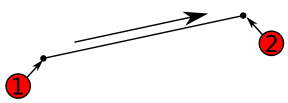
Symbols

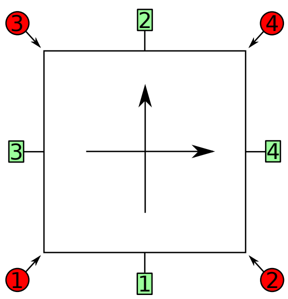
T	transformation factor
$\boldsymbol{T}$	transformation matrix
$\boldsymbol{r}$	coordinate vector
r	coordinate r_1
s	coordinate r_2
t	coordinate r_3

Topologies

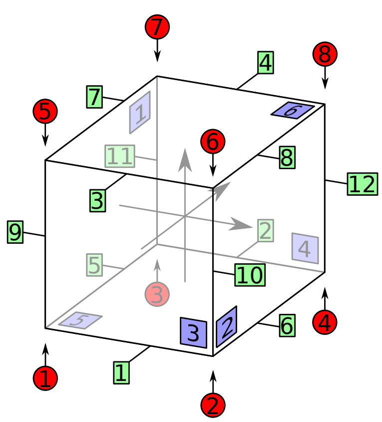
Line



Quad



Hex



Contact

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p-FEM transformations
- on the mapping between *supports* -

The one dimensional case is easy. Suppose we have a line. Its *support*, or *local parameterization* with the variable r may be mapped to or from

- another line with the same or exactly opposite parameterization by a variable r'
- one of the two nodes, which themselves - due to their *zero dimensionality* - have no parameterization. Here the idea begins: A pseudo coordinate r' is introduced for the node, which is alway equal to 1.

Now all entities share a parameterization of the same dimension, an a general mapping rule can be established:

$$r' = T\,r, \quad r = T^{-1} r', \quad \frac{\partial(\cdot)}{\partial r} = T\,\frac{\partial(\cdot)}{\partial r'}, \quad \frac{\partial(\cdot)}{\partial r'} = T^{-1}\,\frac{\partial(\cdot)}{\partial r}$$

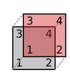
For the transformation factor T, we have:

- $T = 1$ if mapping between two coinciding lines
- $T = -1$ if mapping between two lines with opposite parameterizations
- $T = -1$ if mapping between a line and its first node
- $T = 1$ if mapping between a line and its second

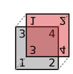
For *quadrilaterals*, we have a parameterization with two coordinates r and s. Mapping rules of the same type can be established, where all edges are parameterized by one *real* coordinate r' and one *pseudo coordinate* $s' = 1$. With $\boldsymbol{r} = (\,r\;\, s\,)^{\mathrm{T}}$ and $\boldsymbol{r}'$ respectively, we get:

$$\boldsymbol{r}' = \boldsymbol{T}\,\boldsymbol{r}, \quad \boldsymbol{r} = \boldsymbol{T}^{-1}\boldsymbol{r}', \quad \begin{pmatrix} \frac{\partial(\cdot)}{\partial r} \\ \frac{\partial(\cdot)}{\partial s} \end{pmatrix} = \boldsymbol{T}^{\mathrm{T}} \begin{pmatrix} \frac{(\cdot)}{\partial r'} \\ \frac{(\cdot)}{\partial s'} \end{pmatrix}, \quad \begin{pmatrix} \frac{\partial(\cdot)}{\partial r'} \\ \frac{\partial(\cdot)}{\partial s'} \end{pmatrix} = \boldsymbol{T}^{-\mathrm{T}} \begin{pmatrix} \frac{(\cdot)}{\partial r} \\ \frac{(\cdot)}{\partial s} \end{pmatrix}$$

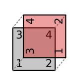
For mappings from one quad to another we have the following cases:

$\boldsymbol{T} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 

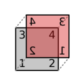
if the quads coincide

$\boldsymbol{T} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ 

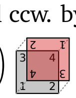
if r'-quad is mirrored about $s = 0$

$\boldsymbol{T} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ 

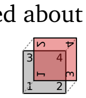
if r'-quad is rotated ccw. by $90\circ$

$\boldsymbol{T} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ 

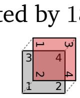
if r'-quad is mirrored about $r = 0$

$\boldsymbol{T} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ 

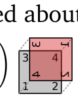
if r'-quad is rotated by $180\circ$

$\boldsymbol{T} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ 

if r'-quads is mirrored about $s = r$

$\boldsymbol{T} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ 

if r'-quad is rotated cw. by $90\circ$

$\boldsymbol{T} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ 

if r'-quads is mirrored about $s = -r$

For mappings from the quad to the edges we have the following cases:

$\boldsymbol{T} = \begin{pmatrix} \pm 1 & 0 \\ 0 & -1 \end{pmatrix}$ if * edge 1

$\boldsymbol{T} = \begin{pmatrix} 0 & \pm 1 \\ -1 & 0 \end{pmatrix}$ if * edge 3

$\boldsymbol{T} = \begin{pmatrix} \pm 1 & 0 \\ 0 & 1 \end{pmatrix}$ if * edge 2

$\boldsymbol{T} = \begin{pmatrix} 0 & \pm 1 \\ 1 & 0 \end{pmatrix}$ if * edge 4

* *r'-edge coincides with (+) / is oppositely parameterized (-) as*

The equations for two dimensions can be directly extended to three dimensions. See the next page for all possible transformations within *hexahedral supports*.