1: Determine whether each of these biconditional/conditional statements is true or false.

(a) $2 + 2 = 4$ if and only if $1 + 1 = 2$.

(b) $1 + 1 = 2$ if and only if $2 + 3 = 4$.

(c) $1 + 1 = 3$ if and only if monkeys can fly.

(d) $0 > 1$ if and only if $2 > 1$.

(e) If $1 + 1 = 3$, then unicorns exist.

(f) If $1 + 1 = 3$, then dogs can fly.

(g) If $1 + 1 = 2$, then dogs can fly.

(h) If $2 + 2 = 4$, then $1 + 2 = 3$.

2.1: Construct the truth table for each of these compound propositions.

(a) $p \to \neg p$

(b)$p \leftrightarrow \neg p$

(c) $p \oplus (p \vee q)$

(d) $(p \wedge q) \to (p \vee q)$

2.2: Show that each of these conditional statements is a tautology by using truth tables.

(a) $[\neg p \wedge (p \vee q)] \to q$

(b) $[p \wedge (p \to q)] \to q$

3.1: Show that $(p \to q) \land (p \to s)$ and $p \to (q \land s)$ are logically equivalent.

3.2: Show that $(p \land q) \to r$ and $(p \to r) \land (q \to r)$ are not logically equivalent.

4: Determine whether each of these compound propositions is satisfiable.

(a) $(\neg p \vee \neg q) \wedge (p \vee q) \wedge (p \vee \neg q)$

(b) $(p \to q) \wedge (p \to \neg q) \wedge (\neg p \to q) \wedge (\neg p \to \neg q)$

(c) $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$

5: Translate these statements into English, where $K(x)$ is "x is a kangaroo" and $H(x)$ is "x hops" and the domain consists of all animals.

(a) $\forall x(K(x) \to H(x))$

(b) $\forall x(K(x) \land H(x))$

(c) $\exists x(K(x) \to H(x))$

(d) $\exists x(K(x) \land H(x))$

6: Let $Q(x)$ be the statement "$2x + 1 > x$." If the domain consists of all integers, what are these truth values?"

(a) $Q(0)$

(b) $Q(-1)$

(c) $Q(1)$

(d) $\exists x Q(x)$

(e) $\forall x Q(x)$

(f) $\exists x \neg Q(x)$

(g) $\forall x \neg Q(x)$

7: Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

(a) No one is perfect.

(b) Not everyone is perfect.

(c) All your friends are perfect.

(d) At least one of your friends is perfect.

(e) Everyone is your friend and is perfect.

(f) Not everybody is your friend or someone is not perfect.

8: Suppose the domain of the propositional function $P(x, y)$ consists of pairs x and y, where x is 1, 2, or 4 and y is 1, 2, or 4. Write out these propositions using disjunctions and conjunctions.

(a) $\exists x P(x, 3)$

(b) $\forall y P(1, y)$

(c) $\exists y \neg P(2, y)$

(d) $\forall x \neg P(x, 2)$

9: Show that $\forall y P(y) \vee \forall y Q(y)$ and $\forall y (P(y) \vee Q(y))$ are not logically equivalent.