1: The $n \times n$ matrix $\mathbf{A} = [a_{ij}]$ is called a diagonal matrix if $a_{ij} = 0$ when $i \neq j$. Show that the product of two $n \times n$ diagonal matrices is again a diagonal matrix. Give a simple rule for determining this product.

2: Let $\mathbf{A}$ be 2×2 matrix $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Show that if $ad - bc \neq 0$ then $\mathbf{A}^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$.

3: Find the Boolean product of A and B, where $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ **and** $\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$

4: Find the least integer n such that $f(x)$ is $O(x^n)$ for each of these functions.

(a) $f(x) = 2x^2 + x^4 \log x$

(b) $f(x) = 3x^4 + (\log x)^6$

(c) $f(x) = (x^4 + x^2 + 1)/(x^4 + 1)$

(d) $f(x) = (x^3 + 5 \log x)/(x^5 + 1)$

5: Show that if a, b, c, and d are integers, where $a \neq 0$, such that $a|c$ and $b|d$, then $ab|cd$.

6: What are the quotient and the remainder when

(a) 45 is divided by 8?

(b) 777 is divided by 21?

(c) -124 is divided by 19?

(d) -1 is divided by 23?

(e) -2001 is divided by 87?

(f) 0 is divided by 17?

(g) 1,234,567 is divided by 1001?

(h) -100 is divided by 101?

7: Suppose that a and b are integers, $a \equiv 11 \pmod{19}$ and $b \equiv 3 \pmod{19}$. Find the integer c with $0 \le c \le 18$ such that

(a) $c \equiv 13a \pmod{19}$

(b) $c \equiv 8b \pmod{19}$

(c) $c \equiv a - b \pmod{19}$.

(d) $c \equiv 7a + 3b \pmod{19}$.

(e) $c \equiv 2a^2 + 3b^2 \pmod{19}$.

(f) $c \equiv a^3 + 4b^3 \pmod{19}$.

8: Find the prime factorization of each of these integers.

(a) 39

(b) 81

(c) 100

(d) 143

(e) 289

9: What are the greatest common divisors of these pairs of integers.

(a) $2^3 \cdot 3^3 \cdot 5^5$, $2^5 \cdot 3^3 \cdot 5^2$

(b) $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$, $2^{11} \cdot 3^9 \cdot 11 \cdot 17^{14}$

(c) 17, 17^{17}

(d) $2^2 \cdot 7$, $5^3 \cdot 13$

(e) $0, 500$

(f) $2 \cdot 3 \cdot 5 \cdot 7$, $2 \cdot 3 \cdot 5 \cdot 7$