1: Assume $R = \{(a, b)|a$ divides $b\}$ is a relation on the set $\{1, 2, 4, 6\}$

(a) List all the ordered pairs in the relation.

(b) Display the relation graphically, as was done in Example 4 of Sec 9.1 (Page 601).

(c) Display this relation in tabular form, as was done in Example 4 of Sec 9.1 (Page 601).

2: Determine whether the relation R on the set of all number is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if

(a) $x + y = 0$

(b) $x = \pm y$

(c) $x - y$ is a rational number

(d) $x = 2y$

(e) $xy \geq 0$

(f) $xy = 0$

(g) $x = 1$

(h) $x = 1$ or $y = 1$

3: Represent each of these relations on $\{1,2,3,4,5\}$ with a matrix (with the elements of this set listed in increasing order).

(a) $\{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4),(4,5)\}$

(b) $\{(1,1),(1,4),(2,2),(3,3),(4,1),(5,2)\}$

4: List the ordered pairs in the relations on $\{1, 2, 3\}$ corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

5: Draw the directed graph

(a) representing each of the relations from the above exercise.

(b) representing each of the reflexive closures of these relations.

(c) representing each of the symmetric closures of these relations.

6: Which of these relations on the set of all people are equivalence relations?

(a) $\{(a, b)|$ a and b are the same age$\}$

(b) $\{(a, b)|$ a and b have the same parents$\}$

(c) $\{(a, b)|$ a and b share a common parent$\}$

(d) $\{(a, b)|$ a and b have met$\}$

(e) $\{(a, b)|$ a and b speak a common language$\}$

7: Let $G = (V, T, S, P)$ be the phrase-structure grammar with $V = \{0, 1, A, B, S\}$, $T = \{0, 1\}$, and the set of production P consisting of $S \to 0A$, $S \to 1A$, $A \to 0B$, $B \to 1A$, and $B \to 1$.

(a) Show that 10101 belongs to the language generated by G.

(b) Show that 10110 does not belong to the language generated by G.

(c) What is the language generated by G?

8: Find the phrase-structure grammar for each of these languages.

(a) The set consisting of the bit strings 0, 1, and 11.

(b) The set of bit strings containing only 1s

(c) The set of bit strings that start with 0 and end with 1

(d) The set of bit strings that consist of a 0 followed by an even number of 1s

9: Determine whether the string 11011 is in each of these sets.

(a) $\{0, 1\}^*$

(b) $\{1\}^* \{0\}^* \{1\}^*$

(c) $\{11\} \{0\}^* \{011\}$

(d) $\{11\}^* \{01\}^*$

(e) $\{111\}^* \{0\}^* \{1\}$

(f) $\{11, 0\} \{00, 101\}$

10: Construct a deterministic finite-state automaton that recognizes the set of all bit strings beginning with 01.

11: Construct a deterministic finite-state automaton that recognizes the set of all bit strings that contain the string 110.

12: Find the language recognized by the given nondeterministic finite-state automaton.