Contents

Linear Algebra Review X

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Section 5.1 Eigenvectors and Eigenvalues

Definitions

- **eigenvector** - nonzero vector such that $A\mathbf{x} = \lambda\mathbf{x}$ for a scalar λ.

- **eigenvalue** - the scalar λ, for the eigenvector $\mathbf{x}$.

- **eigenspace** - the subspace of $\mathbb{R}^n$ defined by set of all solutions of $(A - \lambda I)\mathbf{x} = 0$ for A and λ.

Eigens from another perspective

khanacademy eigenvalues and eigenvector video

- Eigenvector $(\mathbf{x})$ is a vector scaled (λ) by a transformation, $T(\mathbf{x}) = \lambda\mathbf{x}$, where transformation T is $\mathbf{A}$.

- The transformed vectors make for better basis vectors, make for simpler computations or good coordinate systems.

By Lyudmil Antonov Lantonov 16:35, 13 March 2008 (UTC) - This vector image was created with Inkscape., GFDL, https://commons.wikimedia.org/w/index.php?curid=3698599

Finding Eigenvalues, Eigenvectors, and Eigenspace

- Vector $\mathbf{x}$ is an eigenvector of $\mathbf{A}$ if the resulting vector is a scalar multiple of $\mathbf{x}$. The scalar λ is the eigenvalue.

- λ is an eigenvalue of $\mathbf{A}$ if the columns in $(A - \lambda I)$ are linearly dependent, e.g. has a nontrivial solution.

- Basis of an eigenspace is $(A - 2I)\mathbf{x} = \mathbf{0}$, find by row reduction of the augmented matrix.

Theorem 1

The eigenvalues of a triangular matrix are the entries on its main diagonal.
Key idea is that $(A - \lambda I)\mathbf{x} = 0$, therefore at least one entry on the diagonal of $A - \lambda I$ is zero.

$$
\begin{aligned}
A - \lambda I &= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \\
&= \begin{bmatrix} a_{11} - \lambda & a_{12} & a_{13} \\ 0 & a_{22} - \lambda & a_{23} \\ 0 & 0 & a_{33} - \lambda \end{bmatrix}
\end{aligned}
$$

Figure 1: Lay p. 306

Theorem 2

If $\mathbf{v}_1, \ldots, \mathbf{v}_r$ are eigenvectors that correspond to distince eigenvalues, $\lambda_1, \ldots, \lambda_r$ of an $n \times n$ matrix A, then the set $\{\mathbf{v}_1, \ldots, \mathbf{v}_r\}$ is linearly independent.

Other points of interest

- 0 is an eigenvalue of A if and only if A is not invertible.

Additional Eignevalue Propertives

from Wikipedia

- determinant is equal to the product of the eigenvalues

References

Lay - Linear Algebra and its Applications