1: Convert the decimal expansion of each of these integers to a binary expansion.

(a) 320

(b) 1022

(c) 100633

2: Convert the binary expansion of each of these integers to an octal expansion.

(a) $(1111\ 0110)_2$

(b) $(1010\ 1010\ 1011)_2$

(c) $(111\ 0111\ 0111\ 1111)_2$

(d) $(101\ 0101\ 0101\ 1101)_2$

3: Convert the hexadecimal expansion of each of these integers to a binary expansion.

(a) $(80F)_{16}$

(b) $(135BB)_{16}$

(c) $(ABBA)_{16}$

(d) $(DEFACED)_{16}$

4: Use Fermat's little theorem to find 23^{1041} mod 41.

5: Let $P(n)$ be the statement $1^3 + 2^3 + \cdots + n^3 = (n(n+1)/2)^2$ for the positive integer n.

(a) What is the statement $P(1)$?

(b) Show that $P(1)$ is true, completing the basis step of the proof.

(c) What is the inductive hypothesis?

(d) What do you need to prove in the inductive step?

(e) Complete the inductive step, identifying where you use the inductive hypothesis.

6: Find a formula for

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)}$$

by examining the values of this expression for small values of n.

(a) Show the formula.

(b) Prove the formula you conjectured in part (a) with mathematical induction.

7: Prove that for every positive integer n,
$1 \cdot 2 \cdot 3 \cdot 4 + 2 \cdot 3 \cdot 4 \cdot 5 + \cdots + n(n+1)(n+2)(n+3) = n(n+1)(n+2)(n+3)(n+4)/5$**.**

8: Let $P(n)$ be the statement that $n! < n^n$ where n is an integer greater than 1.

(a) What is the statement $P(2)$?

(b) Show that $P(2)$ is true, completing the basis step of the proof.

(c) What is the inductive hypothesis?

(d) What do you need to prove in the inductive step?

(e) Complete the inductive step.

9: Let $P(n)$ be the statement that a postage of n stamps can be formed using just 4-cent and 7-cent stamps. the parts in this exercise outline an induction proof that $P(n)$ is true for $n \geq 18$.

(a) Show the statement $P(18)$ is true, completing the basis step of the proof.

(b) What is the inductive hypothesis of the proof?

(c) What do you need to prove in the inductive step.

(d) Complete the inductive step for $k \geq 18$. Hint: you may discuss two cases, when there is at least one 7-cent stamp for $P(k)$, and when k is a multiple of 4.