1: Find $f(1)$, $f(2)$, $f(3)$, $f(4)$ and $f(5)$ if f is defined recursively by $f(0) = 2$ and for $n = 1, 2, \ldots$

(a) $f(n+1) = -2f(n)$

(b) $f(n+1) = 3f(n) + 7$

(c) $f(n+1) = f(n)^2 - 2f(n) - 2$

(d) $f(n+1) = 2^{f(n)/2}$

2: Give a recursive definition of the sequence a_n, $n = 1, 2, 3, \ldots$ if

(a) $a_n = 3n - 2$

(b) $a_n = 1 - (-1)^n$

(c) $a_n = (n-1)n$

(d) $a_n = n^2 - 1$

3: Give a recursive definition of

(a) the set of even positive integers.

(b) The set of positive integer powers of 4.

(c) The set of polynomials with integer coefficients.

4: Give a recursive definition of w^i where w is a string and i is a nonnegative integer. (Here w^i represents the concatenation of i copies of the string w.)

5: Solve these recurrence relations together with the initial conditions given.

(a) $a_n = a_{n-1} + 6a_{n-2}$ for $n \geq 2$, $a_0 = 0$, $a_1 = 5$

(b) $a_n = 7a_{n-1} - 10a_{n-2}$ for $n \geq 2$, $a_0 = 3$, $a_1 = 0$

(c) $a_n = 2a_{n-1} - a_{n-2}$ for $n \geq 2$, $a_0 = 3$, $a_1 = 1$

(d) $a_n = -a_{n-2}$ for $n \geq 2$, $a_0 = 0$, $a_1 = -2$. Hint: You may need complex numbers.

6: Find the solution to $a_n = 5a_{n-2} - 4a_{n-4}$ with $a_0 = 3$, $a_1 = 2$, $a_2 = 6$, and $a_3 = 8$.