Write-up

Hypothesis

Merge sort will outperform insertion sort for all but very small lists, with the difference in speed becoming particularly apparent for very large lists. As for just how small the list will have to get for insertion sort to outpace merge sort, I'd guess low double digits.

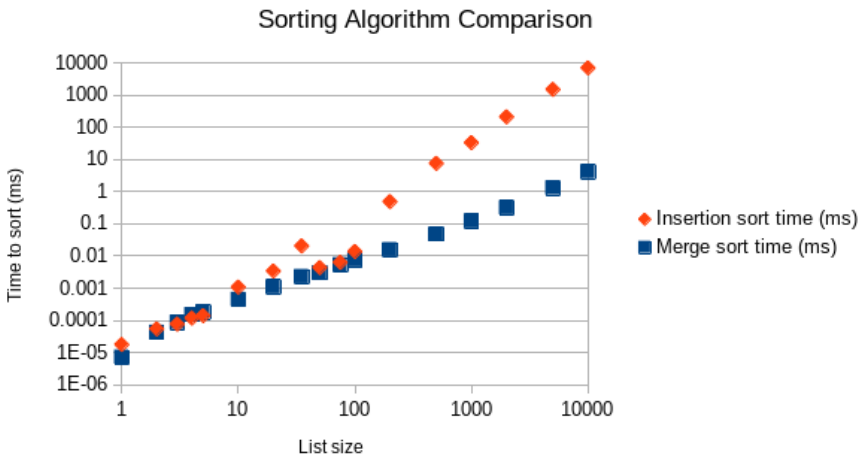
Methods

The code I used to test these algorithms can be found at https://github.com/khuldraeseth/cse431hw4. Clone the repository, move into `p1`, and `stack run -- --output wherever-you-want-the-results.html` to run these tests yourself. Compiles with `-threaded`.

These are my own implementations of insertion sort and merge sort, written in Haskell. Merge sort does not split the list at the middle—rather, it uses the inverse of an interleave (which I have named `unterleave`) that takes `[a,b,c,d,e] ↦ ([a,c,e],[b,d])` as an example for what I expect may be a moderate speedup.

Results

See below for Criterion output. The results are summarized in this plot:



Discussion

Not quite as I had expected. For starters, merge sort is faster in these tests than insertion sort for very small lists as well as sufficiently large ones! This makes sense in retrospect—I have special cases in my merge sort code for empty and singleton cases, so these are just one pattern match from being sorted, rather than a call to `foldr`. Then insertion sort has its time on top, outperforming merge sort for lists of length 3, 4, and 5. The two cross somewhere between 5 and 10, and repeated tests show no consistent particular value.

Interesting is that insertion sort, after being outpaced by an order of magnitude on lists of length 35, makes a resurgence and competes with merge sort again for lengths 50, 75, and 100 before falling off again at 200 and never having a third chance. I'll attribute this to some compile-time optimization.

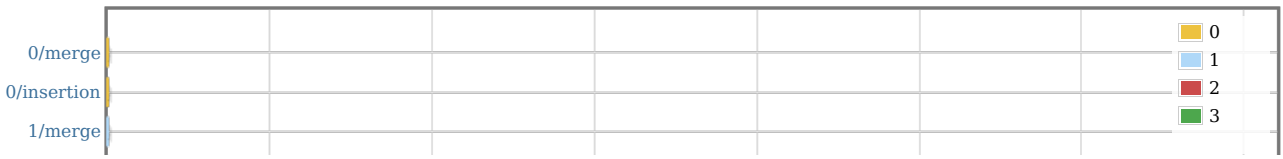
Conclusion

Under the conditions tested, merge sort was faster than insertion sort on lists of all lengths except those between 3 and 10.

criterion performance measurements

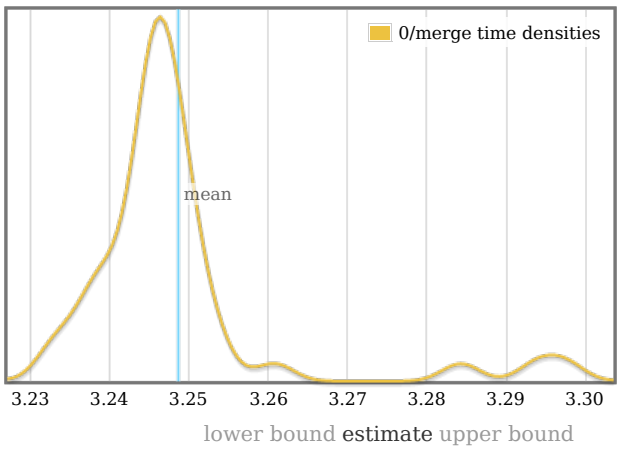
overview

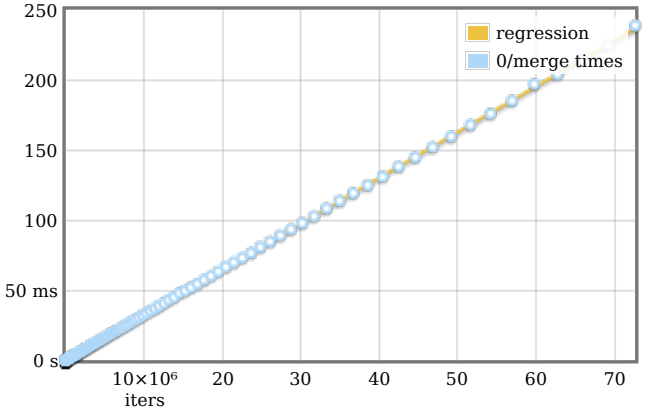
want to understand this report?




4
5
10
20
35
50
75
100
200
500
1000
2000
5000
10000

0/merge

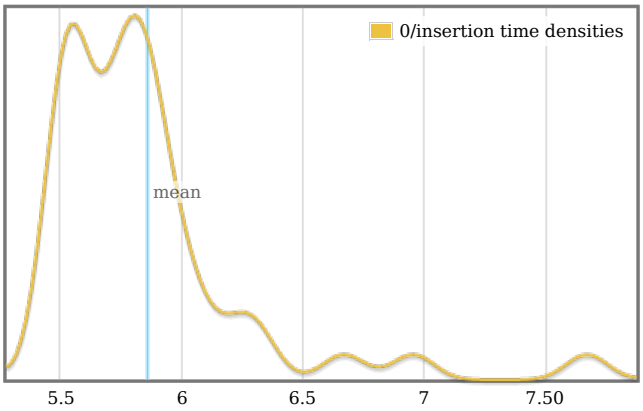

0/merge time densities
mean
3.23
3.24
3.25
3.26
3.27
3.28
3.29
3.30
lower bound estimate upper bound

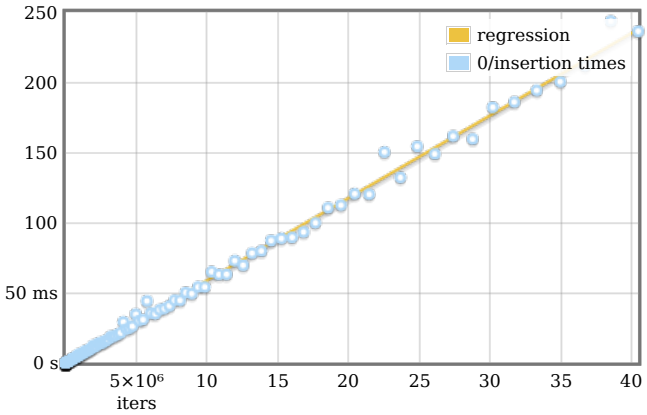

250
200
150
100
50 ms
0 s
regression
0/merge times
10×10⁶
iters
20
30
40
50
60
70

	lower bound	estimate	upper bound
OLS regression	3.24 ns	3.25 ns	3.26 ns
R² goodness-of-fit	1.000	1.000	1.000
Mean execution time	3.25 ns	3.25 ns	3.25 ns
Standard deviation	8.39 ps	13.0 ps	19.2 ps

Outlying measurements have no (0.3%) effect on estimated standard deviation.

0/insertion

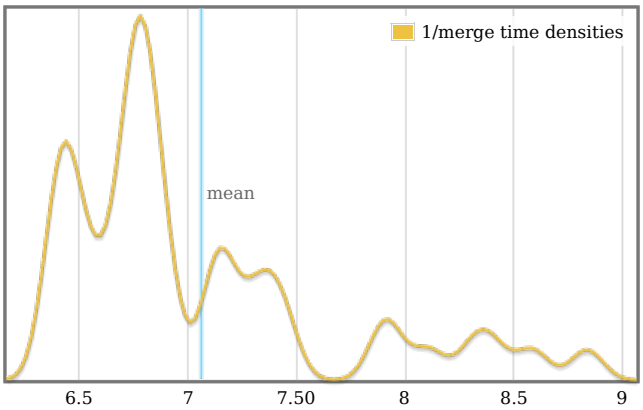


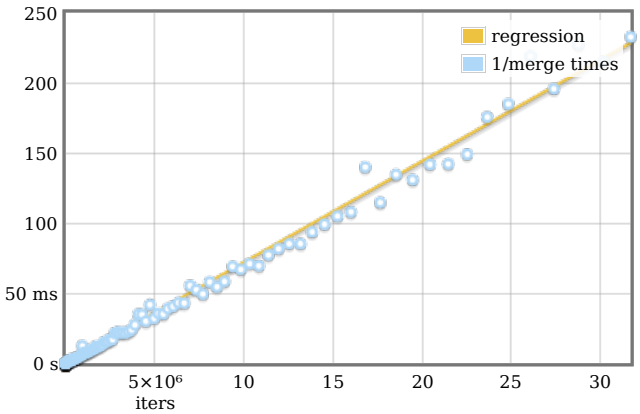


	lower bound	estimate	upper bound
OLS regression	5.77 ns	5.87 ns	6.00 ns
R² goodness-of-fit	0.996	0.998	0.999
Mean execution time	5.76 ns	5.86 ns	5.99 ns
Standard deviation	270 ps	417 ps	639 ps

Outlying measurements have severe (85.7%) effect on estimated standard deviation.

1/merge

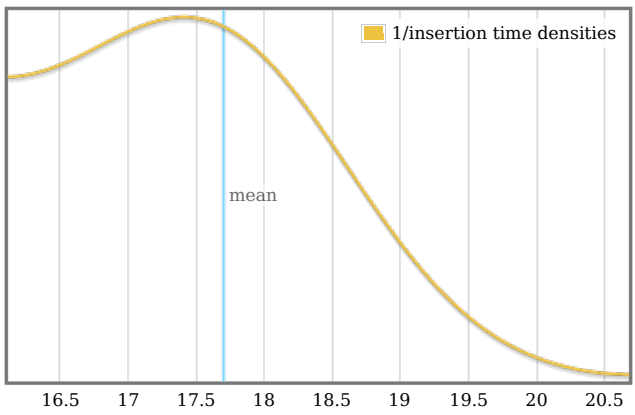


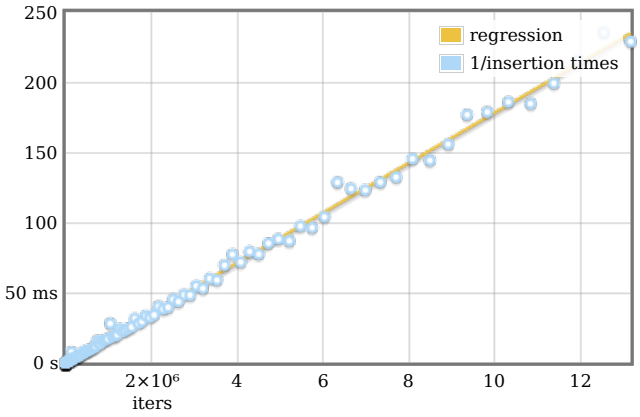


	lower bound	estimate	upper bound
OLS regression	6.94 ns	7.20 ns	7.44 ns
R² goodness-of-fit	0.990	0.993	0.997
Mean execution time	6.92 ns	7.06 ns	7.29 ns
Standard deviation	482 ps	637 ps	822 ps

Outlying measurements have severe (90.5%) effect on estimated standard deviation.

1/insertion

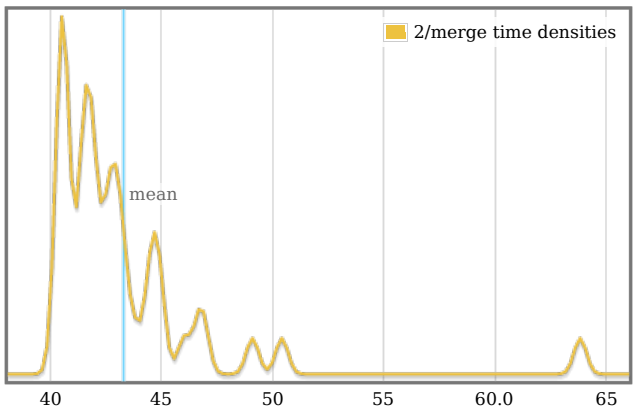


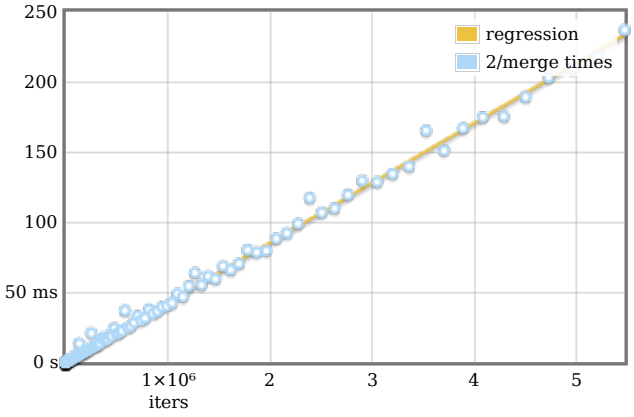


	lower bound	estimate	upper bound
OLS regression	17.5 ns	17.8 ns	18.1 ns
R² goodness-of-fit	0.996	0.998	0.999
Mean execution time	17.4 ns	17.7 ns	17.9 ns
Standard deviation	734 ps	924 ps	1.19 ns

Outlying measurements have severe (75.1%) effect on estimated standard deviation.

2/merge

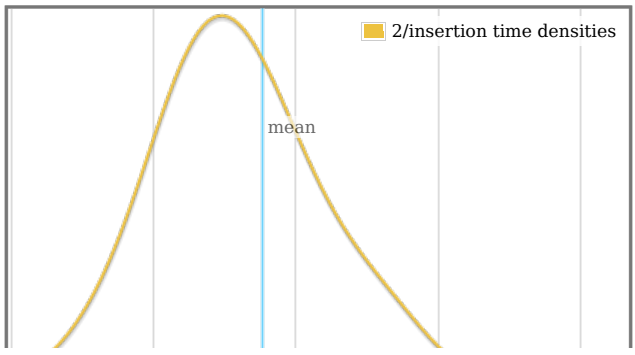


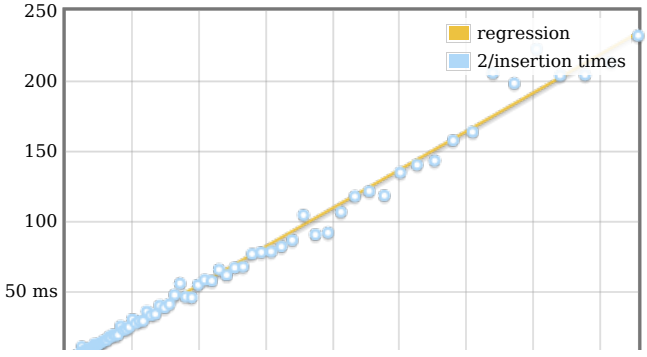


	lower bound	estimate	upper bound
OLS regression	42.1 ns	42.6 ns	43.3 ns
R² goodness-of-fit	0.996	0.998	0.999
Mean execution time	42.5 ns	43.3 ns	45.6 ns
Standard deviation	2.16 ns	3.93 ns	6.72 ns

Outlying measurements have severe (89.7%) effect on estimated standard deviation.

2/insertion

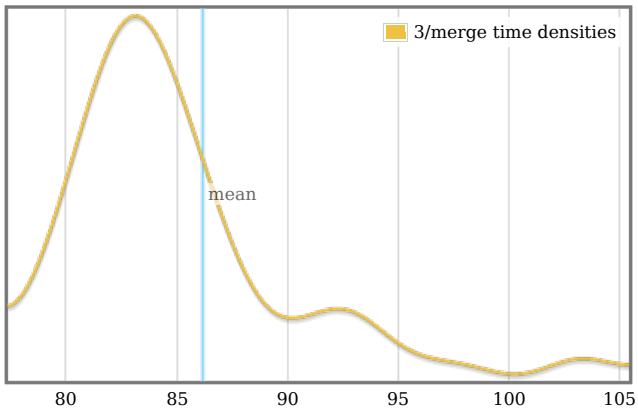


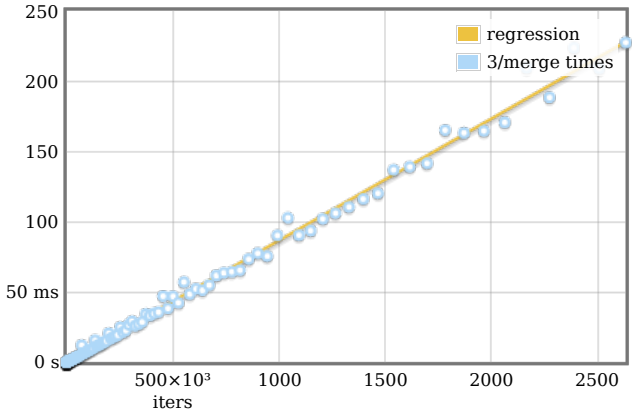


	lower bound	estimate	upper bound
OLS regression	52.9 ns	54.6 ns	56.4 ns
R² goodness-of-fit	0.990	0.993	0.998
Mean execution time	52.8 ns	53.8 ns	55.2 ns
Standard deviation	3.22 ns	4.05 ns	5.39 ns

Outlying measurements have severe (85.2%) effect on estimated standard deviation.

3/merge

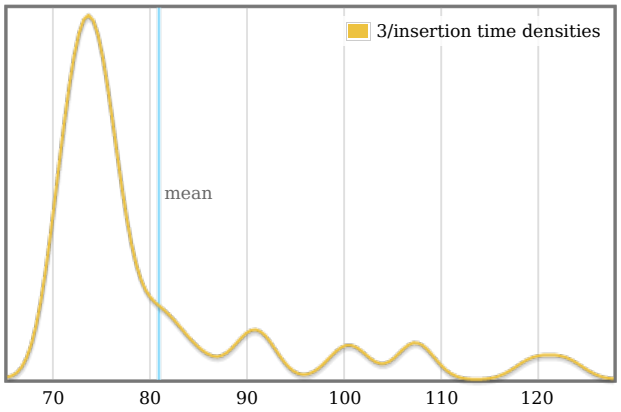


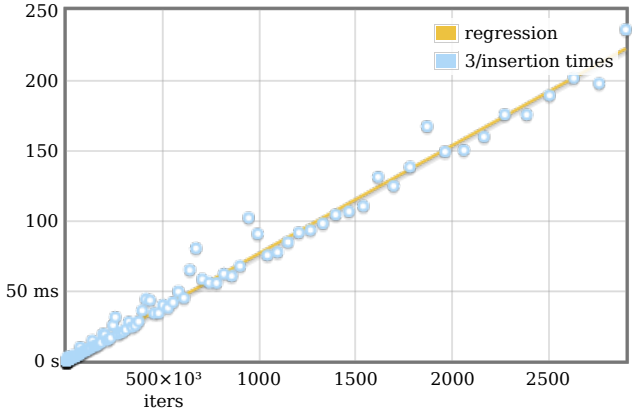


	lower bound	estimate	upper bound
OLS regression	84.5 ns	86.5 ns	88.8 ns
R² goodness-of-fit	0.994	0.996	0.998
Mean execution time	84.3 ns	86.2 ns	88.0 ns
Standard deviation	4.49 ns	5.99 ns	8.07 ns

Outlying measurements have severe (82.7%) effect on estimated standard deviation.

3/insertion

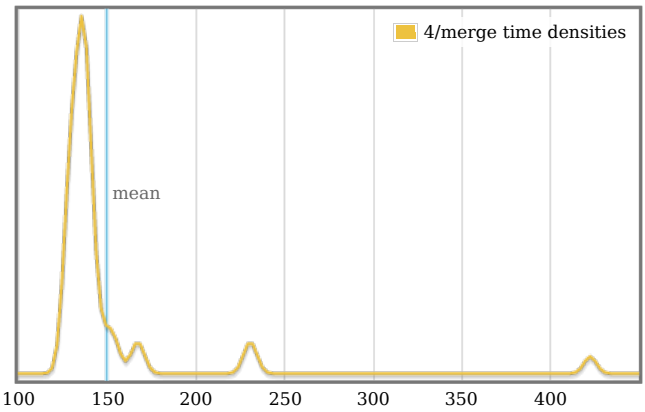


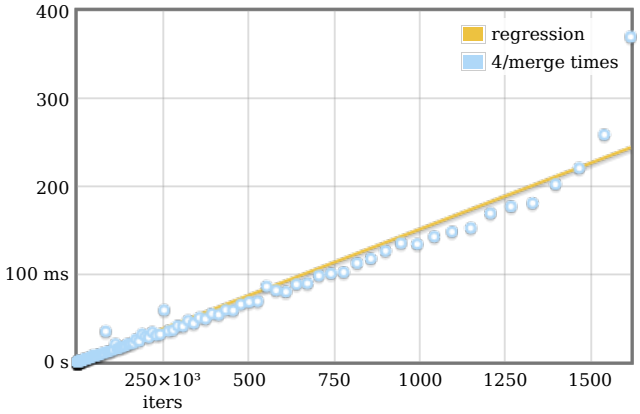


	lower bound	estimate	upper bound
OLS regression	74.8 ns	76.6 ns	78.8 ns
R² goodness-of-fit	0.984	0.991	0.996
Mean execution time	77.8 ns	80.9 ns	85.2 ns
Standard deviation	9.52 ns	13.1 ns	17.8 ns

Outlying measurements have severe (96.4%) effect on estimated standard deviation.

4/merge

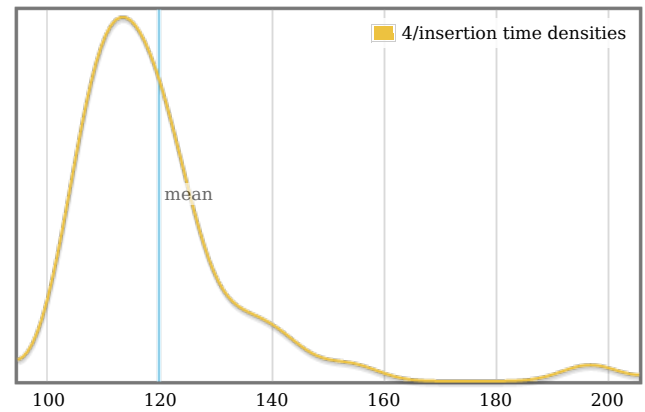


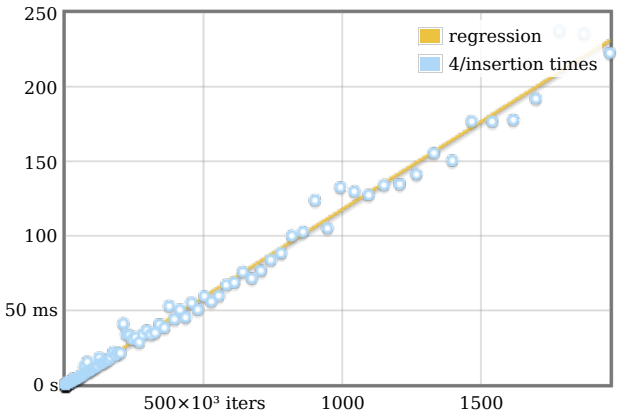


	lower bound	estimate	upper bound
OLS regression	138 ns	151 ns	169 ns
R² goodness-of-fit	0.936	0.961	0.997
Mean execution time	142 ns	149 ns	178 ns
Standard deviation	20.8 ns	47.0 ns	91.4 ns

Outlying measurements have severe (99.1%) effect on estimated standard deviation.

4/insertion

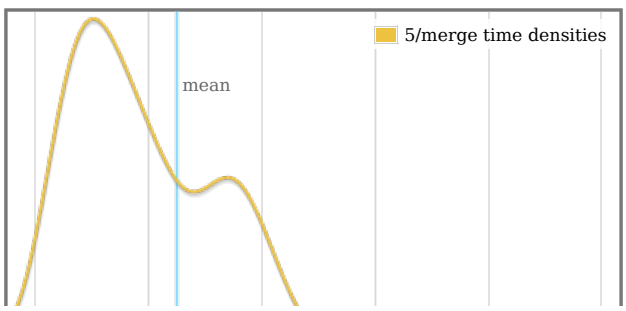


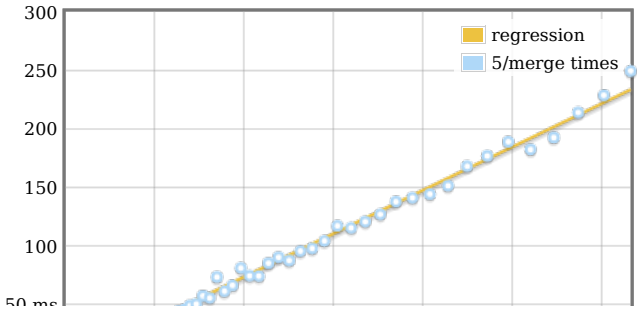


	lower bound	estimate	upper bound
OLS regression	114 ns	118 ns	122 ns
R² goodness-of-fit	0.991	0.993	0.997
Mean execution time	117 ns	120 ns	127 ns
Standard deviation	9.62 ns	15.7 ns	28.0 ns

Outlying measurements have severe (94.6%) effect on estimated standard deviation.

5/merge

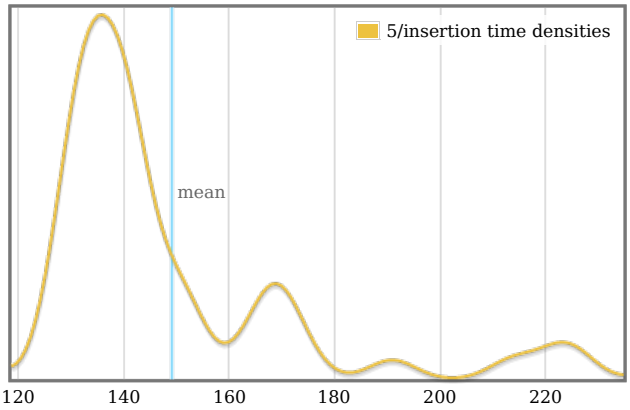


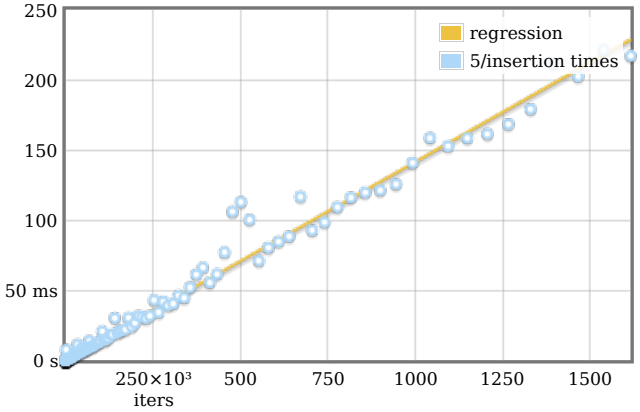


	lower bound	estimate	upper bound
OLS regression	181 ns	184 ns	188 ns
R² goodness-of-fit	0.997	0.998	0.999
Mean execution time	180 ns	182 ns	187 ns
Standard deviation	7.58 ns	10.2 ns	13.9 ns

Outlying measurements have severe (74.1%) effect on estimated standard deviation.

5/insertion

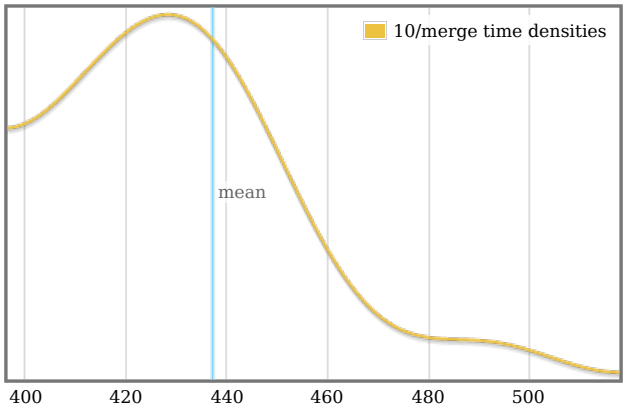


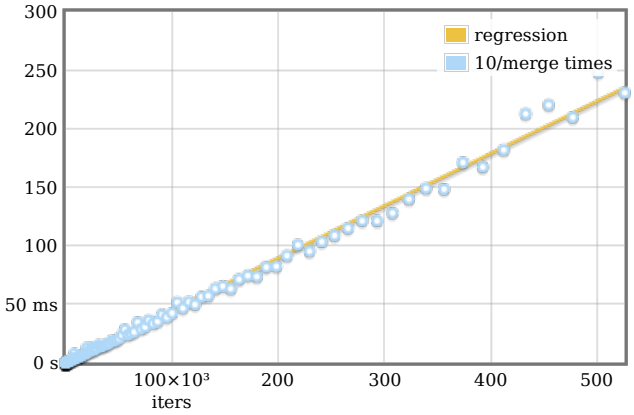


	lower bound	estimate	upper bound
OLS regression	138 ns	141 ns	146 ns
R² goodness-of-fit	0.975	0.987	0.996
Mean execution time	143 ns	149 ns	157 ns
Standard deviation	15.6 ns	24.0 ns	32.5 ns

Outlying measurements have severe (96.2%) effect on estimated standard deviation.

10/merge

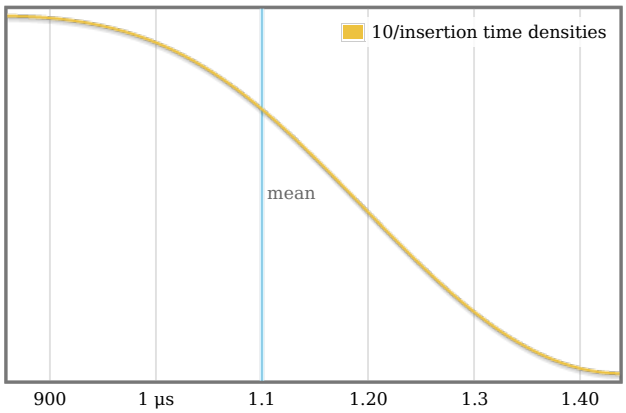


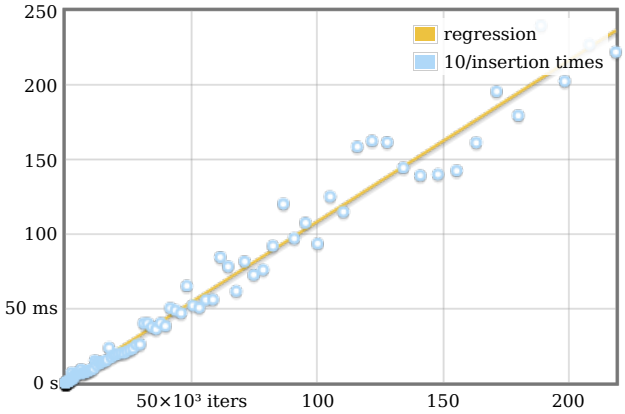


	lower bound	estimate	upper bound
OLS regression	433 ns	447 ns	459 ns
R² goodness-of-fit	0.994	0.996	0.999
Mean execution time	430 ns	437 ns	446 ns
Standard deviation	19.9 ns	26.1 ns	32.7 ns

Outlying measurements have severe (75.2%) effect on estimated standard deviation.

10/insertion

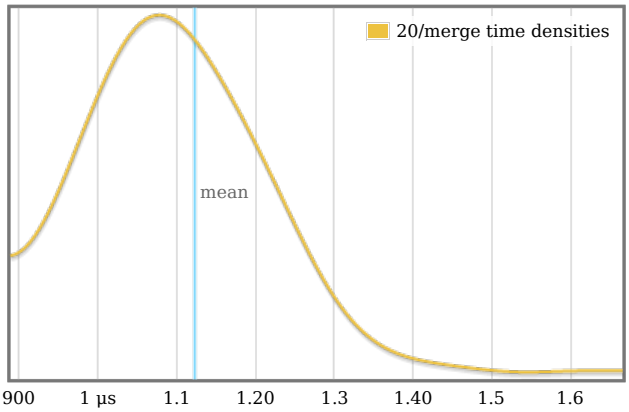


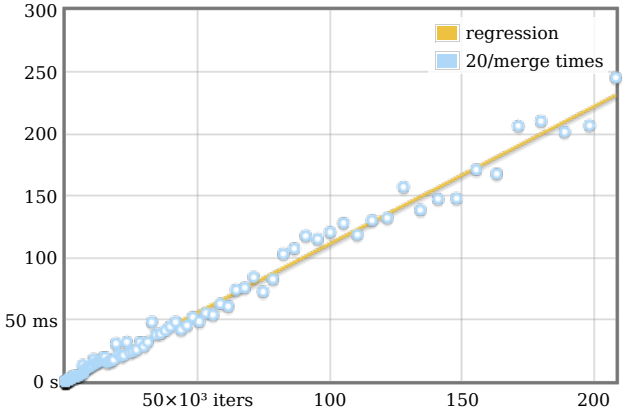


	lower bound	estimate	upper bound
OLS regression	1.04 µs	1.08 µs	1.14 µs
R² goodness-of-fit	0.976	0.983	0.990
Mean execution time	1.06 µs	1.10 µs	1.15 µs
Standard deviation	120 ns	141 ns	165 ns

Outlying measurements have severe (92.9%) effect on estimated standard deviation.

20/merge

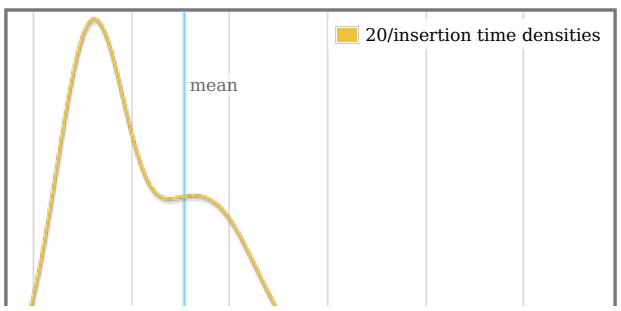


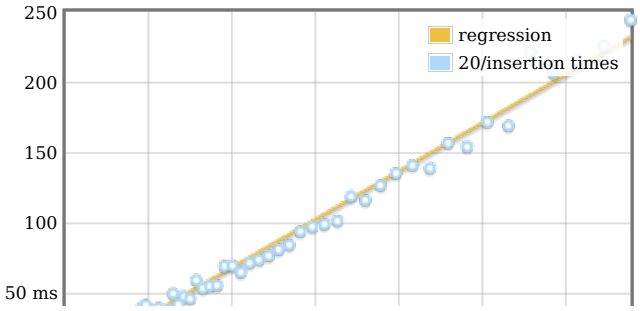


	lower bound	estimate	upper bound
OLS regression	1.07 µs	1.11 µs	1.15 µs
R² goodness-of-fit	0.990	0.992	0.995
Mean execution time	1.08 µs	1.12 µs	1.16 µs
Standard deviation	98.7 ns	133 ns	187 ns

Outlying measurements have severe (91.9%) effect on estimated standard deviation.

20/insertion

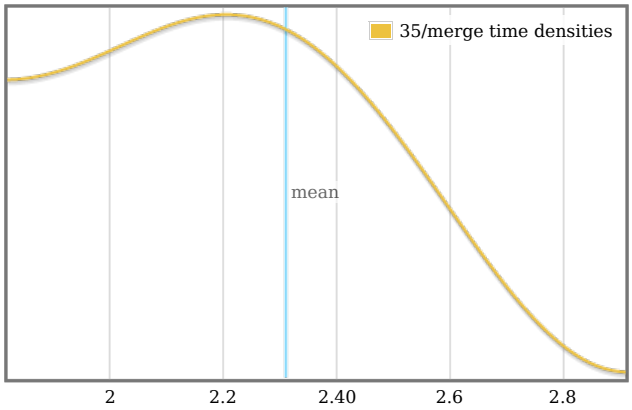


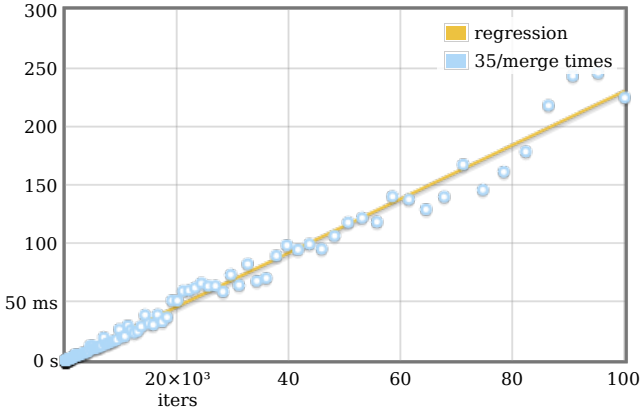


	lower bound	estimate	upper bound
OLS regression	3.32 µs	3.42 µs	3.52 µs
R² goodness-of-fit	0.990	0.994	0.997
Mean execution time	3.30 µs	3.38 µs	3.48 µs
Standard deviation	246 ns	315 ns	409 ns

Outlying measurements have severe (85.8%) effect on estimated standard deviation.

35/merge

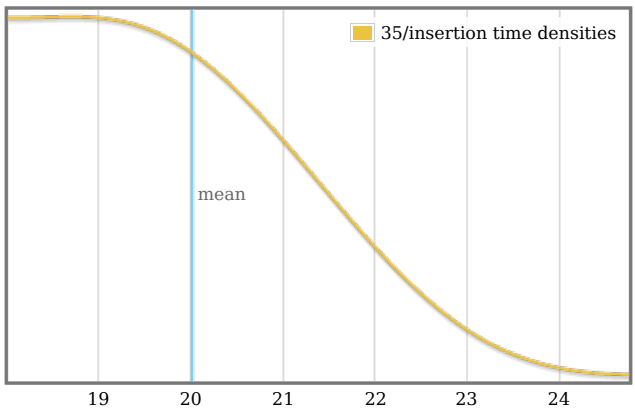


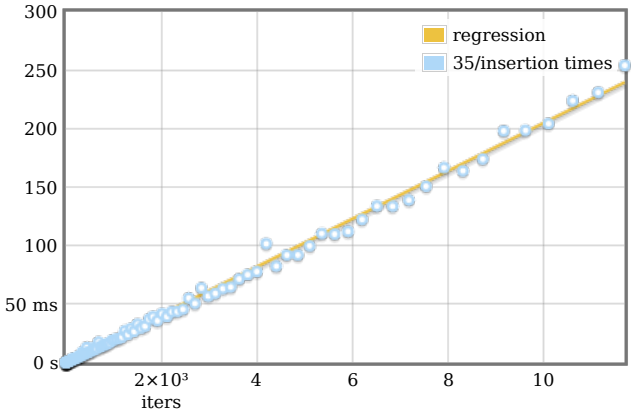


	lower bound	estimate	upper bound
OLS regression	2.20 µs	2.30 µs	2.41 µs
R² goodness-of-fit	0.983	0.987	0.993
Mean execution time	2.22 µs	2.31 µs	2.38 µs
Standard deviation	225 ns	261 ns	298 ns

Outlying measurements have severe (90.7%) effect on estimated standard deviation.

35/insertion

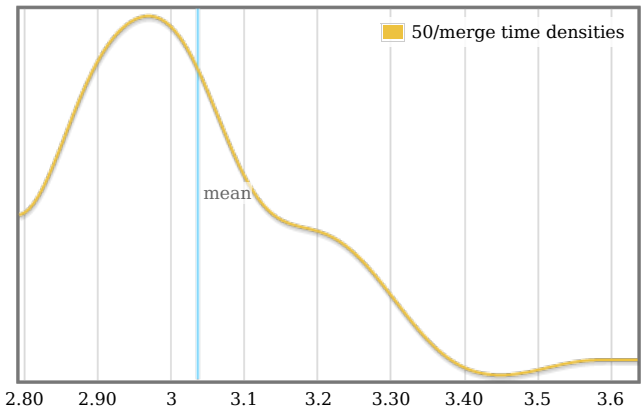


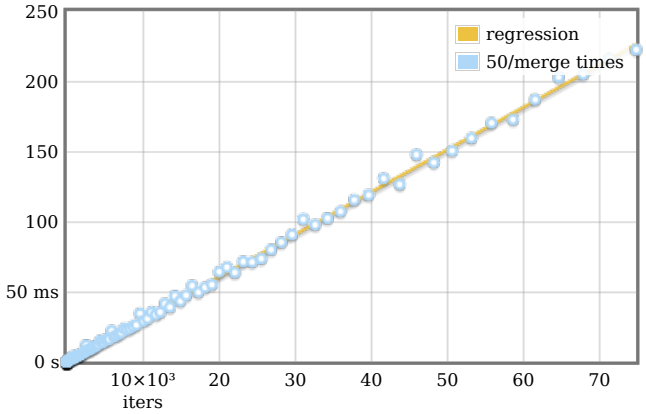


	lower bound	estimate	upper bound
OLS regression	20.0 µs	20.5 µs	20.9 µs
R² goodness-of-fit	0.995	0.997	0.998
Mean execution time	19.6 µs	20.0 µs	20.4 µs
Standard deviation	1.04 µs	1.25 µs	1.88 µs

Outlying measurements have severe (68.7%) effect on estimated standard deviation.

50/merge

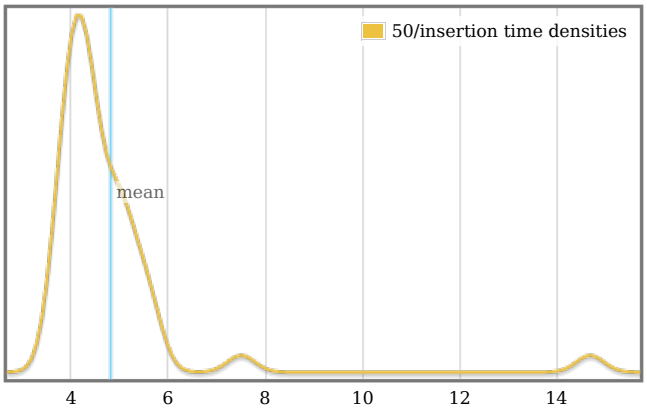


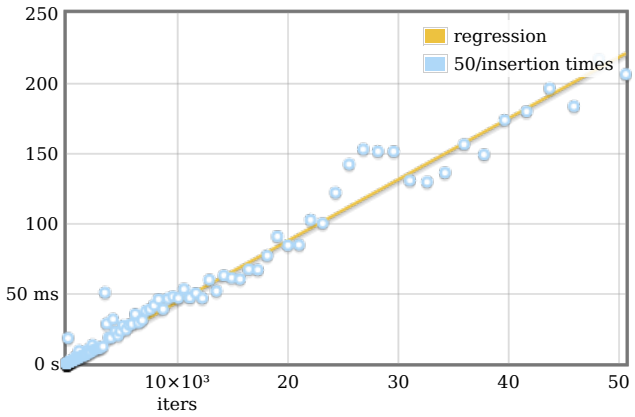


	lower bound	estimate	upper bound
OLS regression	2.99 µs	3.02 µs	3.06 µs
R² goodness-of-fit	0.998	0.999	0.999
Mean execution time	3.00 µs	3.04 µs	3.08 µs
Standard deviation	120 ns	151 ns	204 ns

Outlying measurements have severe (63.5%) effect on estimated standard deviation.

50/insertion

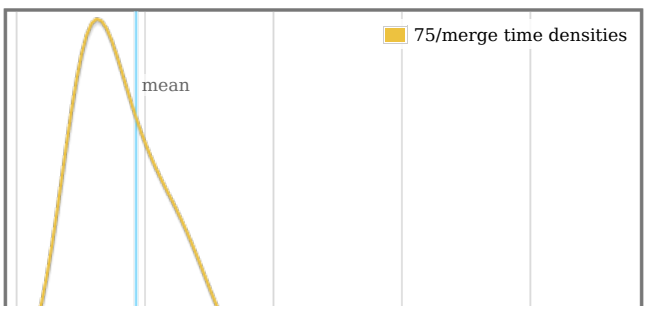


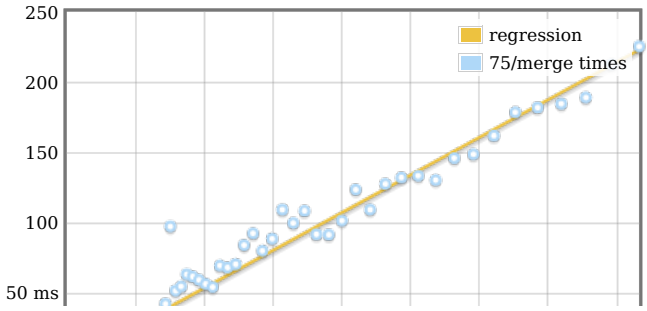


	lower bound	estimate	upper bound
OLS regression	4.21 µs	4.37 µs	4.62 µs
R² goodness-of-fit	0.972	0.981	0.990
Mean execution time	4.52 µs	4.83 µs	5.75 µs
Standard deviation	577 ns	1.64 µs	3.25 µs

Outlying measurements have severe (98.8%) effect on estimated standard deviation.

75/merge

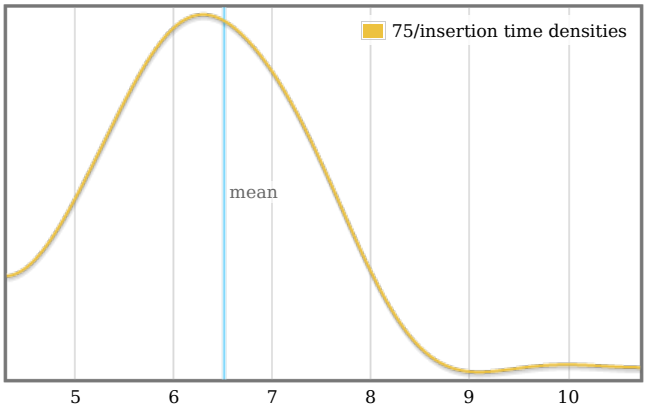


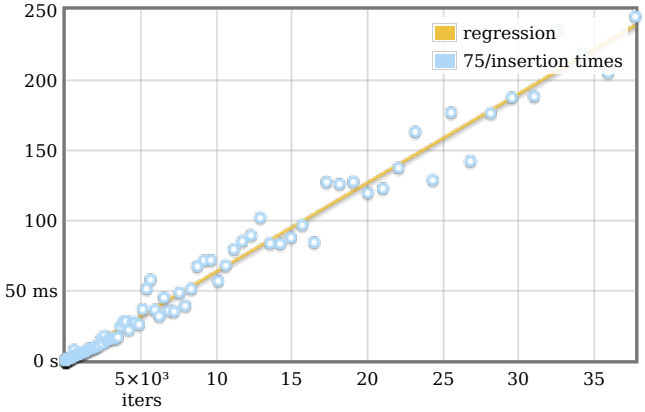


	lower bound	estimate	upper bound
OLS regression	5.25 µs	5.35 µs	5.52 µs
R² goodness-of-fit	0.961	0.983	0.994
Mean execution time	5.59 µs	5.85 µs	6.50 µs
Standard deviation	661 ns	1.33 µs	2.54 µs

Outlying measurements have severe (97.6%) effect on estimated standard deviation.

75/insertion

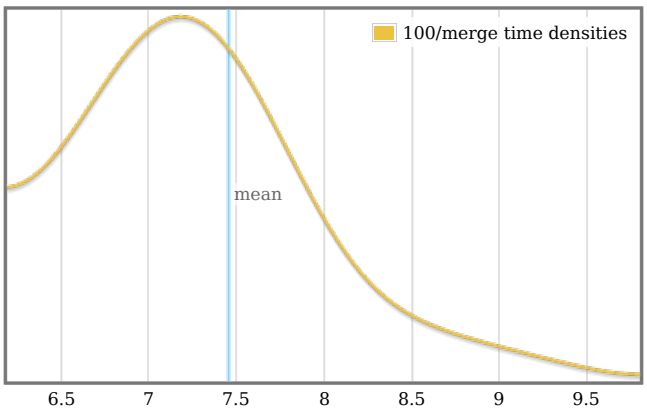


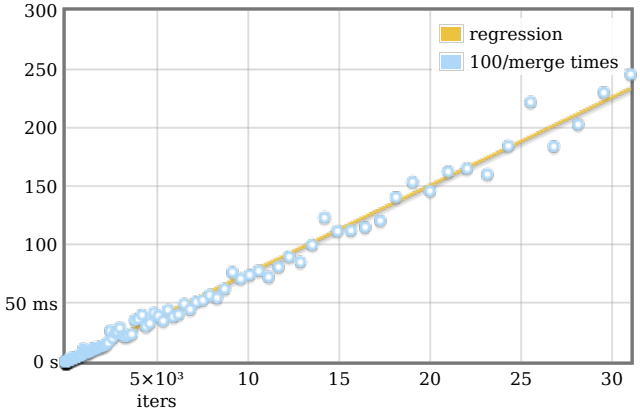


	lower bound	estimate	upper bound
OLS regression	6.05 µs	6.32 µs	6.57 µs
R² goodness-of-fit	0.977	0.985	0.991
Mean execution time	6.15 µs	6.51 µs	6.85 µs
Standard deviation	821 ns	1.11 µs	1.52 µs

Outlying measurements have severe (95.1%) effect on estimated standard deviation.

100/merge

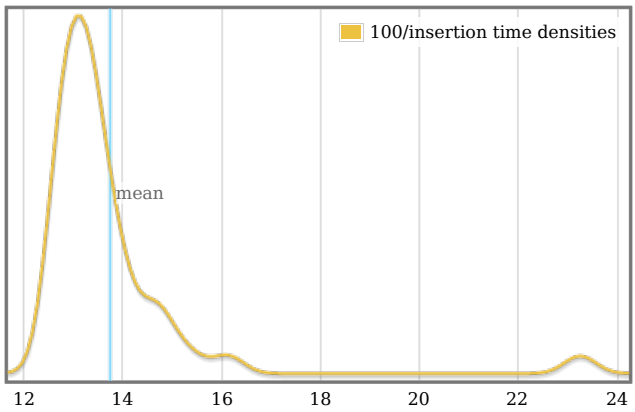


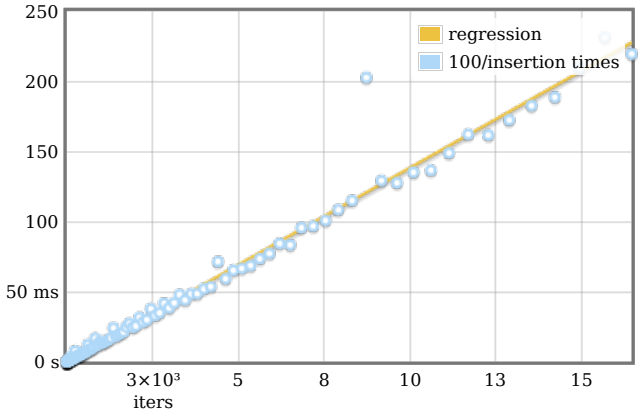


	lower bound	estimate	upper bound
OLS regression	7.26 µs	7.52 µs	7.72 µs
R² goodness-of-fit	0.989	0.992	0.996
Mean execution time	7.28 µs	7.45 µs	7.71 µs
Standard deviation	627 ns	764 ns	936 ns

Outlying measurements have severe (87.0%) effect on estimated standard deviation.

100/insertion

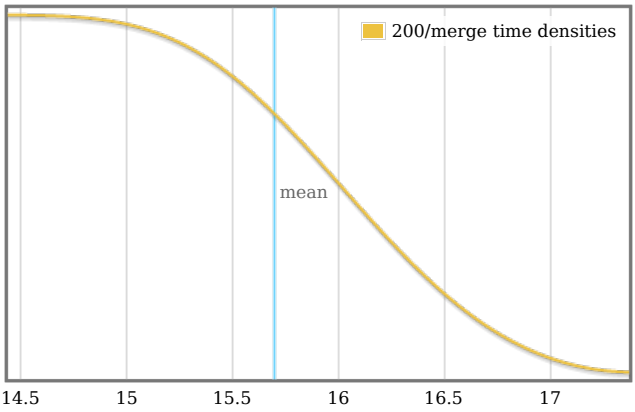


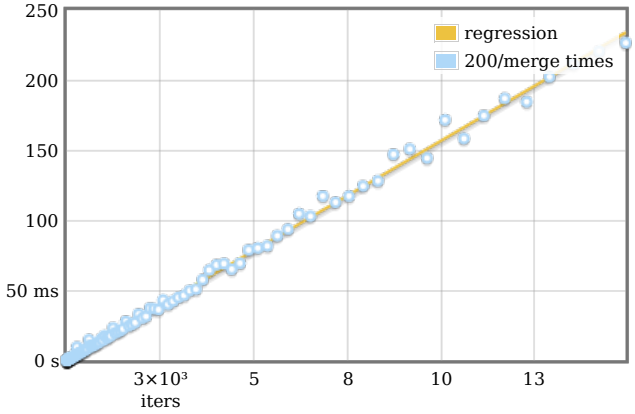


	lower bound	estimate	upper bound
OLS regression	13.4 µs	13.8 µs	14.5 µs
R² goodness-of-fit	0.951	0.981	0.999
Mean execution time	13.4 µs	13.8 µs	14.7 µs
Standard deviation	737 ns	1.70 µs	3.34 µs

Outlying measurements have severe (90.5%) effect on estimated standard deviation.

200/merge

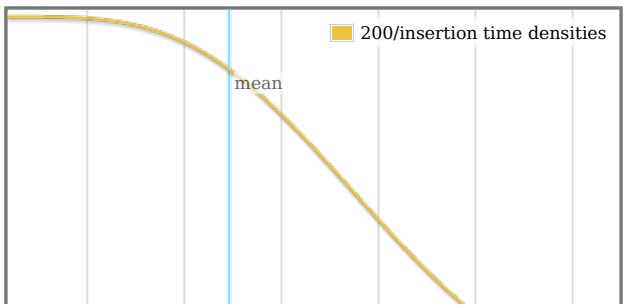


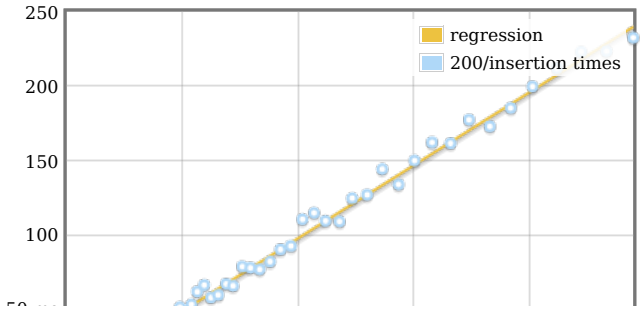


	lower bound	estimate	upper bound
OLS regression	15.5 µs	15.7 µs	15.9 µs
R² goodness-of-fit	0.997	0.998	0.999
Mean execution time	15.5 µs	15.7 µs	16.0 µs
Standard deviation	672 ns	777 ns	897 ns

Outlying measurements have severe (58.5%) effect on estimated standard deviation.

200/insertion

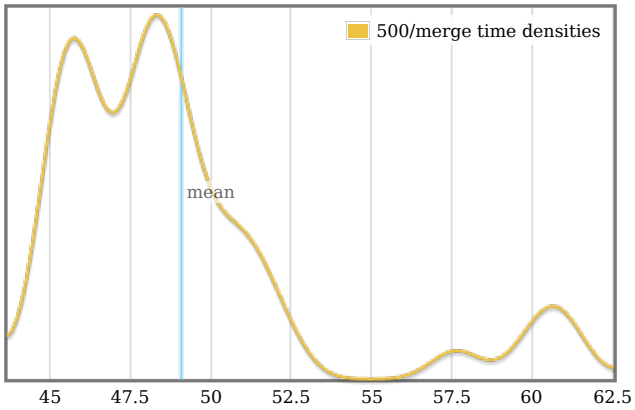


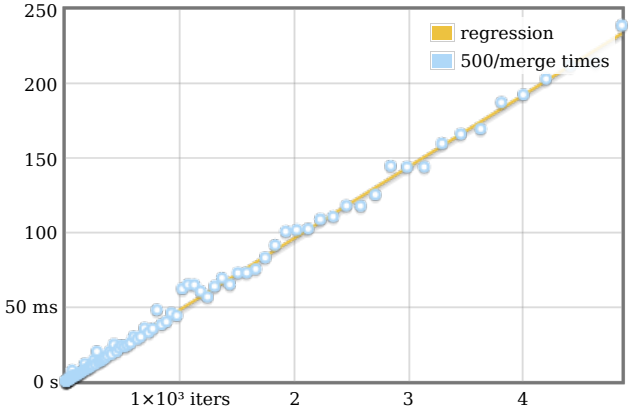


	lower bound	estimate	upper bound
OLS regression	481 µs	489 µs	498 µs
R² goodness-of-fit	0.995	0.996	0.998
Mean execution time	482 µs	489 µs	500 µs
Standard deviation	23.3 µs	28.3 µs	34.3 µs

Outlying measurements have severe (51.4%) effect on estimated standard deviation.

500/merge

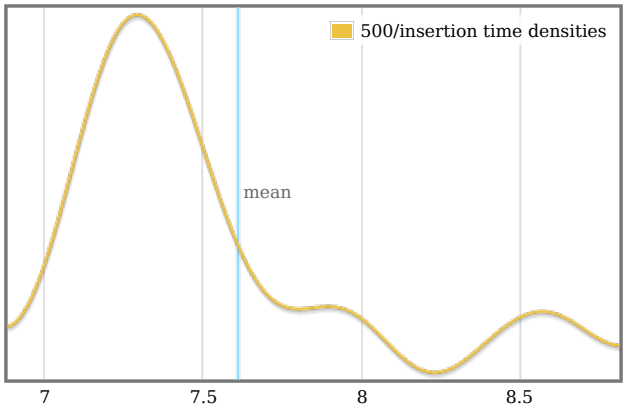


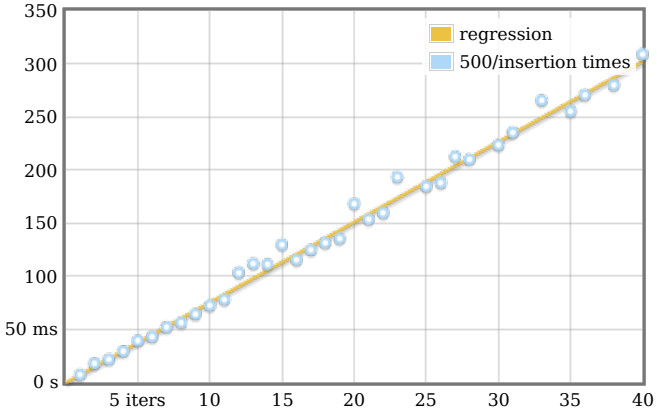


	lower bound	estimate	upper bound
OLS regression	47.5 µs	48.1 µs	48.7 µs
R² goodness-of-fit	0.995	0.997	0.998
Mean execution time	48.1 µs	49.1 µs	50.7 µs
Standard deviation	2.72 µs	4.06 µs	5.60 µs

Outlying measurements have severe (77.7%) effect on estimated standard deviation.

500/insertion

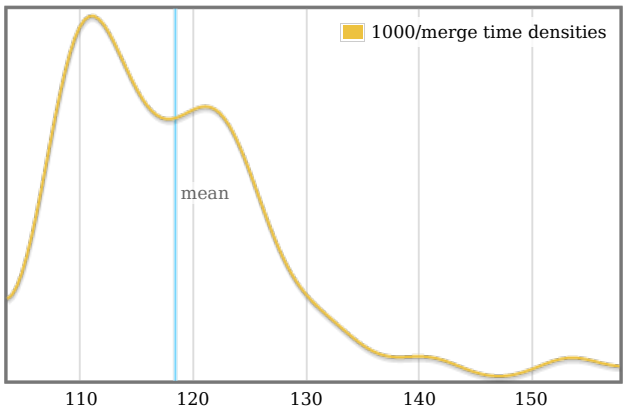


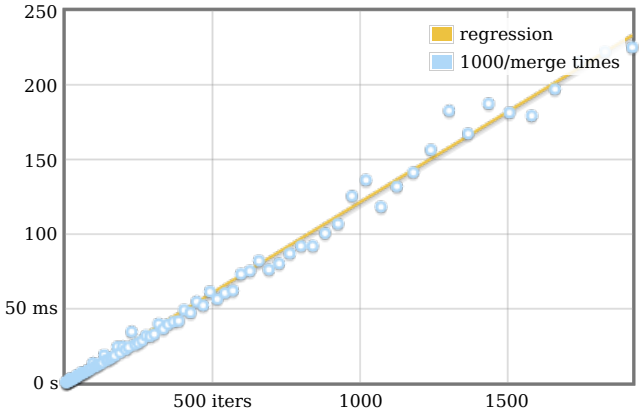


	lower bound	estimate	upper bound
OLS regression	7.36 ms	7.55 ms	7.78 ms
R² goodness-of-fit	0.984	0.991	0.996
Mean execution time	7.47 ms	7.61 ms	7.87 ms
Standard deviation	382 µs	496 µs	617 µs

Outlying measurements have moderate (36.7%) effect on estimated standard deviation.

1000/merge

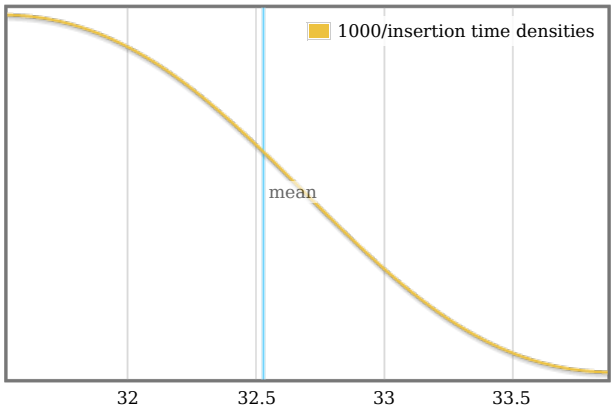


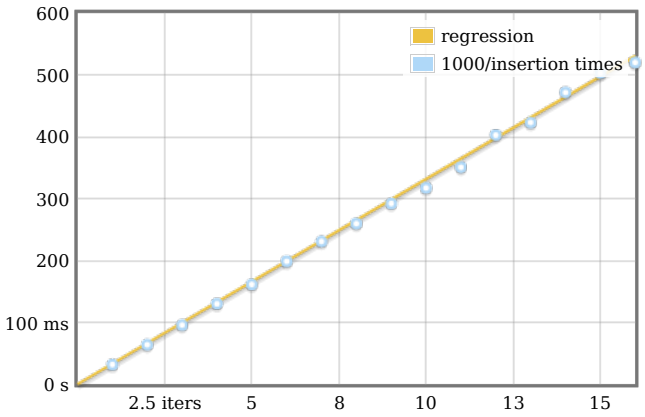


	lower bound	estimate	upper bound
OLS regression	119 µs	121 µs	125 µs
R² goodness-of-fit	0.991	0.994	0.997
Mean execution time	116 µs	118 µs	121 µs
Standard deviation	7.16 µs	9.43 µs	13.4 µs

Outlying measurements have severe (73.1%) effect on estimated standard deviation.

1000/insertion

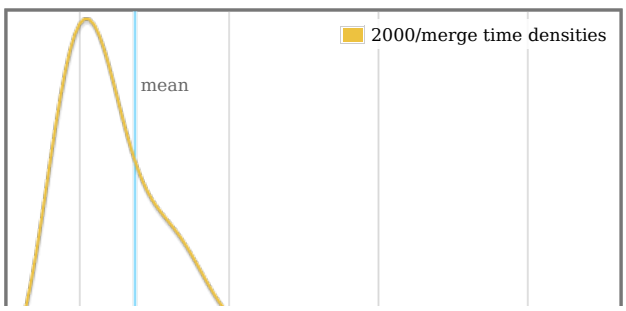


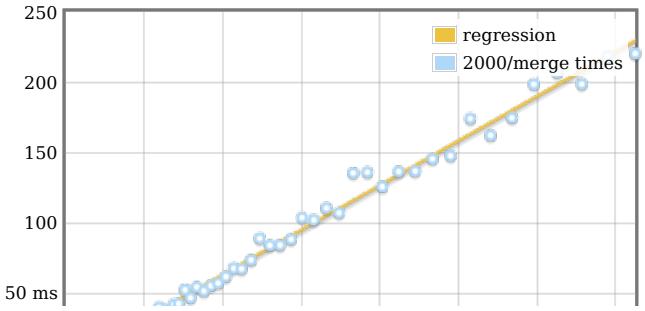


	lower bound	estimate	upper bound
OLS regression	32.5 ms	33.2 ms	33.8 ms
R² goodness-of-fit	0.997	0.998	0.999
Mean execution time	32.3 ms	32.5 ms	32.9 ms
Standard deviation	498 µs	661 µs	866 µs

Outlying measurements have slight (5.9%) effect on estimated standard deviation.

2000/merge

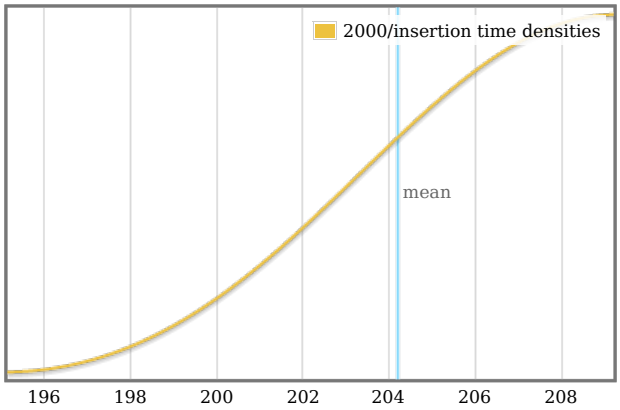


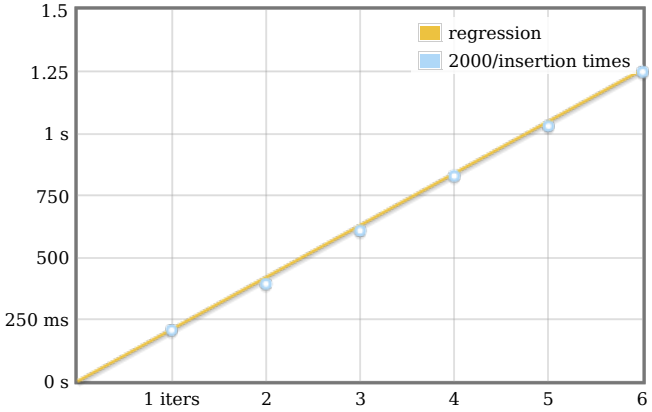


	lower bound	estimate	upper bound
OLS regression	309 µs	316 µs	325 µs
R² goodness-of-fit	0.992	0.994	0.997
Mean execution time	311 µs	318 µs	331 µs
Standard deviation	18.6 µs	30.2 µs	51.2 µs

Outlying measurements have severe (76.3%) effect on estimated standard deviation.

2000/insertion

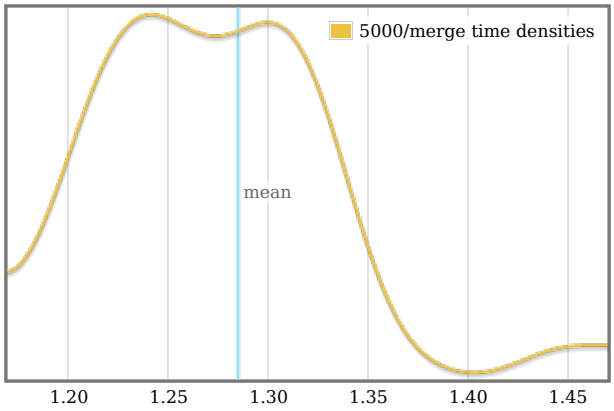


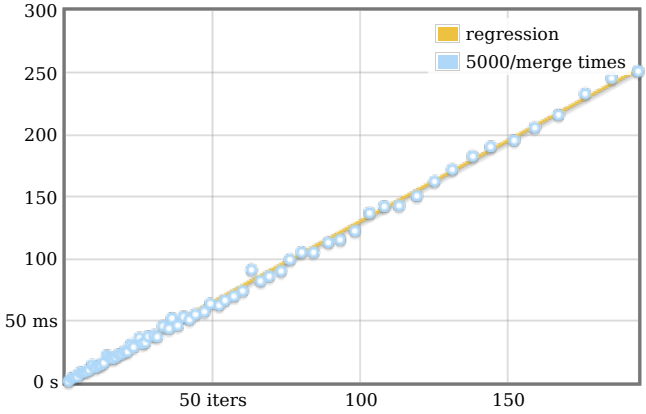


	lower bound	estimate	upper bound
OLS regression	206 ms	210 ms	214 ms
R² goodness-of-fit	0.998	1.000	1.000
Mean execution time	200 ms	204 ms	206 ms
Standard deviation	1.73 ms	4.30 ms	5.86 ms

Outlying measurements have moderate (13.9%) effect on estimated standard deviation.

5000/merge

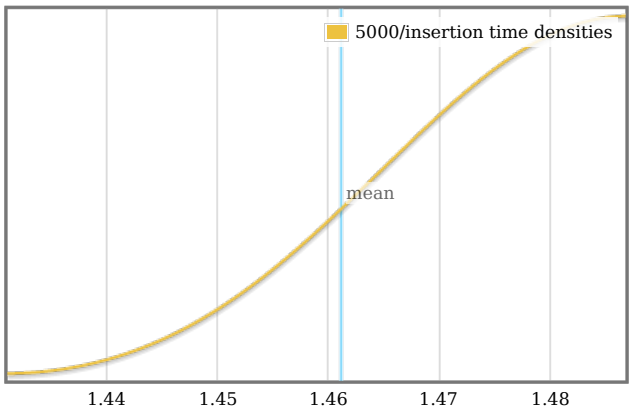


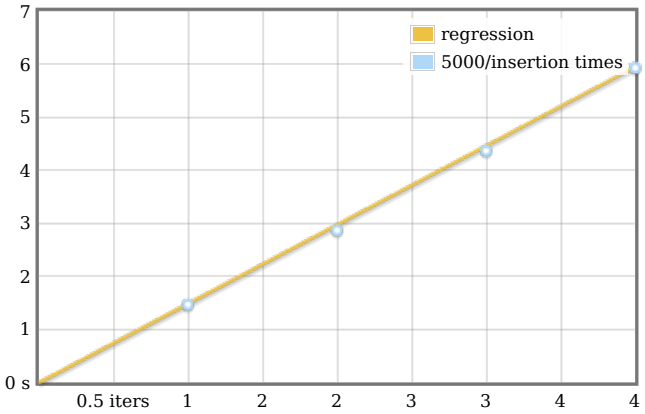


	lower bound	estimate	upper bound
OLS regression	1.29 ms	1.30 ms	1.31 ms
R² goodness-of-fit	0.997	0.998	0.999
Mean execution time	1.27 ms	1.28 ms	1.31 ms
Standard deviation	50.5 µs	64.4 µs	81.9 µs

Outlying measurements have moderate (37.9%) effect on estimated standard deviation.

5000/insertion

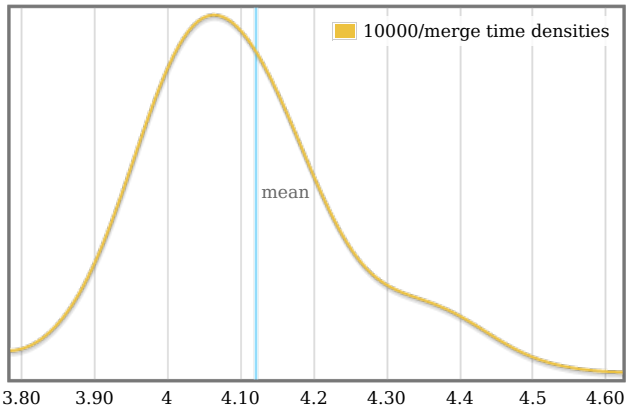


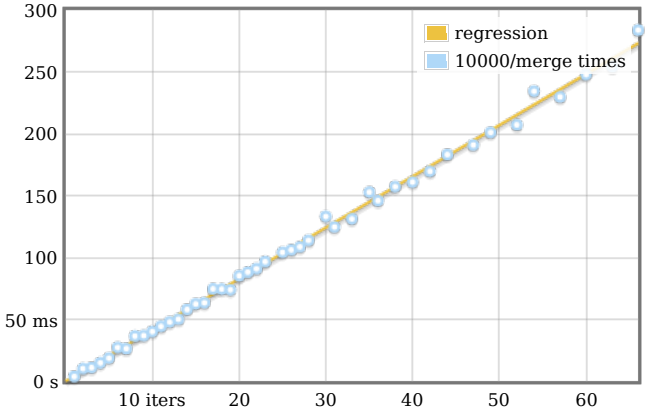


	lower bound	estimate	upper bound
OLS regression	1.40 s	1.49 s	1.56 s
R² goodness-of-fit	0.999	0.999	1.000
Mean execution time	1.44 s	1.46 s	1.47 s
Standard deviation	6.88 ms	19.9 ms	26.8 ms

Outlying measurements have moderate (18.8%) effect on estimated standard deviation.

10000/merge

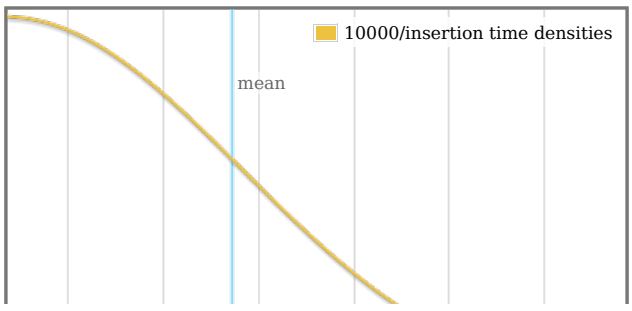


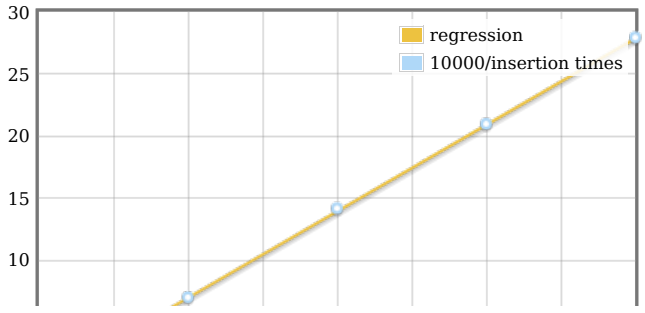


	lower bound	estimate	upper bound
OLS regression	4.05 ms	4.14 ms	4.23 ms
R² goodness-of-fit	0.995	0.997	0.998
Mean execution time	4.08 ms	4.12 ms	4.18 ms
Standard deviation	118 µs	152 µs	213 µs

Outlying measurements have moderate (19.4%) effect on estimated standard deviation.

10000/insertion





	lower bound	estimate	upper bound
OLS regression	6.79 s	6.95 s	7.17 s
R² goodness-of-fit	1.000	1.000	1.000
Mean execution time	6.98 s	7.01 s	7.06 s
Standard deviation	4.54 ms	49.3 ms	62.5 ms

Outlying measurements have moderate (18.8%) effect on estimated standard deviation.

understanding this report

In this report, each function benchmarked by criterion is assigned a section of its own. The charts in each section are active; if you hover your mouse over data points and annotations, you will see more details.

- The chart on the left is a kernel density estimate (also known as a KDE) of time measurements. This graphs the probability of any given time measurement occurring. A spike indicates that a measurement of a particular time occurred; its height indicates how often that measurement was repeated.
- The chart on the right is the raw data from which the kernel density estimate is built. The *x* axis indicates the number of loop iterations, while the *y* axis shows measured execution time for the given number of loop iterations. The line behind the values is the linear regression prediction of execution time for a given number of iterations. Ideally, all measurements will be on (or very near) this line.

Under the charts is a small table. The first two rows are the results of a linear regression run on the measurements displayed in the right-hand chart.

- *OLS regression* indicates the time estimated for a single loop iteration using an ordinary least-squares regression model. This number is more accurate than the *mean* estimate below it, as it more effectively eliminates measurement overhead and other constant factors.
- *R² goodness-of-fit* is a measure of how accurately the linear regression model fits the observed measurements. If the measurements are not too noisy, R² should lie between 0.99 and 1, indicating an excellent fit. If the number is below 0.99, something is confounding the accuracy of the linear model.
- *Mean execution time* and *standard deviation* are statistics calculated from execution time divided by number of iterations.

We use a statistical technique called the bootstrap to provide confidence intervals on our estimates. The bootstrap-derived upper and lower bounds on estimates let you see how accurate we believe those estimates to be. (Hover the mouse over the table headers to see the confidence levels.)

A noisy benchmarking environment can cause some or many measurements to fall far from the mean. These outlying measurements can have a significant inflationary effect on the estimate of the standard deviation. We calculate and display an estimate of the extent to which the standard deviation has been inflated by outliers.

colophon

This report was created using the criterion benchmark execution and performance analysis tool.

Criterion is developed and maintained by Bryan O'Sullivan.