1: Determine whether the function $f(n) = \lceil n/2 \rceil$ from $\mathbb{Z}$ to $\mathbb{Z}$ is one-to-one, and whether it is onto.

2: Find the first five terms of the sequence defined by each of these recurrence relations and initial conditions.

(a) $a_n = 5a_{n-1}$, $a_0 = 2$

(b) $a_n = a_{n-1}^2$, $a_1 = 2$

(c) $a_n = 3a_{n-1} + a_{n-2}$, $a_0 = 1$, $a_1 = 2$

(d) $a_n = na_{n-1} + a_{n-2}^2$, $a_0 = 1$, $a_1 = 1$

(e) $a_n = a_{n-1} - a_{n-2} + a_{n-3}$, $a_0 = 1$, $a_1 = 2$

3: Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if

(a) $a_n = 0$

(b) $a_n = 1$

(c) $a_n = (-4)^n$

(d) $a_n = 2(-4)^n + 2$

4: Suppose that the number of bacteria in a colony doubles every hour.

(a) Set up a recurrence relation for the number of bacteria after n hours have elapsed.

(b) If 100 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours.

5: Show that $\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0$ where $a_0, \cdots, a_n$ is a sequence of real numbers. This type of sum is called telescoping.

6: Use the identity $k(k+1) = \frac{1}{3}k(k+1)((k+2)-(k-1)) = \frac{1}{3}k(k+1)(k+2) - \frac{1}{3}(k-1)k(k+1)$ **and the previous question to compute** $\sum_{k=1}^{n} k(k+1)$**.**

7: Determine whether each of these sets is countable or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.

(a) all bit strings not containing the bit 0

(b) all positive rational numbers that cannot be written with denominators less than 5

(c) the real numbers not containing 0 in their decimal representation

(d) The real numbers containing only a finite number of 1s in their decimal representation

8: Give an example of two uncountable sets A and B such that $A - B$ is

(Multiple answers exist but you only need one example for each.)

(a) finite.

(b) countably infinite

(c) uncountable.

9: Show that the set $\mathbb{Q} \times \mathbb{Q}$ is countable.