Linear Algebra Review II

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Matricies and Linear Regression

What we are working towards. . . .

$$Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i, i = 1, \ldots, N$$

as:

$$\begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{pmatrix} = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \\ 1 & x_N \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{pmatrix}$$

or simply:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

1.3 Vector Equations

Definitions

- vector - a list of numbers
- column vector - one column matrix
- zero vector - all values in a vector are zero
- scalar - real numbers that have magnitude but no direction wikipedia
- span - subset of a vector

Parallelogram Rule for Addition

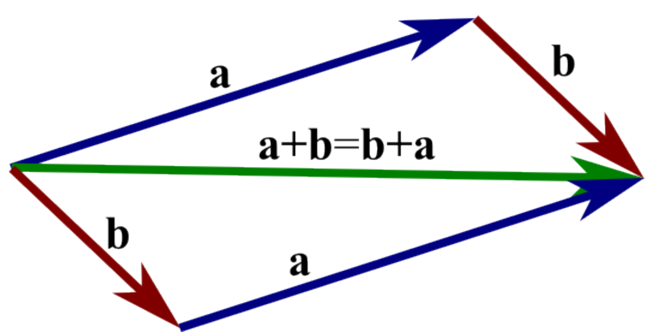

Figure 1: http://i.stack.imgur.com/O6Ved.png

Algebraic Properties of $\mathbb{R}^n$

$\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors, where c and d are scalars

1. $\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u}$
2. $(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{v}+\mathbf{w})$
3. $\mathbf{u}+0=0+\mathbf{u}=\mathbf{u}$
4. $\mathbf{u}+(-\mathbf{u})=0$, where $(-1)\mathbf{u}=-\mathbf{u}$
5. $c(\mathbf{u}+\mathbf{v})=c\mathbf{u}+c\mathbf{v}$
6. $(c+d)\mathbf{u}=c\mathbf{u}+d\mathbf{u}$
7. $c(d\mathbf{u})=(cd)(\mathbf{u})$
8. $1\mathbf{u}=\mathbf{u}$

1.4 The Matrix Equation $A\mathbf{x}=\mathbf{b}$

A is a $m\times n$ matrix, with columns $\mathbf{a}_1$, ..., $\mathbf{a}_n$, and if $\mathbf{x}$ is in $\mathbb{R}^n$,

$$A\mathbf{x}=\begin{bmatrix}\mathbf{a}_1 & \mathbf{a}_2 & \dots & \mathbf{a}_n\end{bmatrix}\begin{bmatrix}x_1\\x_2\\\vdots\\x_n\end{bmatrix}=x_1\mathbf{a}_1+x_2\mathbf{a}_2+\cdots+x_n\mathbf{a}_n$$

Theorem 3

$\mathbf{A}$ is a $m\times n$ matrix with columns $\mathbf{a}_1,\dots,\mathbf{a}_n$, $\mathbf{b}$ is in $\mathbb{R}^m$

The solutions of x (and solution set for the augmented matrix) are all the same.

- $\mathbf{A}x=\mathbf{b}$
- $x_1\mathbf{a}_1+x_2\mathbf{a}_2+\cdots+x_n\mathbf{a}_n=\mathbf{b}$
- $\begin{bmatrix}\mathbf{a}_1 & \mathbf{a}_2 & \dots & \mathbf{a}_n & \mathbf{b}\end{bmatrix}$

Existence of Solutions

Only solution if and only if $\mathbf{b}$ is a linear combination of the columns of A

Theorem 4

$\mathbf{A}$ is a $m\times n$ matrix. The following are all true or false.

1. for all real $\mathbf{b}$, $\mathbf{A}x=\mathbf{b}$ has a solution
2. for all real $\mathbf{b}$, $\mathbf{b}$ is a linear combination of columns in $\mathbf{A}$

3. column of **A** span $\mathbb{R}^m$ - I think this means the columns are real as well

4. **A** has a pivot position in every row.

Computation of $A\mathbf{x}$

Introduction of the identity matrix concept - the identity matrix is key to finding inverse of a matrix

$$I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Figure 2: https://tfetimes.com/c-identity-matrix/

Theorem 5

Assuming the following,

- **A** is a $m \times n$ matrix
- **u** and **v** are real vectors
- c is a scalar

Then

- $\mathbf{A}(\mathbf{u} + \mathbf{v}) = \mathbf{Au} + \mathbf{Av}$

- $\mathbf{A}(c\mathbf{u}) = c(\mathbf{Au})$