
1: How many bit strings are there of length five or less, not counting the empty string?

2: How many strings of seven lowercase English letters are there

- (a) if letters can be repeated?
- (b) if no letter can be repeated?
- (c) that start with z, if letters can be repeated?
- (d) that start with z, if no letter can be repeated?
- (e) that start or end with the letters bo (in that order), if letters can be repeated?

3: How many different functions are there from a set with 8 elements to sets with the following numbers of elements?

(a) 2

(b) 3

4: A bowl contains 5 red balls and 5 blue balls. A woman selects balls at random without looking at them.

- (a) How many balls must she select to be sure of having at least four balls of the same color?
- (b) How many balls must she select to be sure of having at least three blue balls?

5: Find the value of each of these quantities.

(a) $P(5, 3)$

(b) $P(6, 5)$

(c) $P(8, 5)$

6: Find the value of each of these quantities.

(a) $C(5, 0)$

(b) $C(5, 3)$

(c) $C(8, 4)$

7: How many permutations of the letters $ABCDEFGH$ contain

- (a) the string ED ?
- (b) the strings BA and FGH ?
- (c) the strings CAB and BED ?
- (d) the strings BCA and ABF ?

8: The row of Pascal's triangle containing the binomial coefficients $\binom{10}{k}$, $0 \leq k \leq 10$, is: 1 10 45 210 252 210 120 45 10 1. Use Pascal's identity to produce the row immediately following this row of Pascal's triangle.

9: Prove that if n and k are integers with $1 \leq k \leq n$ then $k \binom{n}{k} = n \binom{n-1}{k-1}$,

(a) using a combinatorial proof.

(b) using an algebraic proof based on the formula for $\binom{n}{r}$.