Language Modeling

Introduction to N-grams

Slides taken from

Speech and Language Processing, Martin and Jurafsky, MIT

Probabilistic Language Models

- Today's goal: assign a probability to a sentence
  - Machine Translation:
    - P(**high** winds tonite) > P(**large** winds tonite)
  - Spell Correction
    - The office is about fifteen **minuets** from my house
      - P(about fifteen **minutes** from) > P(about fifteen **minuets** from)
  - Speech Recognition
    - P(I saw a van) >> P(eyes awe of an)
  - + Summarization, question-answering, etc., etc.!!

Why?

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Probabilistic Language Modeling

- Goal: compute the probability of a sentence or sequence of words:

  $$P(W) = P(w_1,w_2,w_3,w_4,w_5 \ldots w_n)$$

- Related task: probability of an upcoming word:

  $$P(w_5|w_1,w_2,w_3,w_4)$$

- A model that computes either of these:

  $P(W)$ or $P(w_n|w_1,w_2 \ldots w_{n-1})$ is called a **language model**.

- Better: **the grammar** But **language model** or **LM** is standard

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How to compute P(W)

- How to compute this joint probability:
  - P(its, water, is, so, transparent, that)
- Intuition: let’s rely on the Chain Rule of Probability

Reminder: The Chain Rule

- Recall the definition of conditional probabilities

$$\mathbf{p(B|A) = P(A,B)/P(A)} \qquad \text{Rewriting:} \quad \mathbf{P(A,B) = P(A)P(B|A)}$$

- More variables:

$$P(A,B,C,D) = P(A)P(B|A)P(C|A,B)P(D|A,B,C)$$

- The Chain Rule in General

$$P(x_1,x_2,x_3,\ldots,x_n) = P(x_1)P(x_2|x_1)P(x_3|x_1,x_2)\ldots P(x_n|x_1,\ldots,x_{n-1})$$

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The Chain Rule applied to compute joint probability of words in sentence

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i \mid w_1 w_2 \ldots w_{i-1})$$

P("its water is so transparent") =

P(its) × P(water|its) × P(is|its water)

P(so|its water is) × P(transparent|its water is so)

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How to estimate these probabilities

- Could we just count and divide?

$$P(\text{the} \mid \text{its water is so transparent that}) = \frac{Count(\text{its water is so transparent that the})}{Count(\text{its water is so transparent that})}$$

- No! Too many possible sentences!
- We'll never see enough data for estimating these

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Markov Assumption

Andrei Markov

- Simplifying assumption:

$$P(\text{the} \mid \text{its water is so transparent that}) \text{ » } P(\text{the} \mid \text{that})$$

- Or maybe

$$P(\text{the} \mid \text{its water is so transparent that}) \text{ » } P(\text{the} \mid \text{transparent that})$$

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N grams

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i \mid w_{i-k} \ldots w_{i-1})$$

- In other words, we approximate each component in the product

$$P(w_i \mid w_1 w_2 \ldots w_{i-1}) \approx P(w_i \mid w_{i-k} \ldots w_{i-1})$$

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Simplest case: Unigram model

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i)$$

Some automatically generated sentences from a unigram model

```
fifth, an, of, futures, the, an, incorporated, a,
a, the, inflation, most, dollars, quarter, in, is,
mass

thrift, did, eighty, said, hard, 'm, july, bullish

that, or, limited, the
```

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Bigram model

- Condition on the previous word:

$$P(w_i \mid w_1 w_2 \ldots w_{i-1}) \approx P(w_i \mid w_{i-1})$$

```
texaco, rose, one, in, this, issue, is, pursuing, growth, in,
a, boiler, house, said, mr., gurria, mexico, 's, motion,
control, proposal, without, permission, from, five, hundred,
fifty, five, yen

outside, new, car, parking, lot, of, the, agreement, reached

this, would, be, a, record, november
```

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N-gram models

- We can extend to trigrams, 4-grams, 5-grams
- In general this is an insufficient model of language
  - because language has **long-distance dependencies**:

  "The computer(s) which I had just put into the machine room on the fifth floor is (are) crashing."

- But we can often get away with N-gram models

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Language Modeling

Estimating N-gram Probabilities

Estimating bigram probabilities

- The Maximum Likelihood Estimate

$$P(w_i \mid w_{i-1}) = \frac{count(w_{i-1}, w_i)}{count(w_{i-1})}$$

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

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An example

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s> I am Sam </s>

<s> Sam I am </s>

<s> I do not like green eggs and ham </s>

$P(\texttt{I} \mid \texttt{<s>}) = \frac{2}{3} = .67$ $\quad P(\texttt{Sam} \mid \texttt{<s>}) = \frac{1}{3} = .33$ $\quad P(\texttt{am} \mid \texttt{I}) = \frac{2}{3} = .67$

$P(\texttt{</s>} \mid \texttt{Sam}) = \frac{1}{2} = 0.5$ $\quad P(\texttt{Sam} \mid \texttt{am}) = \frac{1}{2} = .5$ $\quad P(\texttt{do} \mid \texttt{I}) = \frac{1}{3} = .33$

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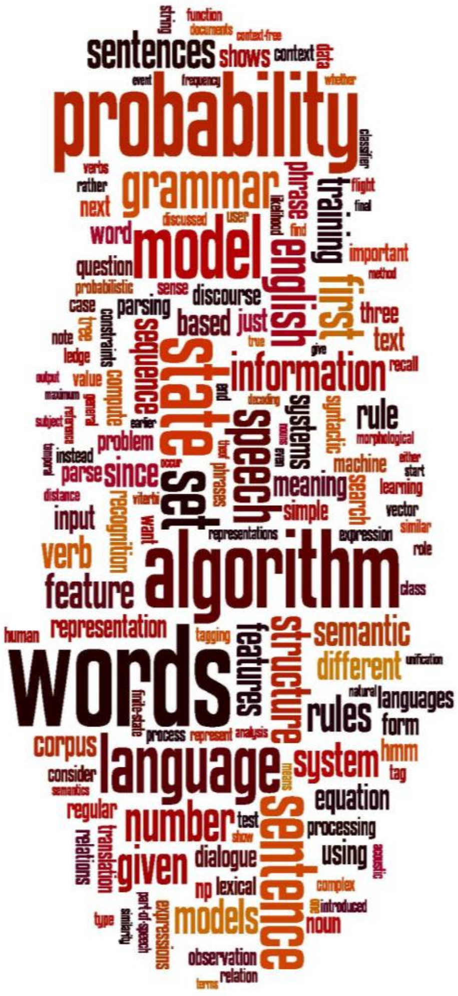
Language Modeling Toolkits

- SRILM
  - http://www.speech.sri.com/projects/srilm/
- KenLM
  - https://kheafield.com/code/kenlm/

Google Book N-grams

- http://ngrams.googlelabs.com/



Text Classification and Naïve Bayes

Naïve Bayes (I)

Naïve Bayes Intuition

- Simple ("naïve") classification method based on Bayes rule
- Relies on very simple representation of document
  - Bag of words

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The bag of words representation

$\gamma($

I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun… It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it to just about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it yet.

$)=c$

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The bag of words representation

$\gamma($

I **love** this movie! It's **sweet**, but with **satirical** humor. The dialogue is **great** and the adventure scenes are **fun**… It manages to be **whimsical** and **romantic** while **laughing** at the conventions of the fairy tale genre. I would **recommend** it to just about anyone. I've seen it **several** times, and I'm always **happy** to see it **again** whenever I have a friend who hasn't seen it yet.

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The bag of words representation: using a subset of words

$\gamma($

x **love** xxxxxxxxxxxxxxxx **sweet**
xxxxxxx **satirical** xxxxxxxxxx
xxxxxxxxxxx **great** xxxxxxx
xxxxxxxxxxxxxxxxxxx **fun** xxxx
xxxxxxxxxxxxx **whimsical** xxxx
romantic xxxx **laughing**
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
xxxxxxxxxxxxxx **recommend** xxxxx
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
xx **several** xxxxxxxxxxxxxxxxx
xxxxx **happy** xxxxxxxxx **again**
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
xxxxxxxxxxxxxxxxx

$)=c$

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The bag of words representation

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great	2
love	2
recommend	1
laugh	1
happy	1
...	...

$)=c$

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Bayes' Rule Applied to Documents and Classes

- For a document *d* and a class *c*

$$P(c \mid d) = \frac{P(d \mid c)P(c)}{P(d)}$$

Naïve Bayes Classifier (I)

$$c_{MAP} = \underset{c \in C}{\operatorname{argmax}}\, P(c \mid d)$$

MAP is "maximum a posteriori" = most likely class

$$= \underset{c \in C}{\operatorname{argmax}}\, \frac{P(d \mid c)P(c)}{P(d)}$$

Bayes Rule

$$= \underset{c \in C}{\operatorname{argmax}}\, P(d \mid c)P(c)$$

Dropping the denominator

Naïve Bayes Classifier (II)

$$c_{MAP} = \underset{c \in C}{\operatorname{argmax}}\, P(d \mid c) P(c)$$

$$= \underset{c \in C}{\operatorname{argmax}}\, P(x_1, x_2, \ldots, x_n \mid c) P(c)$$

Document d represented as features x1..xn

Multinomial Naïve Bayes Independence Assumptions

$$P(x_1, x_2, \ldots, x_n \mid c)$$

- **Bag of Words assumption**: Assume position doesn't matter
- **Conditional Independence**: Assume the feature probabilities $P(x_i|c_j)$ are independent given the class c.

$$P(x_1, \ldots, x_n \mid c) = P(x_1 \mid c) \bullet P(x_2 \mid c) \bullet P(x_3 \mid c) \bullet \ldots \bullet P(x_n \mid c)$$

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Multinomial Naïve Bayes Classifier

$$c_{MAP} = \underset{c \in C}{\operatorname{argmax}} P(x_1, x_2, \ldots, x_n \mid c) P(c)$$

$$c_{NB} = \underset{c \in C}{\operatorname{argmax}} P(c_j) \prod_{x \in X} P(x \mid c)$$

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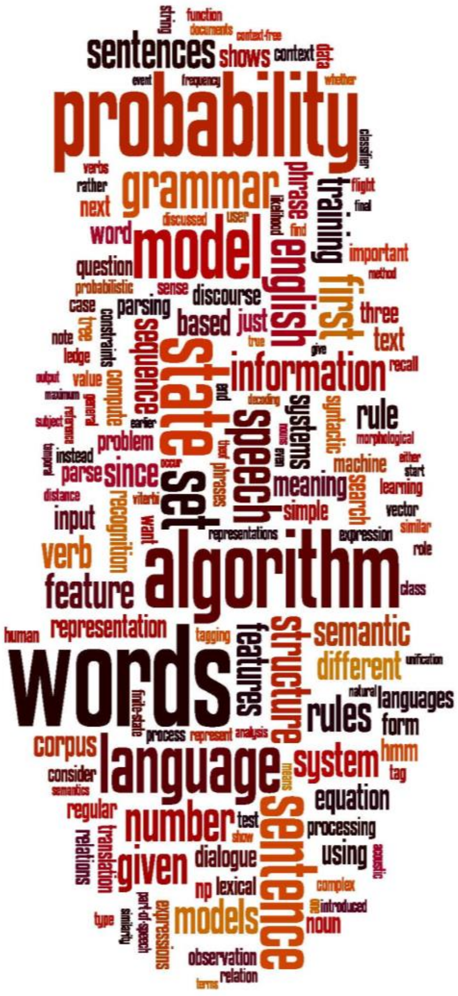
Applying Multinomial Naive Bayes Classifiers to Text Classification

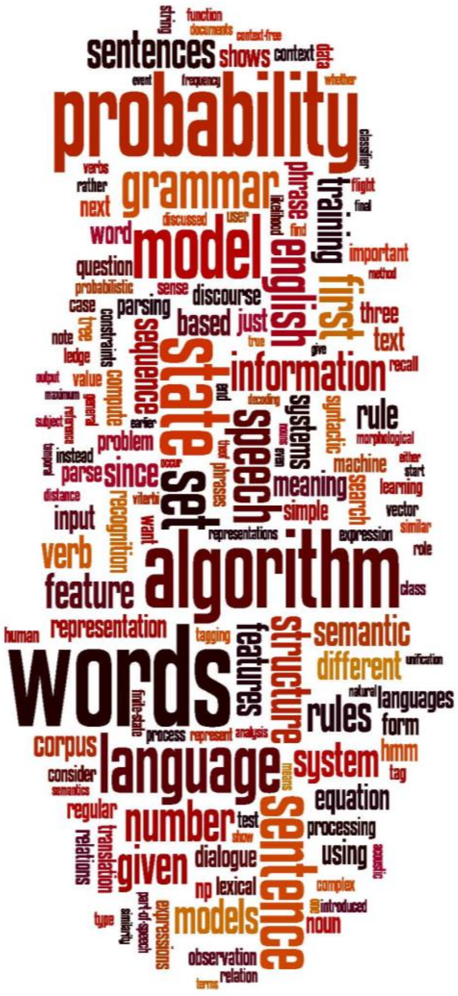
positions $\leftarrow$ all word positions in test document

$$c_{NB} = \underset{c_j \in C}{\operatorname{argmax}} P(c_j) \prod_{i \in positions} P(x_i \mid c_j)$$



Text Classification and Naïve Bayes

Formalizing the Naïve Bayes Classifier



Text Classification and Naïve Bayes

Naïve Bayes: Learning

Learning the Multinomial Naïve Bayes Model

- First attempt: maximum likelihood estimates
  - simply use the frequencies in the data

$$\hat{P}(c_j) = \frac{doccount(C = c_j)}{N_{doc}}$$

$$\hat{P}(w_i \mid c_j) = \frac{count(w_i, c_j)}{\sum_{w \in V} count(w, c_j)}$$

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Parameter estimation

$$\hat{P}(w_i \mid c_j) = \frac{count(w_i, c_j)}{\sum_{w \in V} count(w, c_j)}$$

fraction of times word w_i appears among all words in documents of topic c_j

- Create mega-document for topic *j* by concatenating all docs in this topic
  - Use frequency of *w* in mega-document

Problem with Maximum Likelihood

- What if we have seen no training documents with the word ***fantastic*** and classified in the topic **positive** (***thumbs-up)***?

$$\hat{P}(\text{"fantastic"} \mid \text{positive}) = \frac{count(\text{"fantastic", positive})}{\sum_{w \in V} count(w, \text{positive})} = 0$$

- Zero probabilities cannot be conditioned away, no matter the other evidence!

$$c_{MAP} = \text{argmax}_c \hat{P}(c) \prod_i \hat{P}(x_i \mid c)$$

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Laplace (add-1) smoothing for Naïve Bayes

$$\hat{P}(w_i \mid c) = \frac{count(w_i, c) + 1}{\sum_{w \in V} \left(count(w, c) + 1\right)}$$

$$= \frac{count(w_i, c) + 1}{\left(\sum_{w \in V} count(w, c)\right) + |V|}$$

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Multinomial Naïve Bayes: Learning

- From training corpus, extract *Vocabulary*

- Calculate $P(c_j)$ terms
  - For each c_j in C do

    $docs_j \leftarrow$ all docs with class $= c_j$

$$P(c_j) \neg \frac{|docs_j|}{|\text{total \# documents}|}$$

- Calculate $P(w_k \mid c_j)$ terms
  - $Text_j \leftarrow$ single doc containing all $docs_j$
  - For each word w_k in *Vocabulary*

    $n_k \leftarrow$ # of occurrences of w_k in $Text_j$

$$P(w_k \mid c_j) \neg \frac{n_k + a}{n + a\,|Vocabulary|}$$

Laplace (add-1) smoothing: unknown words

Add one extra word to the vocabulary, the “unknown word” w_u

$$\hat{P}(w_u \mid c) = \frac{count(w_u, c) + 1}{\left(\sum_{w \in V} count(w, c)\right) + |V + 1|}$$

$$= \frac{1}{\left(\sum_{w \in V} count(w, c)\right) + |V + 1|}$$

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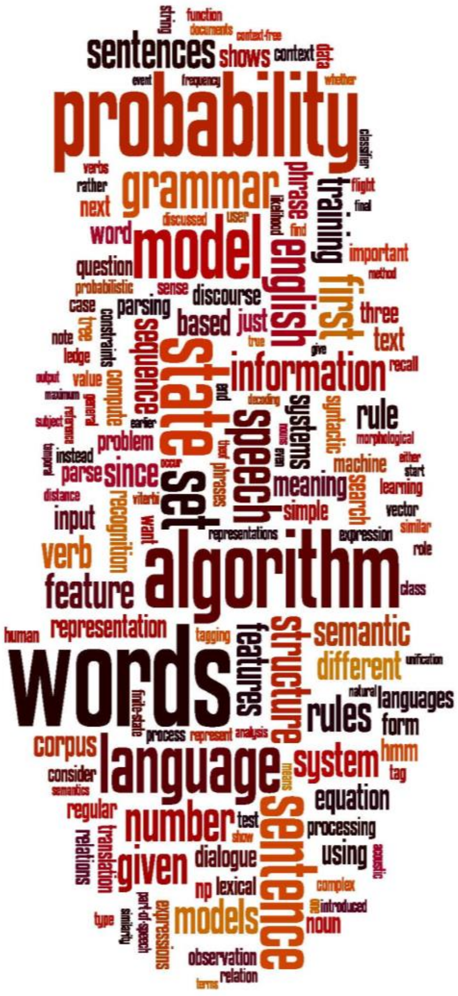

Text Classification and Naïve Bayes

Naïve Bayes: Learning

Text Classification and Naïve Bayes

Multinomial Naïve Bayes: A Worked Example

$$\hat{P}(c) = \frac{N_c}{N}$$

$$\hat{P}(w \mid c) = \frac{count(w,c)+1}{count(c)+|V|}$$

	Doc	Words	Class
Training	1	Chinese Beijing Chinese	c
	2	Chinese Chinese Shanghai	c
	3	Chinese Macao	c
	4	Tokyo Japan Chinese	j
Test	5	Chinese Chinese Chinese Tokyo Japan	?

Priors:

$P(c) = \frac{3}{4}$

$P(j) = \frac{1}{4}$

Choosing a class:

P(c|d5) $\propto$ 3/4 * $(3/7)^3$ * 1/14 * 1/14

$\approx$ 0.0003

P(j|d5) $\propto$ 1/4 * $(2/9)^3$ * 2/9 * 2/9

$\approx$ 0.0001

Conditional Probabilities:

P(Chinese|*c*) = (5+1) / (8+6) = 6/14 = 3/7

P(Tokyo|*c*) = (0+1) / (8+6) = 1/14

P(Japan|*c*) = (0+1) / (8+6) = 1/14

P(Chinese|*j*) = (1+1) / (3+6) = 2/9

P(Tokyo|*j*) = (1+1) / (3+6) = 2/9

P(Japan|*j*) = (1+1) / (3+6) = 2/9

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Naïve Bayes in Spam Filtering

- SpamAssassin Features:
  - Mentions Generic Viagra
  - Online Pharmacy
  - Mentions millions of (dollar) ((dollar) NN,NNN,NNN.NN)
  - Phrase: impress ... girl
  - From: starts with many numbers
  - Subject is all capitals
  - HTML has a low ratio of text to image area
  - One hundred percent guaranteed
  - Claims you can be removed from the list
  - 'Prestigious Non-Accredited Universities'
  - http://spamassassin.apache.org/tests_3_3_x.html

Summary: Naive Bayes is Not So Naive

- Very Fast, low storage requirements
- Robust to Irrelevant Features

  Irrelevant Features cancel each other without affecting results
- Very good in domains with many equally important features

  Decision Trees suffer from *fragmentation* in such cases – especially if little data
- Optimal if the independence assumptions hold: If assumed independence is correct, then it is the Bayes Optimal Classifier for problem
- A good dependable baseline for text classification