PENSION LIABILITY VALUATION MODEL

Analysis of Defined Benefit Pension Obligations

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1. EXECUTIVE SUMMARY

Project Overview:

This project calculates the present value of pension obligations for a hypothetical employee in a defined benefit pension plan. The goal is to demonstrate understanding of actuarial valuation principles including life-contingent cash flows, present value calculations, and sensitivity analysis.

Key Findings:

Under base assumptions, the total pension liability is **$222,095**. This represents the amount an employer would need to set aside today to fund this employee's future pension benefits.

Sensitivity analysis reveals:

- **Discount rate** has the largest impact (±$359,122 for 4% change)
- **Salary growth** has nearly equal impact (±$362,104 for 4% change)
- **Accrual rate, retirement age, and mortality** have much smaller effects

These results align with real-world pension behaviour where interest rates and wage inflation are primary risk drivers.

What This Project Demonstrates:

Through this analysis, I learned how to:

- Build actuarial models incorporating mortality and time value of money
- Perform sensitivity analysis to assess risk
- Apply concepts from actuarial exams (present value, life contingencies)
- Understand why pension plans are so sensitive to economic conditions

This project uses simplified assumptions for educational purposes but follows standard actuarial methodology consistent with professional practice.

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2. BACKGROUND & MOTIVATION

Why This Project:

Pension obligations represent one of the largest liabilities on corporate balance sheets, yet many people don't understand how these liabilities are calculated. After researching actuarial work, I wanted to build a model that demonstrates the core principles of pension valuation.

What I Wanted to Learn:

- How do actuaries value long-term pension promises?
- Why are pensions so sensitive to interest rate changes?
- How do mortality assumptions affect pension costs?
- What makes defined benefit plans so expensive for employers?

This project gave me hands-on experience with concepts I'll encounter in Exam FAM and professional actuarial work.

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3. METHODOLOGY

Model Setup:

I built an Excel model to value pension obligations for a representative employee with these characteristics:

- **Current Age:** 35
- **Retirement Age:** 65
- **Current Salary:** $60,000
- **Pension Formula:** 1.5% × Years of Service × Final Salary

Key Assumptions:

- Salary growth: 3% annually
- Discount rate: 5% (representing expected investment returns)
- Years of service at retirement: 30 years
- Mortality: Simplified increasing rates starting at 1% (age 65)
- Benefits paid annually from age 65 until death (max age 85)

Note on Simplifications:

Real actuarial valuations use detailed SOA mortality tables (like RP-2014), incorporate early retirement and termination probabilities, and model spousal benefits. I've simplified these for a manageable student project, but the core methodology follows professional standards.

Step 1: Final Salary Projection

First, I projected the employee's salary at retirement using compound growth:

Formula:

Final Salary = Current Salary × (1 + growth rate)^years

Final Salary = $60,000 × (1.03)^30 = $145,636

What this shows: Over 30 years, 3% annual raises more than double the salary. This compounding effect is why salary growth assumptions matter so much for pension costs.

Step 2: Annual Pension Benefit

Using the plan formula:

Formula:

Annual Pension = 1.5% × 30 years × $145,575 = $65,536 per year

What this means: The employee receives $65,536 every year in retirement, for life. The employer must fund this obligation.

Step 3: Build Payment Schedule with Mortality

I created a table showing payments from age 65 to 85, incorporating:

Age	Mortality Rate	Survival Probability	Pension Payment	Years from Now	PV Factor	Expected Payment	PV of Expected Payment
65	1.00%	1	$65,536	30	0.231377	$65,536	$15,164
66	1.10%	0.99	$67,502	31	0.2203595	$66,827	$14,726
67	1.21%	0.97911	$69,527	32	0.2098662	$68,075	$14,287
68	1.33%	0.967262769	$71,613	33	0.1998725	$69,269	$13,845

69	1.46%	0.954388502	$73,761	34	0.1903548	$70,397	$13,400
70	1.61%	0.940415299	$75,974	35	0.1812903	$71,447	$12,953
71	1.77%	0.925269817	$78,254	36	0.1726574	$72,406	$12,501
72	1.95%	0.908878098	$80,601	37	0.1644356	$73,257	$12,046
73	2.14%	0.891166635	$83,019	38	0.1566054	$73,984	$11,586
74	2.36%	0.872063687	$85,510	39	0.149148	$74,570	$11,122
75	2.59%	0.851500881	$88,075	40	0.1420457	$74,996	$10,653
76	2.85%	0.829415141	$90,717	41	0.1352816	$75,242	$10,179
77	3.14%	0.805750959	$93,439	42	0.128840	$75,288	$9,700
78	3.45%	0.780463042	$96,242	43	0.1227044	$75,113	$9,217
79	3.80%	0.753519342	$99,129	44	0.1168613	$74,696	$8,729
80	4.18%	0.724904457	$102,103	45	0.1112965	$74,015	$8,238
81	4.59%	0.694623399	$105,166	46	0.1059967	$73,051	$7,743
82	5.05%	0.662705641	$108,321	47	0.1009492	$71,785	$7,247
83	5.56%	0.629209382	$111,571	48	0.0961421	$70,201	$6,749
84	6.12%	0.59422586	$114,918	49	0.0915639	$68,287	$6,253
85	6.73%	0.557883547	$118,365	50	0.0872037	$66,034	$5,758
							$222,095

How I calculated this:

1. **Survival Probability:** Starts at 100% (age 65), decreases each year based on mortality
   - Formula: Survival(age) = Survival(age-1) × (1 - Mortality Rate)
2. **Expected Payment:** Pension amount × Probability of being alive
   - Formula: $65,508 × Survival Probability
3. **PV Factor:** Discount factor for time value of money
   - Formula: 1 / (1.05)^(years from now)

4. **PV of Payment:** Expected payment discounted to present value
    - Formula: Expected Payment × PV Factor

Key Learning: This is life-contingent present value - a core concept in actuarial mathematics. We're combining two types of uncertainty: mortality (will they be alive?) and time value of money (what's a future dollar worth today?).

Step 4: Sum to Get Total Liability

Result:

Total Pension Liability = Sum of all PV of Payments = $222,095

Interpretation: If the employer invests $222,095 today at 5% returns, and mortality follows our assumptions, this amount will exactly fund the promised pension benefits.

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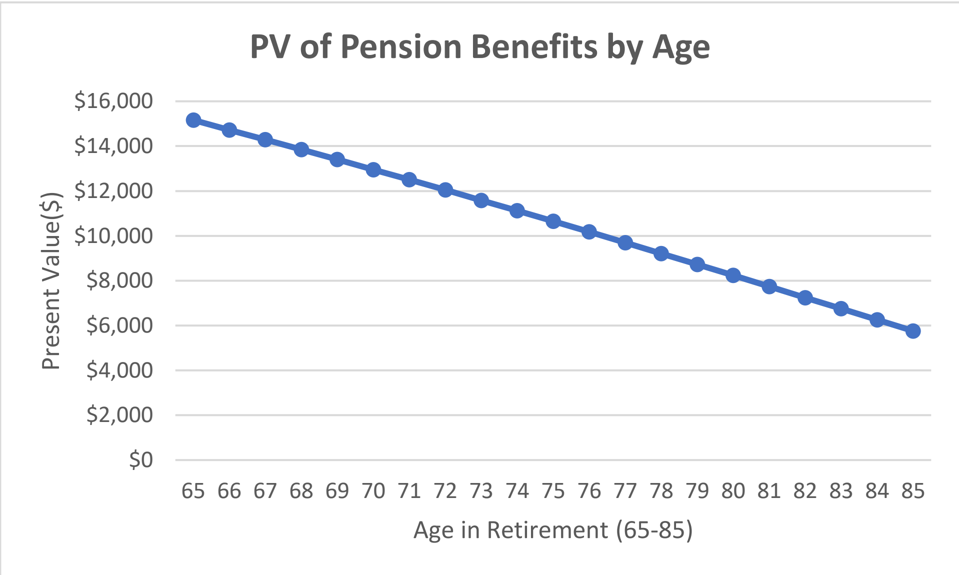
4. RESULTS & ANALYSIS

4.1 Present Value by Age



What the pattern shows:

The present value of pension benefits decreases monotonically from age 65 to 85. This reflects two compounding effects:

- Early retirement (65-72): Nearly 100% survival probability with minimal time discounting → Maximum present value contribution

- Middle retirement (72-80): Declining survival probability combined with moderate time discounting → Moderate present value contribution

- Late retirement (80+): Very low survival probability combined with heavy time discounting → Minimal present value contribution.

This reveals that most pension liability comes from **EARLY RETIREMENT YEARS (ages 65-72)** when the vast majority of **retirees are still alive**. Longevity risk, the possibility that retirees live longer than expected is the primary concern for pension sponsors. If mortality is higher than assumed, liability decreases. If mortality is lower (people live longer), liability increases substantially.

This is why actuaries are so focused on mortality assumptions: small changes in longevity projections can dramatically alter pension costs.

4.2 Sensitivity Analysis - The Key Findings

I tested how the $222,095 liability changes when we vary key assumptions:

Test 1: Discount Rate (3% to 7%)

Discount Rate	Total Liability	Change from Base	% Change
3%	$467,444	$245,349	110.47
4%	$321,121	$99,025	44.59
5% (base rate)	$222,095	$0	0.00
6%	$154,616	-$67,480	-30.38
7%	$108,323	-$113,773	-51.23

What I learned: This is Enormous. A 2% drop in interest rates (5% → 3%) more than doubles the liability! This is why pension plans are so concerned about low interest rate environments.

Real-world connection: When the Federal Reserve lowers rates, corporate pension obligations explode. This is why companies have been freezing defined benefit plans and switching to 401(k)s.

Test 2: Salary Growth Rate (1% to 5%)

Salary Growth	Final Salary	Annual Pension	Total Liability	Change from Base	% Change
1%	$80,871	$36,392	$105,340	-$116,755	-52.57
2%	$108,682	$48,907	$152,992	-$69,103	-31.11
3% (base growth)	$145,636	$65,536	$222,095	$0	0.00
4%	$194,604	$87,572	$322,272	$100,176	45.11
5%	$259,317	$116,692	$467,444	$245,349	110.47

What I learned: Salary growth compounds aggressively. At 5% growth, final salary is 3.2x higher than at 2% growth. Since pension is based on final salary, this directly multiplies the liability.

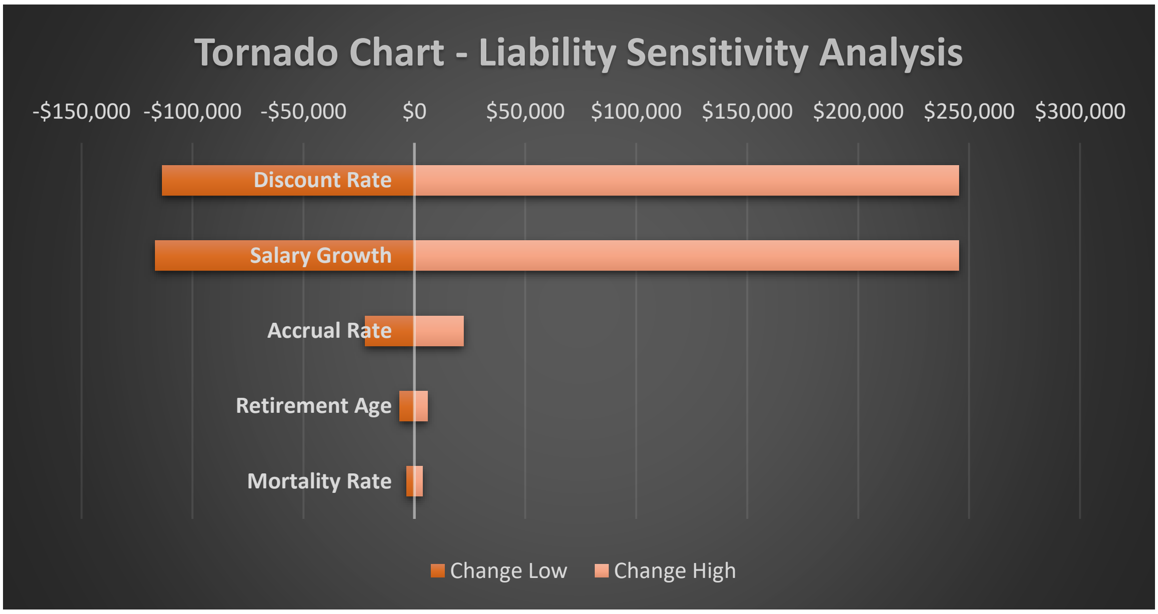
Why this matters: Employers face double risk, if wages grow faster than expected AND interest rates fall, pension costs explode from both directions.

Test 3: Other Assumptions (Smaller Effects)

Assumption	Low Value	Base Value	High Value	Impact on Liability
Accrual Rate	$199,886	$222,095	$244,305	$44,419
Retirement Age	$215,417	$222,095	$227,963	$12,546
Mortality Rate	$218,506	$222,095	$225,791	$7,285

What this shows: While these assumptions matter, they're much less important than discount rate and salary growth. Even doubling the accrual rate (1% → 2%) has less impact than a 2% change in discount rate.

4.3 Tornado Diagram - Visual Summary



The chart clearly shows discount rate and salary growth dwarf other factors. This is consistent with how professional actuaries think about pension risk.

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5. WHAT I LEARNED

Technical Skills:

- Built complex Excel model with multiple linked calculations
- Implemented mortality tables and survival probabilities
- Calculated present values with life contingencies
- Performed comprehensive sensitivity analysis
- Created professional visualizations

Actuarial Concepts:

- **Time value of money:** Why $1 today ≠ $1 in 30 years
- **Life contingencies:** How mortality affects valuations
- **Compound interest:** Why long-duration liabilities are so sensitive to rates
- **Risk assessment:** Using sensitivity analysis to understand uncertainty

Real-World Insights:

1. Why companies stopped offering pensions: The combination of interest rate risk, longevity risk, and salary risk makes defined benefit plans extremely expensive and volatile. A 2% drop in rates can add hundreds of millions to a large company's pension obligation.

2. Why pension accounting matters: Financial statements must reflect pension liabilities accurately. If rates change, companies must adjust their balance sheets, affecting stock prices and credit ratings.

3. Why actuaries exist: Someone needs to quantify these liabilities, assess the risks, and help companies manage them. This isn't just math - it's judgment about assumptions, understanding of economic forces, and communication with CFOs and boards.

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6. LIMITATIONS & FUTURE ENHANCEMENTS

What I Simplified:

This model makes several simplifying assumptions that would need to be addressed in professional work:

Mortality:

- I used a simple increasing mortality rate (1% starting, 10% annual increase)
- Real actuaries use SOA mortality tables (like RP-2014) with detailed age/gender-specific rates

- Professional tables also include mortality improvement trends (people living longer over time)

Demographic Assumptions:

- No early retirement option (real plans have ages 55, 60, 62 options)
- No termination probability (employees leave before retirement)
- No disability assumptions (some employees become disabled)
- No spousal benefits (many pensions include survivor options)

Economic Assumptions:

- Fixed 5% discount rate (real rates vary by year and economic conditions)
- Fixed 3% salary growth (real wages vary by individual and economy)
- No inflation adjustments (some pensions have COLAs)

Model Structure:

- Single employee (real plans have thousands with different ages/salaries)
- Deterministic (real actuaries use stochastic models for risk assessment)
- No asset modelling (didn't look at pension fund investments)

Future Enhancements I'd Like to Add:

If I had more time, I would:

1. Use actual SOA mortality tables for realistic mortality patterns
2. Add early retirement options with different benefit formulas
3. Incorporate stochastic interest rate scenarios (what if rates are volatile?)
4. Model termination probabilities (not everyone stays 30 years)
5. Compare to real pension plan data to validate results
6. Add COLA adjustments to model inflation-linked benefits

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7. CONCLUSIONS

What This Project Achieved:

I successfully built a pension liability valuation model that:

- Calculates present value of long-term obligations

- Incorporates mortality and time value of money
- Performs sensitivity analysis to assess risk
- Produces results consistent with actuarial principles

Key Takeaway:

Pension liabilities are extremely sensitive to economic assumptions, particularly interest rates. A 2% rate change can swing liabilities by 50-100%. This helps me understand:

- Why pension funding is so challenging for companies
- Why actuaries are essential for managing these risks
- Why many companies have moved away from defined benefit plans

Connection to Actuarial Career:

This project gave me hands-on experience with:

- Exam FAM concepts (present value, annuities, life contingencies)
- Professional actuarial work (what pension actuaries actually do)
- Excel modelling (building tools that others can use and understand)
- Risk thinking (identifying and quantifying sources of uncertainty)

While simplified for educational purposes, this analysis follows the same fundamental methodology used by professional actuaries valuing corporate pension plans worth billions of dollars. The experience reinforced my interest in pursuing actuarial science and gave me a deeper appreciation for the complexity of long-term financial obligations.

APPENDICES

Appendix A: Assumptions Summary

EMPLOYEE PROFILE

Current Age:	35
Retirement Age:	65
Current Annual Salary:	$60,000
Years to Retirement:	30

ACTUARIAL ASSUMPTIONS

Salary Growth Rate:	3.0% per year
Discount Rate:	5.0% per year
Pension Accrual Rate:	1.5% per year of service
Starting Mortality Rate (age 65):	1.0%
Mortality Increase:	10% per year
Maximum Age:	85

CALCULATED VALUES

Final Salary at 65:	$145,636
Years of Service:	30
Annual Pension Benefit:	$65,508
Total Present Value:	$222,095

Appendix B: Key Formulas Used

1. Future Value (Salary Projection):

$FV = PV \times (1 + g)^n$

Where: g = growth rate, n = years

2. Present Value Factor:

PV Factor = $1 / (1 + i)^n$

Where: i = discount rate, n = years

3. Survival Probability:

$S(x) = S(x-1) \times (1 - q(x-1))$

Where: q = mortality rate at age

4. Expected Payment:

E(Payment) = Payment Amount × Survival Probability

5. Present Value of Payment:

PV = E(Payment) × PV Factor

Appendix C: References & Learning Resources

- Society of Actuaries (SOA) - Pension Section resources
- Actuarial Standards of Practice (ASOP) No. 4: Measuring Pension Obligations
- ASC 715: Pension Accounting Standards (FASB)
- Bowers et al., "Actuarial Mathematics" - Life contingencies chapters