MODELING FISH DYNAMICS AS TRIPLE PENDULUM

Evan Lutchmidat, Khushant Khurana, Scott Losirisup

AGENDA

Introduction

Background Info

Virtual Work Method

Lagrange Method

Discussion

Lessons Learned

INTRODUCTION

INSPIRATIONS

Development of Subcarangiform Bionic Robotic Fish Propelled by Shape Memory Alloy Actuators

M. Muralidharan* and I.A. Palani

Mechatronics and Instrumentation Laboratory, Mechanical Engineering, Indian Institute of Technology, Indore - 453 552, India

E-mail: muralidharanporthy@gmail.com

Yury L. Karavaev
Assistant Professor
Center for Technologies in Robotics and Mechatronics Components,
Innopolis University,
Kalashnikov Izhevsk State Technical University,
Izhevsk 426069, Russia
e-mail: karavaev_yury@istu.ru

Anton V. Klekovkin
Kalashnikov Izhevsk State Technical University,
Izhevsk 426069, Russia
e-mail: klanvlad@mail.ru

Ivan S. Mamaev[1]
Professor
Kalashnikov Izhevsk State Technical University,
Izhevsk 426069, Russia
e-mail: mamaev@rcd.ru

Valentin A. Tenenev
Professor
Kalashnikov Izhevsk State Technical University,
Izhevsk 426069, Russia
e-mail: v.tenenev@gmail.com

Evgeny V. Vetchanin
Assistant Professor
Ural Mathematical Center,
Udmurt State University,
Izhevsk 426034, Russia
e-mail: eugene186@mail.ru

A Simple Physical Model for Control of a Propellerless Aquatic Robot

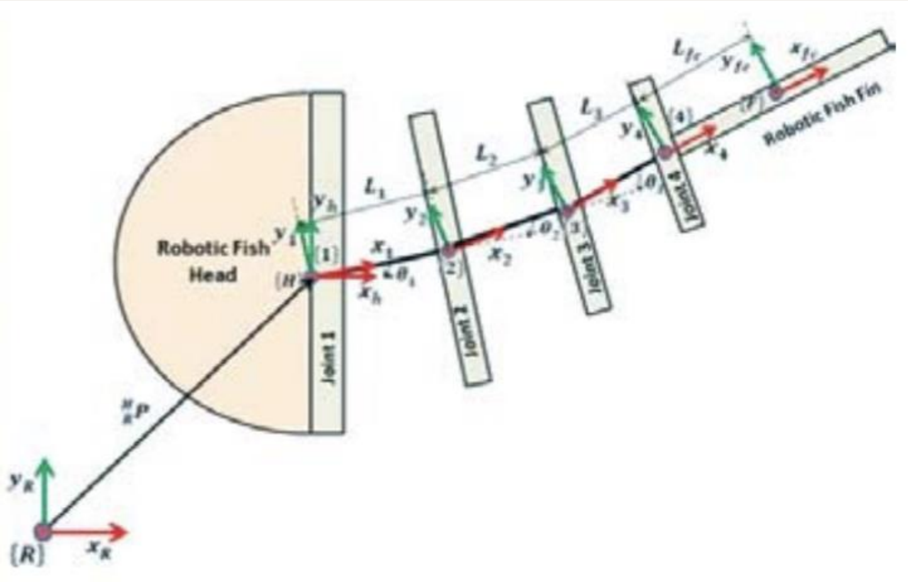
This paper is concerned with the motion of an aquatic robot whose body has the form of a sharp-edged foil. The robot is propelled by rotating the internal rotor without shell deformation. The motion of the robot is described by a finite-dimensional mathematical model derived from physical considerations. This model takes into account the effect of added masses and viscous friction. The parameters of the model are calculated from comparison of experimental data and numerical solution to the equations of rigid body motion and the Navier–Stokes equations. The proposed mathematical model is used to define controls implementing straight-line motion, motion in a circle, and motion along a complex trajectory. Experiments for estimation of the efficiency of the model have been conducted.
[DOI: 10.1115/1.4051240]

Keywords: aquatic robot, propulsion in a fluid, periodic control action, motion planning, identification of parameters of the model

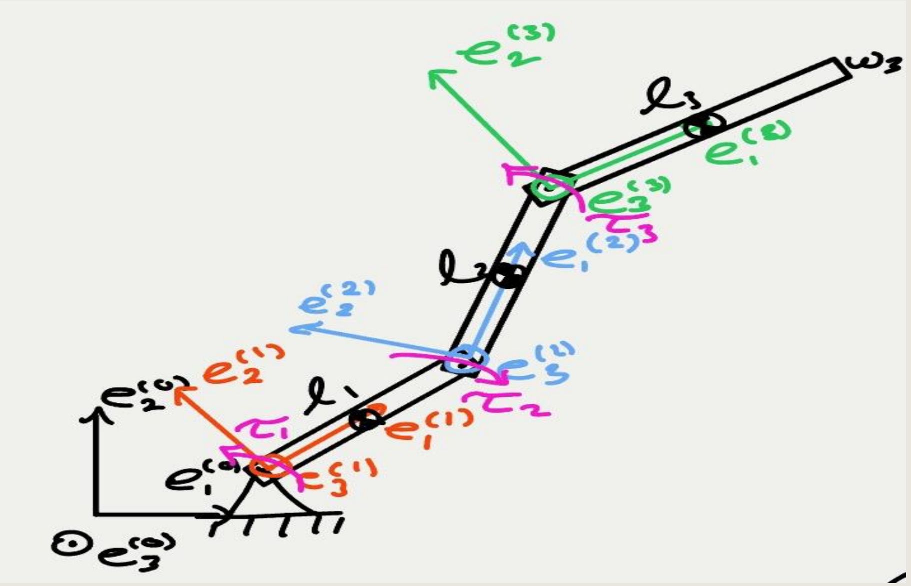
BACKGROUND INFO

MODELLING OF THE FISH

Fixed Compound Pendulum



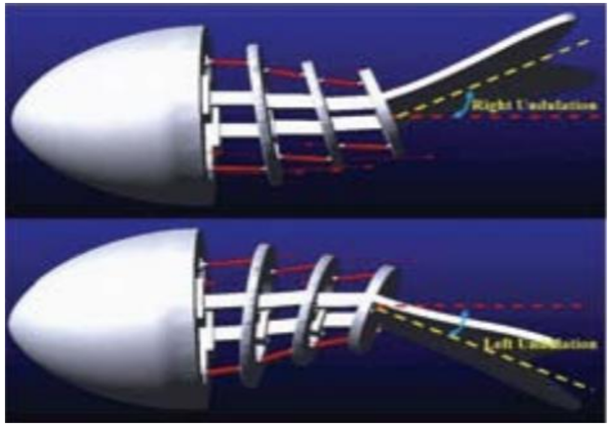
Applied Torque to Each Joint



https://giphy.com/stickers/fish-swim-aanimals-kG1ESrXBMfxg2N20pT

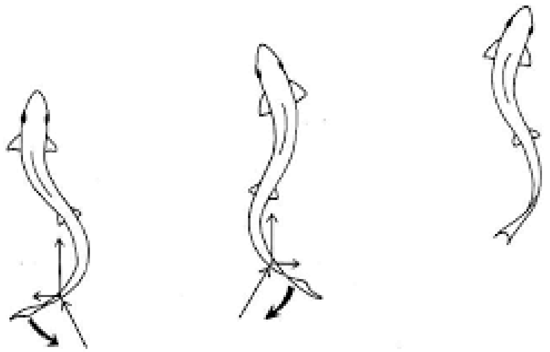
PROBLEM CONSTRAINTS/SETUP

Fixed About The Head

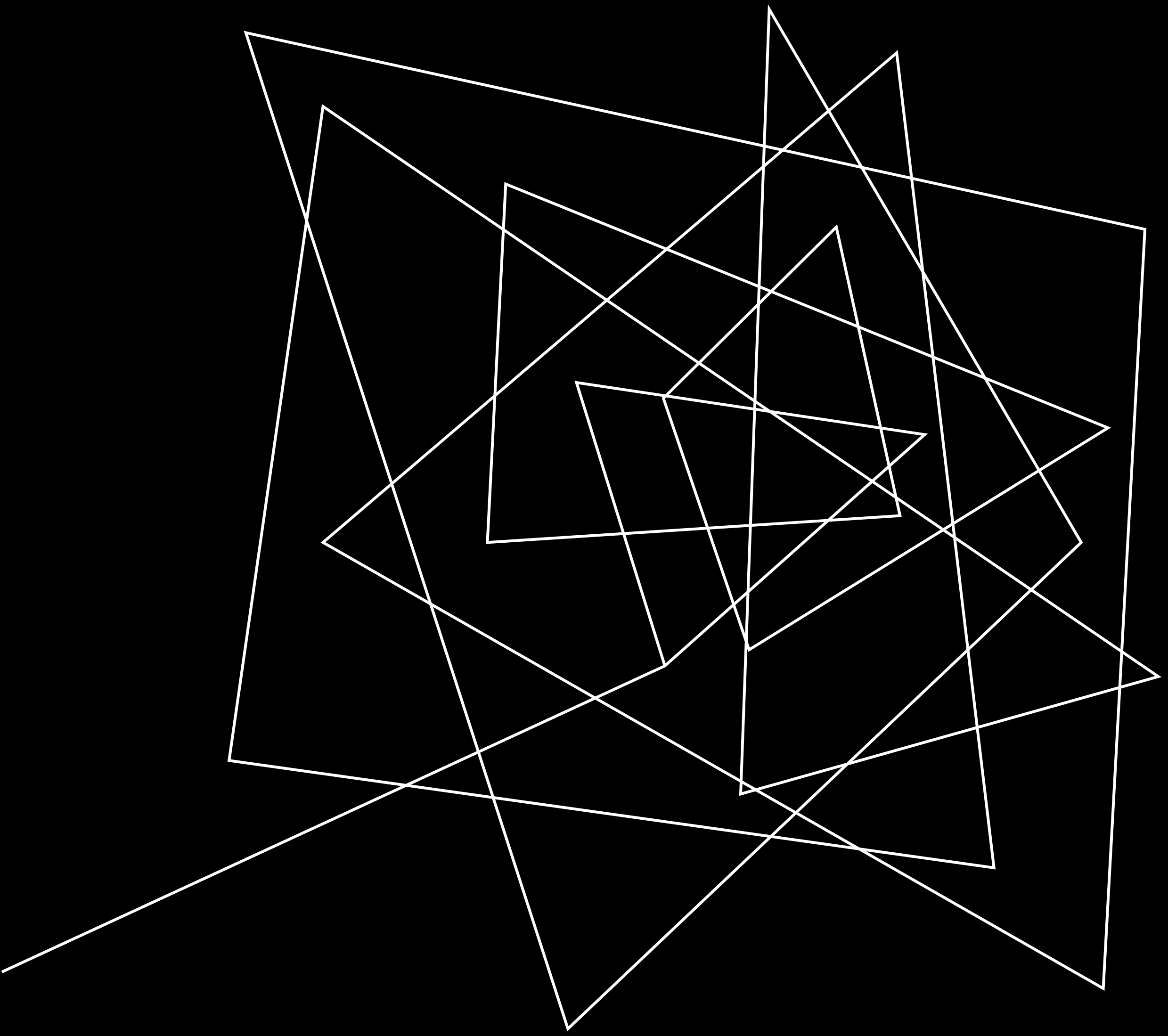


Weight Canceled Out by Buoyant Force

Largest Torque on Tail



http://www.biology-resources.com/drawing-fish-swimming.html



VIRTUAL WORK METHOD & PROJECTIONS

PARAMETERS

Parameter	Magnitude (units)
L1, L2, L3	1
W1, W2, W3	0.1
M1, M2, M3	1
J1_1, J1_2, J2_,1, J2_2, J3_1, J3_2	0
J1_3	(1/12) * M1 * (L1^2 + W1^2)
J2_3	(1/12) * M2 * (L2^2 + W2^2)
J3_3	(1/12) * M3 * (L3^2 + W3^2)
Tau_1, Tau_2, Tau_3	Our forced inputs!
K – spring constant	Variable
B – damping constant	variable

BASIC DEFINITIONS

1. Defining the position vectors for center of mass in respective frames.

rc_1, rc_2, rc_3 = $\begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix}$

2. Find the rotation matrices around z – axis for each transformation.

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Find the angular velocities with respect to each frame.

Omega01_11 = $\begin{bmatrix} 0 & -\sin^2(\theta_1(t))\frac{d}{dt}\theta_1(t) - \cos^2(\theta_1(t))\frac{d}{dt}\theta_1(t) & 0 \\ \sin^2(\theta_1(t))\frac{d}{dt}\theta_1(t) + \cos^2(\theta_1(t))\frac{d}{dt}\theta_1(t) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Omega02_22 = $\begin{bmatrix} 0 & -\frac{d}{dt}\theta_1(t) - \frac{d}{dt}\theta_2(t) & 0 \\ \frac{d}{dt}\theta_1(t) + \frac{d}{dt}\theta_2(t) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Omega03_33 = $\begin{bmatrix} 0 & -\frac{d}{dt}\theta_1(t) - \frac{d}{dt}\theta_2(t) - \frac{d}{dt}\theta_3(t) & 0 \\ \frac{d}{dt}\theta_1(t) + \frac{d}{dt}\theta_2(t) + \frac{d}{dt}\theta_3(t) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

BASIC DEFINITIONS

4. Find linear velocities

VC_0 = $$\begin{bmatrix} -0.5\sin(\theta_1(t))\frac{d}{dt}\theta_1(t) \\ 0.5\cos(\theta_1(t))\frac{d}{dt}\theta_1(t) \\ 0 \end{bmatrix}$$

VC_1 = $$\begin{bmatrix} -0.5\left(\frac{d}{dt}\theta_1(t) + \frac{d}{dt}\theta_2(t)\right)\sin(\theta_1(t)+\theta_2(t)) - \sin(\theta_1(t))\frac{d}{dt}\theta_1(t) \\ 0.5\left(\frac{d}{dt}\theta_1(t) + \frac{d}{dt}\theta_2(t)\right)\cos(\theta_1(t)+\theta_2(t)) + \cos(\theta_1(t))\frac{d}{dt}\theta_1(t) \\ 0 \end{bmatrix}$$

VC_2 =

$$\begin{bmatrix} -\left(\frac{d}{dt}\theta_1(t) + \frac{d}{dt}\theta_2(t)\right)\sin(\theta_1(t)+\theta_2(t)) - 0.5\left(\frac{d}{dt}\theta_1(t) + \frac{d}{dt}\theta_2(t) + \frac{d}{dt}\theta_3(t)\right)\sin(\theta_1(t)+\theta_2(t)+\theta_3(t)) - \sin(\theta_1(t))\frac{d}{dt}\theta_1(t) \\ \left(\frac{d}{dt}\theta_1(t) + \frac{d}{dt}\theta_2(t)\right)\cos(\theta_1(t)+\theta_2(t)) + 0.5\left(\frac{d}{dt}\theta_1(t) + \frac{d}{dt}\theta_2(t) + \frac{d}{dt}\theta_3(t)\right)\cos(\theta_1(t)+\theta_2(t)+\theta_3(t)) + \cos(\theta_1(t))\frac{d}{dt}\theta_1(t) \\ 0 \end{bmatrix}$$

5. Initialize the generalized coordinates

$$\begin{bmatrix} \frac{d}{dt}\theta_1(t) \\ \frac{d}{dt}\theta_2(t) \\ \frac{d}{dt}\theta_3(t) \end{bmatrix}$$

6. Find Jacobian matrix (18 * 3)

BASIC DEFINITIONS

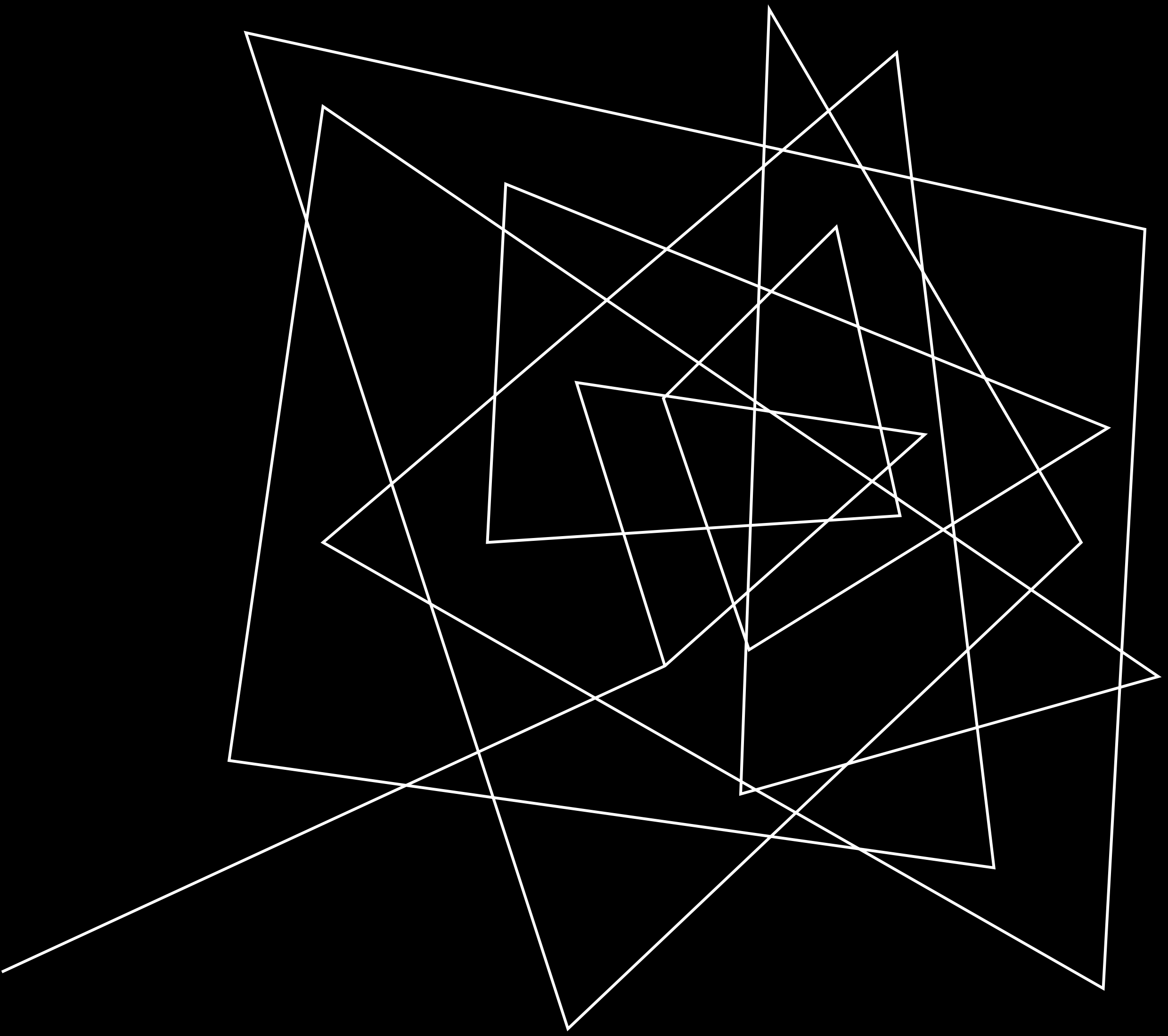
7. Initialize the mass matrix

$$\begin{bmatrix}
m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & m_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & m_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & m_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & m_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & m_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{23} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{33}
\end{bmatrix}$$

8. Initialize the frame rotation matrix

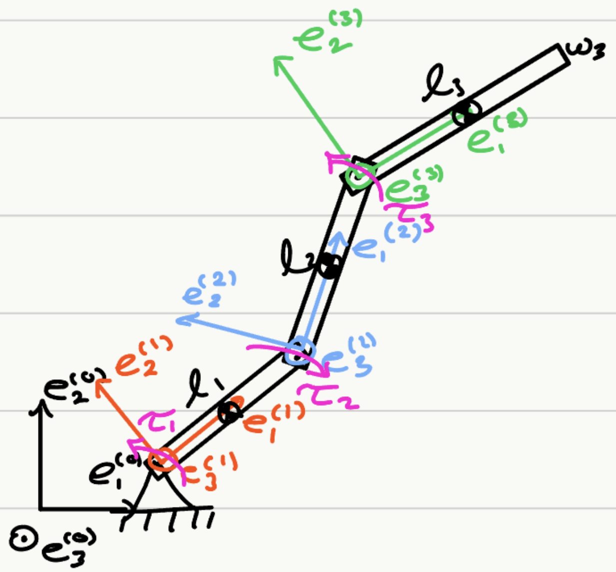
9. Initialize the external forces matrix: G.T

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau_1 - \tau_2 & 0 & 0 & \tau_2 - \tau_3 & 0 & 0 & \tau_3 \end{bmatrix}$$



LAGRANGE METHOD

BEGINNING FROM SCRATCH: LAGRANGE APPROACH



$$e^{(0)} = R^{(0,1)}(\theta_1)\, e^{(1)}$$

$$e^{(1)} = R^{(1,2)}(\theta_2)\, e^{(2)}$$

$$e^{(2)} = R^{(2,3)}(\theta_3)\, e^{(3)}$$

$$T = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 + \frac{1}{2} m_3 V_3^2 + \frac{1}{2} J_1 \omega_1^2 + \frac{1}{2} J_2 \omega_2^2 + \frac{1}{2} J_3 \omega_3^2$$

$$R^{(0,1)}(\theta) = \begin{bmatrix} c(\theta_1) & -s(\theta_1) & 0 \\ s(\theta_1) & c(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \frac{1}{2} \omega_{1/0}^{(1)T} J_c^{(1,1)} \omega_{1/0}^{(1)} + \frac{1}{2} \omega_{2/0}^{(2)T} J_c^{(2,2)} \omega_{2/0}^{(2)} + \frac{1}{2} \omega_{3/0}^{(3)T} J_c^{(3,3)} \omega_{3/0}^{(3)}$$
$$+ \frac{1}{2} V_{c_1}^T M_1 V_{c_1} + \frac{1}{2} V_{c_2}^T M_2 V_{c_2} + \frac{1}{2} m_3 V_{c_3}^T M_3 V_{c_3}$$

NEXT, ROTATIONAL KINETIC ENERGY

$$J_c^{(b,b)} = \begin{bmatrix} J_1 & & \\ & J_2 & \\ & & J_3 \end{bmatrix}$$

for all plates:

$$J_1 = \frac{1}{12} m_b l_b^2$$

$$J_2 = \frac{1}{12} m_b w_b^2$$

$$J_3 = J_1 + J_2 = \frac{1}{12} m_b (l_b^2 + w_b^2)$$

$$\mathrm{J}_3 = \begin{bmatrix} 0.0833333333333333 l_3^2 m_3 & 0 & 0 \\ 0 & 0.0833333333333333 m_3 w_3^2 & 0 \\ 0 & 0 & 0.0833333333333333 m_3 \left(l_3^2 + w_3^2\right) \end{bmatrix}$$

$$\tilde{\omega}_{1/0}^{(1,1)} = R^{(1,0)}(\theta_1)\,\dot{R}^{(0,1)}(\theta_1)$$

$$\tilde{\omega}_{2/0}^{(2,2)} = R^{(2,0)}\,\dot{R}^{(0,2)}$$

$$\tilde{\omega}_{3/0}^{(3,3)} = R^{(3,0)}\,\dot{R}^{(0,3)}$$

$$\tilde{\omega}_{1/0}^{(1,1)} = \begin{bmatrix} 0 & -\dot{\theta}_1 & 0 \\ \dot{\theta}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\omega}_{2/0}^{(2,2)} = \begin{bmatrix} 0 & -\dot{\theta}_1 - \dot{\theta}_2 & 0 \\ \dot{\theta}_1 + \dot{\theta}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\omega}_{3/0}^{(3,3)} = \begin{bmatrix} 0 & -\dot{\theta}_1 - \dot{\theta}_2 - \dot{\theta}_3 & 0 \\ \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

NOW, SOLVE FOR TRANSLATIONAL KINETIC ENERGY

$$r_{c_1} = \frac{l_1}{2} e_1^{(1)} \quad r_{c_2} = \frac{l_2}{2} e_1^{(2)} + l_1 e_1^{(1)} \cdot \quad r_{c_3} = \frac{l_3}{2} e_1^{(3)} + l_2 e_1^{(2)} + l_1 e_1^{(1)}$$

$$V_{c_1}^{(0)} = \dot{r}_{c1}^{(0)} = \dot{R}^{(0,1)} \left(\frac{l_1}{2} e_1^{(0)}\right)^T$$

$$v_{c,1}^{(0)} = \begin{bmatrix} -\frac{l_1 \mathrm{s}(\theta_1)\dot{\theta}_1}{2} \\ \frac{l_1 \mathrm{c}(\theta_1)\dot{\theta}_1}{2} \\ 0 \end{bmatrix}$$

$$V_{c_2}^{(0)} = \dot{r}_{c_2}^{(0)} = \dot{R}^{(0,2)} \left(\frac{l_2}{2} e_1^{(0)}\right)^T + \dot{R}^{(0,1)} \left(l_1 e_1^{(0)}\right)^T$$

$$v_{c,2}^{(0)} = \begin{bmatrix} -l_1 \mathrm{s}(\theta_1)\dot{\theta}_1 - \frac{l_2(\dot{\theta}_1+\dot{\theta}_2)\mathrm{s}(\theta_1+\theta_2)}{2} \\ l_1 \mathrm{c}(\theta_1)\dot{\theta}_1 + \frac{l_2(\dot{\theta}_1+\dot{\theta}_2)\mathrm{c}(\theta_1+\theta_2)}{2} \\ 0 \end{bmatrix}$$

$$V_{c_3}^{(0)} = \dot{r}_{c3}^{(0)} = \dot{R}^{(0,3)} \left(\frac{l_3}{2} e_1^{(0)}\right)^T + \dot{R}^{(0,2)} \left(l_2 e_1^{(0)}\right)^T + \dot{R}^{(0,1)} \left(l_1 e_1^{(0)}\right)^T$$

$$v_{c,3}^{(0)} = \begin{bmatrix} -l_1 \mathrm{s}(\theta_1)\dot{\theta}_1 - l_2\left(\dot{\theta}_1+\dot{\theta}_2\right)\mathrm{s}(\theta_1+\theta_2) - \frac{l_3(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)\mathrm{s}(\theta_1+\theta_2+\theta_3)}{2} \\ l_1 \mathrm{c}(\theta_1)\dot{\theta}_1 + l_2\left(\dot{\theta}_1+\dot{\theta}_2\right)\mathrm{c}(\theta_1+\theta_2) + \frac{l_3(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)\mathrm{c}(\theta_1+\theta_2+\theta_3)}{2} \\ 0 \end{bmatrix}$$

NOW, EVERYTHING IS IN PYTHON'S HANDS(?)

Lagrange Approach

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = Q_j^{(nc)}, \qquad \dot{q} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

where $\mathcal{L} = T - V$

$$B^T \cdot G = Q_j^{(nc)} = \sum_{j=1}^{n} \frac{\partial \omega_j}{\partial \dot{q}_i} \cdot \tau_j$$

① $$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_1}\right) - \frac{\partial \mathcal{L}}{\partial q_1} = \frac{\partial \omega_1}{\partial \dot{q}_1} \cdot (\tau_1) + \frac{\partial \omega_2}{\partial \dot{q}_1} \cdot (\tau_2) + \frac{\partial \omega_3}{\partial \dot{q}_1} \cdot (\tau_3)$$

② $$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_2}\right) - \frac{\partial \mathcal{L}}{\partial q_2} = \frac{\partial \omega_2}{\partial \dot{q}_2} \cdot (\tau_1) + \frac{\partial \omega_2}{\partial \dot{q}_2} \cdot (\tau_2) + \frac{\partial \omega_3}{\partial \dot{q}_2} \cdot (\tau_3)$$

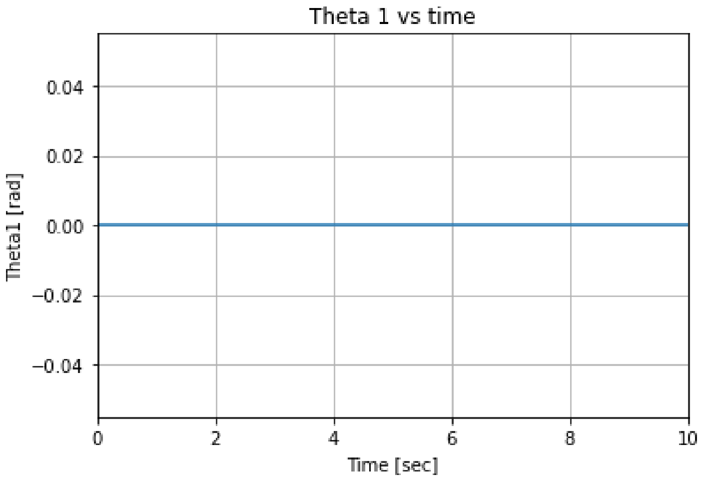
③ $$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_3}\right) - \frac{\partial \mathcal{L}}{\partial q_3} = \frac{\partial \omega_3}{\partial \dot{q}_3} \cdot (\tau_1) + \frac{\partial \omega_2}{\partial \dot{q}_3} \cdot (\tau_2) + \frac{\partial \omega_3}{\partial \dot{q}_3} \cdot (\tau_3)$$

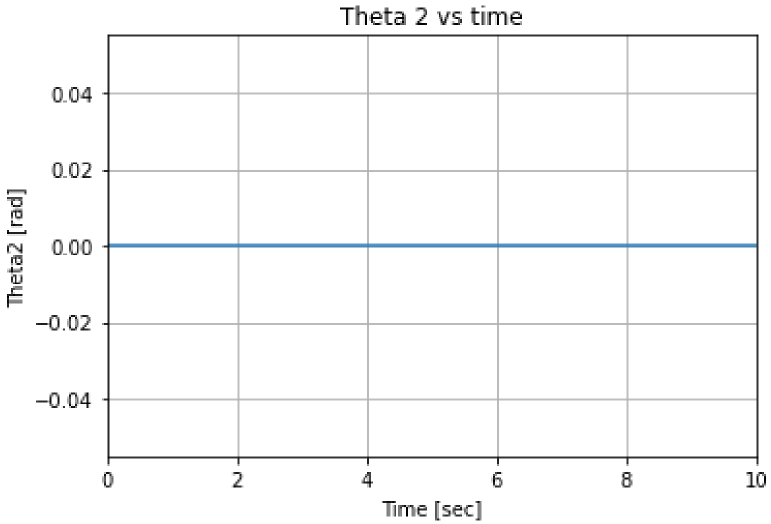
RESULTS AND DISCUSSION

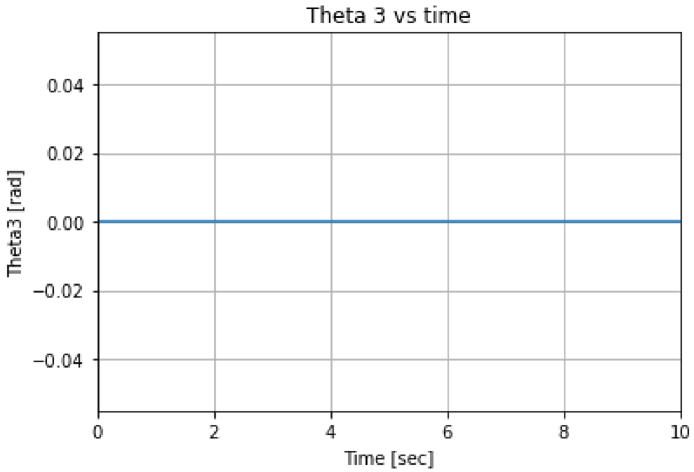
WITH THE IDENTICAL MATRICES DERIVED BY THE TWO METHODS, WE RAN MULTIPLE TESTS TO PREDICT THE MOVEMENT OF A FISH WITH THE CONSTRAINTS

TEST 0 – UNIT TEST

```
def t1(time):
    return 0#.5*np.sin(.5*time)
def t2(time):
    return 0#-40*signal.square(2 * np.pi * 20 * time)
def t3(time):
    return 0#2*np.sin(4*time)
```







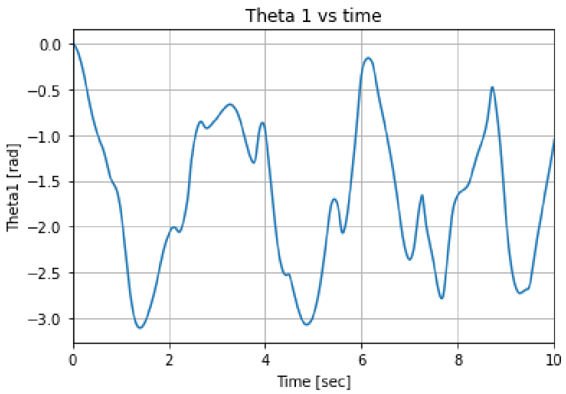
With no torques, the system should be stationary and accordingly the angles don't change from their initial conditions of 0 degrees.

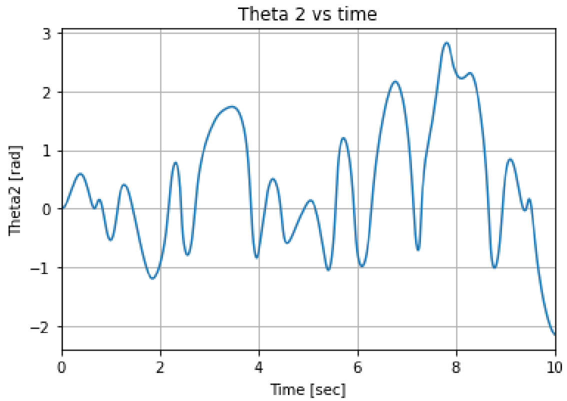
TEST 0 – UNIT TEST

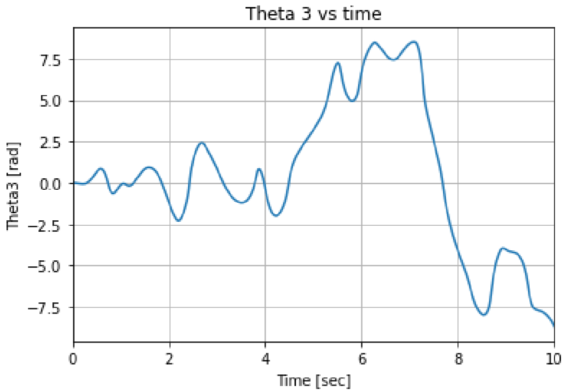
```
W_1 = sp.Matrix([
    [0],
    [0],
    [-9.81]
])

W_2 = sp.Matrix([
    [0],
    [0],
    [-9.81]
])

W_3 = sp.Matrix([
    [0],
    [0],
    [-9.81]
])
```







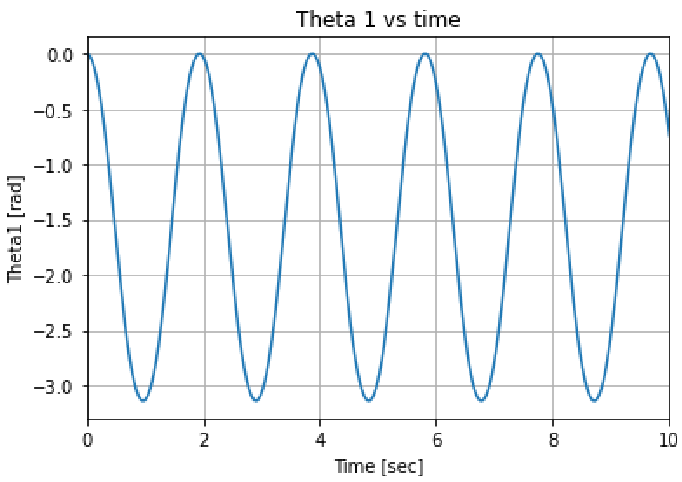
The second test case was giving weight to all linkages and having 0 torques so the linkages can behave as free pendulums. All masses were fixed at 1 and the force was applied in the negative z direction. As expected all the masses behaved like pendulums. Due to the chaotic nature of the system, the above condition gets worse as the number of pendulums increase. Therefore, the length and mass of the second pendulum were made .0001 to see if the first pendulum would still behave like an actual pendulum. The test results are seen in next slide.

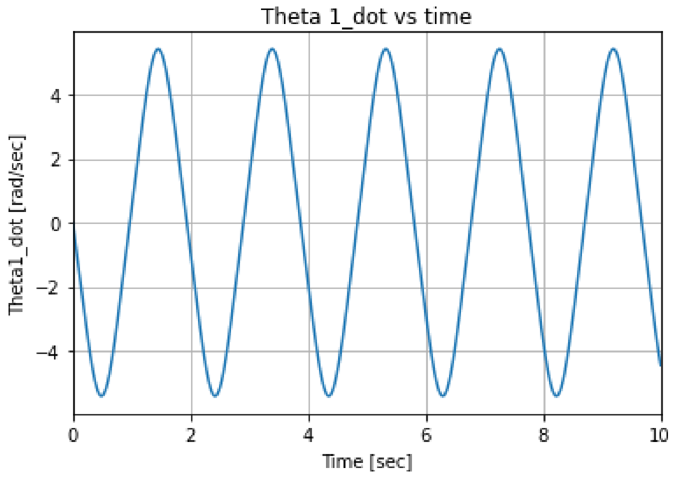
TEST 0 – UNIT TEST

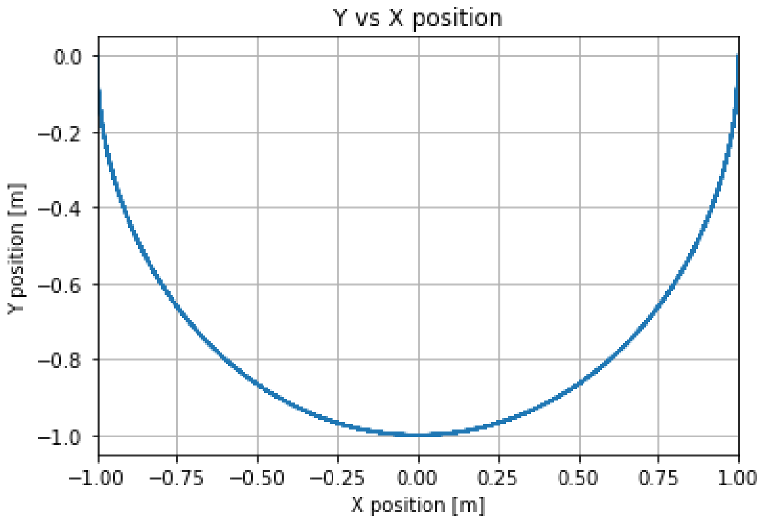
```
W_1 = sp.Matrix([
    [0],
    [0],
    [-9.81]
])

W_2 = sp.Matrix([
    [0],
    [0],
    [-9.81]
])

W_3 = sp.Matrix([
    [0],
    [0],
    [-9.81]
])
```



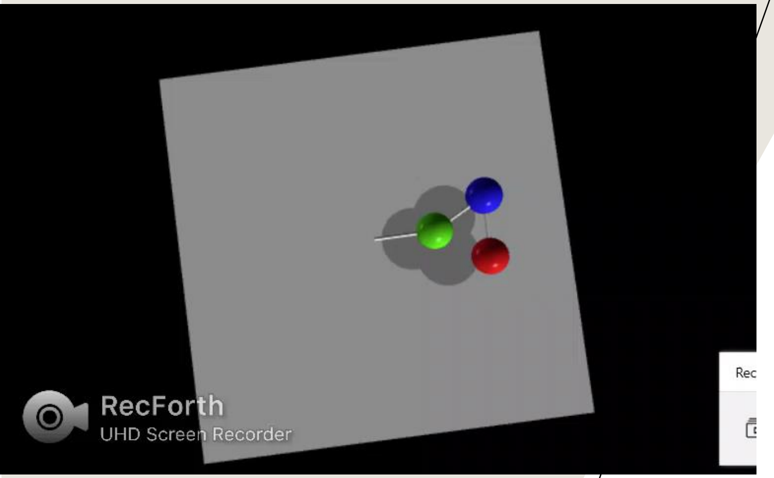


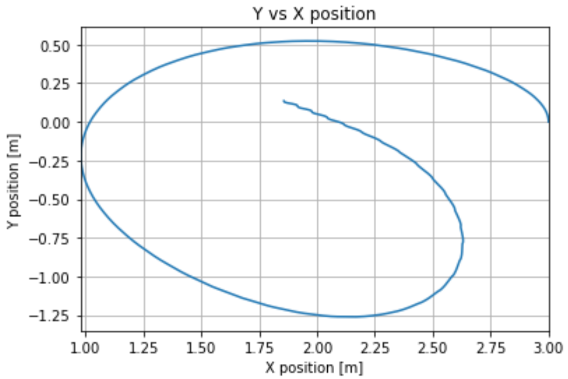


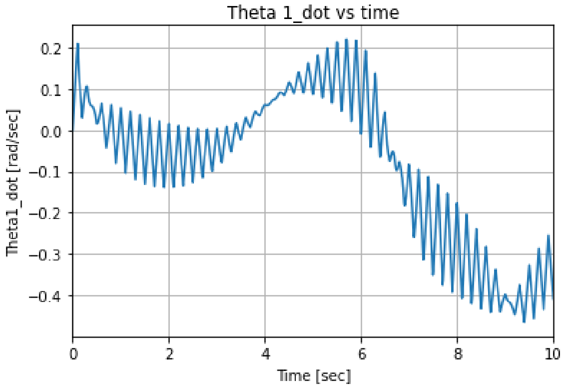
As expected, the first mass behaves exactly like a pendulum when effects of second and thirst pendulum are made negligible.

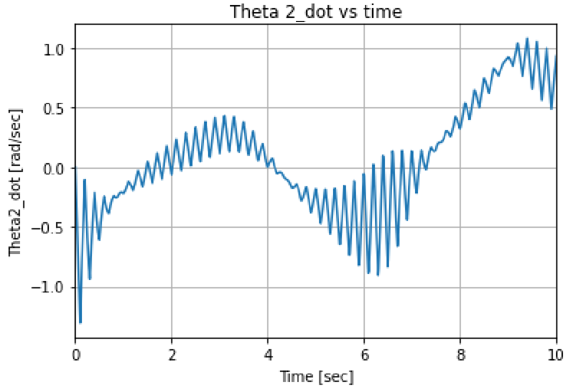
TEST 1 – SINUSOIDAL TORQUES

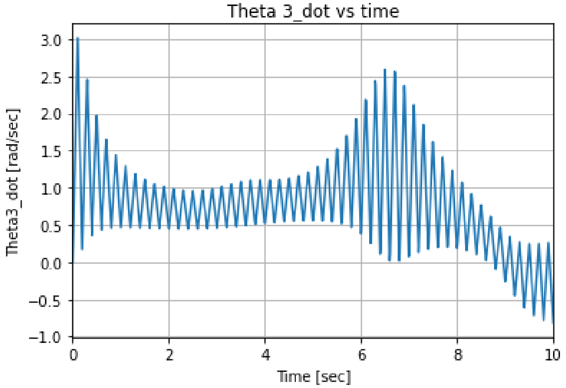
```
def t1(time):
    return 0        #20*signal.square(2 * np.pi * 10 * time)
def t2(time):
    return 0#-40*signal.square(2 * np.pi * 20 * time)
def t3(time):
    return 2*signal.square(2 * np.pi * 5 * time)
```

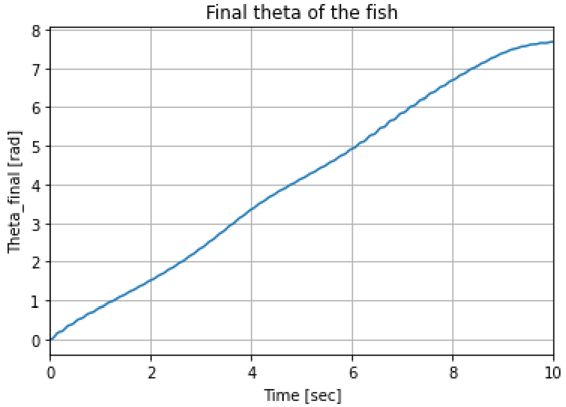








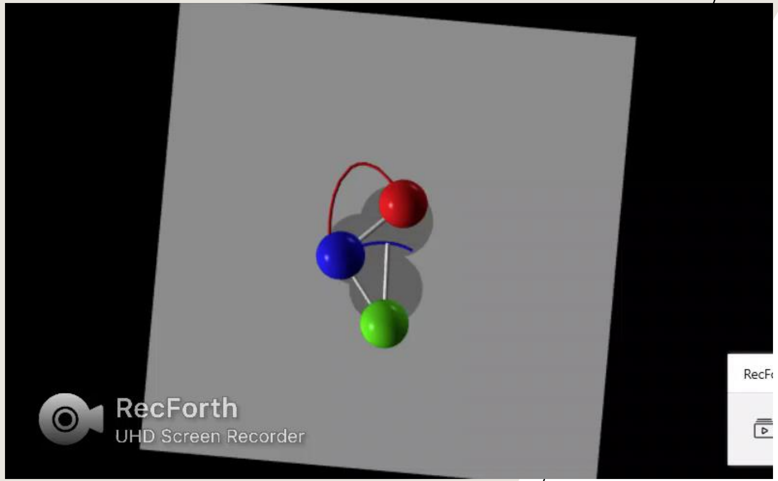


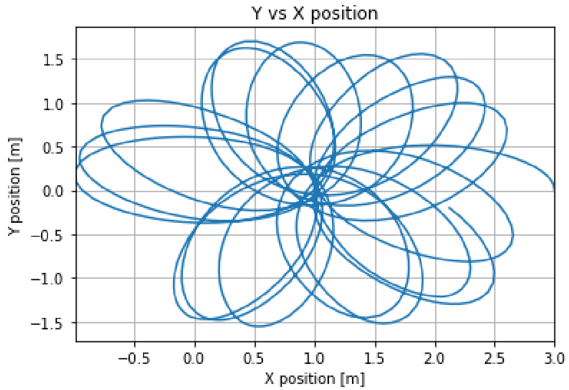


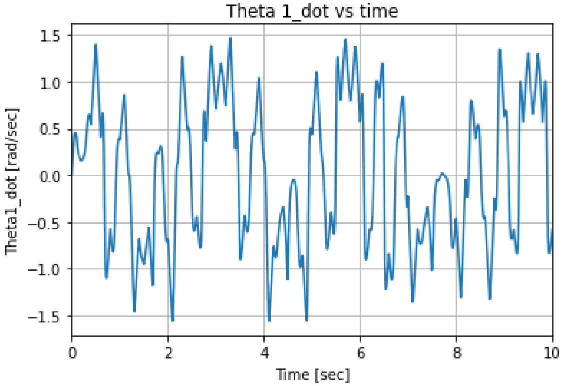
With torque input as a square function, the system has a lot of oscillation in its velocities. Practically this does not happens with a fish since the motion is pretty smooth. So clearly the square input torque is not the way to go. The reason for the smooth y vs x position is because the frequencies are so small that the fish moves really slow (as seen in the video)

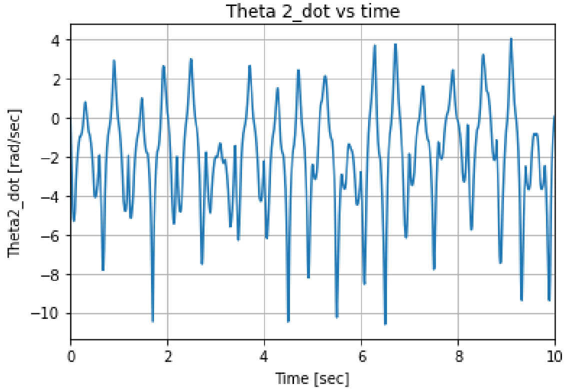
TEST 2 – SQUARE TORUQES

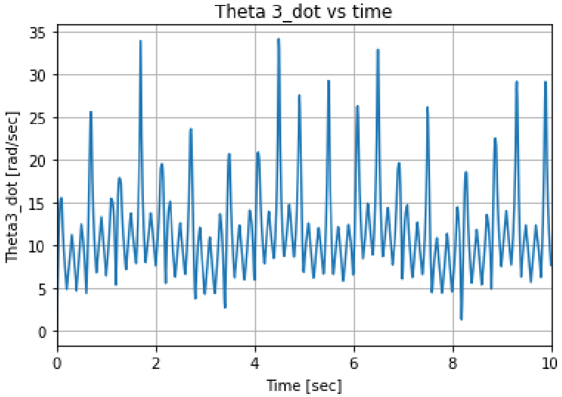
```python
def t1(time):
    return 1*np.sin(.5*time)
def t2(time):
    return 0#-40*signal.square(2 * np.pi * 20 * time)
def t3(time):
    return 2*np.sin(4*time)
```

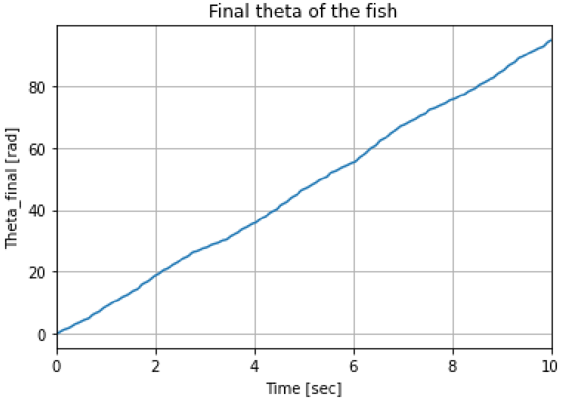










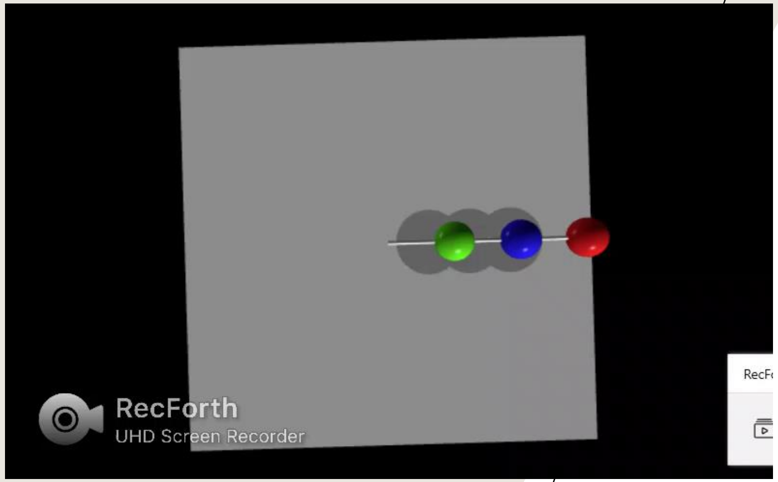


Switching torques to sin waves helps the system to reduce oscillation also making the system smoother. However, the system still overlaps with itself which is not good.

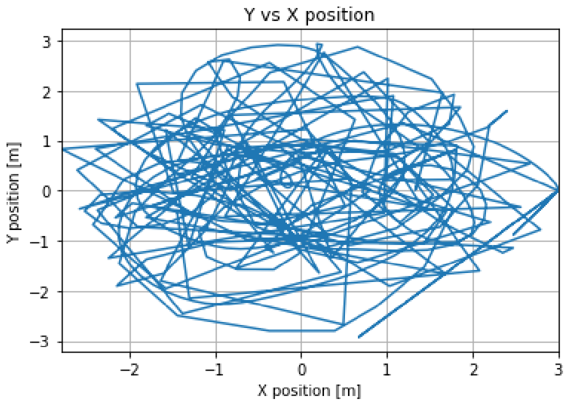
TEST 3 – SINUSOIDAL TORQUES WITH SPRINGS

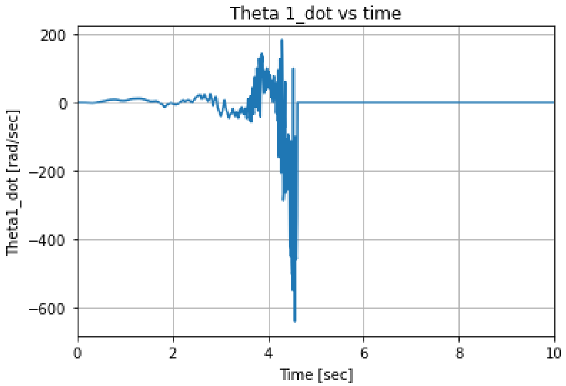
```python
def t1(time):
    return 1*np.sin(.5*time)
def t2(time):
    return 0#-40*signal.square(2 * np.pi * 20 * time)
def t3(time):
    return 2*np.sin(4*time)
```

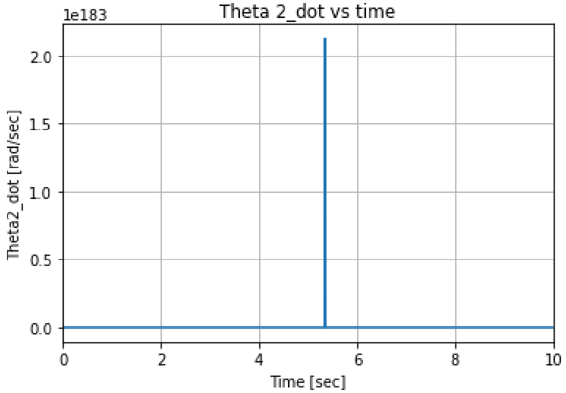
```python
G[11] = tau1-tau2 + k*(theta2 - theta1)*(l1/2) #+ b*(theta2_dot - theta1_dot)*l1/2
G[14] = tau2-tau3 + k*(theta3 - theta2)*(l2/2) #+ b*(theta3_dot - theta1_dot)*l2/2
G[17] = tau3
```

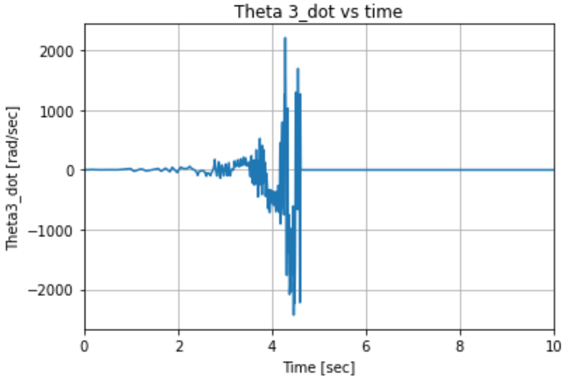


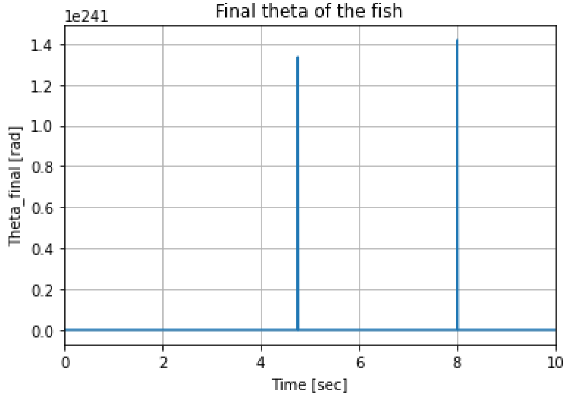
As expected, the springs helped the system in not overlapping. However, the springs energy never dies and makes the system extremely chaotic as time goes on. Clearly the graphs suggest the same because the magnitudes are extreme.







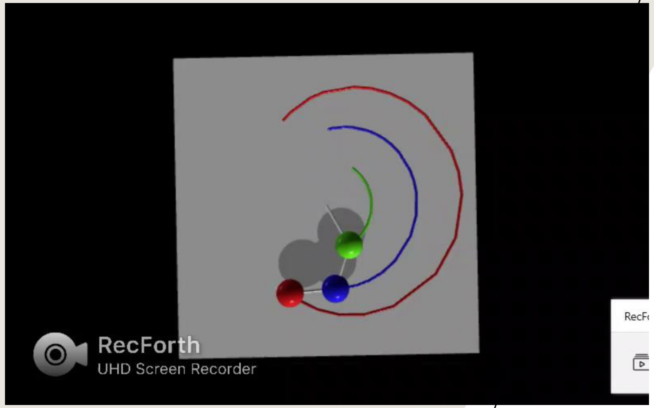


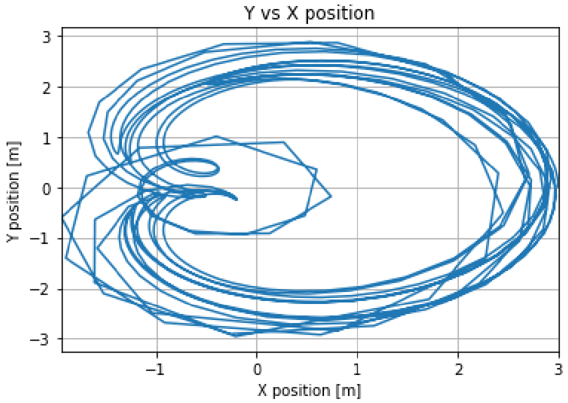


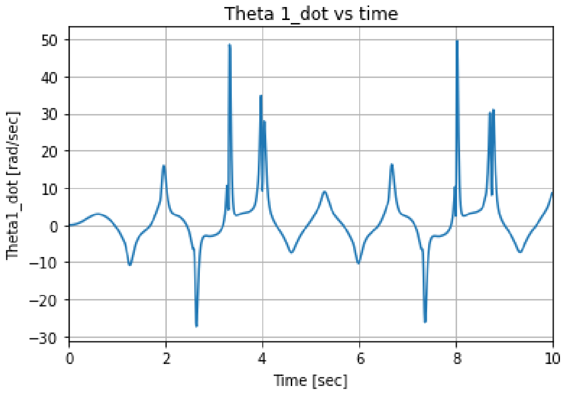
TEST 4 – SIN TORQUES WITH SPRINGS AND DAMPERS

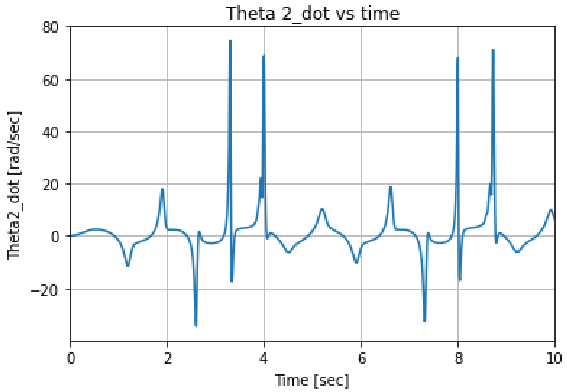
```
def t1(time):
    return 1*np.sin(.5*time)
def t2(time):
    return 0#-40*signal.square(2 * np.pi * 20 * time)
def t3(time):
    return 2*np.sin(4*time)
```

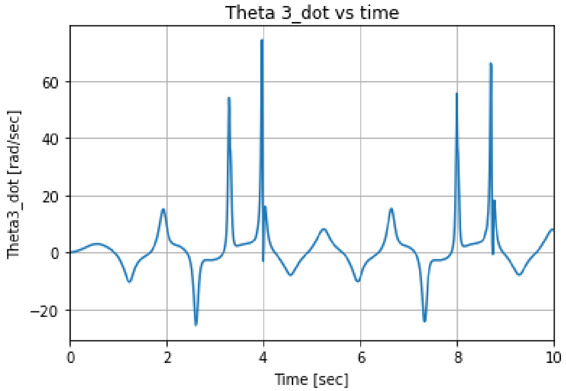
```
G[11] = tau1-tau2 + k*(theta2 - theta1)*(l1/2) + b*(theta2_dot - theta1_dot)*l1/2
G[14] = tau2-tau3 + k*(theta3 - theta2)*(l2/2) + b*(theta3_dot - theta1_dot)*l2/2
G[17] = tau3
```

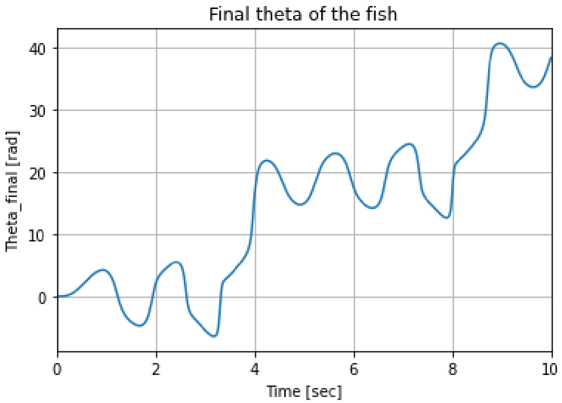












As expected, adding dampers along with springs helps the system loose the energy built up by the springs and cools the system down. In fact, this works best in replicating the motion of the fish. Even though it does

CONCLUSIONS

LESSONS LEARNED / CONCLUSION

1. Torque 3 seems to affect a lot. Any small perturbation catches up with the head of the fish eventually.

2. Higher frequencies (not too high) seem to work best as the switching of torques seem to catch up more efficiently hence preventing the pendulums from interfering.

3. Even with 0 torque on the first and second pendulum, the head of the fish eventually catches up with the oscillation.

4. Sinusoidal inputs are a little better in preventing the body from oscillating a lot in terms of velocities which also makes sense from the actual motion of a fish.

5. Combination of springs and dampers helps a lot for the system to not overlap with each other and loose the accumulated spring energy.