1: How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29$ where each x_i is a nonnegative integer such that

(a) $x_i > 2$?

(b) $x_1 \geq 1, x_2 \geq 1, x_3 \geq 2, x_4 \geq 2, x_5 > 3, and x_6 \geq 3$

(c) $x_1 \leq 4$

(d) $x_1 < 7$ and $x_2 > 7$

2: How many strings of 20-decimal digits are there containing two 0s, four 1s, three 2s, two 3, two 4s, two 5s, two 7s, and three 9s.

3: How many solutions are there to the inequality $x_1 + x_2 + x_3 \leq 10$, where x_1, x_2, and x_3 are nonnegative integers?

4: In bridge, the 52 cards of a standard deck are dealt to four players. How many ways are there to deal bridge hands to four players?

5: What is the probability that a card selected at random from a standard deck of 52 cards is an ace or spade?

6: What is the probability that a seven-card hand contains the ace of hearts?

7: How many ways are there to order the evaluation of the product of n matrices: $M_1 M_2 \dots M_n$? For example, with two matrices, we have $(M_1 M_2) M_3$ and $M_1 (M_2 M3)$.

8: Which is more likely: rolling a total of 9 when two dice are rolled or rolling a total of 9 when three dice are rolled?

9: What is the probability of these events when we randomly select a permutation of $\{1, 2, \ldots, n\}$ where $n \geq 4$?

(a) 1 precedes 3.

(b) 3 precedes 1.

(c) 3 immediately precedes 2.

(d) n precedes 1 and $n - 1$ precedes 2

(e) $n - 1$ precedes 1 and n precedes 2

10: What is the probability of these events when we randomly select a permutation of the 26 lowercase letters of the English alphabet?

(a) The first 13 letters of the permutation are in alphabetical order

(b) a is the first letter of the permutation and z is the last letter

(c) a and z are next to each other in the permutation.

(d) a and b are not next to each other in the permutation.

(e) a and z are separated by at least 23 letters in the permutation.

(f) z precedes both a and b in the permutation

11: Find each of the following probabilities when n independent Bernoulli trials are carried out with probability of success p.

(a) the probability of no successes

(b) the probability of at least one success

(c) the probability of at most one success

(d) the probability of at least two successes

12: Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 200 elements in A_1, 1000 in A_2, and $5,000$ in A_3 if

(a) $A_1 \subseteq A_2$ and $A_2 \subseteq A_3$.

(b) the sets are pairwise disjoint.

(c) There are two elements in common to each pair of sets and one element in all three sets.