of 365 days → show in STK

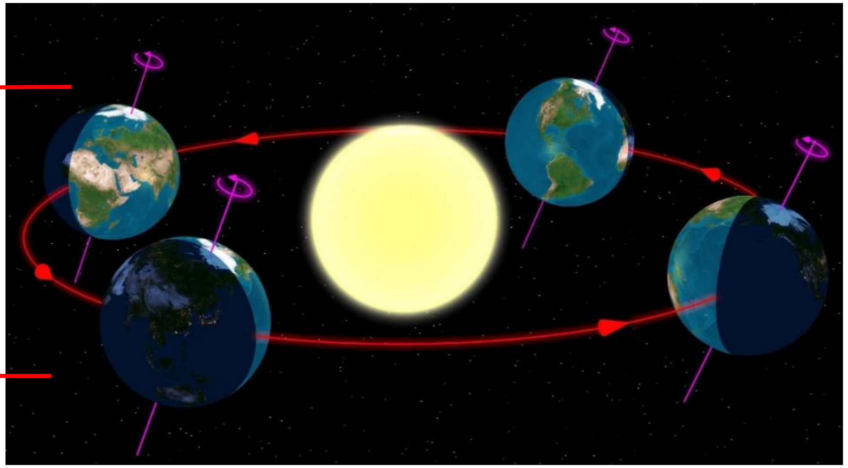


Vernal Equinox (Y)[2]

Beginning of spring (March)

Solstices are the times when the inclination angle of the equatorial plane is at its max.

Equinoxes are the times when the inclination angle of the equatorial is 0



[1]angle formed by the line joining the center of the Earth and the sun with the Earth's equatorial plane

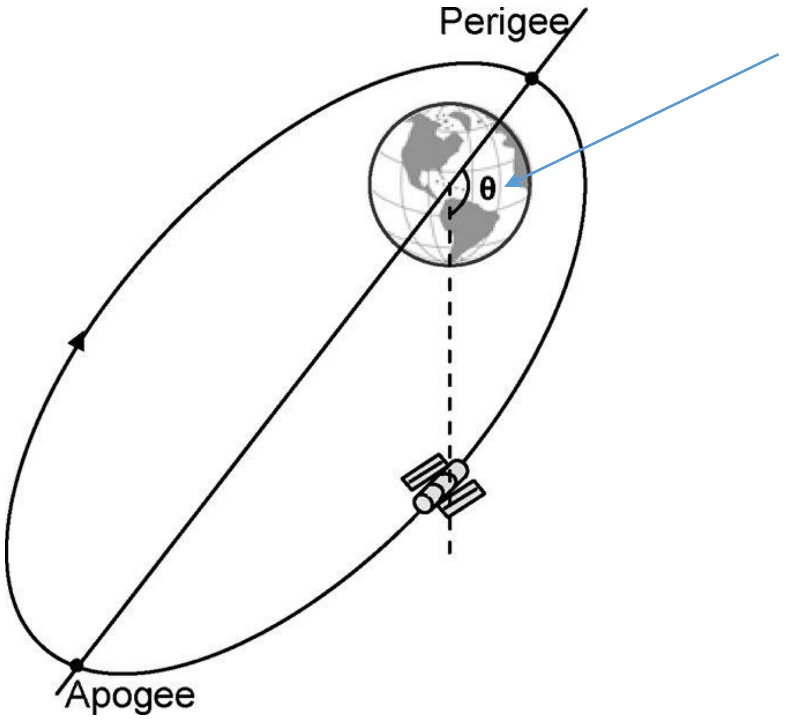
[2]a point at infinity used as a celestial reference for orbits (was in the Aries constellation, now in Pisces)

Orbital Parameters

True anomaly of the satellite

Sat position

- An angle used to indicate the **position of the satellite in its orbit** formed by the line joining the **perigee and the centre of the Earth** with the line joining the **satellite and the centre of the Earth**



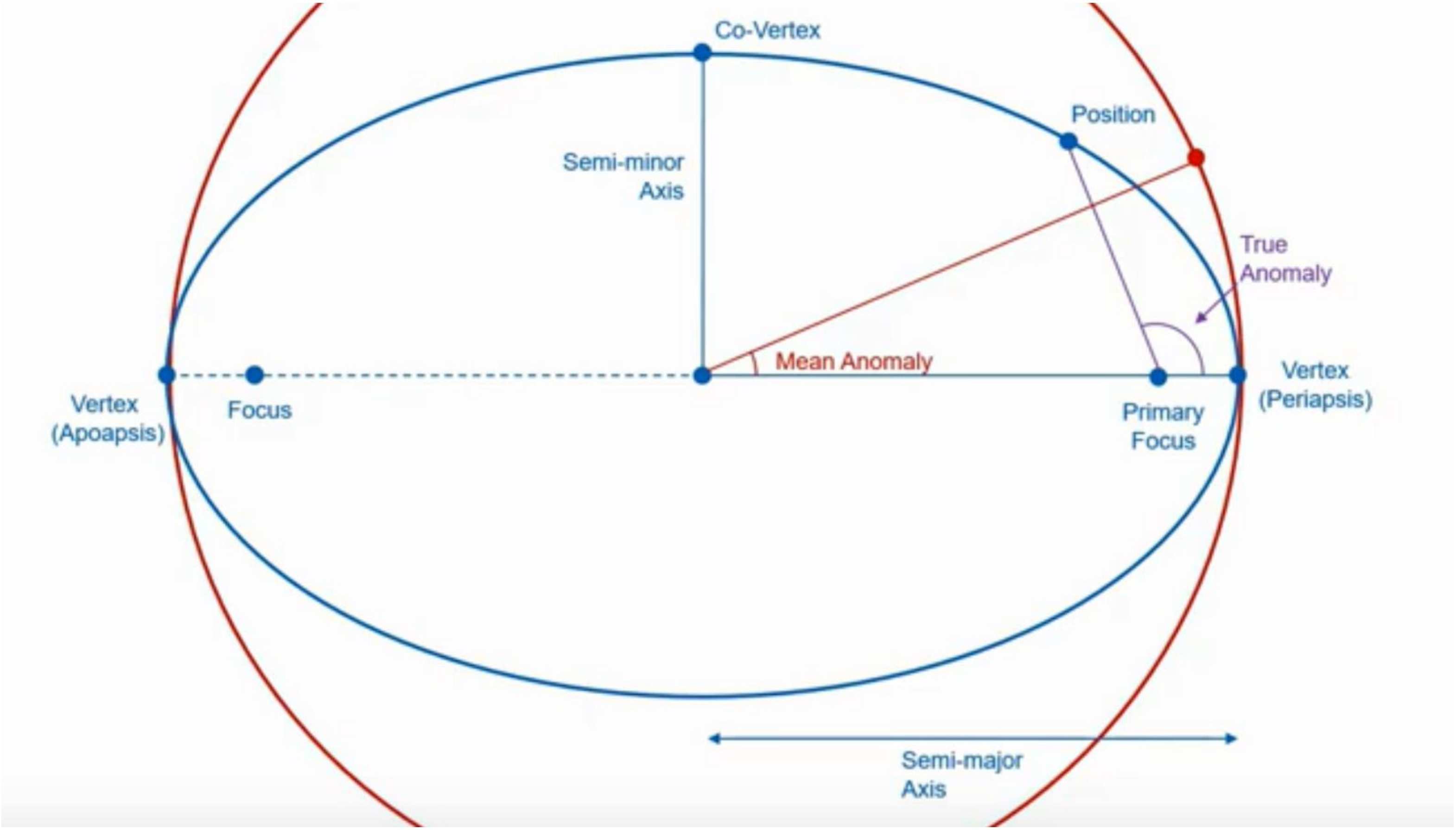
Orbital Parameters

Sat position

True and Mean anomaly

- **True anomaly** is measured from the focus
- **Mean anomaly** is measured from the center

Equal in a circular orbit



Orbital Parameters

Minimal set of Keplerian Parameters

In an **epoch** (expressed as Julian Date: a continuous count of days and fractions of a day since noon on Jan 1st, 4713 BC (in the proleptic Julian calendar)

- Size and shape parameters:
  - **1.** Semi-major axis (a) **or** Semi-minor axis (b)
    - $b = a \cdot \sqrt{1 - e^2}$
  - **2.** Eccentricity (e) **or** Periapsis distance (q) **or** Apoapsis distance (Q):
    - $q = a \cdot (1 - e)$ and $Q = a \cdot (1 + e)$
- Orientation parameters:
  - **3.** Inclination (i)
  - **4.** Longitude of the ascending node (Ω) **or** right ascension of the ascending node (RAAN)
  - **5.** Argument of periapsis (ω)
- Satellite position parameters:
  - **6.** Time of periapsis passage (T or t_0) **or** Mean/True anomaly (M)
    - $M = \frac{2\pi(t-T)}{P}$, where t is the time of interest and P is the orbital period.

Orbital Parameters

Get Intuition Online

Orbital Mechanics - orbital elements visualizer and launch simulator

https://orbitalmechanics.info/

Questions?