Predicting Fuel Economy

Train regression models to predict fuel economy of vehicles.

Load and Prepare Data

```
carData = readtable("carData.csv");
carData = convertvars(carData,vartype("cellstr"),"categorical");

cvpt = cvpartition(carData.FuelEcon,"HoldOut",0.32);
```

```
Warning: The training set does not contain points from all groups.
```

```
carTrain = carData(training(cvpt),:);
carTest = carData(~training(cvpt),:);
```

Linear Models

Plot fuel economy vs engine displacement.

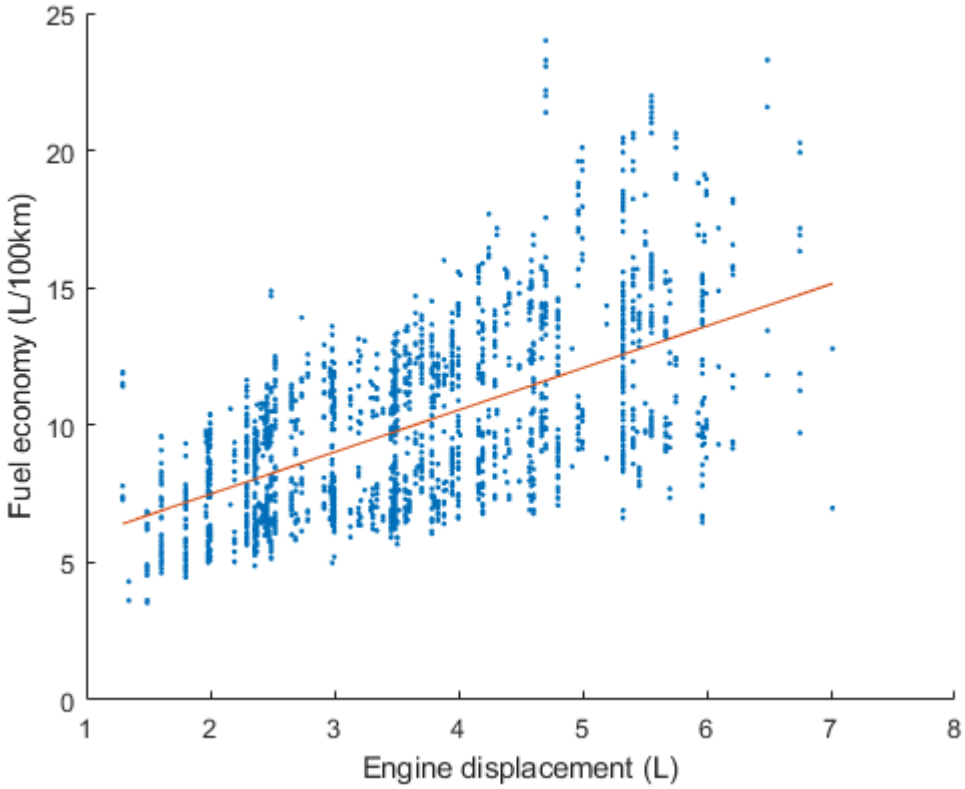
```
clf
scatter(carData.EngDisp,carData.FuelEcon,".")
```

Fit a single-predictor linear model of fuel economy.

```
mdl = fitlm(carTrain.EngDisp,carTrain.FuelEcon);
```

Add model to the plot.

```
x = [min(carData.EngDisp);max(carData.EngDisp)];
y = predict(mdl,x);
hold on
plot(x,y)
hold off
xlabel("Engine displacement (L)")
ylabel("Fuel economy (L/100km)")
```

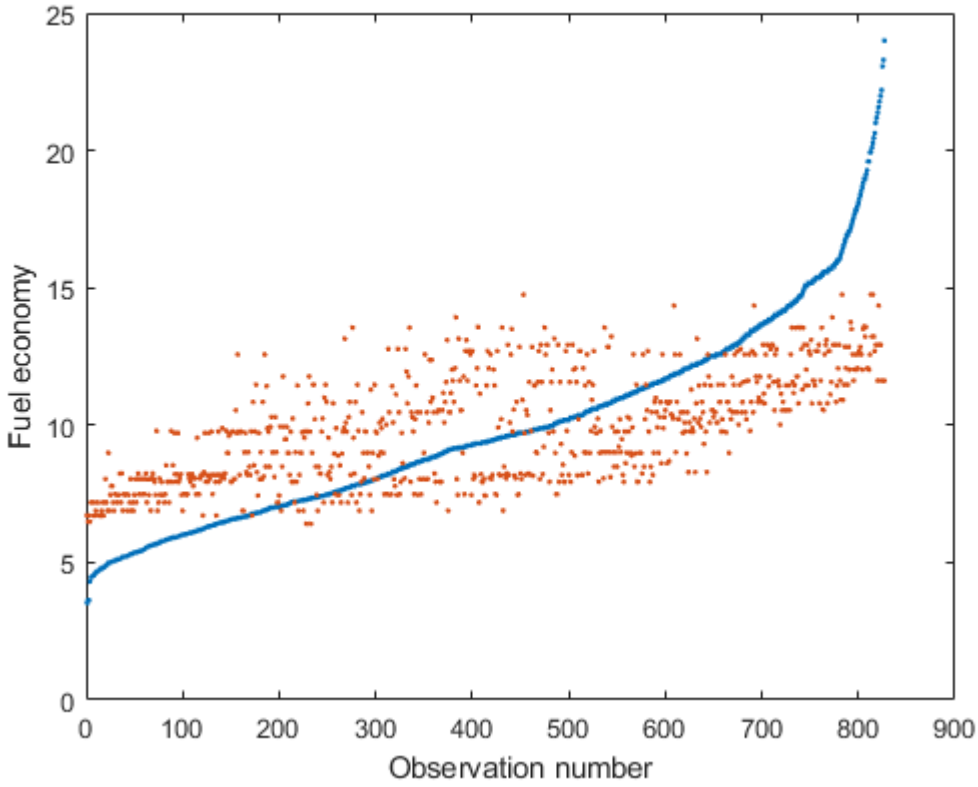


Predict fuel economy at test predictor values.

```
econpred = predict(mdl,carTest.EngDisp);
```

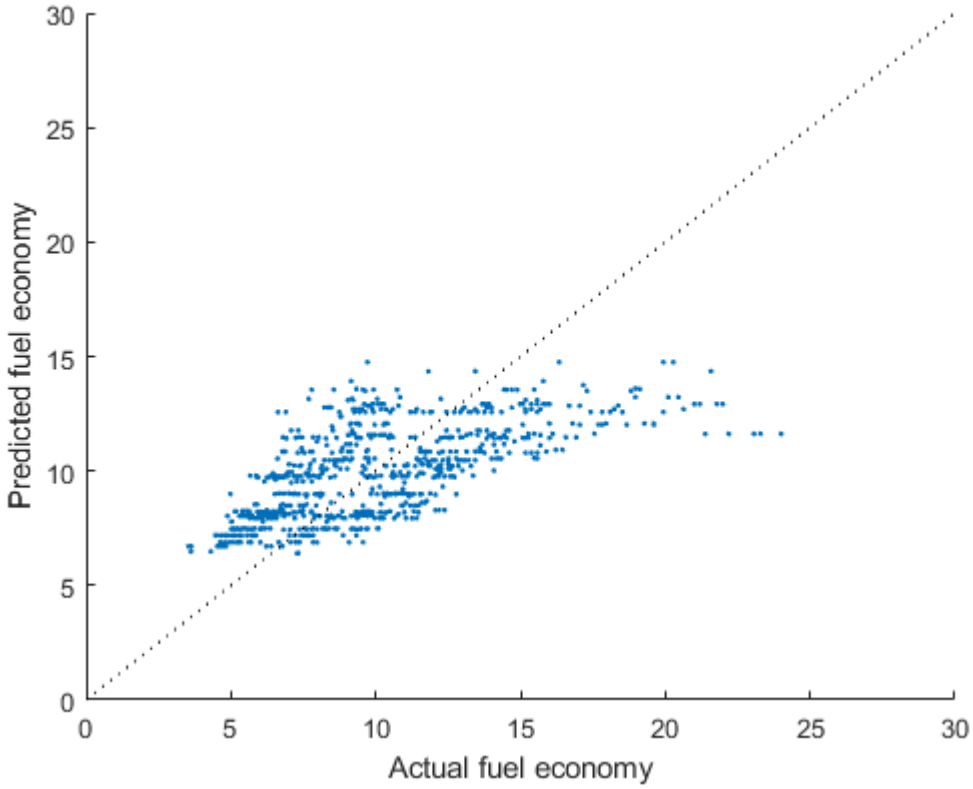
Plot predicted and actual responses vs. observation number.

```
plot(carTest.FuelEcon,".")
hold on
plot(econpred,".")
hold off
xlabel("Observation number")
ylabel("Fuel economy")
```



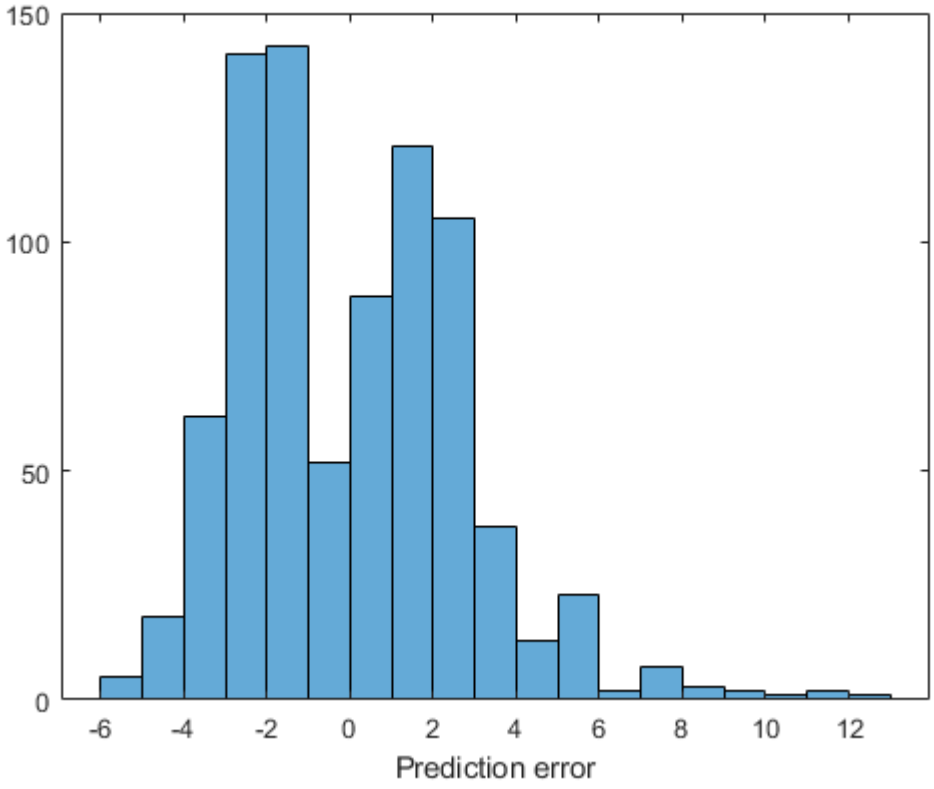
Plot predicted and actual responses against each other.

```
scatter(carTest.FuelEcon,econpred,".")
% add diagonal line
hold on
plot([0 30],[0 30],"k:")
hold off
xlabel("Actual fuel economy")
ylabel("Predicted fuel economy")
```



Plot distribution of errors.

```
err = carTest.FuelEcon - econpred;
histogram(err)
xlabel("Prediction error")
```

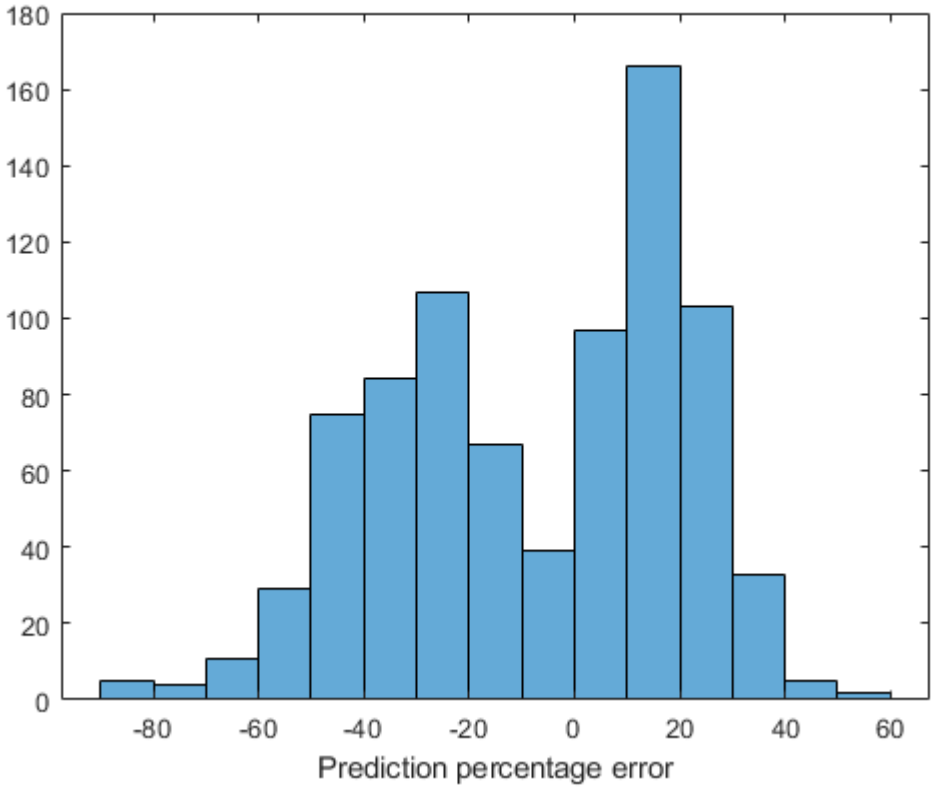



```
MSE = mean(err.^2)
```

```
MSE = 7.5434
```

Plot distribution of percentage errors.

```
percerr = 100*err./carTest.FuelEcon;
histogram(percerr)
xlabel("Prediction percentage error")
```




```
MAPE = mean(abs(percerr))
```

```
MAPE = 24.4363
```

Multivariate Linear Models

Fit a multivariate linear model of fuel economy.

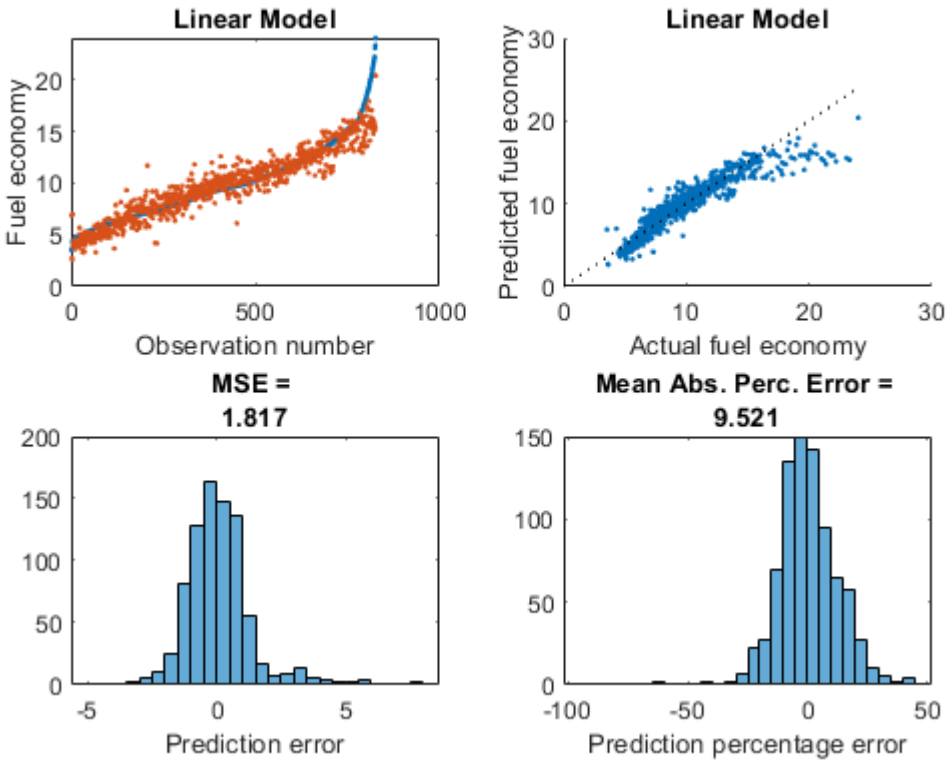
```
mdl = fitlm(carTrain);
```

Predict fuel economy at test predictor values.

```
econpred = predict(mdl,carTest);
```

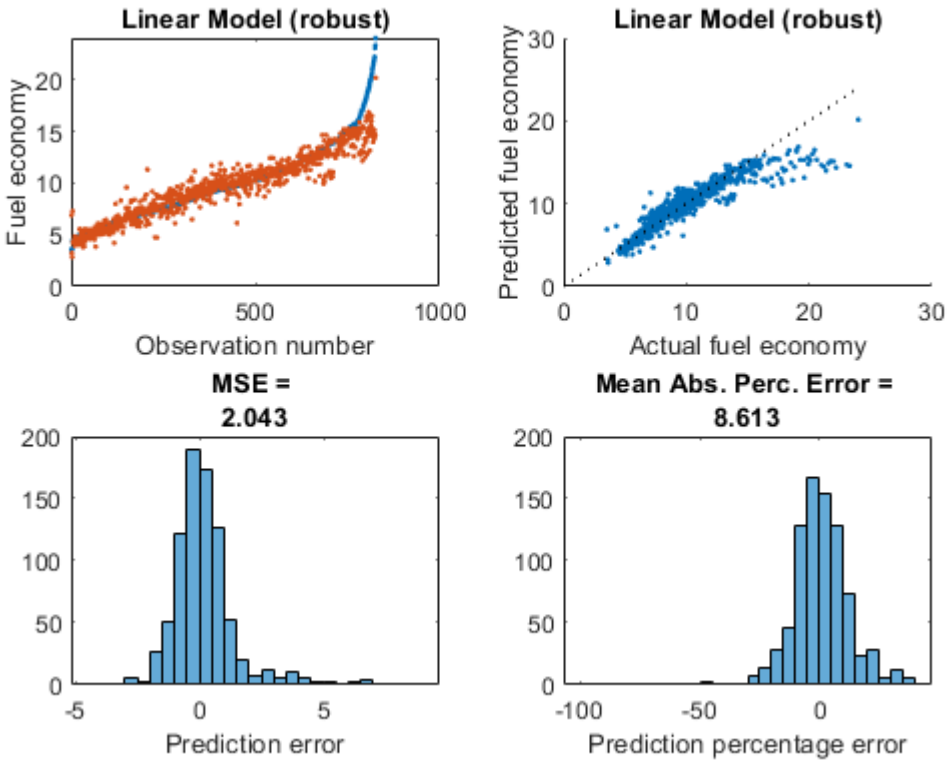
Compare predicted and actual responses.

```
evaluatefit(carTest.FuelEcon,econpred,"Linear Model")
```



Fit and evaluate a multivariate linear model with robust fitting.

```
mdl = fitlm(carTrain,"RobustOpts","cauchy");
econpred = predict(mdl,carTest);
evaluatefit(carTest.FuelEcon,econpred,"Linear Model (robust)")
```



Stepwise Fitting

Perform a stepwise linear fit of fuel economy.

```
mdl = stepwiselm(carTrain);
```

```
1. Adding City_Highway, FStat = 1231.2278, pValue = 6.7117696e-205
2. Adding EngDisp, FStat = 2952.7406, pValue = 0
3. Adding EngDisp:City_Highway, FStat = 227.4625, pValue = 2.005625e-48
4. Adding Weight, FStat = 257.6422, pValue = 3.330078e-54
5. Adding Transmission, FStat = 13.4668, pValue = 1.26677e-34
6. Adding Drive, FStat = 60.9396, pValue = 2.6394e-26
7. Adding Valves_Cyl, FStat = 61.2562, pValue = 8.64365e-15
8. Adding Comp, FStat = 62.836, pValue = 3.98526e-15
9. Adding EngDisp:Transmission, FStat = 6.8602, pValue = 3.1046e-12
10. Adding Transmission:Valves_Cyl, FStat = 9.9204, pValue = 2.5329e-16
11. Adding Drive:Valves_Cyl, FStat = 40.3078, pValue = 7.84208e-18
12. Adding FuelType, FStat = 60.5131, pValue = 1.2546e-14
13. Adding EngDisp:FuelType, FStat = 197.9218, pValue = 1.285022e-42
14. Adding Transmission:FuelType, FStat = 15.9628, pValue = 1.98793e-20
15. Adding Transmission:Weight, FStat = 6.0101, pValue = 2.1814e-12
16. Adding AC, FStat = 53.6907, pValue = 3.62244e-13
17. Adding RatedHP, FStat = 56.2472, pValue = 1.02818e-13
18. Adding RatedHP:Transmission, FStat = 8.2682, pValue = 2.8605e-14
19. Adding RatedHP:City_Highway, FStat = 57.475, pValue = 5.63997e-14
20. Adding AC:FuelType, FStat = 42.5311, pValue = 9.18983e-11
21. Adding Car_Truck, FStat = 21.8792, pValue = 3.13737e-06
22. Adding Weight:Comp, FStat = 24.8658, pValue = 6.7855e-07
23. Adding Weight:FuelType, FStat = 20.4727, pValue = 6.47623e-06
24. Adding Drive:FuelType, FStat = 15.0236, pValue = 3.41553e-07
25. Adding RatedHP:Comp, FStat = 24.4305, pValue = 8.47883e-07
26. Adding RatedHP:Valves_Cyl, FStat = 24.7365, pValue = 7.25124e-07
```

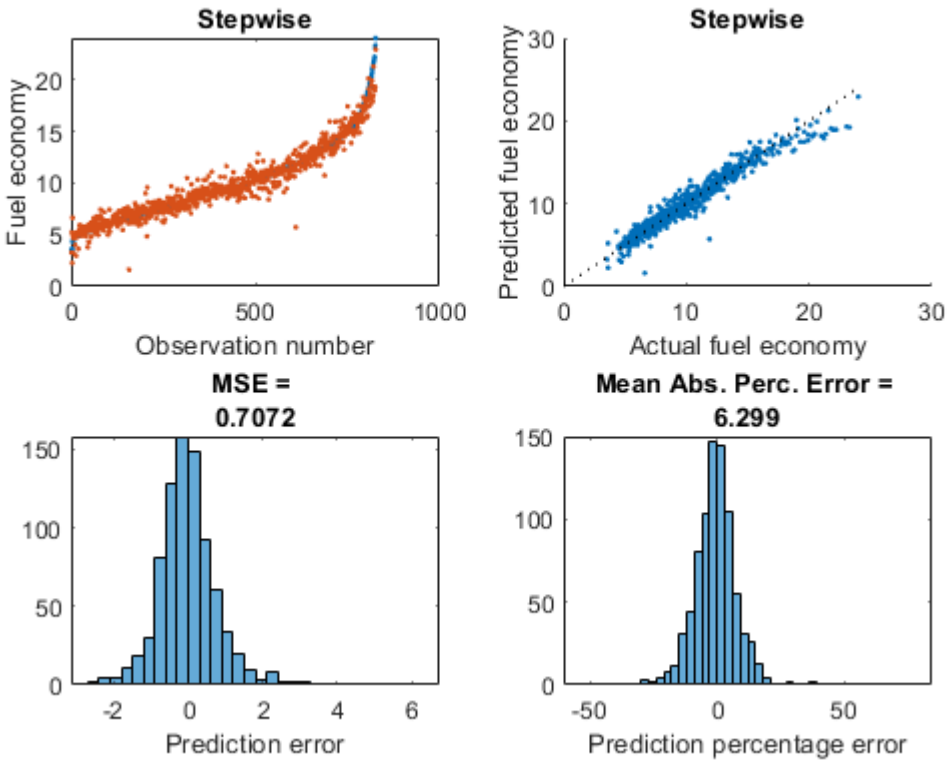
```
27. Adding City_Highway:Valves_Cyl, FStat = 20.2649, pValue = 7.21088e-06
28. Adding EngDisp:Comp, FStat = 20.7543, pValue = 5.60177e-06
29. Adding Car_Truck:AC, FStat = 14.0462, pValue = 0.000184475
30. Adding Comp:City_Highway, FStat = 14.0708, pValue = 0.000182103
31. Adding EVSpeedRatio, FStat = 11.8501, pValue = 0.000590905
32. Adding Transmission:City_Highway, FStat = 2.5265, pValue = 0.001028
33. Adding Transmission:Drive, FStat = 2.3552, pValue = 0.0018335
34. Adding Drive:Comp, FStat = 8.7498, pValue = 0.00016607
35. Adding Weight:City_Highway, FStat = 9.6386, pValue = 0.0019382
36. Adding Weight:Valves_Cyl, FStat = 8.6881, pValue = 0.0032486
37. Adding EngDisp:Weight, FStat = 10.6425, pValue = 0.00112814
38. Adding Transmission:EVSpeedRatio, FStat = 2.8204, pValue = 0.0017856
39. Adding Transmission:AC, FStat = 7.0361, pValue = 2.0846e-07
40. Adding RatedHP:Drive, FStat = 7.2668, pValue = 0.00072157
41. Adding EVSpeedRatio:Valves_Cyl, FStat = 12.6918, pValue = 0.000377997
42. Adding AC:Valves_Cyl, FStat = 6.6939, pValue = 0.0097614
43. Adding RatedHP:AC, FStat = 12.0506, pValue = 0.000531501
44. Adding Comp:FuelType, FStat = 9.1066, pValue = 0.0025871
45. Adding Comp:Valves_Cyl, FStat = 9.983, pValue = 0.0016095
46. Adding EVSpeedRatio:AC, FStat = 10.4937, pValue = 0.00122242
47. Adding EngDisp:RatedHP, FStat = 10.6913, pValue = 0.00109927
48. Adding Car_Truck:Transmission, FStat = 3.1508, pValue = 0.001521
49. Adding FuelType:Valves_Cyl, FStat = 6.6459, pValue = 0.010028
50. Adding Car_Truck:Weight, FStat = 6.2679, pValue = 0.012394
51. Adding Car_Truck:EngDisp, FStat = 21.3149, pValue = 4.21075e-06
52. Adding Comp:EVSpeedRatio, FStat = 6.5893, pValue = 0.01035
53. Adding Weight:EVSpeedRatio, FStat = 6.8214, pValue = 0.0090921
54. Adding PRP, FStat = 4.9981, pValue = 0.025514
55. Adding PRP:FuelType, FStat = 25.5559, pValue = 4.79656e-07
56. Adding AC:PRP, FStat = 8.7497, pValue = 0.0031425
57. Adding Drive:PRP, FStat = 4.1486, pValue = 0.015958
58. Adding Comp:PRP, FStat = 5.1384, pValue = 0.023536
59. Adding Transmission:Comp, FStat = 2.0088, pValue = 0.042109
60. Adding AC:City_Highway, FStat = 4.0297, pValue = 0.044877
61. Adding EVSpeedRatio:City_Highway, FStat = 4.9586, pValue = 0.026102
62. Adding Car_Truck:City_Highway, FStat = 5.0559, pValue = 0.02468
63. Adding Car_Truck:RatedHP, FStat = 4.0523, pValue = 0.044282
64. Removing EngDisp:City_Highway, FStat = 0.0011042, pValue = 0.9735
65. Removing Weight:Comp, FStat = 0.094199, pValue = 0.75895
66. Removing Comp:EVSpeedRatio, FStat = 2.0431, pValue = 0.15309
Warning: Regression design matrix is rank deficient to within machine precision.
```

Predict fuel economy at test predictor values.

```
econpred = predict(mdl,carTest);
```

Compare predicted and actual responses.

```
evaluatefit(carTest.FuelEcon,econpred,"Stepwise")
```

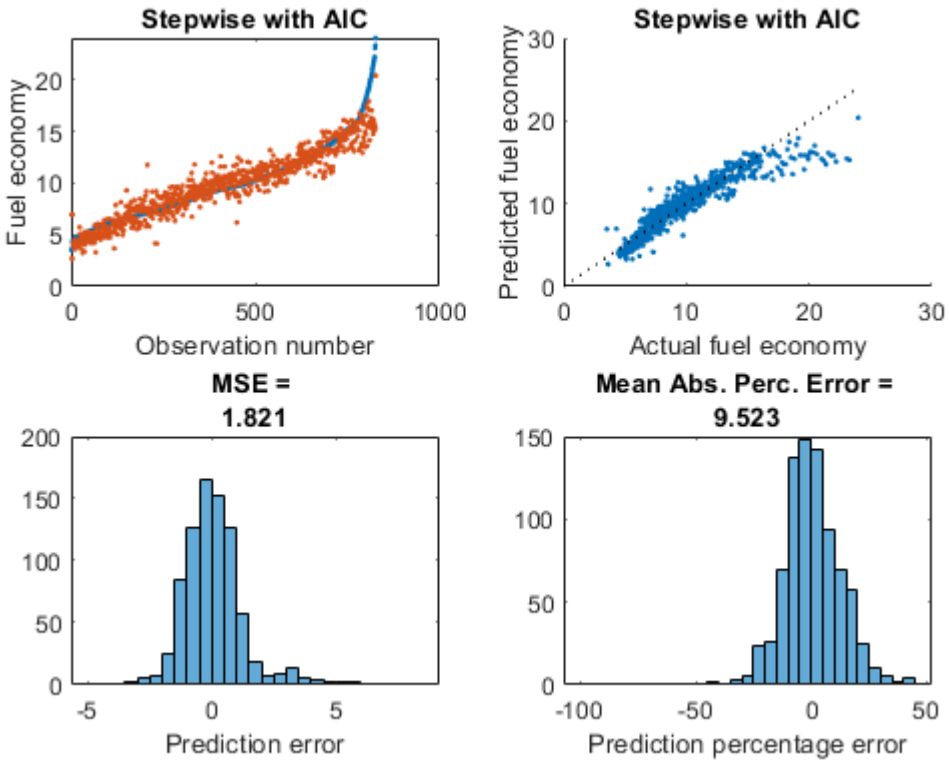


Change criterion and limit to linear models.

```
mdl = stepwiselm(carTrain,"Criterion","aic","Upper","linear");
```

```
1. Adding City_Highway, AIC = 8160.8691
2. Adding EngDisp, AIC = 6427.465
3. Adding Weight, AIC = 6219.5307
4. Adding Transmission, AIC = 6065.9098
5. Adding Drive, AIC = 5964.5861
6. Adding Valves_Cyl, AIC = 5913.6396
7. Adding Comp, AIC = 5868.5743
8. Adding FuelType, AIC = 5835.153
9. Adding Car_Truck, AIC = 5809.0289
10. Adding EVSpeedRatio, AIC = 5778.5294
11. Adding RatedHP, AIC = 5762.2568
12. Adding AC, AIC = 5737.6937
13. Adding AxleRatio, AIC = 5737.2433
```

```
econpred = predict(mdl,carTest);
evaluatefit(carTest.FuelEcon,econpred,"Stepwise with AIC")
```



Regularized Linear Models

Prepare data

Extract response variable.

```
econTrain = carData.FuelEcon(training(cvpt));
econTest = carData.FuelEcon(~training(cvpt));
```

Make dummy variables with n-1 columns for categorical predictors with n categories.

```
minus1 = @(x) x(:,1:end-1);
T = splitvars(convertvars(carData(:,1:end-1),vartype("categorical"),@(x) minus1(dummyvar(x))));
XTrain = T{training(cvpt),:};
XTest = T{~training(cvpt),:};
```

Fit ridge regression model.

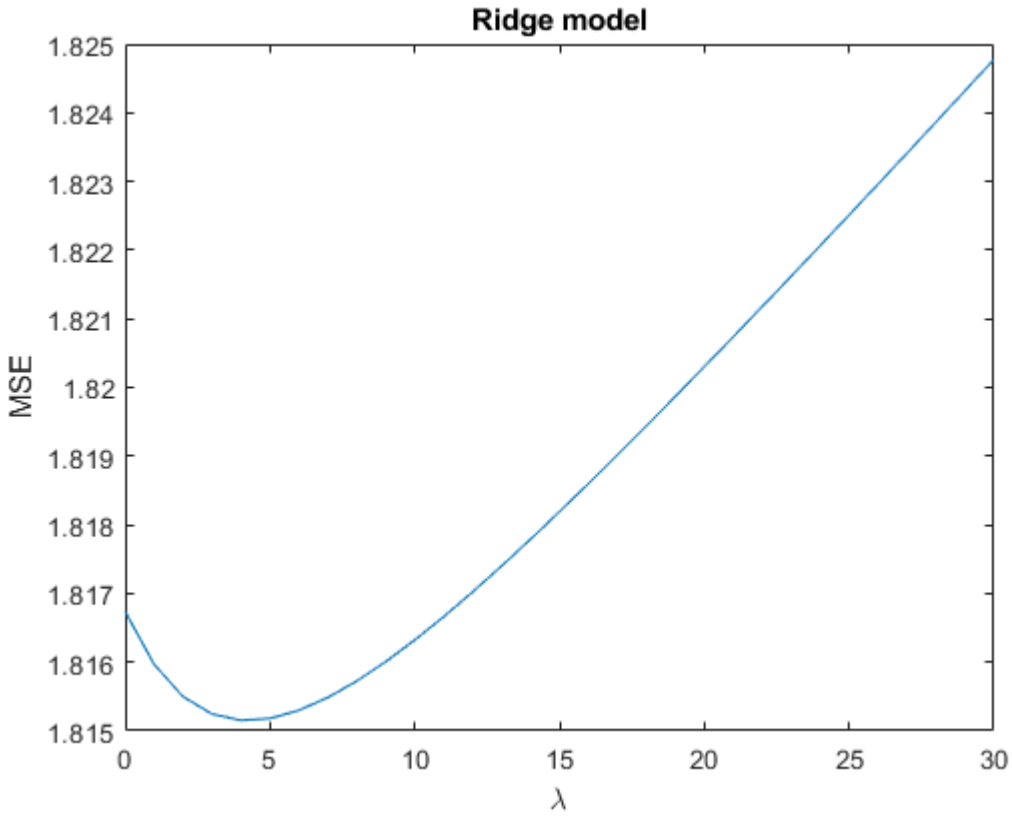
```
lambdaR = 0:30;
bR = ridge(econTrain,XTrain,lambdaR,0);
```

Calculate and plot MSE. Find smallest MSE.

```
econPredR = bR(1,:) + XTest*bR(2:end,:);
err = econPredR - econTest;
MSER = mean(err.^2);
[minMSER,idxR] = min(MSER)
```

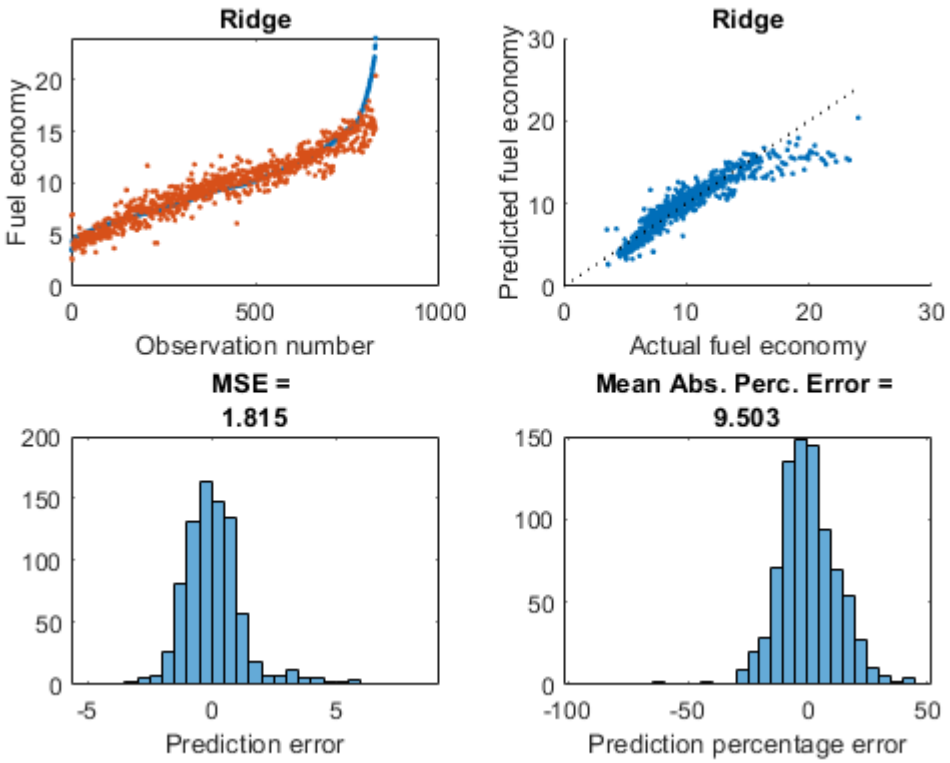
```
minMSER = 1.8152
idxR = 5
```

```
clf
plot(lambdaR,MSER)
xlabel("\lambda")
ylabel("MSE")
title("Ridge model")
```



Compare predicted and actual responses.

```
econpred = econPredR(:,idxR);
evaluatefit(carTest.FuelEcon,econpred,"Ridge")
```



Fit lasso model.

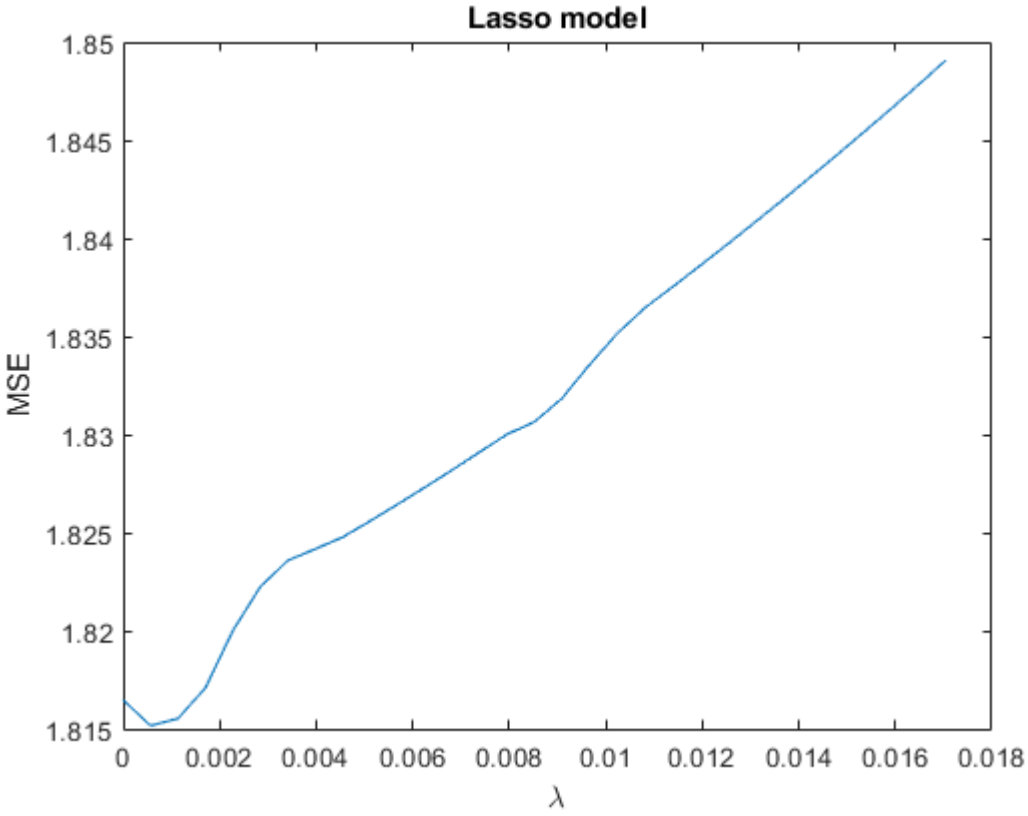
```
lambdaL = lambdaR/length(econTrain);
[bL,fitInfo] = lasso(XTrain,econTrain,"Lambda",lambdaL);
```

Calculate and plot MSE. Find smallest MSE.

```
econPredL = fitInfo.Intercept + XTest*bL;
err = econPredL - econTest;
MSEL = mean(err.^2);
[minMSEL,idxL] = min(MSEL)
```

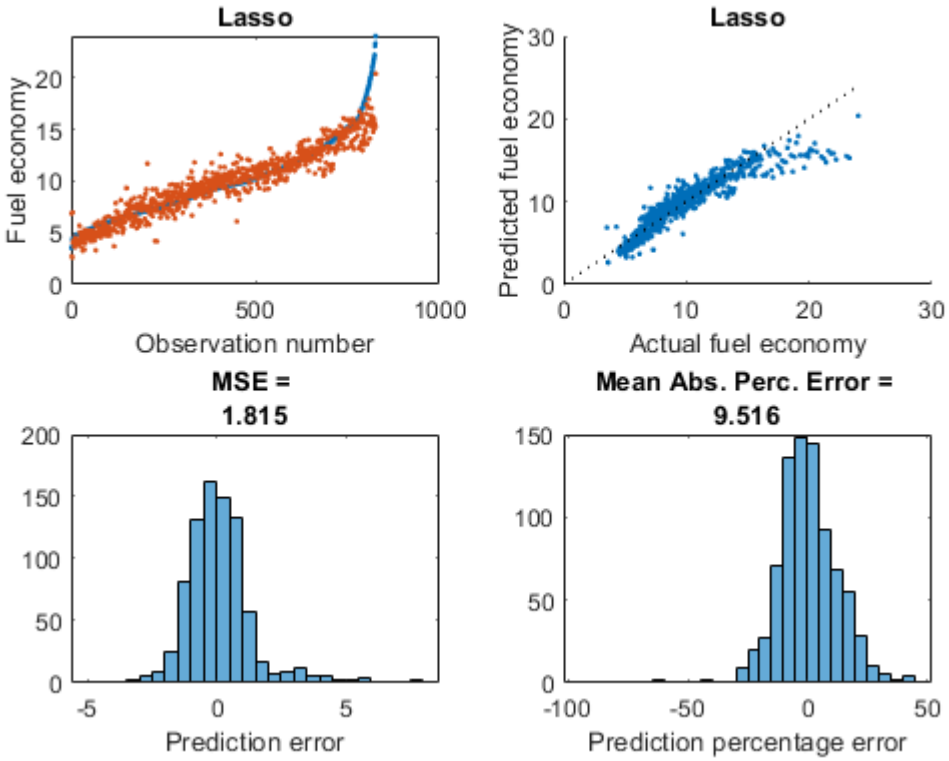
```
minMSEL = 1.8152
idxL = 2
```

```
clf
plot(lambdaL,MSEL)
xlabel("\lambda")
ylabel("MSE")
title("Lasso model")
```



Compare predicted and actual responses.

```
econpred = econPredL(:,idxL);
evaluatefit(carTest.FuelEcon,econpred,"Lasso")
```



SVMs

Fit an SVM regression model of fuel economy.

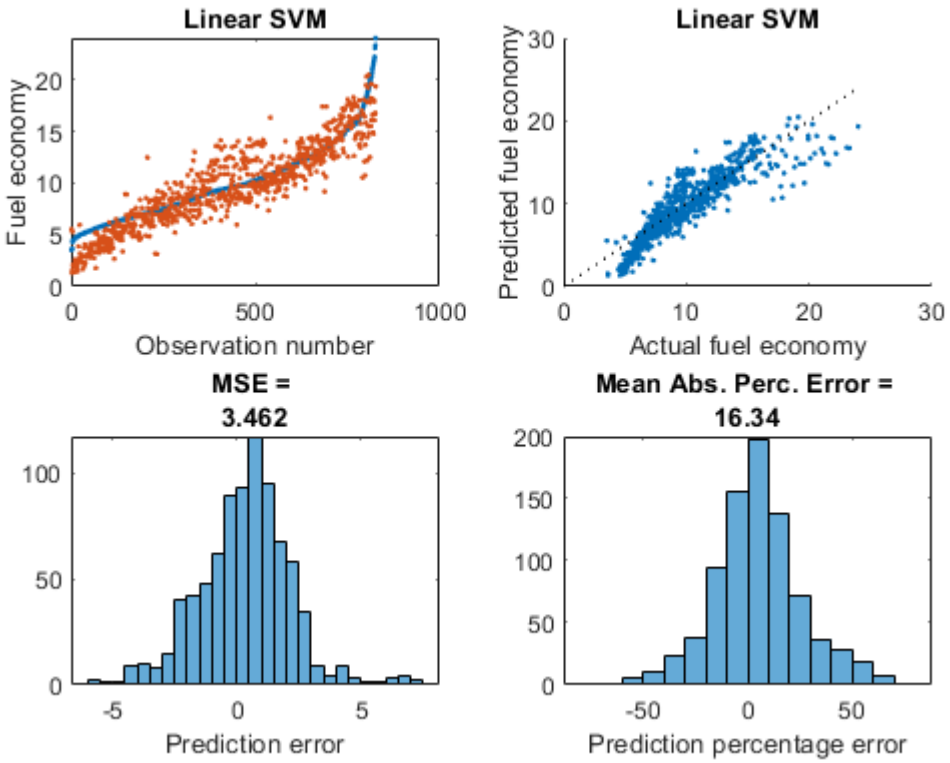
```
mdl = fitrsvm(carTrain,"FuelEcon");
```

Predict fuel economy at test predictor values.

```
econpred = predict(mdl,carTest);
```

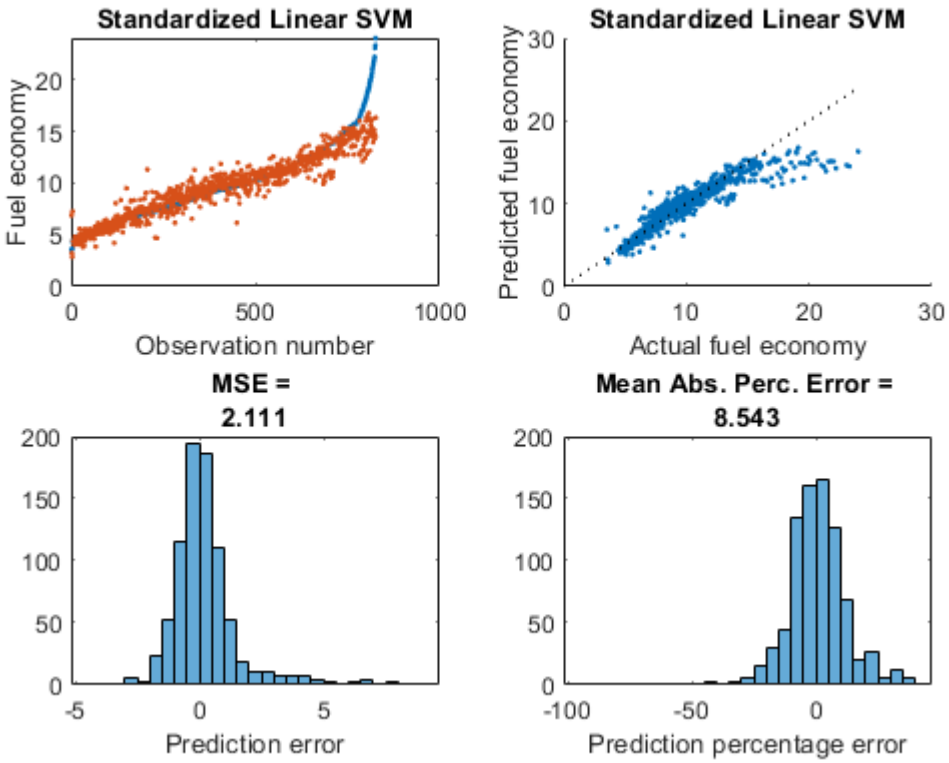
Compare predicted and actual responses.

```
evaluatefit(carTest.FuelEcon,econpred,"Linear SVM")
```



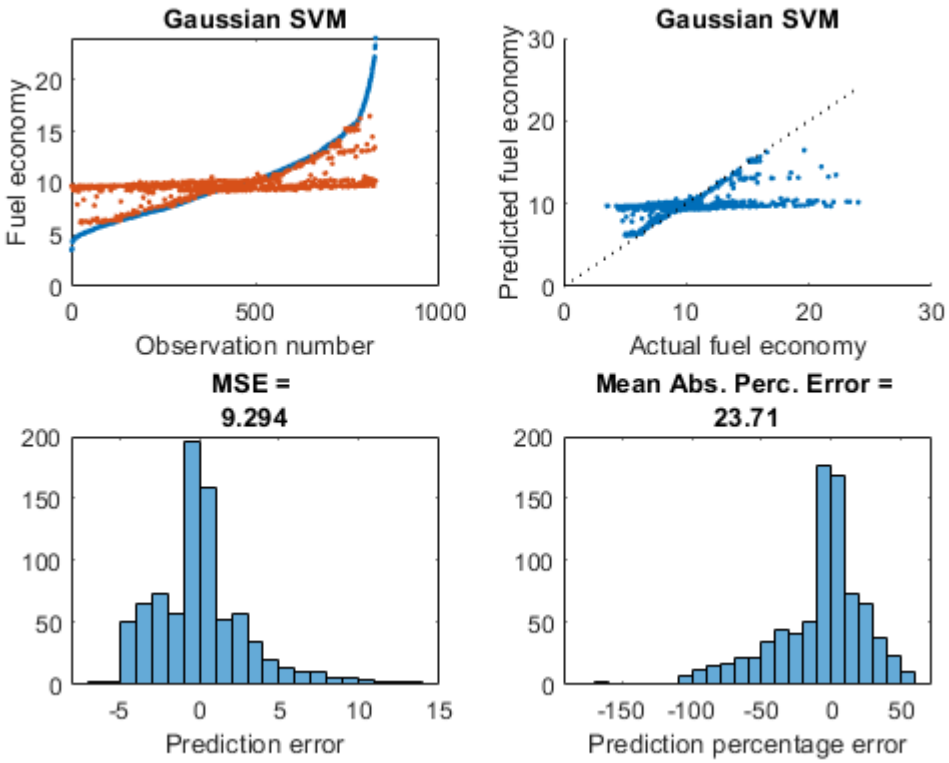
Standardize variables.

```
mdl = fitrsvm(carTrain,"FuelEcon","Standardize",true);
econpred = predict(mdl,carTest);
evaluatefit(carTest.FuelEcon,econpred,"Standardized Linear SVM")
```



Try a different kernel.

```
mdl = fitrsvm(carTrain,"FuelEcon","KernelFunction","gaussian");
econpred = predict(mdl,carTest);
evaluatefit(carTest.FuelEcon,econpred,"Gaussian SVM")
```



Decision Trees

Fit a decision tree regression model of fuel economy.

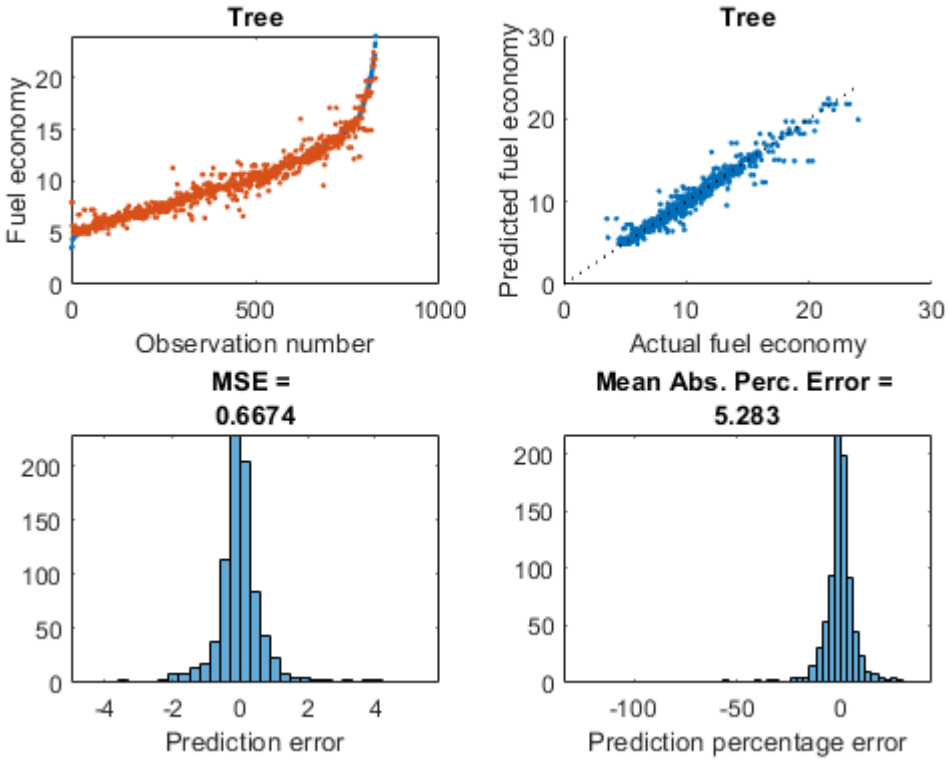
```
mdl = fitrtree(carTrain,"FuelEcon");
```

Predict fuel economy at test predictor values.

```
econpred = predict(mdl,carTest);
```

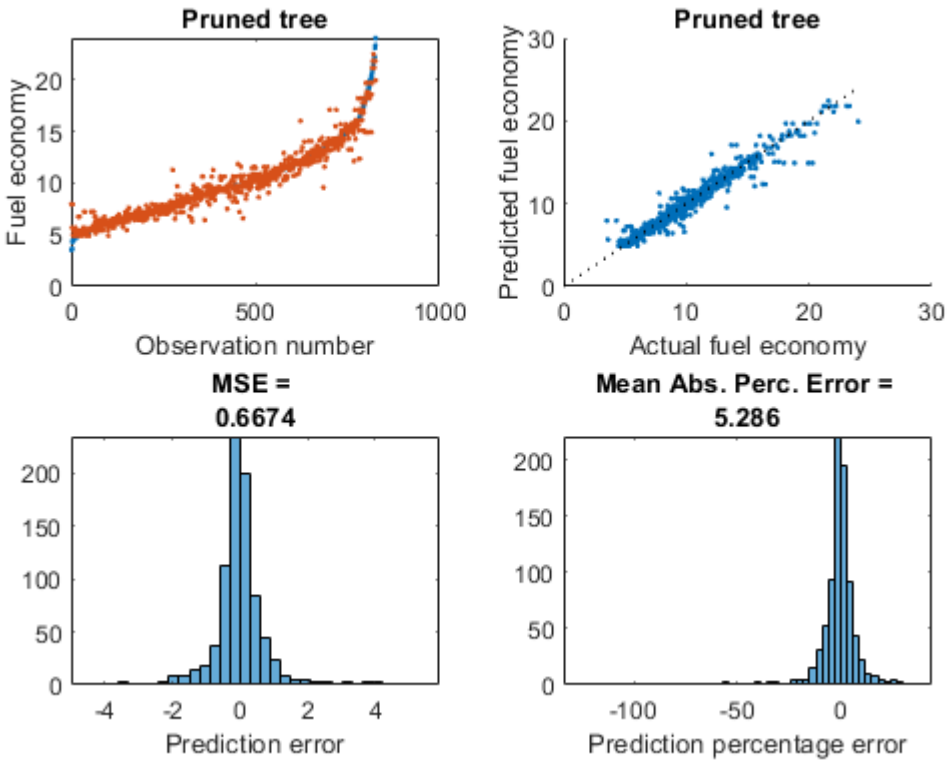
Compare predicted and actual responses

```
evaluatefit(carTest.FuelEcon,econpred,"Tree")
```



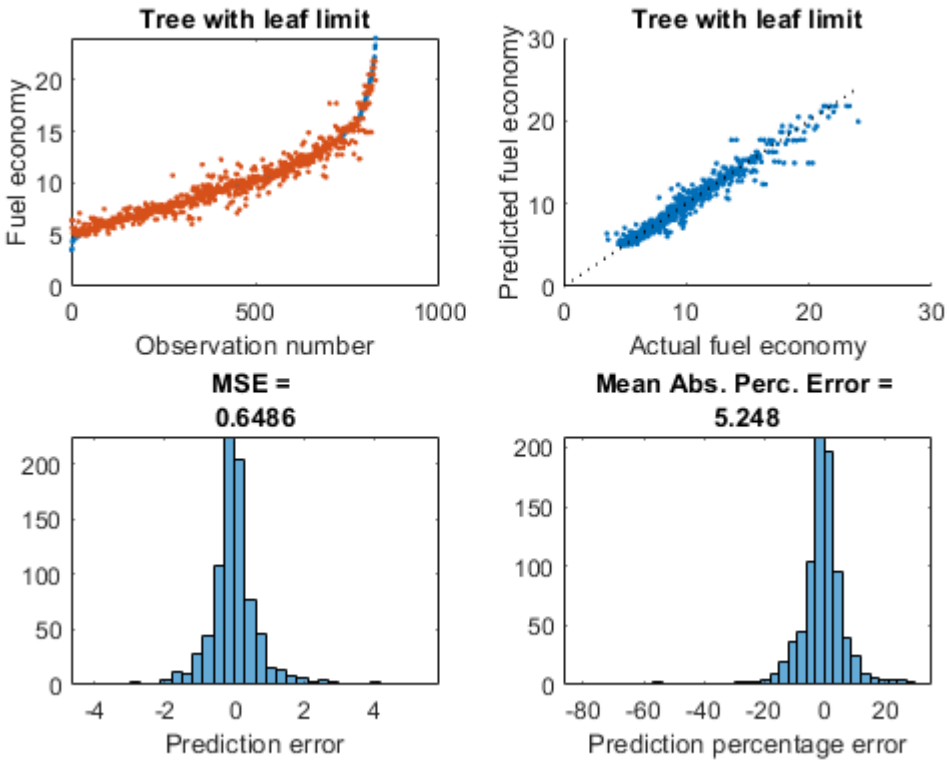
Prune the tree.

```
mdl = prune(mdl,"Level",10);
econpred = predict(mdl,carTest);
evaluatefit(carTest.FuelEcon,econpred,"Pruned tree")
```



Set minimum leaf size.

```
mdl = fitrtree(carTrain,"FuelEcon","MinLeafSize",5);
econpred = predict(mdl,carTest);
evaluatefit(carTest.FuelEcon,econpred,"Tree with leaf limit")
```



Gaussian Process Regression

Fit a Gaussian process model of fuel economy.

```
mdl = fitrgp(carTrain,"FuelEcon");
```

Predict fuel economy at test predictor values.

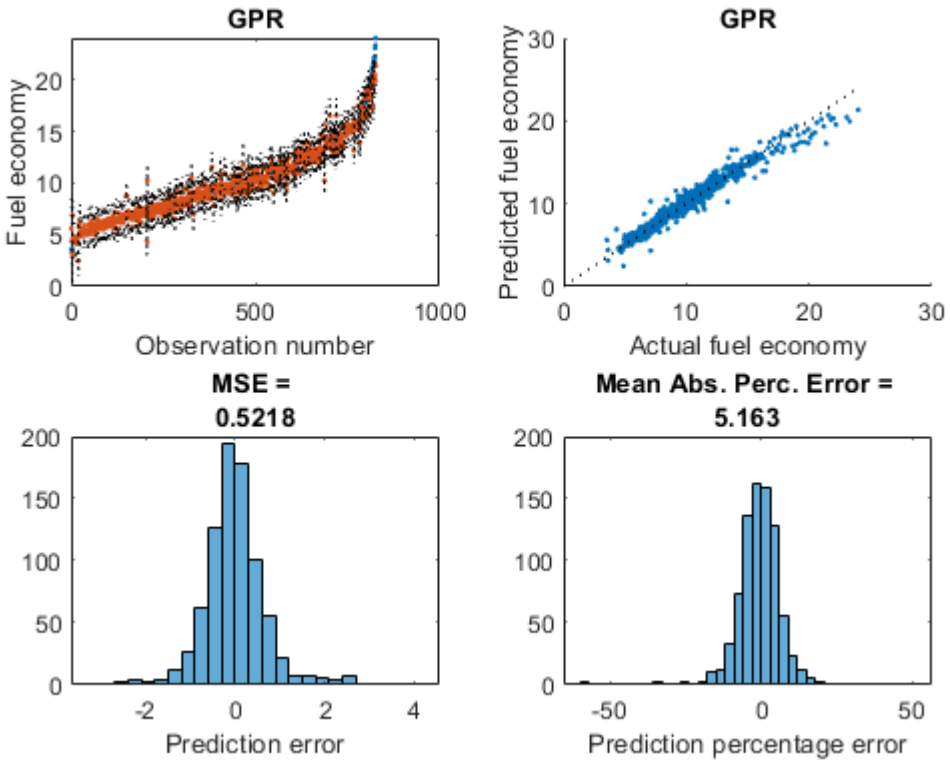
```
[econpred,~,epint] = predict(mdl,carTest);
```

Compare predicted and actual responses.

```
evaluatefit(carTest.FuelEcon,econpred,"GPR")
```

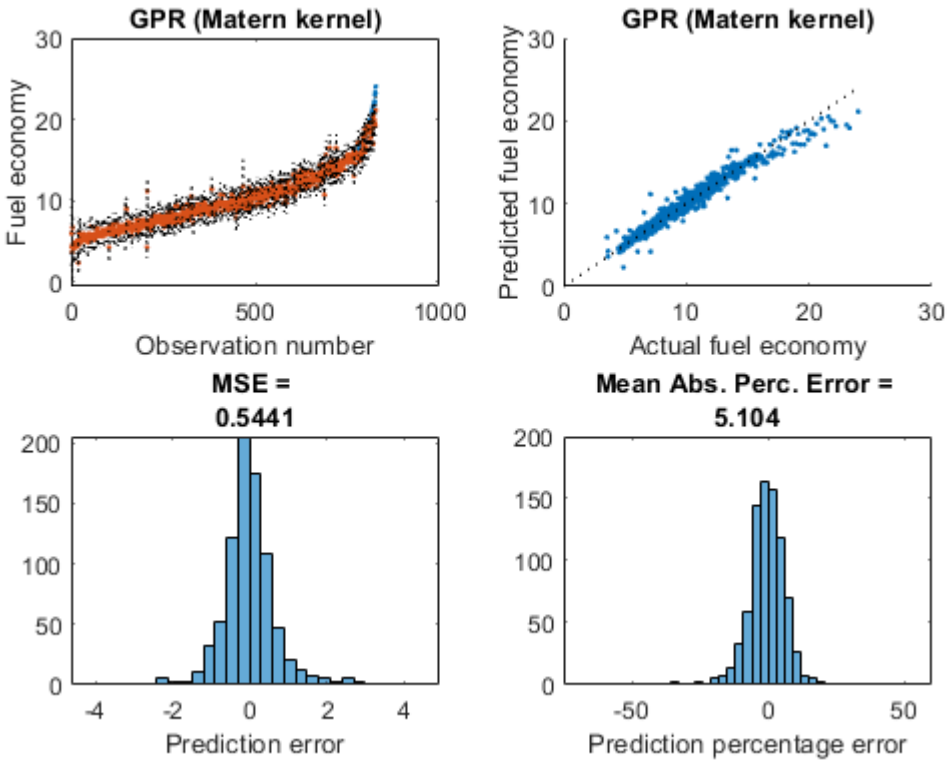
Add intervals to predictions.

```
subplot(2,2,1)
hold on
plot(epint,"k:")
hold off
```



Try a different kernel.

```
mdl = fitrgp(carTrain,"FuelEcon","KernelFunction","matern52");
[econpred,~,epint] = predict(mdl,carTest);
evaluatefit(carTest.FuelEcon,econpred,"GPR (Matern kernel)")
subplot(2,2,1)
hold on
plot(epint,"k:")
hold off
```



Local function: `evaluatefit`

```
function evaluatefit(y,ypred,name)
figure
% Plot against observation number
subplot(2,2,1)
plot(y,".")
hold on
plot(ypred,".")
title(name)
xlabel("Observation number")
ylabel("Fuel economy")

% Plot predicted and actual against each other
subplot(2,2,2)
scatter(y,ypred,".")
% Add 45-degree line
xl = xlim;
hold on
plot(xl,xl,"k:")
title(name)
xlabel("Actual fuel economy")
ylabel("Predicted fuel economy")

% Distribution of errors
subplot(2,2,3)
err = y - ypred;
```

```
MSE = mean(err.^2,"omitnan");
histogram(err)
title(["MSE = ",num2str(MSE,4)])
xlabel("Prediction error")

% Distribution of percentage errors
subplot(2,2,4)
err = 100*err./y;
MAPE = mean(abs(err),"omitnan");
histogram(err)
title(["Mean Abs. Perc. Error = ",num2str(MAPE,4)])
xlabel("Prediction percentage error")
end
```