Standard Linear Model

$E[Y_i] = \mu_i = x_i^T B_i \qquad Y_i \sim N(\mu_i, \sigma^2)$

*note each i independent μ but shared σ^2

Linear Model Extension

1. non-normal response variables
   e.g. exponential family
2. non-linear relationship between response and explanatory variables

$$g(\mu_i) = x_i^T B$$

g is called the link function

Exponential family of Distributions

all distributions can be written as

$$f(y;\theta) = s(y)t(\theta)e^{a(y)b(\theta)}$$

$a, b, s,$ and t are known functions

$$= \exp[a(y)b(\theta) + c(\theta) + d(y)]$$

where $s(y) = \exp d(y)$
$t(\theta) = \exp c(\theta)$

in canonical form if $a(y) = y$
$b(\theta)$ is the natural parameter

parameters $c(\theta)$ and $d(y)$ are nuisance parameters and treated as known

Poisson

$$f(y;\theta) = \frac{\theta^y e^{-\theta}}{y!} = \exp(y\log\theta - \theta - \log y!)$$

Normal

$$f(y;\mu) = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left[-\frac{1}{2\sigma^2}(y-\mu)^2\right] = f(y;\mu) = \exp\left[-\frac{y^2}{2\sigma^2} + \frac{y\mu}{\sigma^2} - \frac{\mu^2}{2\sigma^2} - \frac{1}{2}\log(2\pi\sigma^2)\right]$$

Binomial

$$f(y;\mu) = \binom{n}{y}\pi^y(1-\pi)^{n-y} = \exp\left[y\log\left(\frac{\pi}{1-\pi}\right) + n\log(1-\pi) + \log\binom{n}{y}\right]$$