1: Let $F(x, y)$ be the statement "y can fool x," where the domain consists of all people in the world. Use quantifiers to express each of these statements.

(a) Everybody can fool Fred.

(c) Everybody can fool somebody.

(e) Everyone can be fooled by somebody.

(g) Nancy can fool exactly two people.

(i) No one can fool himself or herself.

2: Express each of these statements using mathematical and logical operators, predicates, and quantifiers, where the domain consists of all integers.

(a) The sum of two positive integers is positive.

(b) The difference of two negative integers is not necessarily negative.

(c) The sum of the squares of two integers is greater than or equal to the square of their sum.

(d) The absolute value of the product of two integers is the product of their absolute values.

3: Express the negation of each of these statements so that all negation symbols immediately precede predicates.

(a) $\forall x \exists y \exists z T(x, y, z)$

(b) $\forall x \exists y P(x, y) \vee \forall y \exists x Q(x, y)$

(c) $\forall x \exists y (P(x, y) \wedge \exists z R(x, y, z))$

(d) $\forall x \exists y (P(x, y) \rightarrow Q(x, y))$

4: What is wrong with this argument? Let $S(x, y)$ be "x is shorter than y." Given the premise $\exists s S(s, Mark)$, it follows that $S(Mark, Mark)$. Then by existential generalization it follows that $\exists x S(x, x)$, so that someone is shorter than him/herself.

5: Identify the error or errors in this argument that supposedly shows that if $\forall x(P(x) \vee Q(x))$ is true then $\forall x P(x) \vee \forall x Q(x)$ is true.

1.	$\forall x(P(x) \vee Q(x))$	Premise
2.	$P(c) \vee Q(c)$	Universal specification from (1)
3.	$P(c)$	Simplification from (2)
4.	$\forall x P(x)$	Universal generalization from (3)
5.	$Q(c)$	Simplification from (2)
6.	$\forall x Q(x)$	Universal generalization from (5)
7.	$\forall x P(x) \vee \forall x Q(x)$	Conjunction from (4) and (6)

6: Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$ and $\forall x((\neg P(x) \wedge Q(x)) \rightarrow \neg R(x))$ are true, then $\forall x(R(x) \rightarrow P(x))$ is also true, where the domains of all quantifiers are the same.

7: Prove that the product of two odd numbers is odd. For this problem and other problems, note that unless we specifically say that a formal proof is needed, you do not have to specify the rules of inference used, and can skip obvious steps.

8: Prove or disprove that if a and b are rational numbers, then a^b is also rational.